# Unit 1: Wave Optics

***Syllabus***

**Interference**

- Introduction to electromagnetic waves and electromagnetic spectrum
- Interference in thin film of uniform thickness (with derivation)
- Interference in thin film wedge shape (qualitative)
- Applications of interference: testing optical flatness, anti-reflection coating

**Diffraction**

- Diffraction of light
- Diffraction at a single slit, conditions for principal maxima and minima, diffraction pattern
- Diffraction grating, conditions for principal maxima and minima starting from resultant amplitude equations, diffraction pattern
- Rayleigh's criterion for resolution, resolving power of telescope and grating

**Polarization**

- Polarization of light, Malus law
- Double refraction, Huygen's theory of double refraction
- Applications of polarization: LCD

***Pre-requisites***

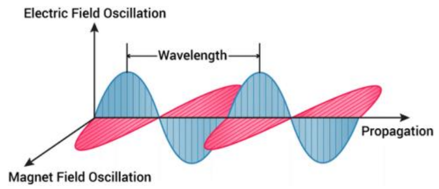
**Part I:** Interference and its types; Conditions for constructive and destructive interference; Definitions: Wavelength, frequency, phase, Phase difference and path difference, Refraction and refractive index; Snell's law for refraction; Stoke's theorem;

**Part II:** Diffraction of light; basic terms: amplitude and intensity, resolution

**Part III:** Polarization of light; refraction of light

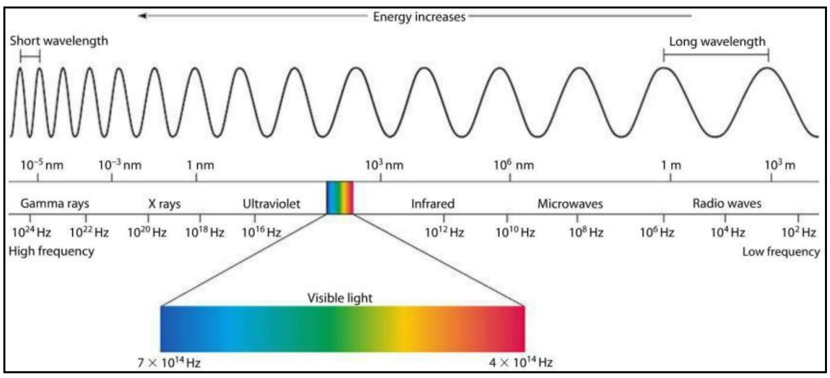
# PART I – Interference of Light

## 1.1.1 Introduction to electromagnetic waves and electromagnetic spectrum




Electromagnetic radiation is created when a charged particle (electron or proton), is accelerated by an electric field. This movement produces oscillating electric and magnetic fields, which travel at right angles to each other. Electromagnetic waves travel with the speed of $3 \times 10^8$ m/s.

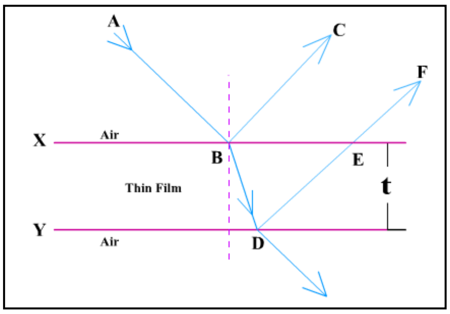
Electromagnetic spectrum consists of span of electromagnetic waves of range of wavelengths and frequencies. The EM spectrum is generally divided into seven regions, in order of decreasing wavelength and increasing energy and frequency. The common designations are: gamma rays, X-rays, ultraviolet (UV), visible light, infrared (IR), microwaves and radio waves.




| Color | Wavelength |
|---|---|
| violet | 380–450 nm |
| blue | 450–495 nm |
| green | 495–570 nm |
| yellow | 570–590 nm |
| orange | 590–620 nm |
| red | 620–750 nm |

***Interference in thin film of uniform thickness [For understanding]***

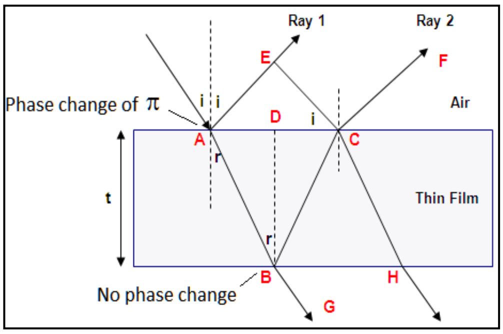
- *A transparent film is said to be thin when its thickness is about the order of one wavelength of visible light; typically the thickness when thickness taken from 4000 Å to 8000 Å.*
- *A thin film may be a thin sheet of a transparent material such as glass, mica, an air film enclosed between two transparent sheets or a soap bubble.*
- *If the thickness of the film is about 10 μm to 50 μm, it is considered to be a thick film.*
- *When light incidents on thin films, a small portion get reflected from the upper surface and a major portion is transmitted into the film.*




- *Within the film light undergoes multiple reflection and reflection due to the division of wavefront and amplitudes.*

## 1.1.2 Conditions for constructive and destructive interference in thin film of uniform thickness

- Consider a transparent thin film of uniform thicknessand the light incidents on it on the top surface.
- Let, t – thickness of the film
  $\mu$ - refractive index of the film
  $\lambda$ - wavelength incident light
  i - angle of incidence on the top surface
  r- angle of refraction inside the film
- The light undergoes partial reflection and partial refraction at the point of incidence. Inside the film, it undergoes multiple reflection and refraction due to division of wavefront and amplitude.The light rays emerging out from the top surface are very close to each other and when observed using eye or a microscope, they produce interference pattern.
- **Change of phase for reflected ray:** The light incident on the top surface of the film at point A and it undergoes partial reflection along *AE* (ray-1). As per Stoke's theorem, the reflected ray undergoesa path change of $\lambda/2$ or phase change of $180^0$as it is reflected from the dense surface.




**Condition for path difference**

- Ray-1 travels in air while ray-2 travels inside the film and travel a distance (*AB*+*BC*) within the film
- **Optical path difference between ray-1 and ray-2**

$\Delta$ = [Distance covered by ray-2 in the film] – [Distance covered by ray-1 in the air]

$\Delta = \mu(AB+BC) - AE$ [as $\mu$=1 for air] --- (1)

Now, $AE = AC\cdot\sin i$

$= 2\cdot AD\cdot\sin i$ [$AC = 2\,AD$]

$= 2\cdot AB\cdot\sin r\cdot\sin i$ [$AD = AB\cdot\sin r$]

$= 2\dfrac{t}{\cos r}\sin r\sin i$ [As $AB = BC = \dfrac{t}{\cos r}$] --- (2)

$= 2\frac{t}{\cos r}\sin r \cdot \mu \sin r$ [As $\mu = \frac{\sin i}{\sin r}$]

$= \frac{2\mu t \sin^2 r}{\cos r}$ --- (3)

Putting the values of *AB*, *BC* and *AE* in equation 1

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r}\sin^2 r = \frac{2\mu t}{\cos r}(1-\sin^2 r) = \frac{2\mu t}{\cos r}\cos^2 r$$

Thus, $\Delta = 2\mu t \cos r$ --- (4)

**Correction in path difference:** Ray-1 is refracted from the top surface of the film (dense medium) at point *A* and hence it undergoes a phase change of $180^0$ which is equivalent to a loss or gain of halfwavelength $\lambda/2$.

Thus, modified condition for path difference is: $\Delta = 2\mu t \cos r - \frac{\lambda}{2}$ ---- (5)

**<u>Condition for Constructive Interference</u>**

Ray-1 and ray-2 undergo constructive interference if the path difference between them is even multiple of $\lambda$ i.e.

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} \quad \text{--- (6)}$$

The integer n=0,1,2,3,…. is known as order of interference.

**<u>Condition for Destructive Interference</u>**

Ray-1 and ray-2 undergo destructive interference if the path difference between them is odd multiple of $\lambda/2$ i.e. $2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t \cos r = (n+1)\lambda$$

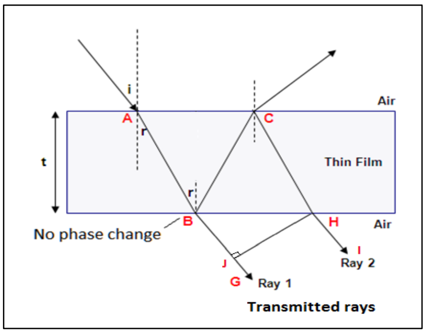
As (*n*+1) is also an integer, we denote (*n*+1) by n only. Subtraction or addition of one full wave to one of the rays does not change the phase relationship of the two interfering waves and $(n+1)\lambda$ can as well be denoted as $n\lambda$

Thus, the condition for destructive interference is $2\mu t \cos r = n\lambda$ --- (7)

[*n*=0, 1, 2, …]

## Interference in thin films of Uniform Thickness [transmitted system]

- For a transmitted system, condition for path difference is $\Delta = 2\mu t \cos r$
- NO MODIFICATION in this condition is required as both rays do not undergo any phase change.




**Condition for path difference in constructive interference**

$\Delta = 2\mu t \cos r = n\lambda$

**Condition for path difference in destructive interference**

$\Delta = 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

From above conditions for transmitted system, it is clear that these conditions are complementary (opposite) to the conditions for reflected system.

***Characteristics of interference in thin film [For understanding]***

The condition for path difference is $\Delta = 2\mu t\cos r - \frac{\lambda}{2}$. The interference and its nature depend on several factors.

