A: 59415

MAHATMA GANDHI MISSION'S

College of Engineering, Nanded.

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Name of Candidate : Deshmukh Shrutika Roll No. : 67

Class : SY-1 (CSE) Branch : CSE

Subject : M-III Date : 6/3/2022

Question	Q.1	Q.2	Q.3	Q.4	Total		
Marks obtained							
Total						Examiner Signature	Invigilator Signature

Q1)

A) $\int_0^\infty t \cdot e^{-3t} \sin(t)\, dt$

$L[\sin t] = \frac{1}{s^2+1}$ $\quad L[e^{-3t} \sin t] = \frac{1}{(s+3)^2+1} = \frac{1}{s^2+1}$

$\int_0^\infty t \cdot e^{-3t} \sin(t)\, dt = (-1)^1 \frac{d}{ds}\left[\frac{1}{(s+3)^2+1}\right]$

$= (-1) \frac{d}{ds}\left[\frac{2(s+3)}{(s+3)^2+1}\right]$

$= (-1)\left[\frac{2(s+3)}{(s+3)^2+1}\right]_0$

$\int_0^\infty t \cdot e^{-3t} \sin(t)\, dt = (-1)\cancel{\left[\frac{2(6)}{36+1}\right] = \frac{-12}{37}}$

$= (-1)\left[\frac{2(3)}{(9+1)^2}\right] = \frac{6}{100}$

B) $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \sin t & t > \pi \end{cases}$

$f(t) = \cos t \left[u(t-0) - u(t-\pi) \right] + \sin \left[u(t-\pi) \right]$

$= \cos t \left[u(t-0) \right] - \cos t\, (u(t-\pi)) + \sin$

$\cancel{\cos \left[u(t-\pi) \right]}$

$= \cos t\, (u(t-0)) + \left[\cos t - \sin t \right] u(t-\pi)$

$a = 0 \qquad\qquad a = -\pi$

$= e^{0} L[\cos t] + e^{-\pi s} L \left[\cos t - \sin t \right]$

$= \frac{s}{s^2+1} + e^{-\pi s} L[\cos t] - e^{-\pi s} L[\sin t]$

$= \frac{s}{s^2+1} + e^{-\pi s} \frac{s}{s^2+1} - e^{-\pi s} \frac{1}{s^2+1}$

$= \cancel{e^{-\pi s}}$

$= \frac{s}{s^2+1} \left[1 + e^{-\pi s} \right] - e^{-\pi s} \left[\frac{1}{s^2+1} \right]$

$= \frac{s}{s^2+1} \left[1 + e^{-\pi s} \right] - e^{-\pi s} \left[\frac{1}{s^2+1} \right]$

c) i) ~~$L^{-1}\left[\dfrac{3s+1}{(s-1)(s^2+1)}\right]$~~

c) i) $L\left[\dfrac{\cos(3t)}{t}\right] \Rightarrow \cos 3\theta = 4\cos 3\theta - 3\cos\theta$

co

$$L[\cos(3t)] = \frac{s}{s^2+9}$$

$$L\left[\frac{\cos(3t)}{t}\right] = \int_s^{\infty} \frac{s}{s^2+9}$$

$$= \Big[\log(s) - \log(s^2+9)\Big]_s^{\infty}$$

$$= \frac{1}{2}\left[\frac{2s}{s^2+9}\right]_s$$

$$= \frac{1}{2}\Big[\log(s^2+9)\Big]_s^{\infty}$$

$$= -\frac{1}{2}\log(s^2+9)$$

ii) $L\left[t^2 e^{-t}\sin(t)\right]$

$L[\sin t] = \dfrac{1}{s^2+1}$ $\qquad L[e^{-t}\sin t] = \dfrac{1}{(s+1)^2+1}$

$$L[t^2 e^{-t}\sin t] = (-1)^2 \frac{d^2}{ds^2}\left[\frac{1}{(1+s)^2+1}\right]$$

