

Unit 4: Semiconductor Physics

Syllabus

- Free electron theory (Qualitative)
- Opening of band gap due to internal electron diffraction due to lattice
- Band theory of solids
- Effective mass of electron
- Density of states
- Fermi Dirac distribution function
- Conductivity of conductors and semiconductors
- Position of Fermi level in intrinsic and extrinsic semiconductors (with derivations based on carrier concentration)
- Working of PN junction on the basis of band diagram
- Expression for barrier potential (derivation)
- Ideal diode equation
- Applications of PN junction diode: Solar cell (basic principle with band diagram) IV Characteristics and Parameters, ways of improving efficiency of solar cell
- Hall effect: Derivation for Hall voltage, Hall coefficient, applications of Hall effect

4.1 Free Electron Theory

Basis of free electron theory

- (a) Free electron theory was developed by Drude and Lorentz in 1900.
- (b) The valence electrons are involved in electrical conduction in metals and alloys.
- (c) Valence electrons become free in solids and move about randomly within the solids in the same way molecules in a gas confined to a container. Hence they are known as free electrons.
- (d) Neglecting the electron-electron interaction and the electron-ion interaction, it is similar to situation that electrons undergo periodic collision with ions in the lattice.

Success of Free Electron Theory

The free electron theory was successful in explaining various properties of metals as below:

- (a) The theory successfully explains Ohm's law.
- (b) **Electrical conductivity of metal:**
The theory successfully explains electrical conductivities of metals. It can be shown that Electrical conductivity $= \sigma = ne\mu$, where n is electron density, e is charge on electron and μ is the mobility of electrons.
- (c) **Thermal Conductivity**
The theory successfully explains thermal conductivities of metals at lower temperatures. From free electron theory, it can be shown that thermal conductivity of metals $K = \frac{k_B n v \lambda}{2}$, where k_B is Boltzmann constant.
- (d) **Relation between electrical and thermal conductivity (Wiedemann-Franz Law)**
The Wiedemann–Franz law states that the ratio of the thermal conductivity (κ) and the electrical conductivity (σ) of a metal is proportional to the temperature (T). From free electron theory it can be shown that $\kappa/\sigma T = 1.11 \times 10^{-8} \text{ W} \cdot \Omega / \text{K}^2$. This number is also known as Lorentz's number.

Limitations of Free Electron Theory

- (a) According to the theory, conductivity of metals is proportional to electron concentration. If this is always true, divalent (cadmium, zinc, etc) and trivalent (aluminum, etc) atoms should have more electrical conductivity than monovalent atoms (copper, silver, etc).
- (b) This model cannot explain the classifications of solids into conductors, semiconductors and insulators.

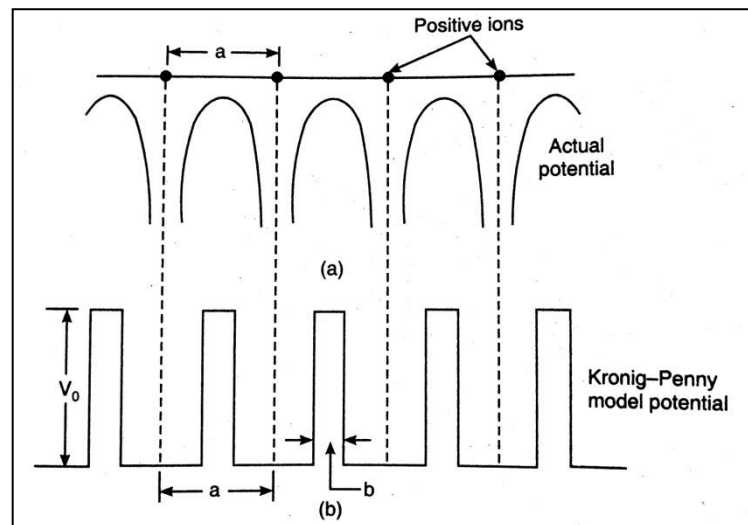
- (c) The theoretical values of mean free path of electrons as calculated from this theory do not agree with experimental values.
- (d) The theoretical values of specific heat and electronic specific heat as calculated from this theory do not agree with experimental values.
- (e) Some metals such as zinc have positive values of Hall effect. This theory could not explain why zinc and other metals have positive value of Hall coefficient.
- (f) Wiedemann-Franz law deviates at low temperature.

4.2 Opening of band gap due to internal electron diffraction

In order to find the allowed energies of electrons in solids, we have to apply Schrodinger's wave equation for an electron in a crystal lattice.

Kronig and Penney Model

1. Kronig and Penney suggested a simplified model where atoms are represented as one dimensional infinite row of rectangular potential well separated by barriers of width 'b'. Each well has width b and depth V_0 .
2. The interatomic spacing is represented as 'a'. Thus, the period of potential is (a+b).
3. When an electron moves through a lattice of positive ions, it experiences varying potentials.



Schrodinger's equation in one dimension

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

Boundary conditions and Schrodinger's equation

In the region $0 < x < a$, potential energy, $V = 0$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0$$

Above equations can be written as

$$\frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0 \text{ for } 0 < x < a,$$

$$\text{where } \alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

Solution using Bloch Theorem

The solution of the Schrodinger's equation for a periodic potential can be found using Bloch theorem, which is

$$\left(\frac{maV_0b}{\hbar^2} \right) \left(\frac{\sin \alpha a}{a} \right) + \cos \alpha a = \cos ka$$

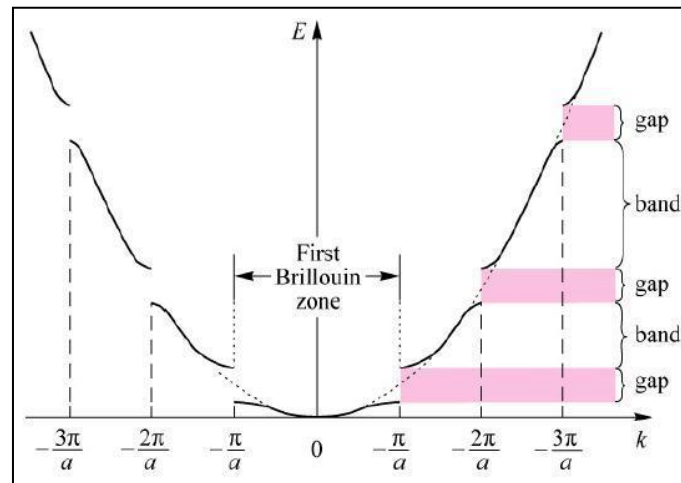
The above equation provides allowed solutions to the Schrodinger's equation. Due to cosine term, the right hand side varies between +1 and -1 and so left hand side is also allowed to vary between these two values. Thus, only certain values of α are possible. As $\alpha = \sqrt{\frac{2mE}{\hbar^2}}$, the energy E is restricted to lie within certain ranges.

Energy vs wave number

The relation between energy E and wave number k is a parabolic. The motion of free electron is interrupted at certain values of k , as shown by the broken curve.

The plot also shows discontinuities in Energy of electron at $k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}$

These gaps correspond to energy bands inside the solids. Thus, the energy level of electron in the crystal lattice is discrete.



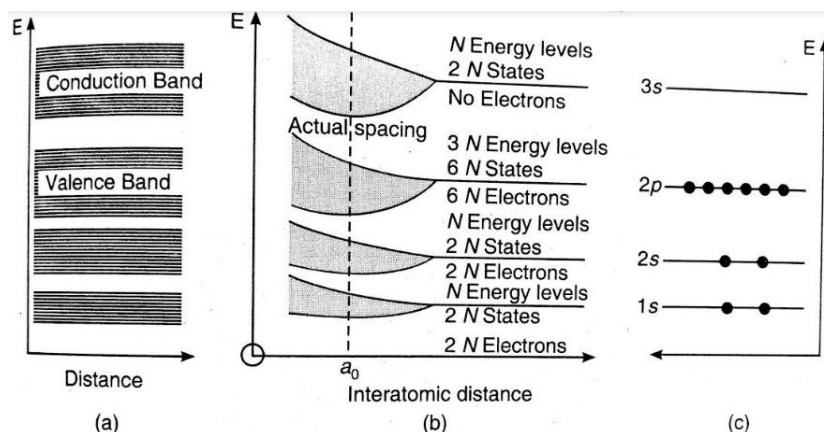
4.3 Band theory of solids

1. Energy level of isolated atom

In a single isolated atom, the energy of electrons is well defined and discrete as shown in figure (c).

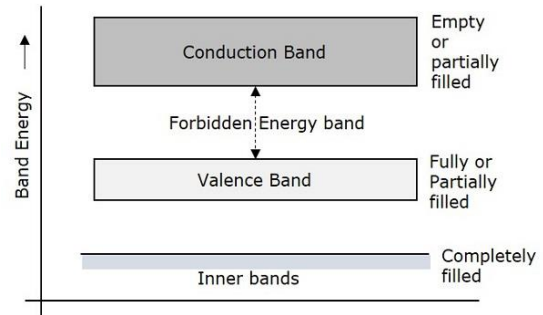
2. Formation of band

- A crystal (i.e. solid) consists of an enormous number of atoms arranged in a period structure. If there are N identical atoms, all of them have identical sets of energy levels.
- Figure (b) shows an atom at the origin of the coordinate system. When the other atoms approach this atom along three directions and stay at distance a_0 , which is lattice constant of the crystal. As atoms approaches each other, the interaction between them increases continuously. Each energy level splits into many distinct levels and form energy bands.
- If figure (b) is sliced at a_0 , it represents the energy band structure of the crystal. Corresponding to each allowed energy level of an isolated atom, there forms an allowed energy band separated by forbidden bands of energy.
- There are number of energy bands in solids but three of them are important to understand the behavior of solids. These energy bands are - Valence band, Conduction band, forbidden band or forbidden gap.