**(a) Thickness of the film**

- <u>If film is too thin</u>: If the film is too thin ($t \approx 0$) the factor $2\mu t\cos r$ is negligible and effective path difference is odd multiple of $\lambda/2$. It satisfies the condition for destructive interference and the film will appear black in reflected light.
- <u>If film is too thick</u>: If the film is too thick factor $\lambda/2$ is negligible. Also the reflected rays (ray-1 and ray-2) will be traveling too away from each other and the film will appear bright due to reflection of the incident light without interference.

**(b) If incident monochromatic light is parallel**

- If incident monochromatic light is parallel, the whole film will appear uniformly dark or uniformly bright, since the film thickness *t* and the angle of refraction *r* are constant. However, the condition of construction interference causes the intensification of the incident color.

**(c) If white light incidents on the film**

- For white light, the optical path difference varies depending on the wavelength and the film will appear color. Those colors will be seen for which the condition of constructive interference will be satisfied.

**(d) Change of angle of incidence**

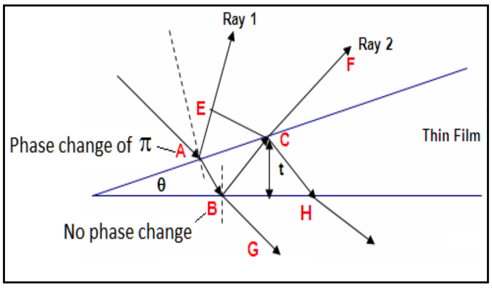
- A change in the angle of incidence of the rays causes a corresponding change in the path difference. The film will appear dark and bright (colored) alternatively.

**(e) Necessity of extended source**

- Extended source is required as light reflected from every point of the film reaches the eye simultaneously and the film can be viewed at a glance.

## 1.1.3 Interference in wedge shaped thin film

Consider a thin film of which the thickness increases from one end to other end (known as wedge-shaped film).If $\theta$ is angle of wedge,




**In reflected system**

**Condition for Constructive Interference**

$$2\mu t\cos(r+\theta) = (2n+1)\frac{\lambda}{2}$$

Where n=0,1,2… is order of interference

**Condition for Destructive Interference**

$$2\mu t\cos(r+\theta) = n\lambda$$

Where n=0, 1, 2… is order of interference

**In transmitted system**

**Condition for constructive interference:**

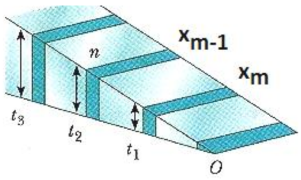
$$\Delta = 2\mu t\cos(r+\theta) = n\lambda$$

**Condition for destructive interference:**

$$\Delta = 2\mu t\cos(r+\theta) = (2n+1)\frac{\lambda}{2}$$

From above conditions for transmitted system, it is clear that these conditions are complementary (opposite) to the conditions for reflected system in wedge shaped film.

***Nature of interference pattern in wedge shaped film [For understanding]***




Along the line of contact a section of the film has same thickness and the locus of the interference pattern along it will be straight line. The fringes are equidistance, straight and parallel to the contact edge of the wedge.

At the contact edge, wedge (and hence the thickness of the air film) is zero. The path difference between the rays at the edge will always be $180^0$ out of phase and the reflected rays will interfere destructively. Thus at the contact edge pattern is always dark.

If, a thin film of refractive index $\mu$, wedge angle $\theta$, is illuminated by a monochromatic light of wavelength $\lambda$, alternate dark and bright fringes are observed. The distance between any two alternate dark or bright bands is band width. The expression for bandwidth is $\beta = \frac{\lambda}{2\mu\theta}$

***Formation of Colors in Thin Films***

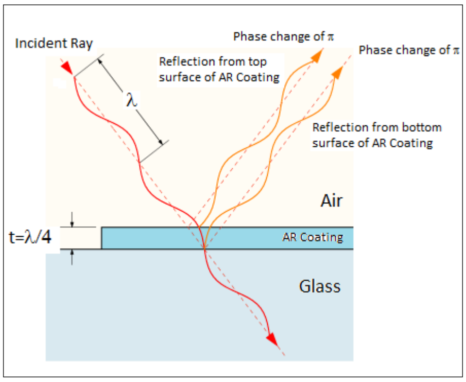
- When a drop of oil or petrol is sprayed over the water especially during rainy days, a thin layer of oil will appear on the water surface. Both the top and bottom surfaces of this oil film can reflect light.
- When ordinary light incident on the film; the optical path difference will vary from one color to other color. Accordingly, the film will appear colored, the color being that of the rays which interfere constructively. White light consists of continuous range of wavelengths. The colors which satisfy condition of maxima are visible with maximum intensity. Hence the film will show different colors when viewed at different angles.

## 1.1.4 Applications of Interference

### (I) Antireflection (AR) / non-reflective coating

Anti-reflection coating is generally done on the optical instruments such as camera lens or telescope to reduce the reflection of light and to improve the efficiency. While designing antireflective coating, a wavelength range must be specified for which the reflection is to be achieved; usually it is done for mid-range wavelengths of visible spectrum (i.e. green).

**Condition for refractive index**




Consider a thin film coated over a glass. The film is enclosed in two mediums: one is air and other is substrate.

Refractive index of the film $=\mu_{film}$
Refractive index of air $=\mu_{air}$
Refractive index of the substrate $=\mu_{glass}$

If $\mu_{air}<\mu_{film}<\mu_{glass}$, the film is a denser medium for air, but it is a rare medium as compared to the substrate.
Example: $MgF_2$:$\mu_{MgF2}$ (=1.38) >$\mu_{air}$(=1) <$\mu_{glass}$ (~1.3-1.5)

**Phase change on reflection**

- If a light of wavelength λ incidents over the coating it is reflected from the top surface of the film and undergoes a phase change of $180^0$ as the coating is a denser medium for air.
- The light travel trough coating and gets reflected from the bottom surfaces of the coating and undergoes $180^0$ phase change as substrate is a denser medium for the coating. This is equivalent to shifting both wavelengths by λ/2.

**Condition for minimum thickness**

Consider, λ - be the wavelength of incident light, for which cancellation is to done
*t*- the thickness of the film
*i* - angle of incidence and
*r* - is the angle of refraction

Then path difference between the two reflected rays is $=2\mu_f t\cos r$

For normal incidence: *i*=0, *r*=0 and cos*r*=1 $=2\mu_f t$

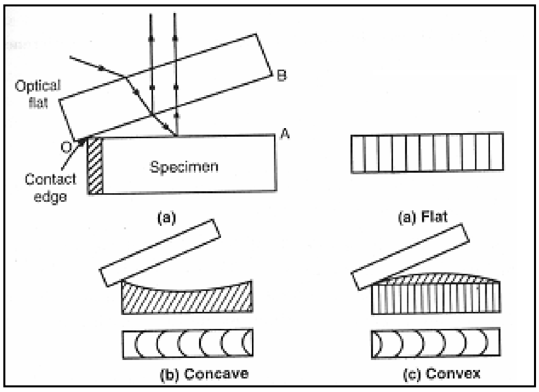
For destructive interference, the path difference should beλ/2

i.e., $2\mu_f t = \frac{\lambda}{2}$

or,$t_{min} = \frac{\lambda}{4\mu_f}$

Under these conditions, the light reflected from the top and bottom surfaces of the film interfere destructively and the two reflected rays cancel out each other and the reflection of light is minimized.

## (II) Testing Optical Surface for Flatness




- The two surfaces – one is optically flat (standard plate) and other is to be tested (test plate) to flatness are kept inclined at a small angle (let $\theta$). The situation is similar to the formation of thin film of air wedge between the two surfaces.
- A monochromatic light of wavelength λ is allowed to fall incident on the optical flat surface.
- The resultant interference pattern consists of alternate dark and bright bands. If the two plates are perfectly flat, spacing between them (i.e. bandwidth$\beta = \frac{\lambda}{2\mu\theta}$) is constant and lines are straight and parallel as shown in figure (a).
- If the test plate is not perfectly optically flat (i.e. either concave or convex) the interference fringes do not remain flat and parallel as shown in figures (b) and (c).

# Questions on Interference

### 6 marks

1. A thin film of uniform thickness is illuminated by monochromatic light. Obtain the conditions for darkness and brightness of the film as observed in reflected light.
2. Draw a neat and labeled diagram and write the conditions for constructive and destructive interference in case of thin film of (a) uniform thickness, (b) wedge shape. Why the interference pattern in the reflected and transmitted system is complementary
3. Write the expressions for path difference between the waves reflected from a wedge shaped film. State the conditions for maxima and minima. Explain the application of wedge shaped thin film for testing the flatness of a glass plate.***[Oct 19, 5m]***

### 3/4 marks

1. Explain in brief (a) Electromagnetic waves (b) Electromagnetic spectrum
2. Draw diagram showing interference in reflected light in a thin wedge shaped film. Write down the mathematical conditions for maximum and minimum intensity of light in reflected system.
3. Explain with diagram how interference Principle is used to design anti reflection coating. Derive an expression for its thickness.

