INV L.T.

$L^{-1}[\bar{F}(s)] = f(t)$

$L[1] = \frac{1}{s}$

1) $L^{-1}\left[\frac{1}{s}\right] = 1$

$L[e^{at}] = \frac{1}{s-a}$

2) $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$

$L[e^{-at}] = \frac{1}{s+a}$

3) $L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$

$L[t^{n-1}] = \frac{(n-1)!}{s^n}$

4) $L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$

$L[\sin at] = \frac{a}{s^2+a^2}$

5) $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin(at)$

$L[\cos at] = \frac{s}{s^2+a^2}$

6) $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$

$L[\sinh t] = \frac{a}{s^2-a^2}$

7) $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\sinh t$

$L[\cosh t] = \frac{s}{s^2 - a^2}$

8) $L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh t$

$L[e^{at}t^n] = \frac{(n-1)!}{(s-a)^n}$

9) $L^{-1}\left[\frac{1}{(s-a)^n}\right] = \frac{e^{at}\,t^{n-1}}{(n-1)!}$

first shifting

~~10)~~

$L[a^t] = L[e^{\log a \cdot t}] = \frac{1}{s-\log a}$

10) $L^{-1}\left[\frac{1}{s-\log a}\right] = a^t$

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L^{-1} by partial fraction method

(1) $\dfrac{2s+3}{(s-a)(s-b)(s-c)}$ $\dfrac{A}{s-a} + \dfrac{B}{s-b} + \dfrac{C}{s-c}$

(2) $\dfrac{2s^2+s}{(s-a)(s-b)^2}$ $\dfrac{A}{s-a} + \dfrac{B}{s-b} + \dfrac{C}{(s-b)^2}$

(3) $\dfrac{s^3}{(s-a)(s^2+bs+c)}$ $\dfrac{A}{s-a} + \dfrac{Bs+C}{s^2+bs+c}$

Q) $L^{-1}\left\{\dfrac{s^2+2s-4}{(s-5)(s^2+9)}\right\}$ (PYQ :))

$$\frac{s^2+2s-4}{(s-5)(s^2+9)} = \frac{A}{s-5} + \frac{Bs+C}{s^2+9}$$

$$s^2+2s-4 = A(s^2+9) + (Bs+C)(s-5)$$

~~A = 1~~ $A+B = 1$

$9A - 5C = -4$

$\boxed{C = 2}$

$9A - 10 = -4$

$9A = 6$

$\boxed{A = \frac{2}{3}}$

$\boxed{B = 1 - \frac{2}{3} = \frac{1}{3}}$

$A=$ $\quad A=1$

$A+B=0$

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$$L^{-1}\left\{\frac{\frac{2}{3}}{s-5}+\frac{\frac{1}{3}s+2}{s^2+3^2}\right\}$$

Ans $\frac{2}{3}e^{5t}+\frac{1}{3}\cos(3t)+2\times\frac{1}{3}\sin(3t)$

L^{-1} of trigo & log

$$L^{-1}\left[\frac{d}{ds}(F(s))\right]=-tf(t) \qquad L[tf(t)]=-\frac{d}{ds}F(s)$$

Q) $L^{-1}\{F(s)\}$, $F(s)=\log\left(\frac{s^2+1}{s(s+1)}\right)$

$$\cancel{\frac{d}{ds}(F(s))=\frac{1}{\frac{s^2+1}{s(s+1)}}}$$

s^2+s

$$F(s)=\log(s^2+1)-\log(s(s+1))$$

$$\frac{d}{ds}(F(s))=\frac{2s}{s^2+1}-\frac{1}{s(s+1)}*(2s+1)$$

$$L^{-1}\left[\frac{d}{ds}F(s)\right]=2L^{-1}\left\{\frac{s}{s^2+1}\right\}-2L^{-1}\left\{\frac{s}{s+1}\right\}-L^{-1}\left\{\frac{1}{s(s+1)}\right\}$$

$$-tf(t)=2\cos t-2e^{-t}-L^{-1}\left\{\frac{1}{s}-\frac{1}{s+1}\right\}$$

Ans $f(t)=\frac{2\cos t-2e^{-t}-1+e^{-t}}{-t}$

Q) $L^{-1}[\bar{F}(s)]$

$$\bar{f}(s) = \cot^{-1}\left(\frac{s+3}{2}\right)$$

$$\frac{d}{ds}\bar{F}(s) = \frac{-1}{1+\left(\frac{s+3}{2}\right)^2}$$

$$\frac{d}{ds}\bar{f}(s) = \frac{-4}{4+(s+3)^2}$$

$$L^{-1}\left[\frac{d}{ds}\bar{F}(s)\right] = 2L^{-1}\left[\frac{-\cancel{4}\,2}{(s+3)^2+4}\right]$$

$$-tf(t) = -2e^{-3t}\sin 2t$$

Ans $$f(t) = \frac{2e^{-3t}\sin(2t)}{t}$$

covolution

if $L[f(t)] = \bar{f}(s)$ & $L[g(t)] = \bar{g}(s)$

then
$$L\left[\int_0^t f(t)\, g(t-u)\, du\right] = \bar{F}(s)\,\bar{g}(s)$$

$$\therefore L^{-1}[\bar{F}(s)\,\bar{g}(s)] = \int_0^t f(u)\, g(t-u)\, du$$

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(i) $\Rightarrow$ using convolution theorem

$$L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$$

$\Rightarrow$ Let
$\bar{F}(s) = \frac{1}{\cancel{s}(s+1)(s+2)}$ | $\bar{g}(s) = \frac{1}{s}$