Valence Band

- The valence band is energy band formed by grouping the range of energy levels of the valence electrons that are engaged in covalent bonding.
- In the solids, valence electrons are not bound to any particular atom therefore, the valence band arises from splitting of the valence energy levels. It is common to the entire solid.



Conduction band:

- Conduction band corresponds to the energy values of free electrons that have broken their valence bonds and hence become free to move in the solid. It is formed by grouping the range of energy levels of the free electrons. It is also common to the entire solid.
- The bottom of the conduction band represents the smallest energy that the electron must possess to become free.

Forbidden energy gap/bandgap:

- Energy gap between top of the valence band and bottom of the conduction band is known as forbidden gap and corresponding energy is called band gap energy.
- In solids, electrons cannot stay in forbidden gap because there is no allowed energy state in this region.
- Forbidden gap is the major factor for determining the electrical conductivity of a solid. The classification of materials as insulators, conductors and semiconductors is mainly depends on forbidden gap.

Classification of solids on the basis of band theory [For understanding]

Concept of energy bands helps us in understanding the classification of solids into three groups. Nature of the energy bands determines whether the solid is conductor, insulator or semiconductor. According to band theory, the electrical conductivity of a solid is characterized by the energy gap E_g that separates valence band and conduction band. Ability of electrical conduction is decided by the order of magnitude of the energy gap E_g .

(a) Metals/Conductors

- In some solids a large numbers of free electrons are randomly moving in the entire solid that belong to conduction band. These free electrons also form a temporary bond with closest ion and are said to belong to valence band.
- Thus valence bands and conduction bands overlap and bandgap energy is zero. Electrons in the valence band can easily move to conduction band.
- Such solids are called conductors. The examples are metals such as copper, aluminum, silver, gold etc. Concentration of Free electrons is about 10^{28} electrons/ m^3 .
- When an electrical field is applied, these large numbers of electrons easily readily jump into conduction band and current flows in a large quantity.
- The resistivity of conductors is in between 10^{-8} to 10^{-6} ($\Omega\cdot m$). The conductivity is typically of the order of 10^7 (mho/m).

(b) Semiconductors

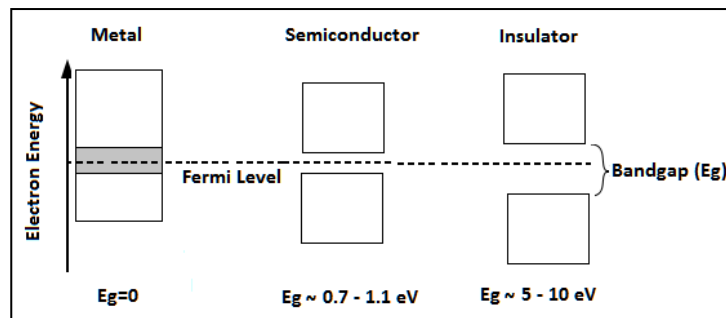
- In some solids, band gap is narrow and it is of the order of 2 eV or less. Valence electrons acquire a small amount of energy from the vibrations of atoms and can move from valence band to the conduction band. Thus conduction band is partially filled.
- If a potential is applied across the material, a moderate number of electrons in valence band

jumps to conduction band. Hence current flows in a modest proportion.

- Such solids are called semiconductors e.g. silicon, germanium, etc. Free electron density is about 10^{14} electrons/m³ which is very less as compared to metals.
- The resistivity of semiconductors varies from 10^{-4} to 10 ohms-m which depend on several factors such as doping concentration, temperature, etc. The conductivity is typically varies from 10^{-6} to 10^4 (mho/m).

(b) Insulators

- In some solids band gap energy is very wide (>5 eV). The electrons are tightly bounded to atoms and it would require very large amount of energy to raise an electron from valence band to conduction band. Very few electrons can get this large amount of energy at ambient temperature. Hence there are only a few electrons present in the conduction band.
- When potential difference is applied across them a negligible current flows and solid exhibits very low electrical conductivity.
- Such solids are called insulators e.g. glass, wood, mica, etc. The free electron density is negligible.
- The resistivity of insulators vary from 10^{12} to 10^{16} (Ω -m) and conductivity is typically 10^{-12} (mho/m).



4.4 Effective mass of electron

Effective mass of electron

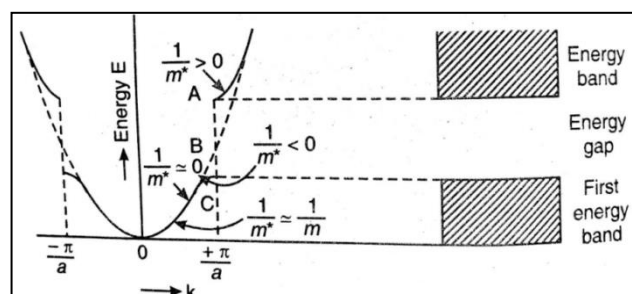
An isolated electron freely moving in vacuum has a well-defined mass. It is generally assumed that mass of an electron in solid is same as mass of a free electron. However, experimentally measured values indicate that in some solids electron mass is larger while for other it is slightly smaller than the free electron mass. This **experimentally determined electron mass is called as effective mass of the electron and it is denoted by m^*** .

Reason for variation of mass of electron

The reason for deviation of mass of electron inside the solid is due to the interactions between the drifting electrons and atoms in solid. Effective mass of electron inside the solid largely depends on its position.

Considering electron as a wave packet having a group velocity v_g , the expression for the

effective mass is derived as $m^* = \frac{\hbar^2}{d^2E/dk^2}$



- When energy of free moving electron is minimum, it is near the bottom of the conduction band. In this situation its effective mass is nearly identical to the mass of free electron. Therefore, in this region $m^* \approx m$.

- (ii) At the point of inflection B, the derivative $\frac{d^2E}{dk^2} = 0$. Hence in these regions $m^* \approx \infty$. It means that an external field cannot exert any action on the motion of the electron in this region.
- (iii) Near the top of the allowed band, the derivative $\frac{d^2E}{dk^2} < 0$. Therefore, the effective mass of electron m^* occupying levels near the top of the band is negative.

Significance of effective mass of electron

The concept of effective mass provides a satisfactory description of the charge carriers in crystal. The crystals such as alkali metals have partially filled energy band. Conduction takes place mainly through electrons. However, in crystals for which the energy band is nearly full, the negative charge and negative mass vacancies may be considered as positive charge and positive mass particle called holes. It also explains the origin of positive Hall coefficient in certain metals such as zinc.

4.5 Density of states

The density of states gives the number of allowed electron (or hole) states per volume at a given energy. It can be derived from basic quantum mechanics. **Density of states function is defined as the number of available states per unit volume per unit energy interval centered around E .** It represents the number of states that could be occupied by charge carriers.

The number of states lying in the range of energies between E and $E + dE$ is given by

$$g(E)dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE$$

The function $g(E)$ is known as density of states function.

Resistivity and conductivity [For understanding]

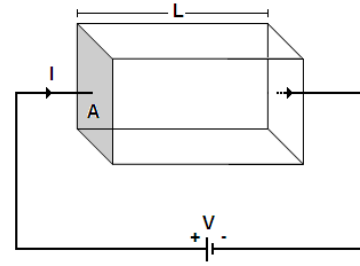
- If a potential difference V is applied across a solid, it creates an electric field E in the solid is given by $E = \frac{V}{L}$, where L is the length along which charge carriers move in the solid
- The current I passing across an area A is defined as the net charge Q transported through the area per unit time i.e. $I = \frac{Q}{t}$
- The magnitude of the current flowing through the material is governed by Ohm's law i.e. $I = \frac{V}{R}$
- The resistance R is given by, $R = \rho \frac{L}{A}$, where ρ is known as resistivity and its unit is ohm-meter
- The reciprocal of resistivity is known as conductivity i.e. $\sigma = \frac{1}{\rho} = \frac{L}{RA}$. The unit of conductivity is (ohm-meter)⁻¹ or mho and is called as siemens

4.6 Electrical conductivity in metals

The conductivity of a material can be related to the number of charge carriers present in the material.

Let us consider a rectangular block of length L and cross-sectional area A . Let n be the concentration of free electrons available in it.

Then,



Total number of electrons in solids (N) = (electron concentration) \times (volume)

Thus, $N = nAL$

Total charge present in the solid (Q) = (total number of electrons) \times (charge)

Thus, $Q = Ne = neAL$

The current through the solid, $I = \frac{Q}{t} = \frac{neAL}{t}$ --- (2)

Or $I = neAv_d$

Where, v_d is the average drift velocity of electrons

The current density is given by $J = \frac{I}{A}$

Or $J = \frac{neAv_d}{A}$

Or $J = nev_d$

Now $I = \frac{V}{R}$

But $R = \rho \frac{L}{A}$, Hence $I = \frac{VA}{\rho L} = \frac{1}{\rho} \frac{V}{L} A$

Or $I = \sigma EA$ --- (3)

Now $J = \frac{I}{A} = \frac{\sigma EA}{A} = \sigma E$ --- (4)

Thus, $J = \sigma E$ --- (5)

This equation is known as point form or microscopic form of Ohm's law

From equations (2) and (5) $J = nev_d = \sigma E$

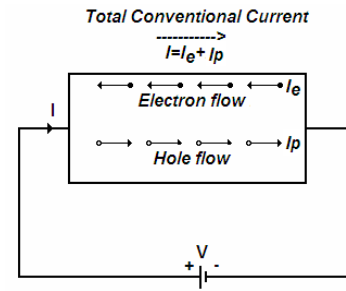
Thus, $\sigma = \frac{nev_d}{E} = ne\mu$ where $\mu = \frac{v_d}{E}$

Thus, conductivity can be written as $\sigma = ne\mu$ --- (6)

The quantity μ is known as mobility of electrons. It indicates the ease with which electrons move in a solid. It is defined as the drift velocity per unit electric field. The unit of mobility is $\text{m}^2/\text{V.s}$. In metals it is of the order of $10^{-3} \text{ m}^2/\text{Vs}$ and in semiconductors it is of the order of $10^{-1} \text{ m}^2/\text{V.s}$.