$$= \frac{d^2}{ds^2}\left[\frac{d}{ds}\left(\frac{1}{(s+1)^2+1}\right)\right]$$

$$= \frac{d}{ds}\left[\frac{1\,(2(s+1)) - (s+1)^2+1}{((s+1)^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{\cancel{2s}+\cancel{1} - (s^2+\cancel{2s}+\cancel{1})+1}{((s+1)^2+1)^2}\right]$$

$$= \frac{d}{ds}\left[\frac{1-s^2}{((1+s)^2+1)^2}\right]$$

$$= \cancel{(1-s^2)\left[2((1+s)^2)+1\right.}$$

$$= \frac{(1-s^2)(2((1+s)^2+1)) - ((1+s)^2+1)^2\,(-2s)}{((1+s)^2+1)^4}$$

$$= \frac{1-s^2(2((1+s^2+2s)+1)) - ((1+2s+s^2)+1)^2\,(-2s)}{((1+s)^2+1)^4}$$

$$= 1-s^2\left[2(2+s^2+2s)\right] - \left[2+2s+s^2\right]^2(-2s)$$

$$= \frac{1-s^2\left[4+2s^2+4s\right] + 2s\left[2+2s+s^2\right]^2}{\left[(1+s)^2+1\right]^4}$$

$$= \frac{1-s^2\left[4+2s^2+4s\right] + 2s\left[2+2s+s^2\right]^2}{(2+2s+s^2)^4}$$

Q2) A) $L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$

$$\frac{3s+1}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{(s^2+1)}$$

$$3s+1 = A(s^2+1) + Bs+C(s-1)$$

$s=1$

$$3+1 = A(2) + Bs+C(0)$$
$$4 = A(2)$$
$$\boxed{A=2}$$

$$3s+1 = As^2+A + Bs(s-1) + C(s-1)$$
$$= As^2+A + Bs^2 - Bs + Cs - C$$

equ. s^2

$$0 = A+B$$
$$\boxed{B=-2}$$

equ. const.

$$1 = A-C$$
$$1 = 2-C$$
$$-1 = -C \qquad \boxed{C=1}$$

$$= \frac{2}{s-1} + \frac{-2s+1}{(s^2+1)}$$

$$= \frac{2}{s-1} + \frac{1}{(s^2+1)} - \frac{2s}{(s^2+1)}$$

$$= 2.e^t + \sin(t) - 2\cos t$$

Q $y''(0) + 2y' + 5y = e^{-t}\sin(t)$

$y(0) = 0, \quad y'(0) = 1$

$y'(s) = s.\bar{y}(s) - y(0)$

$y''(s) = s^2\bar{y}(s) - sy(0) - y'(0)$

$L[y''(t)] + 2[y'(t)] + 5[y(t)] = L[e^{-t}\sin t]$

$s^2\bar{y}(s) - sy(0) - y'(0) + 2s\bar{y}(s) - 2y(0)$

$+ 5\bar{y}(s) = \dfrac{1}{(s+1)^2+1}$

$s^2\bar{y}(s) - 0 - 1 + 2s\bar{y}(s) - 0 + 5\bar{y}(s) = \dfrac{1}{(s+1)^2+1}$

$s^2\bar{y}(s) + 2s\bar{y}(s) + 5\bar{y}(s) = \dfrac{1}{(s+1)^2+1} + 1$

$\bar{y}(s)[s^2+2s+5] = \dfrac{1+(s+1)^2+1}{(s+1)^2+1}$

$= \dfrac{(s+1)^2+2}{(s+1)^2+1}$

$= \dfrac{s^2+2s+1+2}{(s+1)^2+1}$

$\bar{y}(s) = \dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$

$s^2+23+3 = \dfrac{As+B}{s^2+2s+2} + \dfrac{Cs+D}{s^2+2s+5}$

$$= AS+B(s^2+2s+5) + (CS+D)(s^2+2s+2) = s^2+2s+3$$

$$s^2+2s+3 = \left[AS^3+2AS^2+5SA+BS^2+2BS+B5\right] + \left[CS^3+2CS^2+2CS+DS^2+2DS+2D\right]$$