# Numerical on Interference

**Formulae:**

1. Thin film of uniform thickness:
   Condition for darkness - $2\mu t\cos r = n\lambda$
   Condition for brightness - $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$
2. Anti-reflection coating: minimum thickness - $t_{min} = \frac{\lambda}{4\mu_{film}}$

**<u>Thin film of uniform thickness</u>**

***Example:*** *A parallel beam of light 622 nm incident on a glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.*

**Solution** $\lambda$ = 622 nm = 622 x $10^{-9}$ m, r=60°, μ=1.5, t=?

Formula: $2\mu t\cos r = n\lambda$

For smallest thickness, n=1

$$t = \frac{\lambda}{2\mu\cos r} = \frac{622\times10^{-9}}{2\times1.5\times\cos(60)} = 4.146\times10^{-7}\text{m} = 4146\ A^0$$

***Example:*** *A soap film of μ=4/3 and thickness 1.5x$10^{-4}$ cm is illuminated by white light incident at angle $45^0$, The light reflected by it is examined by a spectroscope in which it is found a dark band corresponding to wavelength of 5000 $A^0$. Calculate the order of interference band.*

**Solution** t=1.5x$10^{-4}$ cm, $\lambda$=5000A°= 5000x$10^{-8}$ cm, i=45°, μ=4/3=1.33, n=?

$$\mu = \frac{\sin i}{\sin r} \text{ or } \sin r = \frac{\sin i}{\mu} = \frac{\sin(45)}{1.33} = 0.5316$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.5316)^2} = 0.8469$$

Formula: $2\mu t\cos r = n\lambda$

$$n = \frac{2\mu t\cos r}{\lambda} = \frac{2\times1.33\times1.54\times10^{-4}\times0.8469}{5000\times10^{-8}} = 6.93$$

As "n" can only be an integer, the completed order for n will only be considered

Thus, n = integer (6.93) = 6

***Example:*** *White light falls at an angle $45^0$ on a parallel soap film of refractive index 1.33. At what minimum thickness of the film will it appear bright yellow for wavelength 5900 $A^0$ in the reflected light?*

**Solution** $\lambda$=5900A°=5900x$10^{-10}$ m, i=45°, $\mu$=1.33, t=?

$\mu = \frac{\sin i}{\sin r}$ or $\sin r = \frac{\sin i}{\mu} = \frac{\sin(45)}{1.33} = 0.5316$

$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.5316)^2} = 0.8469$

Formula: $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

For smallest thickness, n=0

$2\mu t \cos r = \frac{\lambda}{2}$

$t = \frac{\lambda}{4\mu t \cos r} = \frac{5900\times10^{-10}}{4\times1.33\times0.8469} = 1.309\times10^{-7}m = 1309\ A^0$

***Example:*** *A soap film having refractive index 1.33, and thickness 5x$10^{-5}$ cm is viewed at an angle of $35^0$ from normal. Find the wavelengths of the light in the visible spectrum that will be absent from the reflected light.*

**Solution** $\lambda$ = 5x$10^{-5}$ cm, i=$35^0$,

Formula: $2\mu t \cos r = n\lambda$

Now $\mu = \frac{\sin i}{\sin r}$

$r = \sin^{-1}\left(\frac{\sin i}{\mu}\right) = \sin^{-1}\left(\frac{35}{1.33}\right) = 25.53$

For minima, $2\mu t \cos r = n\lambda$ or $\lambda = \frac{2\mu t \cos r}{n} = \frac{2\times1.33\times5\times10^{-5}\times\cos(25.53)}{n}$

For n=1, $\lambda_1 = 1.2\times10^{-4}cm = 12000\ A^0$

For n=2 $\lambda_2 = 0.6\times10^{-4}cm = 6000\ A^0$

For n=3 $\lambda_3 = 0.4\times10^{-4}cm = 4000\ A^0$

For n=4 $\lambda_4 = 0.3\times10^{-4}cm = 3000\ A^0$

Thus, wavelengths 6000 $A^0$, and 4000 $A^0$ from visible spectrum will be absent

***Example:*** *A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle of $30^0$ from the normal, $8^{th}$ dark band is seen. Determine the thickness of the film. [Given: RI of oil = 1.46, $\lambda$=5890 AU]* ***[Oct 19, 4m]***

**Solution** $\lambda$ = 5890AU = 5890 x $10^{-10}$ m, i=30°, $\mu$=1.5, t=?

$\mu = \frac{\sin i}{\sin r}$ or $\sin r = \frac{\sin i}{\mu} = \frac{\sin(30)}{1.46} = 0.324$

$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.324)^2} = 0.9460$

Formula: $2\mu t \cos r = n\lambda$

For smallest thickness, n=1

$t = \frac{\lambda}{2\mu \cos r} = \frac{5890\times10^{-10}}{2\times1.46\times0.9460} = 2.132\times10^{-7}\text{m} = 2132\ A^0$

## <u>Wedge shaped thin film</u>

***Example:*** *Interference fringes are produced with monochromatic light falling normally on a wedge shape film of refractive index 1.4. The angle of wedge is 10 sec of an arc and distance between two successive fringes is 0.5 cm. What is the wavelength of light used?*

**Solution** $\theta$=10 sec, $\beta$=0.5 cm, $\mu$=1, $\lambda$=?

Formula: $\beta = \frac{\lambda}{2\mu\theta}$

$\theta = 10$ sec $= \frac{10}{3600}\times\frac{\pi}{180}$rad = 4.848×$10^{-5}$rad

$\lambda = 2\times\beta\times\theta = 2\times0.5\times4.848\times10^{-5} = 4.848\times10^{-5}$cm = 4848 $A^0$

***Example:*** *Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 $A^0$. Calculate the angle of the wedge in seconds of an arc.*

**Solution** $\beta$=1 mm = 0.1 cm, $\mu$=1.52, $\lambda$=5893 $A^0$, $\theta$=?

Formula: $\beta = \frac{\lambda}{2\mu\theta}$

$\theta = \frac{\lambda}{2\mu\beta} = \frac{5893\times10^{-8}}{2\times1.52\times0.1} = 1.938\times10^{-4}$(rad)

$\theta = 1.938\times10^{-4}\times\frac{180}{\pi}\times3600$ sec $= 40$ sec

***Example:*** *A beam of monochromatic light of wavelength 5820 $A^0$ falls normally on a glass wedge of wedge angle of 20 second of an arc. If the reflective index of glass in is 1.5, find the number of dark interfering fringes per cm of the wedge length.*

**Solution** $\lambda$=5820 $A^0$ = 5820x$10^{-8}$ cm, $\theta$=20 sec, $\mu$=1.5

Formula: $\beta = \frac{\lambda}{2\mu\theta}$

$\theta = 20$ sec = $\frac{20}{3600}\times\frac{\pi}{180}$ = 9.696×$10^{-5}$rad

$\beta = \frac{\lambda}{2\mu\theta} = \frac{5820\times10^{-8}}{2\times1.5\times9.696\times10^{-5}} = 0.2$ cm

Thus, in 1cm, there will be $\frac{1 \text{ cm}}{2}$ = fringes

## Antireflection Coating

***Example:*** *A monochromatic beam of light of wavelength 5893 $A^0$ is incident normally on the top of a glass which is coated by transparent material $MgF_2$ having RI 1.38. Calculate smallest thickness of the $MgF_2$ layer which will act as a non reflecting surface.*

**Solution** $\lambda$=5893 A°= 5893x$10^{-10}$ m, $\mu_{film}$=1.38,

Formula: $t_{min} = \frac{\lambda}{4\mu_{film}}$

$t_{min} = \frac{5893\times10^{-10}}{4\times1.38} = 1.067\times10^{-7}m = 1067\,\dot{A}$

***Example:*** *A glass of refractive index 1.5 is to be coated with a transparent material of refractive index 1.2, so that the reflection of light of wavelength 6000 $A^0$ is eliminated by interference. What is the requirement of the thickness of the coating?*

**Solution** $\lambda$=6000A°=6000x$10^{-10}$ m, $\mu_{glass}$=1.5, $\mu_{film}$=1.2,

Formula: $t_{min} = \frac{\lambda}{4\mu_{film}} = \frac{6000\times10^{-10}}{4\times1.2} = 1.250\times10^{-7}m = 1250\,\dot{A}$

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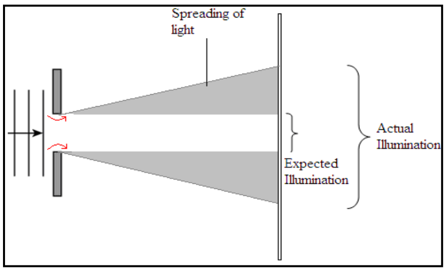
# PART II – Diffraction of Light

## 1.2.1 Diffraction of light




**Diffraction:** If an obstacle of size comparable with wavelength of light is placed in the path of light, the light rays bend around corners/edges of the obstacle. The light waves spreads into geometrical shadow of the obstacle. This phenomenon is known as diffraction.

### *Condition for diffraction*

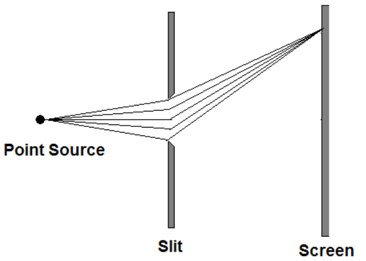
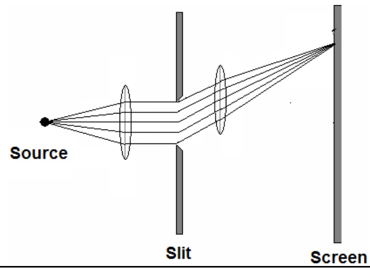
Light waves are very small in wavelength, i.e. from $4 \times 10^{-7}$ m to $7 \times 10^{-7}$ m. If size of obstacle is near to this limit ($a \sim \lambda$) only then process of diffraction is significant. This is because at this dimensions the opening will tend to behave as a point source and will give out secondary wavefronts in all directions.