4.7 Electrical conductivity in semiconductors

Let us consider a sample of semiconductor across which a dc voltage source is connected. If a potential difference is applied across semiconductor, it generates an electric field. This causes electrons to move towards the positive terminal and holes towards the negative terminal of the source.



Let, V – applied potential difference
 E – electrical field generated
 I – total current
 I_e – current due electrons drifting in the conduction band
 I_h – the other consists of holes drifting in the valance band (I_p)
 v_e – drift velocity of electrons
 v_h – drift velocities of holes
 n_e – concentrations of electrons
 n_h – concentration of holes

The current due to electrons $I_e = n_e e A v_e$
 The current due to holes $I_h = n_h e A v_h$

Now, Total current I
 $I = I_e + I_h$
 $I = n_e e A v_e + n_h e A v_h$
 $I = e A (n_e v_e + n_h v_h)$

If μ_e and μ_p are mobilities of electron and holes $\mu_e = \frac{v_e}{E}$ and $\mu_h = \frac{v_h}{E}$
 Hence $v_e = \mu_e E$ and $v_h = \mu_h E$

Thus, Total Current
 $I = e A (n_e \mu_e E + n_h \mu_h E)$
 $I = e A E (n_e \mu_e + n_h \mu_h)$ --- (1)

The total current density is J is given by $J = I/A$, where A is cross-sectional area of the sample
 Thus, $J = \frac{I}{A} = \frac{e A E (n_e \mu_e + n_h \mu_h)}{A}$

And, $J = e E (n_e \mu_e + n_h \mu_h)$ --- (2)

If σ is the total conductivity then, $J = \sigma E$ --- (3)

Comparing equations (2) and (3) $\sigma = e (n_e \mu_e + n_h \mu_h)$ --- (4)

This expression gives the electrical conductivity of a typical semiconductor. In general, it is linked to the variation of mobility with temperature is too small and the large variation in electrical conductivity is because of large variation of electron concentration with temperature.

Case (a) Intrinsic semiconductor

For an intrinsic semiconductor, the concentration of electrons and holes is equal.

Thus, $n_e = n_h = n_i$ (say)

Thus, conductivity of intrinsic semiconductor $\sigma_i = e n_i (\mu_e + \mu_h)$

Case (b) N-type semiconductor

For a n-type semiconductor, the concentration of electrons is much greater than the concentration of holes i.e. $n_e \gg n_h$. The majority charge carriers are electrons and the contribution of holes is neglected due to very small value.

Thus, conductivity of N-type semiconductor $\sigma_e = en_e\mu_e$

Case (c) P-type semiconductor

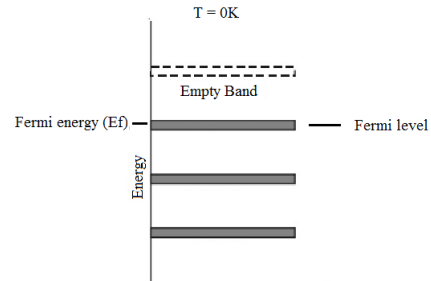
For a p-type semiconductor, the concentration of holes is much greater than the concentration of electrons i.e. $n_h \gg n_e$. The majority charge carriers are holes and the contribution of electrons is neglected due to very small value.

Thus, conductivity of P-type semiconductor $\sigma_h = en_h\mu_h$

4.8 Fermi energy and Fermi-Dirac distribution function

Fermions: Fermions are particles which have half-integer spin (like $1/2, 3/2, \dots$) and are indistinguishable particles. Fermions obey the Pauli Exclusion Principle and therefore cannot co-exist in the same state at same location at the same time. Fermions include electrons, protons.

Fermi Energy: In a conductor at absolute zero temperature, the electrons fill the available states starting from the lowest energy level. Therefore, all the levels with an energy less than a certain value $E_f(0)$ will be filled with electrons, whereas the levels with $E > E_f(0)$ will remain vacant. The energy $E_f(0)$ is known as the Fermi energy and corresponding energy level is known as Fermi level. Thus, Fermi level is the highest filled energy level at 0K.



Fermi-Dirac Distribution Function

The Fermi function $f(E)$ gives the probability that the energy state E will be occupied by electron at temperature T . It is derived as:

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

Where,

$f(E)$ – Fermi function

E – energy of given state to be occupied

E_f – Fermi energy

T – temperature (in Kelvin) for the available state

k – Boltzmann's constant, $k = 1.3806503 \times 10^{-23} \text{ J/K}^0$

Case I: At $T = 0\text{K}$, for $E < E_f$ (for the energy levels located below Fermi energy)

The quantity $(E - E_f)$ in the equation is negative and the argument of the exponential function is $-\infty$.

$$\text{Thus, } f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$$

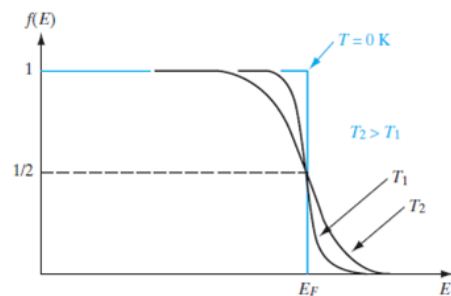
$f(E) = 1$ implies that all the levels below E_f are occupied by electrons.

Case II: At $T = 0\text{K}$, for $E > E_f$ (for the energy levels located above Fermi energy)

The quantity $(E - E_f)$ in the equation is positive and the argument of the exponential function is $+\infty$.

$$\text{Thus, } f(E) = \frac{1}{1 + e^{+\infty}} = \frac{1}{1 + \infty} = 0$$

$f(E) = 0$ implies that all the levels above E_f are vacant.



Case III: At $T = 0\text{K}$, for $E = E_f$ (for the energy levels equal to Fermi energy)

The quantity $(E - E_f)$ in the equation is zero. Thus, $f(E) = \frac{1}{1 + e^{0/0}}$ is indeterminate.

Thus at Fermi level at $T = 0\text{K}$, the occupation of electron has indeterminate value ranging between zero and one.

Case IV: At $T > 0\text{K}$, for $E = E_f$ (for the energy levels equal to Fermi energy above $T = 0\text{K}$)

$$\text{The quantity } (E - E_f) \text{ in the equation is zero. Thus, } f(E) = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2} = 0.5$$

It implies that the probability of occupancy of Fermi level at any temperature above 0K is 0.5. Thus, **Fermi energy is the average energy possessed by the electrons which participate in conduction process in conductors at temperature above 0K.**

4.9 Position of Fermi level in intrinsic semiconductor

The Fermi function $f(E)$ gives the probability that the energy state (E) will be occupied by electron at temperature (T). It is given as $f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$ --- (1)

Where, $f(E)$ – Fermi function

E – energy of given state

E_f – Fermi energy

T – Temperature (in Kelvin) for the available state

k – Boltzmann's constant, $k=1.3806503 \times 10^{-23}$ J/K⁰

For this derivation it is assumed that:

1. The widths of valance band and conduction band are small compared to the forbidden energy gap E_g
2. All energy levels in conduction band have energy E_c , and all energy levels in valence band have energy E_v .

Let n_c - number of electrons in conduction band

n_v - number of electrons in valence band

N - total number of electrons in both conduction and valence bands

$$\text{Thus, } N = N_c + N_v \quad \text{--- (2)}$$

$$\text{The probability that there are } n_c \text{ electrons occupy energy state } E_c \text{ is } P(E_c) = \frac{n_c}{N} \quad \text{--- (3)}$$

According to F-D distribution function, the probability of electron in conduction band occupying energy level E_c will be given by

$$f(E_c) = \frac{1}{1+e^{(E_c-E_f)/kT}} \quad \text{--- (4)}$$

As equations (3) and (4) represents the same probability i.e. $P(E_c) = f(E_c)$

$$\text{Or } \frac{n_c}{N} = \frac{1}{1+e^{(E_c-E_f)/kT}}$$

$$\text{Or } n_c = \frac{N}{1+e^{(E_c-E_f)/kT}} \quad \text{--- (5)}$$

$$\text{Similarly for valence band } n_v = \frac{N}{1+e^{(E_v-E_f)/kT}} \quad \text{--- (6)}$$

$$\text{From equations (2) } N = n_c + n_v \quad \text{--- (7)}$$

Putting values of n_c and n_v from (5) and (6)

$$N = \frac{N}{1+e^{(E_c-E_f)/kT}} + \frac{N}{1+e^{(E_v-E_f)/kT}}$$

$$\text{Or } 1 = \frac{1}{1+e^{(E_c-E_f)/kT}} + \frac{1}{1+e^{(E_v-E_f)/kT}}$$

$$\text{Or } \left[1 + e^{\frac{E_c-E_f}{kT}}\right] \left[1 + e^{\frac{E_v-E_f}{kT}}\right] = 1 + e^{\frac{E_c-E_f}{kT}} + 1 + e^{\frac{E_v-E_f}{kT}}$$