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c) L^{-1} by convolution. $\frac{s}{(s^2+1)(s^2+4)}$

$f(s) = \frac{s}{s^2+1}$ $g(s) = \frac{1}{s^2+4}$

$L^{-1}[f(s)] = \cos(t)$ $L^{-1}[g(s)] = \sin(2t)$

by convolution theorem

$$\int_0^\infty f(t)\, g(t) = \int_0^\infty f(u)\, g(t-u)$$

$f(u) = \cos u$; $g(t-u) = \sin 2(t-u)$

$$= \int_0^\infty \cos u \cdot \sin 2(t-u)\, du$$

$$= \frac{1}{2}\int_0^\infty 2\cos u \cdot \sin(2t-2u)\, du$$

$$= \frac{1}{2}\int_0^t \sin(2t-2u+u) + \sin(u-(2t-2u))$$

2

$$\frac{1}{2}\int_0^t \sin(2t-u)+\sin(2t+3u)\,du$$

$$=\frac{1}{2}\left[\int_0^t \sin(2t-u)du+\int_0^t \sin(2t+3u)\,du\right]$$

$$=\frac{1}{2}\left[\left[\cos(2t-u)\right]_0^t+\left(\frac{\cos(2t+3u)}{3}\right)_0^t\right]$$

$$=\frac{1}{2}\left[(\cos t-\cos 2t)\right]+\left[\frac{-\cos t}{3}+\frac{\cos 2t}{3}\right]$$

$$=\frac{1}{2}\left[\left(\cos t-\frac{\cos t}{3}\right)+\left(\frac{\cos 2t-\cos 2t}{3}\right)\right]$$

$$=\frac{1}{2}\left[\frac{2\cos t}{3}+\left(-\frac{2\cos 2t}{3}\right)\right]$$

$$=\left[\frac{\cos t}{3}-\frac{\cos 2t}{3}\right]$$

Q3)

A) . Parsvels identity

$$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$$

$$F_c[e^{-at}] = \frac{1}{x^2+a^2} \qquad F_c[e^{-bt}] = \frac{1}{x^2+b^2}$$

by P.I

$$\int_0^\infty F_c(s)\, G_c(s) = \int_0^\infty f(x)\, g(x)\, ds$$

$$= \int_0^\infty e^{-at} \cdot e^{-bt}\, dt$$

$$\frac{2}{\pi}\int_0^\infty \frac{a}{x^2+a^2} \cdot \frac{b}{x^2+b^2}\, dx = \int_0^\infty e^{-(a+b)t}\, dt$$

$$= \frac{e^{-(a+b)t}}{a+b}\, dt$$

$$\frac{2ab}{\pi}\int_0^\infty \frac{dx}{(x^2+a^2(x^2+b^2)} = \frac{1}{a+b}$$

$$\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab}\left[\frac{1}{a+b}\right]$$

B) $f(x) = \begin{cases} 1-x^2 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$

$$F(s) = \int_{-\infty}^\infty f(x)\, e^{isx}\, dx$$

$$= \int_{-\infty}^\infty (1-x^2)\,(e^{isx})\, dx$$

$e^{isx} =$
$\cos(sx) +$
$i\sin(sx)$

4

$$= \int_{-1}^{1} (1-x^2) \left[\cos sx + i \sin sx \right]$$

$$= \int_{-1}^{1} (1-x^2) \cdot \cos sx + i \underbrace{\int_{-1}^{1} (1-x^2) \sin sx}_{0}$$