Diffraction can be illustrated by as shown in figure. Inner portion (white) represents the portion expected to be illuminated by light, while outer portion (shadowed) represents the portion actually illuminated by light.

## Types of Diffraction

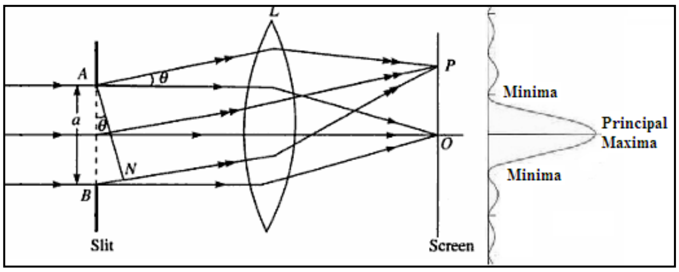
Diffraction phenomenon can be classified into two types:

1. Fresnel diffraction (also known as near-field diffraction)
2. Fraunhofer diffraction (also known as far-field diffraction)

| Sr. | Fresnel diffraction | Fraunhofer diffraction |
|---|---|---|
| 1 |   |   |
| 2 | Point source or screen or both are at finite distance from the obstacle | Source and screen are effectively at infinite distance from the obstacle |
| 3 | Rays striking the obstacle are non-parallel and corresponding wavefront is circular or elliptical | Rays striking the obstacle are parallel and corresponding wavefront is plane |
| 4 | At laboratory level, required conditions can be achieved without using lenses and no modification in incident wavefront is necessary | At laboratory level, required conditions are achieved using lenses which make rays parallel |
| 5 | Mathematical analysis is complicated as different rays undergo diffraction at different angles at the slit | Mathematical analysis is comparatively easier as diffracted rays are parallel. |
| 6 | Angular inclinations of rays i.e. diffraction angle is important | Lateral distances of source and screen are important |

## 1.2.2 Fraunhofer Diffraction at a single slit

### Part-A: Conditions for Amplitude and Intensity



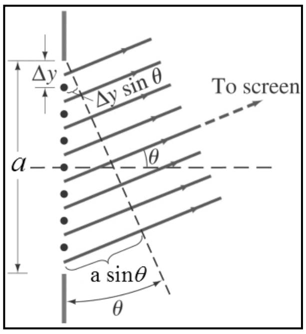

Consider monochromatic light incidents on a narrow slit AB. Let,
$\lambda$- wavelength of light
a – width of the slit

If width of the slit is comparable to the wavelength of light ($a \approx \lambda$), light rays undergo significant diffraction. Diffracted rays are focused on the screen using a convex lens and diffraction pattern is observed on the screen.

**Principal maxima at point O:** Rays diffracted along the direction of incident rays are focused at point *O*. All the secondary wavelets from *AB* reach *O* in same phase will interfere constructively and there is maximum intensity at *O*. This point is known as principal/central maxima.

**Interference at point P:** Rays which are diffracted at an angle $\theta$ are focused at point *P*. Intensity of light at point *P* depends upon path difference between the focused rays.

**Construction of phasor diagram**

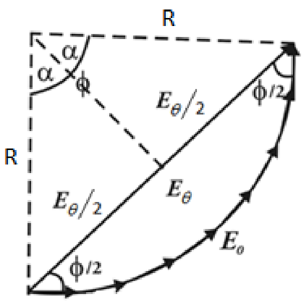



The slit *AB* is considered to be divided further into *N* smaller slits each of width $\Delta y$. Thus, $a = \sum \Delta y$. The two rays coming out of adjoining slits differ in the path and phase difference. If $\Delta y$ is width of the slit and $\theta$ be the angle of diffraction, then between two adjacent rays,

Path difference $:\Delta y = \sin\theta$

Phase difference $:\Delta\emptyset = \frac{2\pi}{\lambda}\Delta y \sin\theta$

Phase difference between the extreme rays is equal to addition of the phase difference of all the rays. As $a = \sum \Delta y$, phase difference between extreme rays: $\Delta\emptyset = \sum \Delta y \sin\theta = a\sin\theta$




Resultant amplitude at any point on the screen is obtained by adding phase difference between the rays meeting at that point. A light ray can be represented by a short phasor. As there are infinite slits, phasor diagram tends to a smooth curve as shown in figure.

Let, $E_0$ - resultant amplitude at $\theta$=0 (central maxima)
$E_\theta$ - resultant amplitude at angle of diffraction $\theta$
$\phi$ - total phase difference when added up
*R* - arc radius

From the geometry of the figure, $\sin\frac{\emptyset}{2} = \frac{E_\theta/2}{R}$ or $E_\theta = 2R\sin\frac{\emptyset}{2}$

Now as $\emptyset = \frac{E_0}{R}$ or $R = \frac{E_0}{\emptyset}$ $\qquad E_\theta = 2\frac{E_0}{\emptyset}\sin\frac{\emptyset}{2}$

Or $\qquad E_\theta = E_0\frac{\sin\emptyset/2}{\emptyset/2}$

Thus, resultant amplitude of waves that are diffracted at an angle $\theta$ and strikes the screen is given by

$$E_\theta = E_0 \frac{\sin\alpha}{\alpha} \qquad \text{--- (4)}$$

$$\text{Where, } \alpha = \frac{\emptyset}{2} = \frac{\pi}{\lambda} a \sin\theta \qquad \text{--- (5)}$$

As intensity is directly proportional to square of the amplitude,

$$I_\theta = E_\theta^2 \qquad \text{(Assuming constant of proportionality one)}$$

$$I_\theta = E_0^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

Thus, resultant intensity of waves that are diffracted at an angle $\theta$ and strikes the screen is given by

$$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{--- (6)}$$

Where, $I_\theta$ is amplitude of resultant of waves diffracted at angle $\theta$ and $I_0 = E_0^2$ is maximum amplitude.

## Part-B: Conditions for Maxima and Minima

### (a) Condition for principal maximum

Resultant amplitude in diffraction pattern of a single slit is given by

$$E_\theta = E_0\left(\frac{\sin\alpha}{\alpha}\right) = \frac{E_0}{\alpha}\sin\alpha = \frac{E_0}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\cdots\right]$$

$$= E_0\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\cdots\right]$$

To obtain maximum intensity we should have $E_\theta = E_0$ i.e. $\theta$ should bezero

Now, $\alpha = \frac{\pi}{\lambda} a \sin\theta = 0$

As , π, *a* and λ are constants $\sin\theta = 0$ or $\theta = 0$

Thus, maximum value of $E_\theta$ is $E_0$ and corresponding point on the screen is known as principal maximum and is obtained at $\theta$=0. This means principal maximum is formed by the parts of the secondary wavelets which <u>travel normally to the slit</u>. Position of principal maxima is at the centre of the diffraction pattern.

### (a) Condition for minima

For the minimum intensity, intensity of the diffraction pattern must be zero i.e.

$$I_\theta = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

This is possible when $\sin\alpha$=0 and $\alpha \neq 0$ as $\alpha$=0 will satisfy the condition for principal maxima

The values of $\alpha$ which satisfies the condition are $\alpha = m\pi$, where *m* = ±1, ±2, ±3, …

Now $\alpha = \frac{\pi}{\lambda} a \sin\theta = m\pi$

Thus the condition for minima is $a\sin\theta = m\lambda$ where, *m* = ±1, ±2, ±3, … --- (7)

The value of integer *m*=0 is not possible as for that value $\theta$ becomes zero, which corresponds to the principal maximum. Equation (7) represents the positions for minima on either side of the principal maxima in the diffraction pattern.

## (c) Intensities of Secondary Maxima

Expression for the resultant intensity is $I_\theta = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$

As we get secondary maxima in between two minima, the phase difference between them varies by an amount $\alpha = \pm(2n+1)\frac{\pi}{2}$

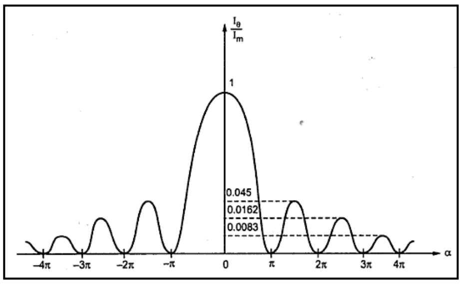
From equation (8), $\frac{I_\theta}{I_0} = \left[\frac{\sin(2n+1)\frac{\pi}{2}}{(2n+1)\frac{\pi}{2}}\right]^2 = \left[\frac{1}{(2n+1)\frac{\pi}{2}}\right]^2$

For the first order maxima, *n*=1 $\frac{I_\theta}{I_0} = \left[\frac{1}{3\frac{\pi}{2}}\right]^2 = \frac{4}{9\pi^2} = 0.045$

For the first order maxima, *n*=2 $\frac{I_\theta}{I_0} = \left[\frac{1}{5\frac{\pi}{2}}\right]^2 = \frac{4}{25\pi^2} = 0.0162$

For the second order maxima, *n*=3 $\frac{I_\theta}{I_0} = \left[\frac{1}{7\frac{\pi}{2}}\right]^2 = \frac{4}{49\pi^2} = 0.0083$

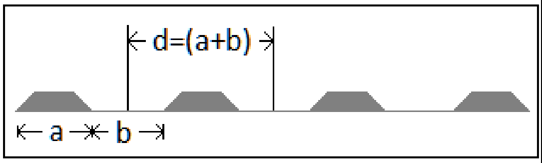
Using above equations we can plot the intensity distribution curve.