Solving this we get

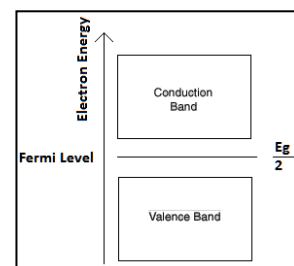
$$e^{(E_c+E_v-2E_f)/kT} = 1$$

$$\text{Taking logarithm of both sides } \frac{E_c+E_v-2E_f}{kT} = 0$$

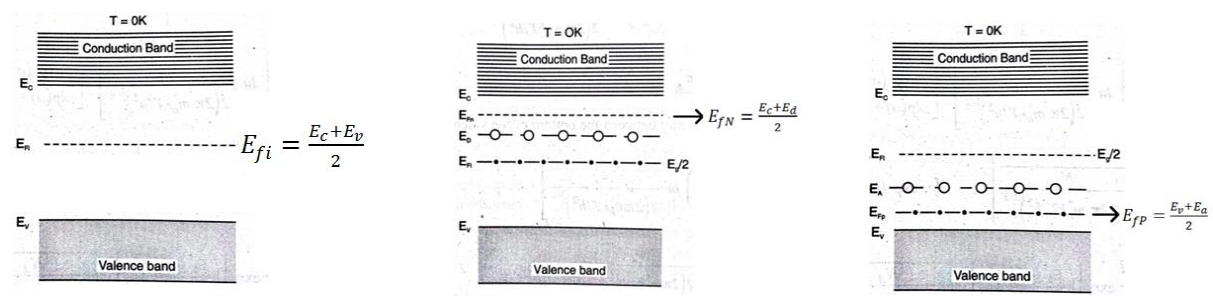
$$\text{Or } E_c + E_v - 2E_f = 0$$

$$\text{Or } E_{fi} = \frac{E_c + E_v}{2} \quad \text{--- (8)}$$

Thus, Fermi level in intrinsic semiconductors is exactly in the middle of the forbidden energy gap i.e. it indicates a reference level indicating the distribution of electrons and holes in intrinsic semiconductors.



Position of Fermi Level in Extrinsic Semiconductors (at 0K)



N-type semiconductor

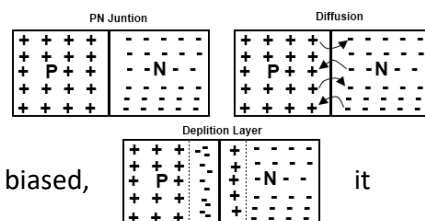
- An intrinsic semiconductor is doped with a pentavalent donor impurity atom.
- The energy levels of donor atoms are very close to bottom of unfilled conduction band.
- The electrons donated by donor atoms enter into conduction band.
- The average energy of electrons increases. Thus, Fermi level shifts towards conduction band.
- At $T=0K$, E_{fN} lies midway between the donor levels and the bottom of the conduction band
 i.e. $E_{fN} = \frac{E_c + E_d}{2}$

P-type Semiconductor

- An intrinsic semiconductor is doped with a trivalent impurity atom and its energy levels are very close to the top of filled valence band
- After doping, a deficiency of an electron is produced and it creates a hole in the valence band.
- Thus, there is less number of electrons available in valence band the average energy of electrons in valence band decreases. As a result Fermi level shifts down towards the valence band.
- At $T=0K$, Fermi level lies midway between the acceptor levels and the top of the valence band. Thus, $E_{fP} = \frac{E_v + E_a}{2}$

PN Junction diode

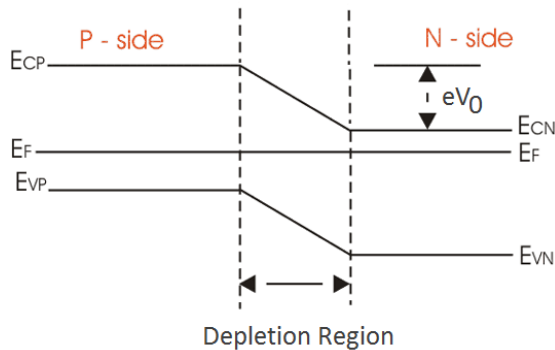
- A p-n junction diode is two-terminal semiconductor device, which allows the electric current in only one direction.
- If the diode is forward biased, it allows the electric current flow. On the other hand, if the diode is reverse biased, blocks the electric current flow.
- When a junction is made between these materials, the electrons would tend to move from n-type material to p-type material, which is known as **diffusion**
- The combining of electrons and holes depletes the holes in the p-region and the electrons in the n-region near the junction. This region is known as **depletion region**.
- After formation of depletion region, the movement of electrons and holes across junction stops and a potential barrier is created.



4.10 Working of PN junction on the basis of band theory

I. Non Equilibrium State

A p-n junction diode is constructed by joining a P and N type semiconductors. When P and N type semiconductors are isolated from each other they are said to be in non-equilibrium state. Fermi level of n-type semiconductor is located close to the bottom of the conduction band. Fermi level of p-type semiconductor is located close to the top of the valence band.

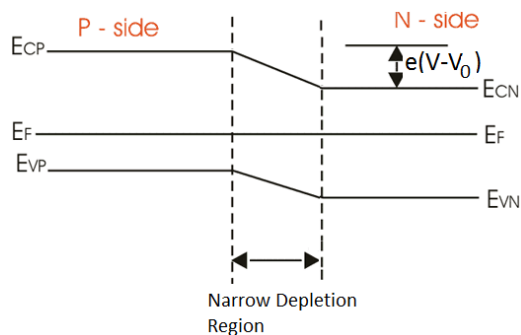


II. At thermal equilibrium / zero bias

Formation of depletion region

When the two semiconductors are joined, the electrons from the n-side are diffused to the p region and they form a layer of negative charge on the p-side. Thus, the Fermi level of N region decreases.

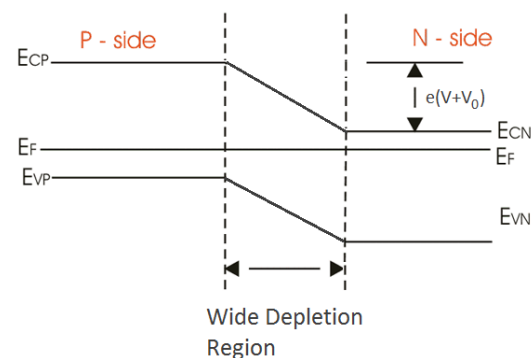
The electrons leaving N region, enters into P region and the holes from p-side diffused to the N region. Thus, the Fermi level of P region increases.



At equilibrium

When equilibrium achieved, the movement of electrons and holes across the junction stops and a depletion region is formed. The band edges in the two regions shift themselves so that Fermi levels are properly aligned and attain equilibrium. The Fermi level in N region shifts down by an energy $e.V_0$ whereas Fermi level in P region shifts upward by the same energy $e.V_0$, where V_0 is the potential barrier across the junction.

After equilibrium is attained, the majority electrons from N region face the potential barrier across the junction. Thus they cannot cross the junction and no current flows.



II. P-N junction under forward bias

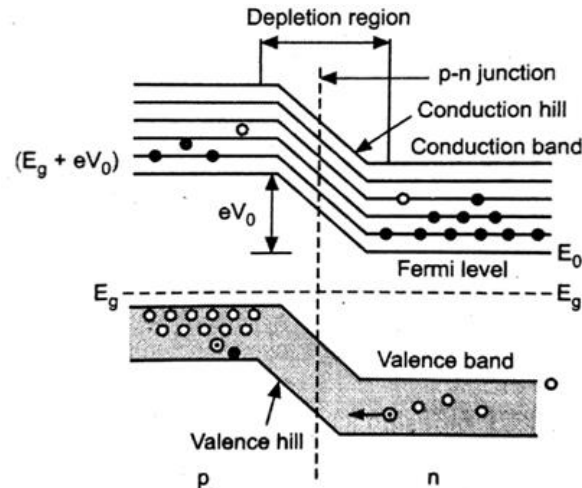
An external potential difference V is applied to diode in forward bias. This voltage should be greater than potential barrier of V_0 . Now electrons from the source are added to N region. Hence potential in N region rises by $e(V_0 - V)$. The Fermi level of P region does down. Electrons in N region now face a lower potential barrier. The width of potential barrier is reduced. Hence electrons from N region cross the junction easily to enter into P region and electrons flow increases. This constitutes forward bias current. This current increases with increase in forward bias.

III. P-N junction under reverse bias

An external potential difference V is applied to diode in reverse bias. Due to this electrons are removed from N region. Hence potential in N region decreases by $e(V_0+V)$. Electrons enter into P region and hence potential in this region rises. Electrons in N region now face a higher potential barrier. The width of potential barrier is increased. Hence electrons from N region cannot cross the junction easily to enter into P region and electron flow decreases. However, small leakage current flows through junction which constitutes reverse bias current.

4.11 Expression for barrier potential of a diode

The internal potential barrier V_0 for a diode can be calculated from the electron concentrations in p and n regions.



The Fermi function $f(E)$ gives the probability that the energy state (E) will be occupied by electron at temperature (T). It is given as $f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$ --- (1)

Where, $f(E)$ – Fermi function

E – energy of given state

E_f – Fermi energy

T – Temperature (in Kelvin) for the available state

k – Boltzmann's constant, $k=1.3806503 \times 10^{-23}$ J/K⁰

- Let
- n_n – electron concentration in the conduction band on the n-side, and
 - n_p – hole concentration in the conduction band on the n-side, and
 - N_c – average concentration of electrons in conduction band of p-side and n-side
 - E_g – band gap energy
 - E_f – Fermi energy
 - k – Boltzmann constant
 - T – Absolute temperature in Kelvin

Using Fermi function, the probability of occupation of electrons (n_n) out of average concentration of electrons in conduction band (N_c) in Fermi level (E_f) is given by

$$n_n = \frac{N_c}{1+e^{(E_g-E_f)/kT}} \quad \text{--- (2)}$$

When number of particles is very small compared to the available energy levels, the probability of an energy state being occupied by more than one electron is small. In this situation, $(E_g-E_f) \gg 3kT$.

Due to this, now, $(1 + e^{(E_g-E_f)/kT}) \approx e^{(E_g-E_f)/kT}$.