$$= 2\int_{0}^{1} (1-x^2) \cos sx$$

$$= \int_{-1}^{1} \left[(1-x^2)\left(\frac{\sin sx}{s}\right) - (-2x)\left(\frac{-\cos sx}{s^2}\right) + (-2)\left(\frac{-\sin sx}{s^3}\right) \right]_0^1$$

$$= 2\left[0 - (-2)\left(\frac{-\cos s}{s^2}\right) + \frac{2 \cdot \sin sx}{s^3} - (0) \cdot - 20 + 0 \right]$$

$$= 2\left[\frac{-2\cos s}{s^2} + \frac{2\sin s}{s^3} \right]$$

$$F(s) = 4\left[\frac{\sin s - s \cdot \cos s}{s^3} \right]$$

by inverse

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(s) e^{-isx} dx$$

$$= \frac{1}{2\pi} \int_{\infty}^{\infty} 4 \left[\frac{\sin s - s \cdot \cos s}{s^3} \right] (\cos sx - i \sin sx)$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} 4 \left[\frac{\cancel{\cos} \sin s - s \cos s}{s^3} \right] \cos s x$$

$$= \frac{\cancel{1} \cdot 2}{\cancel{2}\pi} \int_{-\infty}^{\infty} \cancel{2}\, 4 \left[\frac{\sin s - s \cos s}{s^3} \right] \cos s x$$

$$\frac{\pi}{4}(1 - x^2) = 4 \int_{-\infty}^{\infty} \left[\frac{\sin s - s \cos s}{s^3} \right] \cos (x/2)\, dx$$

$$\left(\frac{1 - \pi}{4} \right)$$

$$\frac{\pi}{4} \cdot \frac{3\pi}{4} = \frac{3\pi}{16}$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \left[\frac{\sin s - s \cdot \cos s}{s^3} \right] \cos s x$$

$$= \frac{4}{\pi} \int_{-\infty}^{\infty} \frac{\sin s - s \cos s}{s^3} \cos s x$$

$$(1 - x^2) \frac{\pi}{4} =$$

$$\frac{3\pi}{16} = \int_{0}^{\infty} \frac{s \cdot \cos s - \sin s}{s^3} \cos (x/2)\, dx$$

$$\therefore \int_{0}^{\infty} \frac{s \cdot \cos s - \sin s}{s^3} \cos (x/2)\, dx = -\frac{3\pi}{16}$$

6

c) $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \\ 0 & x > 2 \end{cases}$

→ F.S.T

$$F_s(s) = \int_0^\infty f(t) \cdot \sin(st)\, dt$$

$$= \int_0^1 x \cdot \sin(sx)\, dx + \int_1^2 (2-x) \sin(sx)\, dx + 0$$

$$= \left[x\left(\frac{-\cos sx}{s}\right) - 1 \cdot \frac{-\sin(sx)}{s^2} \right]_0^1$$

$$+ \left[(2-x)\left(\frac{-\cos(sx)}{s}\right) - (-1)\left(\frac{-\sin(sx)}{s^2}\right) \right]_1^2$$

$$= \left[x\left(\frac{-\cos sx}{s}\right) + \frac{\sin(sx)}{s^2} \right]_0^1 +$$

$$\left[(2-x)\left(\frac{-\cos(sx)}{s}\right) - \frac{\sin(sx)}{s^2} \right]_1^2$$

$$\Rightarrow \left[\frac{-\cancel{\cos s}}{s} + \frac{\cancel{\sin s}}{s^2} - 0 - 0 \right] +$$

$$\left[0 - \frac{\sin(2s)}{s^2} + \frac{\cancel{\cos s}}{s} + \frac{\cancel{\sin s}}{s^2} \right]$$

$$= \frac{2\sin s - \sin(2s)}{s^2}$$

$$\Rightarrow \frac{2\sin s - \sin(2s)}{s^2}$$

D) F.C.I.F $f(x) = \begin{cases} \cos x & x < \pi/2 \\ 0 & x > \pi/2 \end{cases}$