Thus, at the centre of the diffraction pattern, we have central maxima and moving away from central maxima, the consecutive maxima decreases in intensities.

***Plane Diffraction Grating [For understanding]***

A diffraction grating consists of a large number of parallel, very closely spaced equidistant slits and is used to separate light of different wavelengths with high resolution.




Diffraction grating is used as a tool for measuring atomic spectra in both laboratory instruments and telescopes. Diffraction gratings can have as many as 1,00,000 apertures per inch. Each of these openings diffracts the light beam. The ruling is generally done with a fine diamond point.

If *a* is the width of the ruling (slit) and *b* is the opaque space, then *a*=*b* i.e. the spacing on grating are equidistant. Distance between the two slits i.e. *d*=(*a*+*b*) is known as grating element or grating period.

If there are *N* number of rulings on the grating in 1 inch of the grating width, then

Grating Element = $d = (a+b) = \frac{1\,inch}{N}$

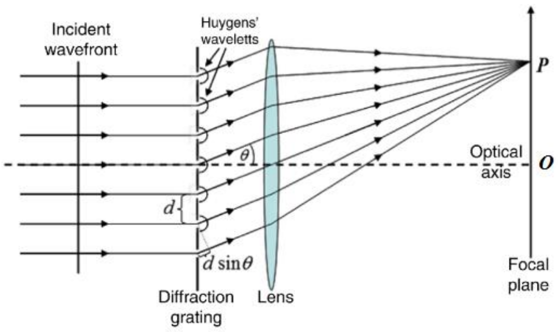
## 1.2.3Diffraction at Plane Diffraction Grating

Consider monochromatic light incidents normally at diffraction grating. The diffracted rays are focused onto a screen using a lens.

Let, λ - wavelength of the light

N – Number of slits on the grating

d = (a+b) – Grating element




**Principal maxima at point O**

The parallel and undiffracted rays travel the same path difference.They undergo constructive interference resulting into brightness. This bright spot is at the centre of the diffraction pattern and is known as central maxima or principal maxima.

**Diffraction pattern at point P**

At each slit, rays are diffracted at angle $\theta$ and are focused at point *P* on the screen. Point *P* will have constructive or destructive interference depending upon the path difference.

Path difference and phase difference between any two adjoining slits is given by

Path difference : $d\sin\theta$

Phase difference : $\frac{2\pi}{\lambda}d\sin\theta$

Let, phase difference = 2γ

Thus, $2\gamma = \frac{2\pi}{\lambda}d\sin\theta$,

And, $\gamma = \frac{\pi}{\lambda}d\sin\theta$ --- (1)

**Amplitude and intensity due to *N* slits**

From the theory of diffraction due to single slit, the amplitude of the resultant waves diffracted at angle $\theta$ is given by $E_\theta = E_0\frac{\sin\alpha}{\alpha}$ --- (2)

Where, $\alpha = \frac{\pi}{\lambda}a\sin\theta$

For a grating, using mathematical analysis and from phasor diagram, resultant amplitude and intensity of the diffracted waves at an angle $\theta$ due to *N* slits is calculated as below:

Resultant amplitude : $E_{\theta N} = E_0\frac{\sin\alpha}{\alpha}\frac{\sin N\gamma}{\sin\gamma}$ --- (3)

And resultant intensity : $I_{\theta N} = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\gamma}{\sin\gamma}\right)^2$ --- (4)

## <u>(a) Condition for principal maxima</u>

At point O, the rays meet in phase and produces constructive interference.

Thus, the path difference for maxima: $d\sin\theta = m\lambda$ where, m=0,1,2,3..

From equation (1) $\gamma = \frac{\pi}{\lambda}d\sin\theta$

From above two equations $\gamma = \frac{\pi}{\lambda}m\lambda = m\pi$

To obtain the expression for intensity due to *N* number of slits we have to put the value $\gamma = m\pi$ in equation (8), but it leads to indeterminate quantity as numerator and denominator becomes zero.

Thus, to obtain the required condition, we apply L'Hospital's rule

$$\lim_{\gamma\to m\pi}\frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma\to m\pi}\frac{\mathrm{d}(\sin N\gamma)}{\mathrm{d}(\sin\gamma)} = \lim_{\gamma\to m\pi}\frac{N\cos N\gamma}{\cos\gamma} = \frac{N\cos Nm\pi}{\cos m\pi} = \pm N$$

Therefore, the equation for intensity reduces to: $I_{\theta N} = N^2 I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$

Thus, the intensity of principal maxima increases with number of slits on diffraction grating *N*. The intensity of principal maxima is greatest while on either side of it, for other maxima, intensity goes on decreasing.

In terms of path difference, the condition for maxima is

$d\sin\theta = m\lambda$ or $\gamma = m\pi$, where, *m*=0,1,2,3..

For *m*=0, we get principal maxima

For *m*=1, we get first principal maxima

For *m*=2, we get second principal maxima and so on.

## (b) Condition for minima

The expression for intensity for diffraction due to N slits is: $I_{\theta N} = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\gamma}{\sin\gamma}\right)^2$

For minima, $I_{\theta N}$ =0.

This is possible either when $\sin\alpha = 0$

But, $\alpha \neq 0$ as it will lead us to the condition of constructive interference.

Thus, $\sin N\gamma = 0$ but $\sin\gamma \neq 0$ (due to indeterminent condition)

This is possible when $N\gamma = m\pi$, where *m* is an integer

The value of $m \neq N, 2N, 3N, \ldots$ as for these values of *m*, $\gamma = 0, 2\pi, 3\pi, \ldots$ and as it will lead to the condition of maxima.

Thus for minima we should have $N\gamma = m\pi$ [$m \neq 0, N, 2N, 3N, \ldots$]

Or $\gamma = m\pi/N$

Putting the value of $\gamma$ $\frac{\pi}{\lambda} d\sin\theta = \frac{m\pi}{N}$

$$d\sin\theta = \frac{m\lambda}{N} \quad [m \neq 0, N, 2N, 3N, \ldots]$$

Thus the minima are obtained at $d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, \ldots$

<u>Intensity distribution curve</u>

Condition for maxima $d\sin\theta = m\lambda$ ($m$=0, ±1, ±2, ….)

Principal maxima $m$=0 and $d\sin\theta = 0$ or $\theta = 0$

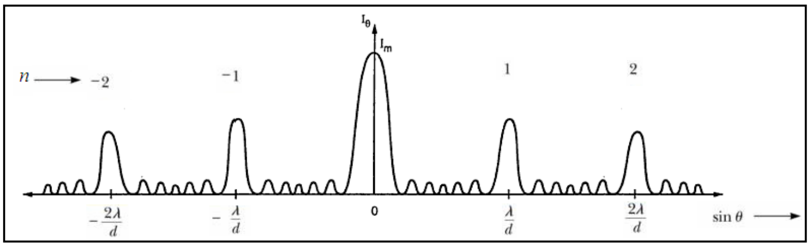
First Maxima $m$=1 and $d\sin\theta = \pm\frac{\lambda}{d}$

Second Maxima $m$=2 and $d\sin\theta = \pm\frac{2\lambda}{d}$

<u>Number of Minima & Maxima</u>

The equation for maxima is: $d\sin\theta = m\lambda$, [$m$=0,1,2,3..]

The equation for minima is: $Nd\sin\theta = m\lambda$ [$m \neq 0, N, 2N, 3N, \ldots$]




In the equation of maxima, *m*=0 gives principal maximum of zero order, *m*=1 gives principal maximum of first order. While m=1,2,3,….(N-1) gives the minima. Then *m*=N gives principal maximum of first order. Thus between zero order and first order principal maximum, there are (N-1) minima. Similarly there are (N-1) minima between first order principal maximum and second order principal maximum. Therefore, between any two (N-1) minima, there are (N-2) secondary maxima.

<u>Conclusions from equation of grating</u> $d\sin\theta = m\lambda$ (m=0, ±1, ±2, ….)

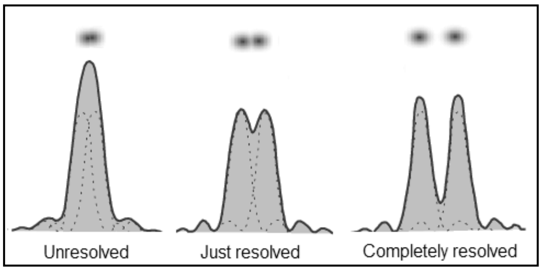
- If the width of the slits (lines) is smaller, the diffraction pattern is wider. This is because as *d* decreases, angle of diffraction $\theta$increases thereby separating the maxima and minima
- If white light incident on a grating it separates it into different colors. This is because white light is a mixture of different colors which will follow different conditions of diffraction.

### 1.2.4 Rayleigh's criterion for resolution, resolving power of telescope and grating

When two objects (generally point sources) are very close to each other, it may not be possible two separate them. The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called resolving power. It is defined as the reciprocal of smallest angle subtended at the objective by two point objects, which can just be distinguish as separate.

When a beam of light from a point object passes through the objective of a telescope, the lens acts like a circular aperture and produces a diffraction pattern instead of a point image. This diffraction pattern is bright disc surrounded by alternate dark and bright rings known as Airy's disc. If there are two point objects lying close to each other, two diffraction patterns are produced, which may overlap on each other and it may be difficult to distinguish them separate.