Thus,
$$n_n = \frac{N_c}{e^{(E_g - E_f)/kT}} = N_c e^{-(E_g - E_f)/kT}$$

It can be written as
$$n_n = N_c \exp[-(E_g - E_f)/kT] \quad (1)$$

Due to the barrier potential V_0 , the band gap energy on p-region increases by $(E_g + eV_0)$. The electron concentration on p-side can be written as

$$n_p = N_c \exp[-\{(E_g + eV_0) - E_f\}/kT] \quad (2)$$

Dividing the two equations, we get
$$\frac{n_n}{n_p} = \exp \frac{eV_0}{kT} \quad (3)$$

Taking log of both sides,
$$\ln \frac{n_n}{n_p} = \frac{eV_0}{kT}$$

$$\text{OR} \quad V_0 = \frac{kT}{e} \ln \frac{n_n}{n_p} \quad (4)$$

If p_p is concentration of holes in p-region, above equation can be written as

$$V_0 = \frac{kT}{e} \ln \frac{n_n p_p}{n_p p_p} \quad (5)$$

Let N_D – concentration of donor atoms

N_A – Concentration of acceptor atoms

At room temperature, all the impurities are ionized, hence: $n_n = N_D, p_p = N_A$

Also, $n_p p_p = n_i^2$

Thus, equation (5) can be written as
$$V_0 = \frac{kT}{e} \ln \frac{N_D N_A}{n_i^2}$$

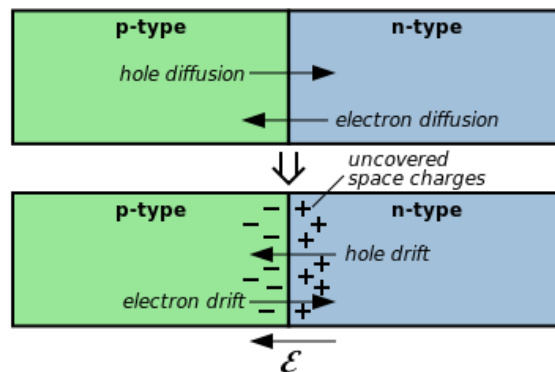
The factor $\frac{kT}{e}$ is of the dimensions of voltage and is denoted by V_T . Thus $V_0 = V_T \ln \frac{N_D N_A}{n_i^2}$

Significance of barrier potential equation

Thus, barrier potential in a junction diode depends on the equilibrium concentration of the impurities in p and n regions and does not depend on the charge density in the depletion region.

4.12 Ideal diode equation

The ideal diode equation is an equation that represents current flow through an ideal p-n junction



Unbiased diode

(a) Current due to Diffusion

When P and N regions are brought together, due to concentration difference, holes in p-region diffuse to n-region and electrons in n-region are diffused to p-region.

Let, $(J_e)_{DF}^{UB}$ – electron diffusion current density of unbiased diode

$(J_h)_{DF}^{UB}$ – hole diffusion current density of unbiased diode

(b) Current due to drift

Due to potential barrier, electrons reaching the edge of the junction on p-side are accelerated by electric field into n-region and similarly, holes reaching the edge of the junction on n-side are accelerated into p-region.

Let, $(J_e)_{DR}^{UB}$ – electron drift current density of unbiased diode

$(J_h)_{DR}^{UB}$ – hole drift current density of unbiased diode

Forward biased diode

When the diode is forward biased with potential V_F , the potential barrier is lowered by an amount of energy eV_F and the probability of a majority carrier crossing the junction and diffusion current density is increased by a factor of $\exp\left(\frac{eV_F}{kT}\right)$

Thus, electron diffusion current density in forward bias = $(J_e)_{DF}^{FB} = (J_e)_{DF}^{UB} \times \exp\left(\frac{eV_F}{kT}\right)$

And hole diffusion current density in forward bias = $(J_h)_{DF}^{FB} = (J_h)_{DF}^{UB} \times \exp\left(\frac{eV_F}{kT}\right)$

Under equilibrium

(hole diffusion current density) = (hole drift current density)

$$(J_h)_{DF}^{FB} = (J_h)_{DF}^{UB} \times \exp\left(\frac{eV_F}{kT}\right) = (J_h)_{DR}^{UB} \times \exp\left(\frac{eV_F}{kT}\right)$$

(electron diffusion current density) = (electron drift current density)

$$(J_e)_{DF}^{FB} = (J_e)_{DF}^{UB} \times \exp\left(\frac{eV_F}{kT}\right) = (J_e)_{DR}^{UB} \times \exp\left(\frac{eV_F}{kT}\right)$$

The drift current density components have not changed and have the same magnitude as in equilibrium case.

Let, $(J_e)_{NET}$ - Net electron current density of unbiased diode

$(J_h)_{NET}$ - Net hole current density of unbiased diode

Therefore, the net hole current density across the forward bias junction is

$$(J_h)_{NET} = (J_h)_{DF}^{FB} - (J_h)_{DR}^{UB}$$

$$(J_h)_{NET} = (J_h)_{DR}^{UB} \times \exp\left(\frac{eV_F}{kT}\right) - (J_h)_{DR}^{UB} = (J_h)_{DR}^{UB} \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right]$$

Similarly,

$$(J_e)_{NET} = (J_e)_{DR}^{UB} \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right]$$

Let

J_{Total} = Total current density across the forward bias junction

$$J_{Total} = (J_h)_{NET} + (J_e)_{NET}$$

$$J_{Total} = (J_h)_{DR}^{UB} \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right] + (J_e)_{DR}^{UB} \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right]$$

$$J_{Total} = \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right] \left[(J_h)_{DR}^{UB} + (J_e)_{DR}^{UB} \right]$$

$$\text{Let, } J_0 = \left[(J_h)_{DR}^{UB} + (J_e)_{DR}^{UB} \right]$$

$$\text{Or } J = J_0 \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right]$$

$J=I/A$ is current density. Thus, above equation can be written as

$$I = I_0 \left[\exp\left(\frac{eV_F}{kT}\right) - 1 \right] \quad \text{Where, } I=J/A \text{ and } I_0=J_0/A$$

The above equation can generally be written as $I = I_0(e^{\frac{eV}{kT}} - 1)$ which is called diode equation.

Where, V denotes voltage ($V=V_F$ is for forward bias and $V=-V_R$ for reverse bias) and

I_0 denotes reverse saturation current.

Significance of diode equation

Diode current equation expresses the relationship between the current flowing through the diode as a function of the voltage applied across it.

Photovoltaic Effect

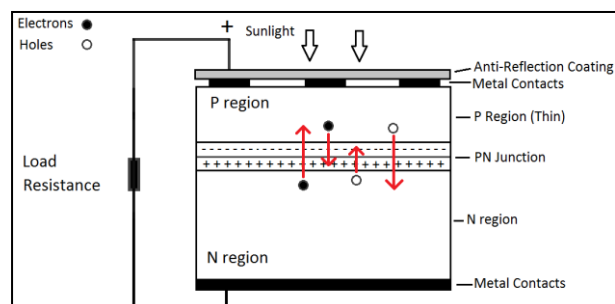
- Photovoltaic effect is a process in which two dissimilar materials in close contact produce an electrical voltage when light or other radiant energy incident on it.
- In crystals such as silicon or germanium, electrons are usually not free to move from atom to atom. When light incident on crystal, the photons provides energy to some electrons to become free from their bound condition. These free electrons cross the junction between two dissimilar crystals more easily in one direction than in the other. This generates a negative charge and negative voltage in one region as compared to other region.
- The photovoltaic effect can continue to provide voltage and current as long as light continues to fall on the two materials.

4.13 Application of PN Junction diode: Solar cell

A solar cell or photovoltaic cell works on the principle of photovoltaic effect. Solar cells convert solar energy into electrical energy. Solar cells can be designed to work in UV, visible or IR region of the electromagnetic spectrum.

Construction of solar cell

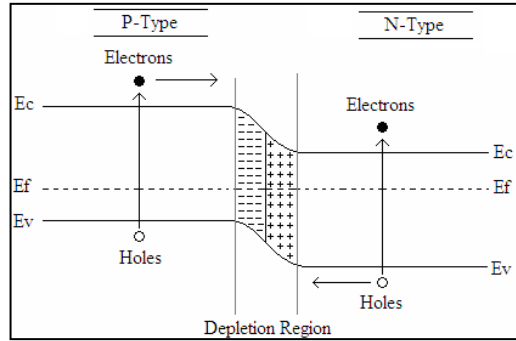
- A thin layer of p-type semiconductor material is doped over an n-type substrate forming a P-N junction.
- The thickness of the p-layer is very small.
- Electrical contacts are made over the top and bottom sides.
- To enhance the transmission of light into the material, an anti-reflection coating is given over the p-type layer.



Working of solar cell

- When the sunlight falls on the upper p-region, it generates the electron-hole pairs in both P and N regions.
- Thickness of p-layer is small. Hence the probability of recombination of electrons and holes is less and they immediately reach the PN junction.

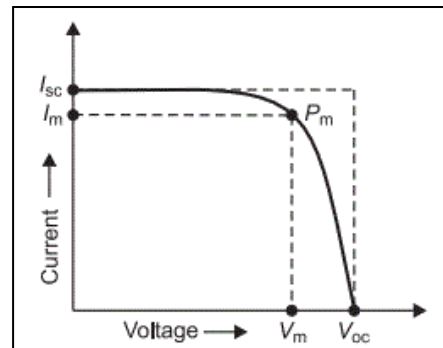
- At the junction, barrier potential attracts electrons from p-region into n-region and holes from n-region into p-region. This leads to an increase in the number of holes on p-region and electrons in the region.
- The accumulation of charges on the two sides of the junction produces a voltage or EMF known as photo EMF. It is known as the open circuit voltage and is proportional to the intensity of the incident light as well as size of the illuminated area.
- If an external circuit is connected across the solar cell terminals, a current flows through the circuit.



