

Sub: M-III

Class: All SY

Marks: 60, Time 3Hr.

Note: Attempt any five

Attempt the following: (12 \times 4 = 48 M)

A) By using L.T. Evaluate $\int t e^{-3t} \sin ct \, dt$

B) Express in terms of Heaviside's unit step function & hence find its L.T. where

$$f(t) = \begin{cases} \cos ct & 0 < t < \pi \\ \sin ct & t > \pi \end{cases}$$

C) Find i) $L\left[\frac{\cos(3t)}{t}\right]$

ii) $L[t^2 e^{-t} \sin ct]$

Attempt any three of the following! [Each 4M]

A) Find $L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$

B) Using L.T. solve $y'' + 2y' + 5y = e^{-t} \sin ct$
 $y(0) = 0, y'(0) = 1$

C) Find L^{-1} by Convolution $\frac{s}{(s^2+1)(s^2+4)}$

D) Find i) $L[t^2 u(t-4) - \cosh ct \, \delta(t-2)]$

ii) $L[\cos ct \, u(t-\frac{\pi}{4}) - \sinh ct \, \delta(t-5)]$

Attempt any three of the following [Each 4M]

A) Using Parseval's identity for cosine transform evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$

B) Find F.T. of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1 \end{cases}$
 & hence evaluate

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$$

CO's

CO 1
Evaluation

CO 2
Analysis

Apply
& Evaluate

CO 2
Evaluation

CO 2
Analysis
& Evaluation

CO 2
Apply &
Evaluation

CO 3

CO 3
Evaluation

CO 3
Evaluation

- c) Find Fourier sine transform of $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$
- d) Find Fourier cosine integral representation of $f(x) = \begin{cases} \cos(x) & 0 < x < \pi/2 \\ 0 & x > \pi/2 \end{cases}$

Q.4) Attempt any three [104]

A) Using Method of separation of variable

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u \quad \text{given that } u(x, 0) = 6e^{-3x}$$

B) Find the temp in bar of length 2 unit whose ends are kept at zero temp & lateral surface insulated if the initial temp is $\sin(\frac{\pi x}{2}) + 3 \sin(\frac{5\pi x}{2})$

c) Form a P.D.E by eliminating functions $z = f(x-it) + g(x-it)$

d) Solve $\tan(x)p + \tan(y)q = \tan(z)$

Q.5)

Attempt any three [105] (Each 4M)

A) If $f(z) = (x^2 + axy + by^2) + i(cu^2 + dxy + y^2)$ is Analytic function find a, b, c, d

B) Find analytic function whose real part is

$$u = e^x [x \cos y - y \sin y]$$

c) State & prove C-R eqn in cartesian form

d) P.T. $u = x^2 - y^2 - 2xy - 2x + 3y$ is Harmonic & hence find v

Q.6

Attempt any three (106) (Each 4M)

A) Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$ where C is $|z+1-i|=2$

B) By Cauchy's integral formula, evaluate

$$1) \oint_C \frac{\cos(\pi z)}{z^2-1} dz \quad \text{over rectangle of vertices } 2 \pm i, -2 \pm i$$

$$2) \oint_C \frac{\sin^2 z}{(z-\pi/6)^3} dz \quad \text{where } C \text{ is } |z|=1$$

c) By Cauchy's Theorem evaluate $\oint_C \frac{e^z}{(z-2)} dz$ where $C: |z|=3$

d) By Residue Theorem evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)} dz$ $C: |z|=2$