**Rayleigh's criterion:**
According to Rayleigh's criterion, two nearby images are said to be resolved if they are separated by at least a certain minimum distance so that the position of diffraction central maximum of the first image coincides with the first diffracted minimum of the second image in any order of diffraction and vice versa.

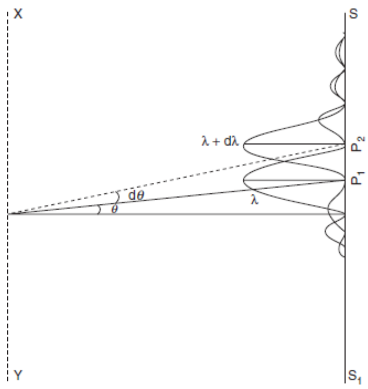



Due to diffraction, object appears as a spot surrounded by rings. In actual practice, the image of a distant point source of light such as a star is a circular disk surrounded by a few progressively fainter rings. The diffraction pattern consists of a central bright disk (known as **Airy disk**) surrounded by bright and dark rings. The diffraction pattern obtained is known as Airy pattern. If

The first minimum for the diffraction pattern of a circular aperture of diameter d, assuming Fraunhofer condition, is given by $\sin\theta = 1.22\frac{\lambda}{d}$

#### Resolving power of a grating

Consider that a grating is illuminated normally by a light consisting of two wavelengths $\lambda$and $\lambda+d\lambda$. After getting diffracted through diffraction grating each light will produce its own pattern on the screen.
According to Rayleigh's criterion for the resolution of two sources is that the central maximum of the single slit interference pattern of one source falls on the first minimum of the pattern of the second source.
It can be shown that Resolving Power = $\frac{\lambda}{d\lambda} = n \times N$




Thus, resolving power of grating is directly proportional to
(a) order of spectrum *n*, and(b) total number of lines on grating *N*

# Questions on diffraction

### 6 marks

1. Derive the expression for resultant amplitude and resultant intensity between the diffracted waves in Fraunhofer diffraction due to a single slit. ***[Oct 19,6m]***
2. Explain Fraunhoffer diffraction at a single slit and obtain the condition for principal maxima and minima. Draw the intensity distribution curve.
3. For a plane diffraction grating, starting from the equations of resultant amplitude and intensity, derive conditions for maxima and minima of the diffraction pattern.

### 3/4 marks

1. What is diffraction? What are its types? ***[Oct 19, 2 m]***
2. What is diffraction of light? Explain in brief Fraunhoffer diffraction.
3. For a single slit diffraction pattern, starting from the condition of resultant amplitude derive the condition for minima.
4. For a single slit diffraction pattern, starting from the condition of resultant intensity derive the relation between central maximum and first two secondary maximum.
5. For a grating, starting from the condition of resultant amplitude and intensity, derive the condition for principal maximum.
6. For a grating, starting from the condition of resultant amplitude and intensity, derive the condition for minima.
7. What is Raileigh's criterion for resolving power? Explain it in brief.

# Numericals on Diffraction

**Formulae:**

1. Diffraction at a single slit: $a\sin\theta = m\lambda$
2. Grating element: d = 1/N
3. Diffraction at a grating: $d\sin\theta = m\lambda$

**<u>Diffraction at Single Slit</u>**

***Example:*** *A slit of width 2µm is illuminated by light of wavelength 6500 $A^0$. Calculate the angle at which the first minimum will be observed.*

**Solution:** λ = 6500 $A^0$ = 6500x$10^{-8}$ cm, a = 2 µm = 2x$10^{-4}$ cm

Formula: For single slit $a\sin\theta = m\lambda$

For first minima, m=1

$$2\times10^{-4}\times\sin\theta = 6500\times10^{-8}$$

$$\sin\theta = \frac{6500\times10^{-8}}{2\times10^{-4}} = 0.325$$

$$\theta = \sin^{-1}(0.325) = 18.96^0$$

***Example:*** *A monochromatic light of wavelength 5500 Å incidents normally on a slit of width 2x$10^{-4}$ cm. Calculate the angular position of first and second minimum.*

**Solution:** λ = 5500 Å = 5500 x $10^{-8}$ cm, a = 2x$10^{-4}$ cm

Formula: $a\sin\theta = m\lambda$

$$\sin\theta = \frac{m\lambda}{a} = m\times\frac{5500\times10^{-8}}{2\times10^{-4}}$$

$$\sin\theta = m\times0.275$$

For first principal maximum, m=1, $\sin\theta = 0.275$, θ = 15.96$^0$

For second principal maximum, m=2, $\sin\theta = 2\times0.275$ θ = 33.36$^0$

***Example:*** *Find the half angular width of the central maximum in the Fraunhoffer diffraction pattern of a slit of width 7.07x$10^{-5}$ cm, when illuminated by a light of wavelength 5000 $A^0$.*

**Solution:** λ = 5000 $A^0$ = 5000x$10^{-8}$ cm, a = 7.07x$10^{-5}$ cm

Formula: For single slit $a\sin\theta = m\lambda$

As central maxima is sandwich between two minima, m=1

$7.07\times10^{-5}\times\sin\theta = 1\times5000\times10^{-8}$

$\sin\theta = \frac{5000\times10^{-8}}{7.07\times10^{-5}} = 0.7072$

$\theta = \sin^{-1}(0.7072) = 45^0$

***Example:****Find the half angular width of the central maximum in the Fraunhoffer diffraction pattern of a slit of width $12\text{x}10^{-5}$ cm, when illuminated by a light of wavelength 6000 $A^0$.* ***[Oct 19,3m]***

**Solution:** $\lambda$ = 6000 $A^0$ = $6000\text{x}10^{-8}$ cm, a = $12\text{x}10^{-5}$ cm

Formula: For single slit $a\sin\theta = m\lambda$

As central maxima is sandwich between two minima, m=1

$12\times10^{-5}\times\sin\theta = 1\times6000\times10^{-8}$

$\sin\theta = \frac{6000\times10^{-8}}{12\times10^{-5}} = 0.5$

$\theta = \sin^{-1}(0.5) = 30^0$

***Example:****A single slit Fraunhoffer's diffraction pattern is formed using white light. For what wavelength of light does the second minimum coincide with third minimum of light of wavelength 4000 A°.*

**Solution:** for [$\lambda_1$ = 4000 $A^0$→ third minima] = [$\lambda$2=? → second minima]

Formula: For single slit $a\sin\theta = n\lambda$

Third minima for $\lambda_1$ and second minima of $\lambda_2$ coincides at angle of diffraction $\theta$

For $\lambda_1$ - $a\sin\theta = 3\lambda_1$, For $\lambda_2$ - $a\sin\theta = 2\lambda_2$

Thus, $3\lambda_1 = 2\lambda_2$

$\lambda_2 = \frac{3\lambda_1}{2} = \frac{3\times4000}{2} = 6000\ A^0$

Third minima of 4000$A^0$ and second minima of 6000$A^0$ will coincide

## Diffraction at Grating

***Example:*** *How many lines per cm are there on the surface of a plane transmission grating which gives 1st order of light of wavelength 6000AU at an angle of diffraction $30^0$.*

**Solution** $\lambda$ = 6000 $A^0$ = $6000\text{x}10^{-8}$ cm

Formula: $d\sin\theta = m\lambda$

For first maxima, *m*=1

Number of slits/cm = $\frac{1}{\mathrm{d}} = \frac{\sin\theta}{\lambda} = \frac{\sin 30}{6000\times10^{-8}} = 8333$ lines/cm

***Example:*** *In a grating, the angle of diffraction for the second order principal maximum for the light of wavelength $5\text{x}10^{-5}$ cm is $30^0$. Calculate the number of lines per centimeter of the grating surface.*

**Solution:** $\lambda$ = $5\text{x}10^{-5}$ cm, Angle of diffraction, $\theta$=$30^0$, Number of order, n=2

Formula: $d\sin\theta = m\lambda$

$d = \frac{\lambda}{\sin\theta} = \frac{2\times5\times10^{-5}}{\sin(30)} = \frac{10\times10^{-5}}{0.5} = 20\times10^{-5} = 2\times10^{-4}$

No. of lines / cm = $\frac{1}{d} = \frac{1}{2\times10^{-4}} = 5000$ lines/cm

***Example:*** *A plane diffraction grating has 5000 lines/cm. What is the highest order spectrum that is visible with light of wavelength 6000 $A^0$.*

**Solution** $\lambda$ = 6000 $A^0$ = $6000\text{x}10^{-8}$ cm, N'=5000 lines/cm

Formula: $d\sin\theta = m\lambda$

For maximum*m*, sin$\theta$ = maximum = 1

d = 1/N' = 1/5000 cm

$$m = \frac{d\sin\theta}{\lambda} = \frac{1}{5000} \times \frac{1}{6000\times10^{-8}} = 3.33$$

As n can be an integer only integral part of 3.33 is considered i.e. 3

***Example:****A laser light of wavelength 6328 AU falls normally on a grating which is 2 cm long. The first order spectrum is observed at an angle of $20^0$. Find the total number of slits on grating.*

**Solution:** $\lambda$ = 6328 AU = $6328\times10^{-8}$ cm, $\theta = 20^0$, m=1

Formula: $\mathrm{d}\sin\theta = \mathrm{m}\lambda$

Number of slits/cm = $\frac{1}{\mathrm{d}} = \frac{\sin\theta}{\lambda} = \frac{\sin 20}{6328\times10^{-8}} = 5404.86$ lines/cm