I-V Characteristics of a Solar Cell

Typical I-V characteristics of the solar cell can be studied with a simple circuit as shown in the figure.

1. **Short circuit current (I_{sc}):** Initially when load resistance R_L is kept minimum, the maximum current flows in the circuit. When the load is zero ($R_L \rightarrow 0$), maximum current is known as Short Circuit Current.
2. **Variation of I with V**
As load resistance is increased, the voltage across the load resistance R_L increases and current starts decreasing.
When the load is maximum ($R_L \rightarrow \infty$), the current is very small and the voltage across the load is maximum.
3. **Open Circuit Voltage (V_{oc}):** Ideally when the circuit is open, voltage is maximum and current is zero. This maximum voltage is Open Circuit Voltage
4. **The Fill Factor:**
Fill factor of solar cell determines the maximum power from a solar cell. It is defined as the ratio of maximum useful power of solar cell to the ideal maximum power. The Fill Factor (FF) is essentially a measure of quality of the solar cell.
Maximum useful power obtained experimentally : $P_{max} = I_m \times V_m$
Ideal power output from the cell : $P_{ideal} = I_{sc} \times V_{oc}$



Where, I_m and V_m are the values of maximum current and voltage obtained experimentally.

$$\text{Fill Factor} = \frac{\text{Maximum Useful Power}}{\text{Ideal Power}} = \frac{P_{max}}{P_{ideal}} = \frac{I_m \times V_m}{I_{sc} \times V_{oc}}$$

The maximum useful power obtained experimentally is always less than the ideal maximum power; the value of fill factor is less than one. Usually the value of fill factor is around 60-70%.

5. **Efficiency of solar cell:** Efficiency of solar cell is defined as the ratio of energy output from the solar cell to input energy from the sun.

$$\eta = \frac{I_m \times V_m}{\text{Incident Light Power}}$$

Usually the voltage developed by a solar cell is of the order of 0.6V and the efficiency of conversion is about 15-21% for silicon cells.

Advantages of solar cell

1. Solar cells obtain energy from the sun and transform this into usable electricity.
2. Renewable energy - The energy can be used both to generate electricity and heat either through solar PV system or solar thermal system.
3. Economy-friendly energy – Once the installation is made, solar cell can be operated at almost no cost (except maintenance).
4. Environmentally friendly energy - Solar cells produce energy without pollution
5. Long term energy - PV systems often have a long life and a good durability.
6. Solar cells can be installed virtually anywhere; in a field to on a roof of building

Limitations of solar cell

1. The main disadvantage of solar energy is the initial cost and installation is much higher.
2. Most types of solar cell require large areas to achieve average efficiency.
3. The efficiency of solar cells is affected due to weather, air pollution

Applications of solar cell

1. Solar cells are largely used for providing electricity to rural and remote areas where grid installation is difficult.
2. Solar cells are used for powering telecommunications system in rural and remote areas such as transmission of signals, radio, TV sets, emergency telephone system, etc.
3. Solar energy is used extensively to pump water from wells and rivers to villages for domestic consumption and irrigation of crops.
4. In the space, solar energy is used to power the satellites and space vehicles.
5. Many consumer products such as watches, toys, calculators, emergency power systems use solar cells.

Ways to improve efficiency of solar cell

The solar cells developed using crystalline silicon technology and are used for commercial purpose have typical efficiencies ranging from 15 to 21%. There are number of ways for improving efficiency of solar cells. The efficiency of solar cell could be up to 45% depending on the technology used for its development. Some of the ways are discussed as below:

Fill factor and shape of solar cell

Fill factor refers to the utilization of available surface area. Typically it ranges from about 70 to 90%. Fill factor of solar panels depends on their shape. For example, round or moon shaped solar cell have lower fill factor; square cells have more fill factor. By choosing proper shape, efficiency of solar cell can be improved.

Solar Cells Glazing

For long life, solar cells must be protected from the external factors such as rain, snow, bird dropping etc. For glazing the solar panel, polycarbonate or low-iron glass is generally used due to high optical transmissivity (around 90%).

Solar Panel Orientation

For highest output, solar panels must be perpendicular to the sun's rays. However, for roof-top solar panels their orientation is fixed. For other types of fixed installations, the azimuth is oriented to the south and tilt adjusted for the winter period.

Solar tracker

Solar tracker is the rotation of the panel so that it always directly faces the sun. However, larger the array, more difficult will be the operation. The optimum tilt angle changes slowly as the earth rotates on its axis

Light Concentrators

Solar panel output power may be increased to almost 50% via a light concentrator such as a Fresnel lens or mirror. However, installing concentrators for a large array of solar cells and orientation of the mirror creates an additional tracking problem.

Solar Charge Controls

Since the solar panel may not have the correct voltage to charge a battery, it must be controlled via solar charge controller to prevent battery overcharge. The excess power needs to be dissipated using heat sink.

MPPT Controller

MPPT stands for Maximum Power Point Tracking. MPPT control does not turn the excess power into heat but it turns it into additional charge current. The control senses both input voltage and current and then adjust output to maximize power transfer.

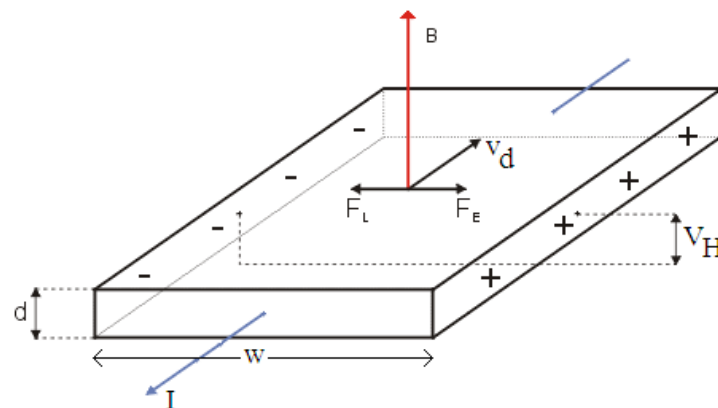
4.14 Hall effect and Hall coefficient

Hall Effect

Hall effect is generation of a transverse electric field in a solid material when it carries an electric current and is placed in a magnetic field that is perpendicular to the current.

Explanation

Let electric current is flowing through a slab of conductor or semiconductor along its length. A magnetic field is applied to the slab perpendicular direction of the current flow along its thickness. Due to the force of magnetic field (Lorentz force), the charge carriers are deflected towards opposite edges of the slab. This generates electric field and develops potential difference across the edges of the slab i.e. along the width of the slab. This electric field is perpendicular to the direction of flow of current and magnetic field.



Consider a slab of a conductor or semiconductor as shown in figure. Let

- w - width of the slab
- d - thickness of the slab
- I - current flowing through the slab
- n - charge concentration (no. of charge carriers per unit volume)
- q - charge on the carriers
- v_d - drift velocity of charge carriers
- A - cross-sectional area of the slab

Cross sectional area of the semiconductor $A = wd$

The current through the semiconductor

Thus,

$$I = nqAv_d$$

$$v_d = \frac{I}{nqA} = \frac{I}{nqwd} \quad \text{--- (1)}$$

(a) Force due to magnetic field

Due to current I , electrons move along length of the solid. When magnetic field of strength B is applied perpendicular along thickness of the slab, charge carriers experiences a force known as Lorentz force. Due to this Lorentz force, charge carriers are deflected towards edges of the slab.

The magnitude of the force is given by

$$F_L = Bqv_d \quad \text{--- (2)}$$

Charge carriers are now concentrated along the edges of the slab. Thus, an electric field created along edges of the slab, say E_H .

(b) Force due to electric field

Electric field developed across the edges of E_H , exerts an attractive force on charge carriers. It exerts a force on charge carriers in opposite direction.

Force due to electric field on the charge carriers is given by

$$F_E = qE_H \quad \text{--- (3)}$$

(c) Under the equilibrium of forces due to magnetic and electric field

Now, charge carriers are under the influence of two forces. One force due to magnetic field i.e. F_L and second due to electric field F_E . Both forces are in opposite directions. Under equilibrium, if magnitude of both forces is are equal and net force on charge carrier is zero.

Thus,

Substituting the values from (2) and (3)

Or

$$F_L = F_E$$

$$Bqv_d = qE_H$$

$$E_H = Bv_d \quad \text{--- (4)}$$

If V_H is potential difference generated along the width of the slab due to the electric field E_H , then,

$$E_H = \frac{V_H}{w}$$

Putting the value of E_H in equation (4)

$$\frac{V_H}{w} = Bv_d$$

Or

$$V_H = Bv_d w$$

$$\text{As, } v_d = \frac{I}{nqwd}$$

Or

$$V_H = B \frac{I}{nqwd} w$$

$$V_H = \frac{BI}{nqd} \quad \text{--- (5)}$$

Or

$$V_H = \frac{1}{nq} \frac{BI}{d} \quad \text{--- (6)}$$

Let the quantity $\frac{1}{nq} = R_H$. This quantity is known as Hall coefficient.

Thus, Hall Voltage is given by

$$V_H = R_H \frac{BI}{d} \quad \text{--- (7)}$$

Hence, magnitude and direction of Hall voltage is dependent on magnitude and direction of current and magnetic field. Also, Hall voltage is inversely proportional to thickness of the slab.

Applications of Hall Effect

(a) Determination of type of semiconductor

The Hall voltage developed across the edges of the semiconductor is given by $V_H = R_H \frac{BI}{d}$ or Hall coefficient $R_H = V_H \frac{d}{BI}$

By measuring Hall voltage, the value of Hall coefficient R_H can be determined.

If value of R_H is positive then type of semiconductor is p-type. If R_H is negative the type of semiconductor is n-type.