Taking I.L.T on both side

$L^{-1}[\bar{F}(s)] = L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right]$ | $L^{-1}[\bar{g}(s)] = L^{-1}\left[\frac{1}{s}\right]$

$F(t) = \cancel{1 - e^{-t}}$
$e^{-t} - e^{-2t}$ | $g(t) = \cancel{e^{-t}} \cdot 1$

~~by cor~~

$f(u) = e^{-u} - e^{-2u}$ | $g(t-u) = 1$

by convolution

$$L^{-1}[\bar{F}(s)\,\bar{g}(s)] = \int_0^t F(u)\,g(t-u)\,du$$

$$= \int_0^t (e^{-u} - e^{-2u})\,du$$

$$= \left[\frac{e^{-u}}{-1} - \frac{e^{-2u}}{-2}\right]_0^t$$

Ans $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right] = \left[-e^{-t} + \frac{e^{-2t}}{2}\right] - \left[-1 + \frac{1}{2}\right]$

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Q) using convolution find

$$L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right]$$

$$= L^{-1}\left[\bar{F}(s)\ \bar{g}(s)\right]$$

$$\bar{F}(s) = \frac{s}{s^2+2^2} \qquad \bar{g}(s) = \frac{s}{s^2+2^2}$$

$$L^{-1}\left[\bar{F}(s)\right] = \cos(2t) \qquad L^{-1}\left[\bar{g}(s)\right] = \cos(2t)$$

$$f(u) = \cos(2u) \qquad g(t-u) = \cos(2(t-u))$$

by convolution

$$L^{-1}\left[\bar{F}(s)\bar{g}(s)\right] = \int_0^t F(u)\, g(t-u)\, du$$

$$= \frac{1}{2}\int_0^t 2\cos(2u)\cdot\cos(2t-2u)\, du$$

$$= \frac{1}{2}\int_0^t \left[\cos(2t) + \cos(2t+4u)\right] du$$

$$= \frac{1}{2}\int_0^t \cos(2t)\, du + \int_0^t \cos(2t)\cos(4u)\, du - \int_0^t \sin(2t)\sin(4u)\, du$$

$$= \frac{1}{2} t\cos(2t) + \frac{\cos(2t)}{4}\sin(4t) + \sin(2t)\left[\frac{\cos(4\ldots}{-\ldots}\right.$$

d) by convolution

$$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$$

$$\bar{f}(s) = \frac{s}{s^2+1} \qquad \bar{g}(s) = \frac{1}{s^2+4}$$

$$f(t) = \cos(t) \qquad g(t) = \frac{1}{2}\sin(2t)$$

$$f(u) = \cos(u) \qquad g(t-u) = \frac{1}{2}\sin(2t-2u)$$

$$L^{-1}\left(\bar{f}(s)\bar{g}(s)\right) = \int_0^t f(u)\,g(t-u)\,du$$

$$= \frac{1}{4}\int_0^t 2\cos(u).\sin(2t-2u)$$

$+2t+3u$

$$= \frac{1}{4}\int_0^t \sin(2t-u) + \sin(u-2t-2u)\,du$$

$$= \frac{1}{4}\left[\int_0^t [\sin(2t)\cos(u) - \cos(2t)\sin(u)]\,du + \int_0^t [\sin(2t)\cos(3u) + \cos(2t)\sin(3u)]\,du\right]$$

Ans

$$= \frac{1}{4}\left[\sin(2t).\sin(t) + \cos(2t)[\cos(t) - 1] + \frac{\sin(2t)}{3}\sin(3t) - \frac{\cos(2t)}{3}[\cos(3t) - 1]\right]$$

Application of L.T.

// $L[f'(t)] = s\bar{f}(s) - f(0)$

$L[f''(t)] = s^2\bar{f}(s) - sf(0) - f'(0)$

$L[f'''(t)] = s^3\bar{f}(s) - s^2 f(0) - sf'(0) - f''(0)$

// steps

↳ ① Express in given Differentia Equation in Dash Notation

↳ ② Take LT both side

↳ ③ Using inital codn & obtain $\bar{y}(s)$

↳ ④ Take inverse LT & find required $y(t)$.