Total number of slits = N = 2x5404.86 = 10809.7 ≈ 10809 lines

***Example:*** *Monochromatic light from laser of wavelength 6328 $A^0$ incident normally on a diffraction grating containing 6000 lines/cm. Find the angles at which the first and second order maximum are obtained.*

**Solution:** $\lambda$=6328 $A^0$

N =6000 lines/cm, $d = \frac{1}{6000}\,\mathrm{cm}^{-1}$

Formula: $d\sin\theta = n\lambda$

| For first maximum, n=1 | For second maximum, n=2 |
|---|---|
| $\sin\theta = \frac{n\lambda}{d} = \frac{1\times 6328\times10^{-8}}{1/6000} = 0.3796$<br>$\theta = \sin^{-1}(0.3796) = 22.30^0$ | $\sin\theta = \frac{n\lambda}{d} = \frac{2\times 6328\times10^{-8}}{1/6000} = 0.7593$<br>$\theta = \sin^{-1}(0.7593) = 49.40^0$ |

***Example:*** *What is the longest wavelength that can be observed in the third order for a transmission grating having 7000 lines/cm. Assume normal incidence.*

**Solution:** N' =7000 lines/cm, n=3

Formula: For maxima $d\sin\theta = n\lambda$

d = 1/N' = 1/7000 cm

For longest wavelength, $sin\theta = 1$

$\lambda = \frac{d}{n} = \frac{1}{7000\times3} = 4.762\times10^{-5}$cm

Thus, $\lambda$ = 4762 $A^0$

***Example:*** *When parallel waves of monochromatic light of wavelength 5790 $A^0$, fall normally on a grating 2.54 cm wide, the first order spectrum is produced at an angle $19.994^0$, from the normal. Calculate the total number of lines on the grating.*

**Solution** $\lambda$ = 5790 $A^0$ = $5790\times10^{-8}$ cm

Width of grating = 2.54 cm

Angle of diffraction, $\theta$=$19.994^0$

Number of order, n=1

Formula: $d\sin\theta = n\lambda$

$d = \frac{\lambda}{\sin\theta} = \frac{5790\times10^{-8}}{\sin(19.994)} = 1.693\times10^{-4}$cm

No of lines/cm = $\frac{1}{d} = \frac{1}{1.693\times10^{-4}} = 5905.38$

No. of lines/cm can be integer only thus, N=5905

As width of the grating is 2.54 cm, Total number of lines on grating = 2.54 x 5905 = 14998.7 ≅ 14998

***Example:****The sodium yellow doublet has wavelengths 5890 AU and 5896 AU. What should be the resolving power of grating to resolve these lines?*

**Solution:**

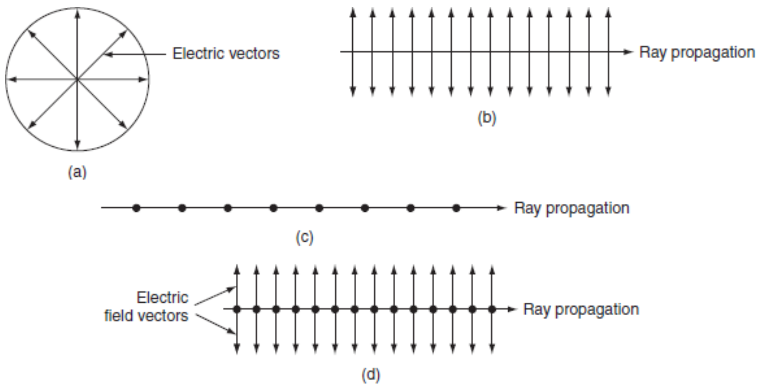
Mean wavelength = $\lambda$ = (5890+5896)/2 = 5893 AU

Difference in wavelength = d$\lambda$ = 5896 – 5890 = 6 AU

Thus, resolving power of grating required to resolve the lines = $\frac{\lambda}{d\lambda} = \frac{5896}{6} = 982$

# PART III – Polarization of Light

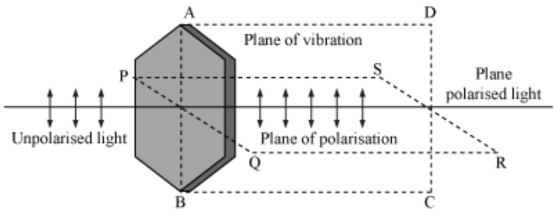
## 1.3.1 Unpolarized and Polarization of light




1. **Unpolarized light:** The light emitted by a source consists of waves whose planes of vibration are randomly oriented about the direction of propagation. Such a light is known as **unpolarized light**.
2. **Linearly polarized light:** The light wavers in which the vibrations of light (of E or H component) occur in a single plane. Figure (b) vertically polarized light (c) horizontally polarized light
3. Partially polarized light: It is a combination of unpolarized light waves and polarized waves, it is said to be partially polarized light, Figure (d).

**Plane of vibration and plane of polarization**

When unpolarized light incident on polarizer, the vibrations parallel to the crystallographic axis are transmitted and those perpendicular to the axis are absorbed. Thus, the transmitted light is linearly polarized.



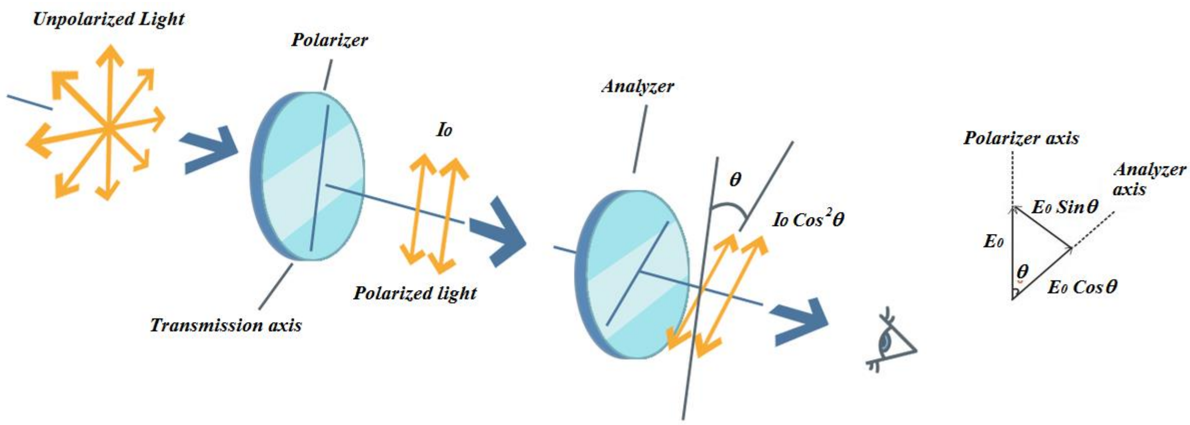

The plane which contains the crystallographic axis and vibrations transmitted from the polarizer (containing E vector) is called **plane of vibration**.

The plane which is perpendicular to the plane of vibration (containing H vector) is called **plane of polarization**.

## 1.3.2 Malus law

**Law of Malus:**

When unpolarized light is passed through a polarizer, the light waves vibrating parallel to the axis of polarizer are transmitted through the polarizer. The light becomes linearly polarized. This light is then passed through an analyzer. The intensity of the light passing through an analyzer is proportional to cosine square of the angle between the axis of polarizer (or plane of the vibration of the light) and the axis of the analyzer. This law is known as Malu’s cosine square law or Malus law.




When a beam of unpolarized light incidents on a polarizer, it is linearly polarized and let its amplitude is $E_0$ and intensity be $I_0$. This polarized light now falls over the analyzer.

Let $\theta$ is the angle between axes of the polarizer and analyzer. The analyzer splits the light into two components $E_0\, cos\,\theta$ and $E_0\, sin\,\theta$. The component $E_0 \sin\theta$ cannot pass through the analyzer as it makes large angle with the analyzer axis. Another component of amplitude $E_0 \cos\theta$ can pass through analyzer.

As intensity is directly proportional to square of the amplitude, the intensity of light transmitted through the analyzer is given by

$$I = (\mathrm{E}_0 \cos\theta)^2 = E_0^2 \cos^2\theta$$

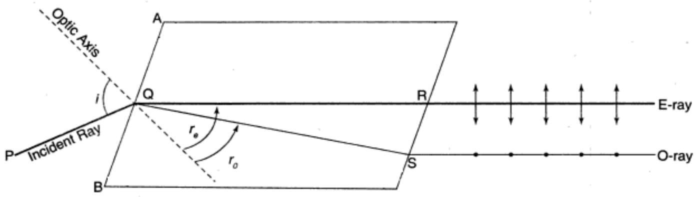
Or $$I = I_0 \cos^2\theta, \text{ where } I_0^2 = E_0^2$$

The above law is known as Malu's law.

Thus, the intensity of the light transmitted by the analyzer is maximum when the transmission axes of the polarizer and analyzer are parallel i.e. when $\theta=0^0$ or $180^0$ and the intensity of the light transmitted by the analyzer is minimum (ideally zero) when $\theta=90^0$ or $270^0$. During one full rotation of the analyzer, two positions of maximum intensity and two position of minimum intensity are obtained.

### 1.3.3 Double refraction

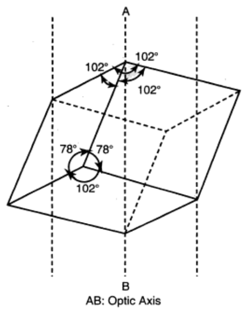
- The change in the path of the light when it moves at an angle from one transparent substance through another is known as refraction.
- An anisotropic material is a material which does not behave the same way in all directions.
- Some transparent anisotropic crystals cause light to bend or refract in two different directions, causing light to divide into two rays. This is called double refraction or birefringence. The crystals are said to be birefringent. The two rays that are produced in double refraction are linearly polarized in mutually perpendicular directions.