(b) Determination of charge carrier concentration

Hall coefficient is given by $R_H = V_H \frac{d}{BI}$

This value of Hall coefficient is also equal to $R_H = \frac{1}{nq}$

Hence, charge carrier concentration $n = \frac{1}{R_H q}$

Thus, if value of R_H is known, the charge carrier concentration can be found.

(c) Determination of charge carrier mobility

The electrical conductivity is given by $\sigma = nq\mu$

where μ is mobility of charge carriers

Thus, $\mu = \frac{\sigma}{nq}$

As $R_H = \frac{1}{nq}$ $\mu = \sigma R_H$

Thus, the charge mobility can be determined using Hall coefficient.

Questions on Semiconductor Physics

6 Marks

1. Explain Opening of band gap due to internal electron diffraction.
2. Explain how energy bands are formed inside a solid.
3. Using band theory, explain how solids are classified into conductors, semiconductors and insulators.
4. Using Fermi-Dirac probability distribution function, derive an expression for the position of Fermi energy level in the intrinsic semiconductor. *[Dec 19, 6m]*
5. Derive the expression for conductivity of intrinsic and extrinsic semiconductors.
6. Explain the working of PN junction diode in (a) zero bias (b) forward bias (c) reverse bias on the basis of energy level diagram
7. What is photovoltaic effect? Explain construction and working of solar cell. Draw IV characteristics of solar cell and define fill factor.
8. Explain Hall effect with figure. Derive the equation of Hall voltage and Hall coefficient. *[Dec 19, 6m]*

3/4 Marks

1. Derive the expression for barrier potential for a PN junction.
2. Derive the ideal diode equation for a PN junction. *[Dec 19, 4m]*
3. Explain in brief how free electron theory explains electrical conductivity and thermal conductivity of solids, and relation between electrical and thermal conductivity of solids (Wiedemann–Franz law). What are the limitations of this theory?
4. Explain terms (a) valence band (b) conduction band (c) band gap energy.
5. Explain in brief concept of effective mass of electron. *[Dec 19, 3m]*
6. Explain in brief density of states.
7. What is Fermi level? Explain Fermi-Dirac probability distribution function specifying the meaning of each symbol.
8. What is Fermi level? Show the position of Fermi level in P-type semiconductor at $T=0K$ and $T>0 K$

9. Write the formula for the Fermi Dirac probability distribution function. Draw in the same figure the Fermi Dirac probability versus electron energy at $T=0$ K, T_1 and T_2 (where $T_2 > T_1 > 0$ K). Explain the significance of the figure.
10. What is Fermi energy in a semiconductor? With the help of a labeled diagram, show the position of Fermi level in case of a diode that is connected in forward bias.
11. Draw a neat and labeled diagram showing Fermi energy level in case of a P-type and N-type semiconductor at 0K.
12. What is photovoltaic effect? Draw IV characteristics of solar cell and define fill factor. [Dec 19, 3m]
13. State the advantages, limitations and applications of solar cell.
14. State any four measures to improve efficiency of solar cell. [Dec 19, 4m]

Numerical on: Solid State Physics

Formulae:

1. Band gap energy = $E_g = hv = \frac{hc}{\lambda}$
2. Conductivity of intrinsic semiconductor = $\sigma = ne(\mu_e + \mu_h)$
3. Resistivity of intrinsic semiconductor = $\rho = \frac{1}{ne(\mu_e + \mu_h)}$
4. Resistivity of doped semiconductor = $\rho = \frac{1}{n_a e \mu_h}$ or $\rho = \frac{1}{n_d e \mu_h}$
5. number of atoms = $\frac{\text{Avogadro number}}{\text{Atomic weight}} \times \text{density}$
6. Mobility = $\mu = \sigma R_H$
7. Barrier potential, $V_0 = \frac{kT}{e} \ln \frac{N_D N_A}{n_i^2}$
8. Ideal diode equation, $I = I_0 (e^{\frac{eV_F}{kT}} - 1)$
9. Hall Voltage: $V_H = \frac{1}{ne} \frac{BI}{d}$

Band gap Energy

Example: Calculate the energy gap of Germanium, given that it is transparent to radiation of wavelength greater than 17760 \AA .

Solution:

$$E_g = hv = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{17760 \times 10^{-10}} = 1.12 \times 10^{-19} \text{ J} = \frac{1.12 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{0.7 \text{ eV}}$$

Example: Calculate the energy gap of silicon (in eV), given that it is transparent to radiation of wavelength greater than 11000 \AA .

Solution:

$$E_g = hv = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{11000 \times 10^{-10}} = 1.80 \times 10^{-19} \text{ J} = \frac{1.80 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{1.13 \text{ eV}}$$

Conductivity of a conductor and semiconductor

Example: Calculate the number of acceptors to be added to a germanium sample to obtain the resistivity of $8 \text{ } \Omega\text{-cm}$ [Given $\mu = 1600 \text{ cm}^2/\text{volt-sec}$]

Solution:

Data: $\rho = 8 \text{ } \Omega\text{-cm}$, $\mu_h = 1600 \text{ cm}^2/\text{V-s}$

$$\begin{aligned} \text{Resistivity } \rho &= \frac{1}{n_a e \mu_h} \\ \therefore n_a &= \frac{1}{\rho e \mu_h} = \frac{1}{8 \times 1.6 \times 10^{-19} \times 1600} = 4.876 \times 10^{14} / \text{cc} \end{aligned}$$

Example: An N type semiconductor is to have resistivity 10 ohm-cm. Calculate the number of donor atoms which must be added to achieve this. Given $\mu_d = 500 \text{ cm}^2/\text{V-s}$

Solution:

Data: $\rho = 10 \text{ } \Omega\text{-cm}$, $\mu_d = 500 \text{ cm}^2/\text{V-s}$, $n_d = ?$

Formula: $\sigma_n = n_d \mu_d e$ or $\sigma_n = \frac{1}{\rho_n} = n_d \mu_d e$

$$\text{Thus, } n_d = \frac{1}{\rho_n \mu_d e} = \frac{1}{10 \times 1.6 \times 10^{-9} \times 500} = 1.248 \times 10^{15} / \text{cc}$$

Example: A sample of intrinsic germanium at room temperature has a carrier concentration of $4.41 \times 10^{22} \text{ cm}^3$. Donor impurity is added in the ratio 1 donor atom per 10^8 atoms/ cm^3 . Determine the resistivity of the material. [$\mu_e = 3800 \text{ cm}^2/\text{V.s}$]

Solution:

Concentration of Ge = $4.41 \times 10^{22} \text{ m}^3$

Concentration of Donor impurity = $n = \frac{4.41 \times 10^{22}}{10^8} = 4.41 \times 10^{14}$

Formula: Resistivity = $\rho = \frac{1}{n e \mu_e}$

$$\text{Resistivity} = \rho = \frac{1}{4.41 \times 10^{14} \times 1.6 \times 10^{-19} \times 3800} = 3.724 \text{ Ohm - cm}$$

Example: A germanium sample has 4.56×10^{22} atoms/cc and a donor impurity in the ratio of one part per 10^{10} is added to create an N type semiconductor. If the mobility of charge carriers is $3900 \text{ cm}^2/\text{V-s}$, find the conductivity of silicon.

Solution:

Data: Density of atoms in germanium = 4.56×10^{22} atoms/cc

Donor impurity = 1 atom/ 10^{10} Si atoms

Mobility of charge carriers = $3900 \text{ cm}^2/\text{V-s}$

Charge carrier density = $n = \frac{4.56 \times 10^{22}}{10^{10}} = 4.56 \times 10^{12} / \text{cc}$

Thus, conductivity = $\sigma = n \cdot e \cdot \mu_e = 4.56 \times 10^{12} \times 1.6 \times 10^{-19} \times 3900 = 2.849 \times 10^{-3} \text{ mho/cm}$

Example: Calculate the conductivity of pure silicon at room temperature when concentration of carriers is 1.6×10^{10} per cc [$\mu_e = 1500 \text{ cm}^2/\text{V-sec}$, $\mu_h = 500 \text{ cm}^2/\text{V-sec}$] [Dec 19, 4m]

Solution:

Given: $\mu_e = 1500 \text{ cm}^2/\text{V-sec}$, $\mu_h = 500 \text{ cm}^2/\text{V-sec}$, $n_i = 1.6 \times 10^{10} / \text{cm}^3$

$$\sigma_i = e n_i (\mu_e + \mu_h)$$

Therefore the conductivity of pure silicon is given by

$$\sigma_i = 1.6 \times 10^{-19} \times 1.6 \times 10^{10} \times (1500 + 500) = 5.126 \times 10^{-6} \text{ mho/cm}$$

Example: Calculate the conductivity of germanium sample if a donor impurity is added to the extent of one part in 10^7 germanium atoms at room temperature. [Given: Atomic weight of germanium: 72.6, Density of Ge: 5.32 gm/cc , Avogadro Number: $6.022 \times 10^{23} \text{ mol}^{-1}$, mobility $\mu = 3800 \text{ cm}^2/\text{V-s}$]

Solution:

The expression for conductivity is given by $\sigma = n \cdot e \cdot \mu_e$

$$\begin{aligned} \text{number of Ge atoms} &= \frac{\text{Avogadro number}}{\text{Atomic weight of Ge}} \times \text{density of Ge crystal} = \frac{6.022 \times 10^{23}}{72.6} \times 5.32 \\ &= 4.412 \times 10^{22} / \text{cc} \end{aligned}$$

There is one donor atom per 10^8 atoms of Ge atoms. Hence

$$n = \frac{4.412 \times 10^{22}}{10^9} = 4.41 \times 10^{13} / \text{cc}$$

The conductivity, therefore is,

$$\sigma = n \cdot e \cdot \mu_e = 4.412 \times 10^{13} \times 1.6 \times 10^{-19} \times 3800 = 0.0268 \text{ mho/cm}$$

Ideal diode equation and equation for barrier potential

Example: Calculate the potential barrier for a germanium PN junction at room temperature (27°) if both p and n regions are doped equally and to the extent of one atom per 10^6 germanium atoms.