$$f(x) = \int_0^\infty A(\lambda)\cos(\lambda t)\,d\lambda$$

$$A(\lambda) = \frac{2}{\pi}\int_0^\infty f(x)\cdot\cos(\lambda x)\,dx = \frac{2}{\pi}\int_0^\infty \cos x\cdot\cos(\lambda x)\,dx$$

$$= \frac{1}{\pi}\int_0^\infty 2\cos x\cdot\cos\lambda x\,dx = \frac{1}{\pi}\int_0^\infty \left[\cos(\lambda+1)x + \cos(1-\lambda)x\right]dx$$

$$= \frac{1}{\pi}\int_0^\infty \left[\frac{\sin(\lambda+1)x}{\lambda+1} + \frac{\sin(\lambda-1)x}{\lambda-1}\right]_0^{\pi/2}$$

$$= \frac{1}{\pi}\left[\frac{\sin(\lambda+1)\pi/2}{\lambda+1} + \frac{\sin(\lambda-1)\pi/2}{\lambda-1}\right]$$

$$A = \frac{1}{\pi}\left[\frac{\cos \pi/2\,\lambda}{\lambda+1} - \frac{\cos \pi/2\,\lambda}{\lambda-1}\right]$$

$$= \frac{1}{\pi}\cos \pi/2\,\lambda\left[\frac{1}{\lambda+1} - \frac{1}{\lambda-1}\right] = \frac{1}{\pi}\cos\pi/2\,\lambda\left[\frac{\lambda-1-\lambda-1}{(\lambda^2-1)}\right]$$

$$= \frac{-2}{\pi}\left[\frac{\cos\pi/2\,\lambda}{(\lambda^2-1)}\right]$$

$$f(x) = \int_0^\infty -\frac{2}{\pi}\left[\frac{\cos\pi/2\,\lambda}{\lambda^2-1}\right]\cos(\lambda t)\,d\lambda$$

$$= -\frac{2}{\pi}\int_0^\infty \frac{\cos\pi/2\,\lambda}{\lambda^2-1}\cos(\lambda t)\,d\lambda$$

8

Q5) a) $f(z) = (x^2+axy+by^2) + i(x^2+dxy+y^2)$,

$u = x^2+axy+by^2$ $\quad$ $v = cx^2+dxy+y^2$

$\frac{\partial u}{\partial x} = 2x+ay+0$ $\quad$ $\frac{\partial v}{\partial x} = 2xc+dy$

$\frac{\partial u}{\partial y} = ax+bz$ $\quad$ $\frac{\partial v}{\partial y} = dx+2y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ $\quad$ $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$2x+ay = dx+2y$ & $ax+bz = -(2xc+dy)$

$\boxed{d=2}$ $\boxed{a=2}$ & $2x+bz = -(2xc+2y)$

$\boxed{b=-2}$ $\quad$ $\boxed{c=-1}$

b) $u = e^x[x\cos y - y\sin y]$

u v

$\frac{\partial u}{\partial x} = e^x[\cos y] - [x\cos y - y\sin y]e^x(1)$

$= e^x[\cos y] - e^x[x\cos y] + y\sin y$

$= e^x\cos y[1-x] + y\sin y$

$x=2$
$y=0$ $\quad =$

$\phi_1(z,0) = e^z + e^z(z) = (z+1)e^z$

$\frac{\partial u}{\partial y} = 0$

milla

$= \int \phi_1\,dz - i\int \phi_2\,dz$

$= \int (z+1)e^z\,dz$

$\Rightarrow (z+1)e^z - (1)e^z$

$= z.e^z + c$

~~$x+iy =$~~ $u+iv = (x+iy)e^{x+iy}$

$= e^x[(x+iy)[\cos y + i\sin y]]$

$u + iv = e^x[x\cos y + y\sin y]$
$\quad + i\,e^x[x\sin y + y\cos y]$

$V = e^x[x\sin y + y\cos y]$

c) State and prove C-R eqn in cartesian form.