- The ray that obeys Snell's law is said to be **ordinary ray** or **o-ray** and another ray that does not obey Snell's law is said to be **extra-ordinary ray** or **e-ray**.
- The identification of e-rays and o-rays exists within the crystal only. Once they come out of the crystal, they behaves as normal light rays.
- Once these rays come out of the crystal their polarization is different. Through a birefringent substance a double image can be seen i.e. one due to o-ray and another due to e-ray.

***Geometry of calcite Crystal [For understanding]***

Chemically, calcite is crystallized calcium carbonate ($CaCO_3$) and it is known as ice-land spar. It is transparent to visible as well as to ultraviolet light. It crystallizes into many forms and can be reduced by cleavage or breakage into rhombohedron. Each of the six faces of this crystal is a parallelogram having angles $102^0$ and $78^0$.




At the two opposite corners of the rhomboheron (A&B), all the three angles of the faces are obtuse. These corners are known as blunt corners of the crystal. A line passing through any one of the blunt corners, making equal angles with each of the three edges, gives the direction of the optic axis. For other remaining six corners, two angles are acute and one is obtuse.

**Optic Axis**

Optic axis is a direction in the crystal and not a unique straight line passing through a set of specific points in the crystal. A ray of light propagating along the optic axis does not undergo double refraction as crystal is symmetric along the optic axis.

## 1.3.4 Huygen's theory of double refraction Polarization of light

The phenomenon of double refraction in uniaxial crystal can be explained using Huygen principle of secondary waves.

**1. Formation of two wavefronts:**

When light wave incidents on a double refracting crystal, the point at the incidence become a source for secondary waves and emit two secondary wavefronts. One wavefront corresponds to extra-ordinary ray (e-ray) while other corresponds to ordinary ray (o-ray).

Let, $c$ – velocity of the light
$v_o$ and $v_e$ – velocities of o-ray and e-ray respectively
$\mu_o$ and $\mu_e$ – refractive indices of o-ray and e-ray respectively

**2. Spherical wavefront for Ordinary ray**

a) The velocity of o-ray ($v_o$) is constant in all directions within the crystal
b) The wavefront corresponding to o-ray is spherical
c) Refractive index $\mu_0 = \frac{c}{v_0}$ has a single value

**3. Elliptical wavefront for Extra-ordinary ray**

a) The velocity of e-ray ($v_e$) varies within the crystal depending upon the direction
b) The wavefront corresponding to e-ray is elliptical
c) Refractive index $\mu_e = \frac{c}{v_e}$ also varies along with the direction of e-ray

**4. Along optic axis**

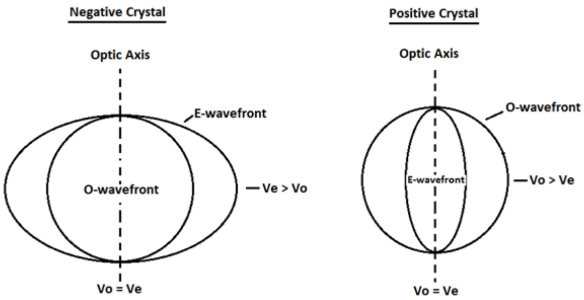
a) Inside the crystal, along the optic axis $v_e = v_o$ and $\mu_o = \mu_e$.
b) When rays are incident along the optic axis, the spherical and ellipsoidal wavefronts touch each other at the points of intersection with the optic axis and double refraction does not occur.

**5. Negative Crystals**

a) In such crystals $v_e > v_o$ and $\mu_o > \mu_e$.
b) Corresponding ellipsoidal wavefront of e-ray is outside the spherical wavefront of o-ray.
c) e.g. Calcite, tourmaline, etc.

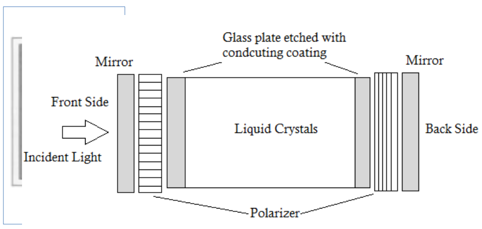
**6. Positive Crystals**

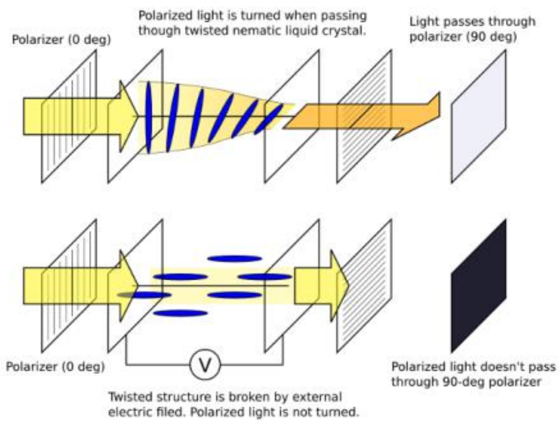
a) In such crystals the $v_o > v_e$ and $\mu_e > \mu_o$.
b) Corresponding ellipsoidal wavefront of e-ray is inside the spherical wavefront of o-ray.
c) e.g. Quartz, ice, etc.




## 1.3.5 Applications of polarization: LCD

- Liquid Crystal Display (LCD) devices are widely used in wrist watches, calculators, clocks, electronic instruments, lap-top computers, video games, TV, etc. The display technology makes the use of polarization.




- An LCD consists of a liquid crystal material is supported between two thin glass plates having transparent conducting coatings on their inner surfaces.
- The conducting coating is etched in the form of a digit or character (figure a). The assembly of glass plates with liquid crystal material is sandwiched between two Polaroid sheets held in crossed field configuration.
- The liquid crystals molecules are arranged in such a way that their long axes undergo a $90^0$ rotation.

**LCD in OFF mode**

- When light is incident on the assembly, the front polarizer polarizes it.
- The linear polarized light is rotated through $90^0$ by the twisted molecular arrangement.
- This polarization is now parallel to rear polarizer and hence the light passes smoothly through rear polarizer.
- A mirror at the back reflects the light which comes out of the front polarizer.

**LCD in ON mode**

- When an external voltage is applied to the device, the molecules lying in the region between electrodes untwist and they align along the field direction.
- Thus, the light does not rotate when it passes through this region.
- The rear polarizer blocks the light and a dark digit or character is seen in that region.

# Questions on polarization

### 5/6 marks

1. What is double refraction? Explain Huygen's theory of double refraction.
2. What is polarized and unpolarized light? Explain how the phenomenon of polarization of light is used in LCD displays.***[Oct 19, 6m]***

### 3/4 marks

1. What is polarized light? Define (a) plane of polarization (b) plane of vibration.
2. State and explain Law of Malus.

# Numericals on Polarization

## Basics of Polarization

***Example:*** *At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light (Data Given: Critical angle for diamond=24.5$^0$).*

**Solution** Critical angle = $i_c$ = 24.5$^0$

Formula: $\mu = \tan i_P$

The relation between refractive index and critical angle is $\mu = \frac{1}{\sin i_c}$

Thus, $\mu = \frac{1}{\sin 24.5^0} = \frac{1}{0.4147} = 2.414$

Now using $\mu = \tan i_P$, we get

$i_P = \tan^{-1}\mu = \tan^{-1}(2.414) = 67.47^0$

Thus, angle of incidence required to produce complete polarization is 67.47$^0$

## Malu's law

***Example:*** *Polarizer and analyzer are kept with their axes parallel for maximum intensity of light to be transmitted by the analyzer. At what angle should either polarizer or analyzer be rotated to reduce the light intensity to (a) 40% and (b) 80% of the original light intensity?*

**Solution**

| (a) For 40% intensity | (b) For 80% intensity |
|---|---|
| $\frac{I}{I_0} = 40\% = \frac{40}{100} = 0.4$<br>$\frac{I}{I_0} = \cos^2\theta$ or $\cos^2\theta = 0.4$<br>$\cos\theta = \pm\sqrt{0.4}$<br>Thus, θ = 50.76$^0$, 129.24$^0$ | $\frac{I}{I_0} = 80\% = \frac{80}{100} = 0.8$<br>$\frac{I}{I_0} = \cos^2\theta$ or $\cos^2\theta = 0.8$<br>$\cos\theta = \pm\sqrt{0.8}$<br>Thus, θ = 26.56$^0$, 153.44$^0$ |

***Example:*** *Polarizer and analyzer are set with their polarizing angles parallel, so that the intensity of the transmitted light is maximum. What will be the percentage of reduction in intensity of the transmitted light when the analyzer is rotated through (a) 30$^0$ and (b) 90$^0$.****[Oct 19, 4m]***

**Solution:**

| (a) At θ=30$^0$ | (b) At θ=90$^0$ |
|---|---|
| $\frac{I}{I_0} = \cos^2\theta = \cos^2(30) = 0.75$<br>Thus, $I = 0.75 I_0 = 75\% I_0$<br>As 75% intensity is transmitted, the reduction in intensity is 25%. | $\frac{I}{I_0} = \cos^2\theta = \cos^2(90) = 0$<br>Thus, $I = 0$ i.e. 100% intensity is blocked.<br>Thus, reduction in the intensity is 0%. |