[Given: Concentration of germanium atoms = $2.4 \times 10^{19} / \text{m}^3$, Carrier concentration of intrinsic germanium is $4.4 \times 10^{28} \text{ atoms/m}^3$, Boltzmann constant = $1.38 \times 10^{-23} \text{ m}^2 \text{kg s}^{-2} \text{K}^{-1}$]

Solution:

$$T = 27^\circ = 300\text{K}$$

As intrinsic Germanium is doped equally with one atom per 10^6 Ge atoms,

$$\text{Concentration of donor and acceptor atoms } N_D = N_A = \frac{4.4 \times 10^{28}}{10^6} = 4.4 \times 10^{22} \text{ atoms/m}^3$$

$$V_0 = \frac{kT}{e} \ln \frac{N_D N_A}{n_i^2} = \frac{1.38 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} \ln \frac{(4.4 \times 10^{22})^2}{(2.4 \times 10^{19})^2} = 0.42 \text{ V}$$

Example: Current flowing in a PN junction is $0.2 \mu\text{A}$ at room temperature (27°) when a large reverse bias voltage is applied. Calculate the current when a forward bias of 0.1 V is applied.

[Given: Boltzmann constant = $1.38 \times 10^{-23} \text{ m}^2 \text{kg s}^{-2} \text{K}^{-1}$]

Solution: $I_0 = 0.2 \mu\text{A} = 2 \times 10^{-7} \text{ A}$, $V = 0.1 \text{ V}$, $T = 27^\circ = 300\text{K}$

$$I = I_0 (e^{\frac{eV}{kT}} - 1) = 2 \times 10^{-7} (e^{0.1/0.0026} - 1) = 9.2 \mu\text{A}$$

Electrical conductivity and Hall coefficient

Example: The Hall coefficient of a specimen of a doped silicon is found to be $3.66 \times 10^{-3} \text{ m}^3/\text{C}$. The resistivity of specimen is $8.93 \Omega\text{m}$. Determine the mobility of the charge carriers.

Solution:

$$R_H = 3.66 \times 10^{-3} \text{ m}^3/\text{C}, \rho = 8.93 \Omega\text{-m}, \mu = ?$$

$$\sigma = \frac{1}{\rho} = \frac{1}{8.93}$$

$$\mu = \sigma R_H = \frac{1}{8.93} \times 3.66 \times 10^{-3} = 0.4098 \times 10^{-3} \text{ cm}^2/\text{V-s}$$

Example: The Hall coefficient of a specimen of a doped silicon is found to be $3.66 \times 10^3 \text{ m}^3/\text{C}$. The resistivity of specimen is $8.93 \times 10^3 \Omega\text{m}$. Determine the mobility of the charge carriers.

Solution:

$$R_H = 3.66 \times 10^3 \text{ m}^3/\text{C}, \rho = 8.93 \times 10^3 \Omega\text{-m}, \mu = ?$$

$$\sigma = \frac{1}{\rho} = \frac{1}{8.93 \times 10^3}$$

$$\mu = \sigma R_H = \frac{1}{8.93 \times 10^3} \times 3.66 \times 10^3 = 0.4098 \text{ cm}^2/\text{V-s}$$

Example: Calculate the mobility of charge carriers in doped silicon whose conductivity is $100 \text{ per } \Omega\text{-m}$ and the Hall coefficient is $3.6 \times 10^{-4} \text{ m}^3/\text{coulomb}$. [Dec 19, 4m]

Solution:

$$\text{Given: } \sigma = 100 (\text{Ohm-m})^{-1}, R_H = 3.6 \times 10^{-4} \text{ m}^3/\text{coulomb}$$

$$\mu = \sigma \times R_H = 100 \times 3.6 \times 10^{-4} = 3.6 \times 10^{-2} \frac{\text{m}^2}{\text{V-s}}$$

Example: The resistivity of doped silicon material is $0.009 \Omega\text{-m}$. The Hall coefficient is $3.6 \times 10^{-4} \text{ m}^3/\text{coulomb}$. Assuming there is only one type of charge carriers, find the mobility and density of charge carriers.

Solution:

$$\text{Mobility } \mu = \sigma R_H = \frac{R_H}{\rho} = \frac{3.6 \times 10^{-4}}{0.009} = 0.04 \frac{\text{m}^2}{\text{V-S}}$$

$$\text{Hall coefficient} = R_H = \frac{1}{ne}$$

$$\text{Thus, } n = \frac{1}{R_H e} = \frac{1}{3.6 \times 10^{-4} \times 1.6 \times 10^{-19}} = 1.736 \times 10^{22} \text{ electrons/cc}$$

Hall Voltage and Hall Coefficient

Example: A strip of copper of thickness $1.5 \times 10^{-4} \text{ m}$ is placed in a magnetic field of 1 T perpendicular to the plane of the strip and a current of 10 A is setup in the strip. What Hall voltage would appear across the width of the strip if concentration of charge carriers in copper is $8.5 \times 10^{28} \text{ electrons/m}^3$.

Solution:

Thickness of copper strip = $1.5 \times 10^{-4} \text{ m}$

Strength of magnetic field = $B = 1 \text{ T}$, Current = 10 A

Concentration of charge carriers = $n = 8.5 \times 10^{28} \text{ electrons/m}^3$

$$V_H = R_H \frac{BI}{d} = \frac{1}{ne} \frac{BI}{d} = \frac{1 \times 10}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-4}} = 4.902 \times 10^{-6} \text{ V}$$

Example: A slab of silicon 2 cm in length, 1.5 cm wide and 2 mm thick is applied with magnetic field of 0.4 T along its thickness. When a current of 75 A flows along the length, the voltage measured across its width is 0.81 mV . Calculate the concentration of mobile electrons in silicon.

Solution:

Given: $V_H = 0.81 \text{ mV} = 0.81 \times 10^{-3} \text{ V}$, $B = 0.4 \text{ T}$, $I = 75 \text{ A}$, $w = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$V_H = \frac{1}{ne} \frac{BI}{d}$$

$$n = \frac{1}{V_H} \frac{BI}{d} = \frac{1}{0.81 \times 10^{-3}} \times \frac{0.4 \times 75}{2 \times 10^{-3}} = 1.851 \times 10^7 \text{ electrons/m}^3$$

Example: A copper specimen having length 1 m , width 1 cm and thickness 1 mm is conducting a current of 1 ampere along its length. The specimen is kept in magnetic field of 1 Tesla along its thickness. It experiences Hall effect and Hall voltage $0.074 \mu\text{V}$ appears along its width. If conductivity of copper is $5.8 \times 10^7 (\text{Ohm-m})^{-1}$, find the hall coefficient and mobility of electrons in the specimen.

Solution:

Given: $l = 1 \text{ m}$, $d = 1 \text{ cm} = 10^{-2} \text{ m}$, $t = 1 \text{ mm} = 10^{-3} \text{ m}$, $B = 1 \text{ Tesla}$, $I = 1 \text{ amp}$,

$V_H = 0.074 \times 10^{-6} \text{ Volts}$, $\sigma = 5.8 \times 10^7 (\text{Ohm-m})^{-1}$

$$V_H = R_H \frac{BId}{A} \Rightarrow R_H = V_H \frac{A}{BId} = \frac{0.074 \times 10^{-6} \times 10^{-2} \times 10^{-3}}{1 \times 1 \times 10^{-2}} = 0.074 \times 10^{-9} \text{ m}^3/\text{Coulomb}$$

$$\sigma = \frac{\mu}{R_H} \Rightarrow \mu = \sigma \times R_H = 5.8 \times 10^7 \times 0.074 \times 10^{-9} = 0.4292 \times 10^{-2} \frac{\text{m}^2}{\text{V-S}}$$

Example: A specimen having length 1.00 cm , width 1.00 mm and thickness 0.1 mm is made to conduct with 1.00 mA current and is placed in a magnetic field of 1.0 Wb/m^2 acting along the thickness. Calculate the Hall voltage in case of (i) N-type semiconductor with Hall coefficient $3.44 \times 10^{-8} \text{ m}^3/\text{C}$ (ii) Aluminium with Hall coefficient of $-0.3 \times 10^{-10} \text{ m}^3/\text{C}$. Which of these materials is more sensitive to Hall effect? Why?

Data: $d = 1 \times 10^{-3} \text{m}$, Area $A = (1 \times 10^{-3} \text{m}) \times (0.1 \times 10^{-3} \text{m}) = 10^{-7} \text{m}^2$, $I = 1 \times 10^{-3} \text{amp}$, $B = 1.0 \text{Wb/m}^2$,

i) For N-type semiconductor, $R_H = -3.44 \times 10^{-8} \text{m}^3/\text{C}$

$$V_H = R_H \frac{BId}{A} = -3.44 \times 10^{-8} \times \frac{1 \times 1 \times 10^{-3} \times 1 \times 10^{-3}}{10^{-7}} = -3.44 \times 10^{-7} \text{V} \\ = -\mathbf{0.344 \mu V}$$

ii) For Aluminium, $R_H = -0.3 \times 10^{-10} \text{m}^3/\text{C}$

$$V_H = R_H \frac{BId}{A} = -0.3 \times 10^{-10} \times \frac{1 \times 1 \times 10^{-3} \times 1 \times 10^{-3}}{10^{-7}} = -0.3 \times 10^{-9} \text{V} \\ = -\mathbf{0.0003 \mu V}$$

Comparing the values of V_H , it can be concluded that N-type semiconductor is more sensitive to Hall effect.