$z = r.e^{i\theta}$

$f(z) = u + iv$

$u + iv = f(r.e^{i\theta})$

$\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r} = f'(r.e^{i\theta}).e^{i\theta}$ -①

$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = f'(r.e^{i\theta})r.e^{i\theta}$ -②

$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = ri\left[\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right]$

$\frac{\partial u}{\partial \theta} = -r.\frac{\partial v}{\partial r}$; $\frac{\partial v}{\partial \theta} = \cancel{r}\, i^2\frac{\partial u}{\partial r}$

$u_\theta = -r\, v_r$; $u_r = \frac{1}{r} v_\theta$

$v_r = \frac{-1}{r} u_\theta$

diff. w.r.t. r

$u_{rr} = \frac{1}{r}.\frac{\partial v}{\partial \theta \partial r} - \frac{\partial v}{\partial \theta}.\frac{-1}{r^2}$

$u_{\theta\theta} = -r.v_r$

$\frac{\partial^2 u}{\partial \theta^2} = -r.\frac{\partial^2 v}{\partial r.\partial \theta}$

$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$

D) $u = x^2 - y^2 - 2xy - 2x + 3y$

$\frac{\partial u}{\partial x} = 2x - 2y - 2$

$\frac{\partial u}{\partial y} = -2y - 2x + 3$

$\phi_1(z,0) = 2z - 2$; $\phi_2(z,0) = -2z$

milnae.

$= \int \phi_1(z,0)dz - i\int \phi_2(z,0)dz$

$= \int 2z - 2\,dz - i\int -2z + 3$

$= \left(\frac{\cancel{2}z^2}{\cancel{2}} - 2z\right) - i\left(\frac{-\cancel{2}z^2}{\cancel{2}} + 3z\right)$

$= (z^2 - 2z) - i(-z^2 + 3z)$

$= (1+i)z^2 - \cancel{5z}\,(2+3i)z$

$u + iv = (1+i)(x+iy)^2$
$\quad - (2+3i)(x+iy)$

$= (1+i)(x^2 - y^2 + 2xy)$
$\quad - (2+3i)(x+iy)$

$= (1+i)(x^2 - y^2 + 2xy) -$
$\quad [2x + 2iy + 3ix - 3y]$

$= x^2 - y^2 + 2xy + ix^2 - iy^2 + 2ixy$
$\quad - 2x - 2iy - 3ix + 3y$

$= (x^2 - y^2 + 2xy - 2x + 3y) + i($
$\quad x^2 - y^2 + 2xy - 2y - 3x)$

Q6)

A) $\oint_c \frac{z+4}{z^2+2z+5} dz$ c is $|z+1-i|=2$

$\rightarrow$ Cauchy's integral

$$\oint \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^n(a)$$

$$\oint_c \frac{z+4}{z^2+2z+5} dz$$

$z^2+2z+5=0$

$z=-1\pm 2i$

$$= \frac{1}{z^2+2z+5} = \frac{1}{(z+1)^2-(2i)^2}$$

$$= \frac{1}{(z+1+2i)(z+1-2i)}$$

$$= \frac{1}{4i}\left[\frac{1}{z+1-2i} - \frac{1}{z+1+2i}\right]$$

$$= \frac{1}{4i}\int \frac{z+4}{z+1-2i} dz - \frac{1}{4i}\int \frac{z+4}{z+1+2i} dz$$