Q) Using LT solve $y''+2y'+5y = e^{-t}\sin t$. & $y(0)=0$ $y'(0)=1$

taking LT both side

$$L[y''(t)] + 2L[y'(t)] + 3L[y(t)] = L[e^{-t}\sin t]$$

$$\underline{s^2}\bar{y}(s) - s\cancel{y(0)}^{0} - y'(0) + 2s\bar{y}(s) - 2\cancel{y(0)}^{0} + 3\bar{y}(s) = \frac{1}{(s+1)^2+1}$$

$$(s^2+2s+3)\bar{y}(s) - 1 = \frac{1}{(s+1)^2+1}$$

$$= \frac{1}{(s+1)^2+1} + 1$$

$$= \frac{(s+1)^2+2}{(s+1)^2+1} = \frac{s^2+2s+3}{s^2+2s+2}$$

$$\bar{y}(s) = \frac{\cancel{(s^2+2s+3)}}{\cancel{(s^2+2s+3)}(s^2+2s+2)}$$

$$\bar{y}(s) = \frac{1}{(s+1)^2+1}$$

taking L^{-1} both side

$$y(t) = L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$$

Ans $$y(t) = e^{-t}\sin(t)$$

Q) Using LT solve,

$$y'' - 3y' + 2y = 12e^{-2t}, \quad y(0)=2, \quad y'(0)=6$$

$$L[y''(t)] - 3L[y'(t)] + 2L[y(t)] = 12\,L[e^{-2t}]$$

$$s^2\bar{y}(s) - sy(0) - y'(0) - 3s\bar{y}(s) + 3y(0) + 2\bar{y}(s) = 12\left(\frac{1}{s+2}\right)$$

$$(s^2-3s+2)\bar{y}(s) - 2s - \cancel{6} + \cancel{6} = 12\left(\frac{1}{s+2}\right)$$

$$(s^2+3s+2)\bar{y}(s) = \frac{12}{s+2} + 2s$$

$$= \frac{12 + 2s^2 + 4s}{s+2} = \frac{2(s^2+2s+6)}{s+2}$$

$$\bar{y}(s) = 2 \times \frac{s^2+2s+6}{(s+2)(s^2+3s+2)}$$

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$$y(t) = 2L^{-1}\left[\frac{s^2+2s+6}{(s+2)^2(s+1)}\right]$$

$$\frac{s^2+2s+6}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\boxed{A = \frac{1-2+6}{1} = 5}$$

B =

$$s^2+2s+6 = A(s+2)^2 + B(s+2)^2 + C(s+1)$$

$L[t^2] = \frac{2!}{s^3}$

$L[t] = \frac{1}{s^2}$

$$A + B = 1$$

$$\boxed{B = -4}$$

$$4A + 4B + C = 6$$

$$\underbrace{20 - 16}_{4} + C = 6$$

$$\boxed{C = 2}$$

$\frac{n}{n!}$

$L[t^n] = \frac{n!}{s^{n+1}}$

$$= 2L^{-1}\left[\frac{5}{s+1} - \frac{4}{s+2} + \frac{2}{(s+2)^2}\right]$$

Ans $y(t) = 2 \times \left[5e^{-t} - 4e^{-2t} + 2e^{-2t}\, t\right]$

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y LT find

$\frac{dx}{dt} - y = e^t$; $\frac{dy}{dt} + x = \sin t$

$x(0) = 1$, $y(0) = 0$

taking LT both side

$L[x'(t)] - L[y(t)] = L[e^{-t}]$: | $L[y'(t)] + L[x(t)] = L[\sin t]$

$s\bar{x}(s) - x(0) - \bar{y}(s) = \frac{1}{s+1}$ | $s\bar{y}(s) - \cancel{y(0)}^{0} + \bar{x}(s) = \frac{1}{s^2+1}$

$s\bar{x}(s) - \bar{y}(s) = \frac{1}{s+1} + 1$ | $\bar{x}(s) + s\bar{y}(s) = \frac{1}{s^2+1}$

$s\bar{x}(s) - \bar{y}(s) = \frac{s+2}{s+1}$ — (1) | $s\bar{x}(s) + s^2\bar{y}(s) = \frac{s}{s^2+1}$ — (2)

(1) — (2)

$\cancel{s\bar{x}(s)} - \bar{y}(s) = \frac{s+2}{s+1}$

$\cancel{s\bar{x}(s)} + s^2\bar{y}(s) = \frac{s}{s^2+1}$

$-(1+s^2)\bar{y}(s) = \frac{s+2}{s+1} - \frac{s}{s^2+1}$

$-\bar{y}(s) = \frac{s+2}{(s^2+1)(s+1)} - \frac{s}{(s^2+1)}$

$\bar{y}(s) = \frac{s}{(s^2+1)} - \frac{s+2}{(s^2+1)(s+1)}$ — (3)

$$\bar{y}(s) = \frac{s(s+1) - (s+2)}{(s^2+1)(s+1)}$$

$$= \frac{s^2 + \cancel{s} - \cancel{s} \cancel{+} 2}{(s^2+1)(s+1)}$$

$$\bar{y}(s) = \frac{(s^2+2)}{(s^2+1)(s+1)}$$

put ③ in ①.

$$\bar{x}(s) - \frac{\cancel{s}}{\cancel{s}(s^2+1)} + \frac{s+2}{s(s^2+1)(s+1)} = \frac{s+2}{s(s+1)}$$

$$\bar{x}(s) = \frac{s+2}{s(s+1)} + \frac{1}{s^2+1} + \frac{s+2}{s(s^2+1)(s^2+1)}$$

Now take inverse LT of both.