

Tut 4.

ICA.

Mixing statistically independent sources.
 now variance of mixture

$$\begin{aligned}
 \text{var}(x) &= \langle (x - \langle x \rangle)^2 \rangle \\
 &= \langle x^2 \rangle - (\langle x \rangle)^2 \\
 &= \langle (\sum_i w_i s_i)^2 \rangle - (\langle \sum_i w_i s_i \rangle)^2 \\
 &= \langle (\sum_{i,j} w_i w_j s_i s_j) \rangle \\
 &\quad - \langle (\sum_{i,j} w_i w_j s_i \langle s_j \rangle) \rangle \\
 &= \sum_{i,j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &\quad + \sum_{i,j \neq j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &= \sum_i w_i^2 (\langle s_i s_i \rangle - \langle s_i \rangle^2) \\
 &\quad + \sum_{i,j \neq j} w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle)
 \end{aligned}$$

s_i & s_j are statistically independent.
 for $i \neq j$

$$\begin{aligned}
 \langle s_i \rangle \langle s_j \rangle &= \langle s_i \rangle \langle s_i \rangle = 0. \\
 \sum \text{var}(s_i) &= 1
 \end{aligned}$$

\Rightarrow To generate mixture from unit variance
 $\text{var}(x) = 1$

$\sum_i w_i^2 = 1 \rightarrow$ Following constraint has to
 be followed & imposed on
 the weights w_i for the mixture to have
 unit variance.