0

$f(z) = z+4$

$|z+1-i|=2$

$a=-1+2i$

$a=-1-2i$

$$= \int \frac{z+4}{z+1-2i} = 2\pi i f(a)$$

$= 2\pi i f(-1+2i)$

$= 2\pi i((-1+2i)+4)$

$= 2\pi i(3+2i)$

$$\int_c \frac{z+4}{z^2+2z+5} dz = \frac{1}{4i}(2\pi i(3+2i))$$

$$= \frac{\pi}{2}(3+2i)$$

B) 2) $\oint_c \frac{\sin^2 z}{(z-\pi/6)^3} dz$ where c is $|z|=1$

$z-\pi/6=0$ $z=\pi/6$ pole of order 3

$|z|=1$ $z=\pi/6$ within $|z|=1$

$f'(z) = 2\sin z \cdot \cos z$

$f''(z) = 2\cos 2z)$

$$\oint \frac{f(z)}{(z-a)^n} dz = \frac{2\pi i}{(n-1)!} f^{n-1} a$$

$a=\pi/6$ $n=3$

$$\oint \frac{\sin^2 z}{(z-\pi/6)^3} dz = \frac{2\pi i}{2!} f^2(a)$$

$$= \frac{2\pi i}{2!} \times 2\cdot\cos(2\pi/6_3)$$

$$= \frac{2\pi i}{\cancel{2}!} \times \cancel{2}\cos(\pi/3)$$

$= 2\pi i \times 1/2 = \pi i$

c) By cauchy's evaluate $\oint \frac{e^z}{(z-2)} dz$, $|z| = 3$

$\oint_c \frac{e^z}{(z-2)} dz$;

$z-2=0$; $\boxed{z=2}$

$z=2$ within $|z|=3$

$\oint_c \frac{f(z)}{z-a} = 2\pi i f(a)$

$a=2$

$= 2\pi i f(e^z)$

$= 2\pi i . e^2$ //

d) By residue theorem evaluate $\oint \frac{2z-1}{z(z+1)(z-3)} dz$ $|z| = 2$

$z=0$; $z=-1$; $z=3$ $\quad f(z) = 2z-1 / z(z+1)(z-3)$

i) Residue at $z_0=0$

$\lim_{z \to z_0} (z-z_0) f(z) = \lim_{z \to z_0} (z-0) f(z)$

$= \lim_{z \to 0} (\cancel{z}) \cdot \frac{(2z-1)}{\cancel{z}(z+1)(z-3)}$

$= \frac{-1}{1(-3)} = \frac{1}{3}$ //

ii) Residue at $z_0 = -1$

$\lim_{z \to z_0} (z-z_0) f(z) = \lim_{z \to -1} (z-(-1)) f(z)$

$= \lim_{z \to -1} (\cancel{z+1}) \frac{2z-1}{z(\cancel{z+1})(z-3)}$

$= \lim_{z \to -1} \frac{2z-1}{z(z-3)}$

$= \frac{-2-1}{-1(-1-3)} = \frac{-3}{4}$

$= 2\pi i \left[1/3 - 3/4 \right]$

$= \frac{-5\pi i}{6}$ //

9. By cauchys

① $\oint_C \frac{\cos(\pi z)}{z^2-1}$ over rectangle of vertices $2 \pm i$, $-2 \pm i$

singular point

$z^2-1=0 \quad z=\pm 1$

$2 \pm i \quad \& \quad -2 \pm i$

$A(2,1) \quad B(2,-1)$

$C(-2,1) \quad D(-2,-1)$

$z=\pm 1$ lies within C

by cauchy's integ. for.

$$\oint \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$$

$$I = \oint_C \frac{\cos(\pi z)}{(z+1)(z-1)}\,dz$$

$$\frac{1}{(z+1)(z-1)} = \frac{A}{(z-1)} + \frac{B}{(z+1)}$$

$z=1 \quad A=1/2$

$z=-1 \quad B=-1/2$

$$= \oint \frac{1}{2}\frac{\cos(\pi z)}{(z-1)}\,dz - \frac{1}{2}\oint \frac{\cos(\pi z)}{(z+1)}\,dz$$

$$= \frac{1}{2}2\pi i f(1) - \frac{1}{2}2\pi i f(-1)$$

$f(z) = \cos(\pi z)$

$$= \pi i(-1) + \pi i = 0$$