DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE
End Semester Examination – May 2019

Course: B. Tech   **Sem: III**

Subject Name: Engineering Mathematics-III   **Subject Code: BTBSC301**

Max Marks: 60   **Date: 28-05-2019**   **Duration: 3 Hr.**

Instructions to the Students:
1. Solve **ANY FIVE** questions out of the following.
2. The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.
3. Use of non-programmable scientific calculators is allowed.
4. Assume suitable data wherever necessary and mention it clearly.

		(Level/CO)	Marks
Q. 1	**Attempt any three.**		**12**
A)	Find $L\{f(t)\}$, where $f(t) = t^2\, e^{-3t} sinhat$	**Understand**	4
B)	Express $f(t)$ in terms of Heaviside's unit step function and hence find its Laplace transform where $f(t) = \begin{cases} cost, & 0 < t < \pi \\ sint, & t > \pi \end{cases}$	**Understand**	4
C)	Find $L\{f(t)\}$, where $f(t) = 2^t \int_0^t \frac{\sin 3u}{u}\, du$	**Understand**	4
D)	By using Laplace transform evaluate $\int_0^\infty e^{-t}\left(\frac{1-cos2t}{t}\right)\, dt$	**Evaluation**	4
Q. 2	**Attempt the following.**		**12**
A)	Using convolution theorem find $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$	**Application**	4
B)	Find $L^{-1}\{\bar{f}(s)\}$, where $\bar{f}(s) = cot^{-1}\left(\frac{s+3}{2}\right)$	**Application**	4
C)	Using Laplace transform solve $y'' - 3y' + 2y = 12e^{-2t}; y(0) = 2, y'(0) = 6$	**Application**	4
Q. 3	**Attempt any three.**		12
A)	Express $f(t) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence deduce that $\int_0^\infty \frac{1-cos\pi\lambda}{\lambda} \sin\pi\lambda\, d\lambda = \frac{\pi}{4}$.	**Evaluation**	4
B)	Using Parseval's identity for cosine transform, prove that $\int_0^\infty \frac{sinat}{t(a^2+t^2)}\, dt = \frac{\pi}{2}\left(\frac{1-e^{-a^2}}{a^2}\right)$	**Application**	4

C) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & if \ |x| \leq 1 \\ 0, & if \ |x| > 1 \end{cases}$. Hence prove that $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) \cos \frac{x}{2} \ dx = -\frac{3\pi}{16}$
    Understand   **4**

D) Find Fourier sine transform of $5e^{-2x} + 2e^{-5x}$
    Understand   **4**

Q. 4   **Attempt the following.**   **12**

A) Form the partial differential equation by eliminating arbitrary function f from $f(x + y + z, x^2 + y^2 + z^2) = 0$
    Synthesis   **4**

B) Solve $xz(z^2 + xy)p - yz(z^2 + xy)q = x^4$
    Analysis   **4**

C) Find the temperature in a bar of length two units whose ends are kept at zero temperature and lateral surface insulated if the initial temperature is $sin\frac{\pi x}{2} + 3 \sin\frac{5\pi x}{2}$.
    Application   **4**

Q. 5   **Attempt Any three.**   **12**

A) If the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic, find the values of the constants a, b, c and d.
    Understand   **4**

B) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.
    Understand   **4**

C) Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$.
    Understand   **4**

D) Prove that the function $u = e^x(x\cos y - y\sin y)$ satisfies the Laplace's equation. Also find the coresponding analytic function.
    Synthesis   **4**

Q. 6   **Attempt ANY TWO of the following.**   **12**

A) Evaluate $\oint_C \frac{z+4}{z^2+2z+5} \ dz$, where C is the circle $|z + 1 - i| = 2$.
    Evaluation   **6**

B) Find the residues of $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.
    Understand   **6**

C) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} \ dz$, where C is the circle $|z| = 3$.
    Evaluation   **6**

*** End ***

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