ADVITIY
IIT Bombay Student Satellite Project

External Disturbance Modelling Report

By

Attitude Determination and Control Subsystem

Department of Aerospace Engineering,

Indian Institute of Technology, Bombay

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Preface
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The validity of ADCS is checked via a simulation environment. Environment Model is one of the models used in simulation. Disturbance modeling is an effort to model the environment out there in space through various mathematical approximations and models. It is used for modelling how the environmental factors affect the attitude of the satellite.

Document Revision Log			
Revision No.	**Date**	**Description of Change**	**Pages Affected**
Original	31/05/18	First Draft	All
1	05/06/18	Relative Velocity in Aerodynamic Torque	8,12
2	25/06/18	Drag Definition changed Residual Magnetic Moment menitioned	1,2,5,6,7,8

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*Here it means that person went through this document completely and verified it completed and had done QA on work related to this document and hence passed it. This should only filled after completion of task and QA on it.

Name: Date:
Designation:
Contact:

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Chapter 1

Introduction and Calculations

Disturbance Modelling refers to quantifying the main disturbances affecting the attitude of the satellite. The disturbances considered in our model are:

- Gravity Gradient Torque
- Aerodynamic Drag Torque
- Solar Radiation Pressure Torque

The magnetic field of the Earth interacts with the residual magnetic dipole of the satellite, which causes a torque around the center of mass of the satellite. The residual magnetic dipole of the satellite is caused by the current running through the wires and PCBs on the satellite. This however isn't considered in our model because we are modelling only external disturbances.

Disturbances will be quantified as torques. For our purposes, we calculate torque about the centre of mass of satellite in the Satellite Body Reference Frame. All vectors used here on are in this frame. Since torque doesn't depend on origin of co-ordinates, we can use geometric centre to be origin of frame while calculating torque, keeping axes aligned to body frame.

1.1 Gravity Gradient

An extended object in a non uniform gravitational field experiences a torque in general due to different forces acting at different parts of the object. Consider an extended general body, let $\vec{R}$ be vector from centre of earth to a general point on that object, $\vec{R}_c$ be vector from centre of earth to centre of mass of body and $\vec{r}$ be vector from centre of mass of satellite to that point. From vector addition it is clear that $\vec{R} = \vec{R}_c + \vec{r}$. By Newton's Law, force acting on that point due to gravity is

$$d\vec{F}_g = \frac{-GM_e dm\vec{R}}{|\vec{R}|^3}$$

The total torque about centre of mass will then be

$$\begin{aligned}
N_G &= \int_B \vec{r} \times d\vec{F}_g \\
&= -\int_B \vec{r} \times (\vec{R}_c + \vec{r})\frac{\mu dm}{|\vec{R}|^3} \\
&= \mu\vec{R}_c \times \int_B \frac{\vec{r}dm}{|\vec{R}|^3} \\
& where\ \mu = GM_e
\end{aligned}$$

The vector $\vec{R}$ can be approximated as

$$\begin{aligned}
|\vec{R}|^{-3} &= |\vec{R}_c + \vec{r}|^{-3} = ((\vec{R}_c + \vec{r}).(\vec{R}_c + \vec{r}))^{-3/2} \\
&= (R_c^2 + r^2 + 2\vec{R}_c.\vec{r})^{-3/2} \\
&= R_c^{-3}(1 + (\frac{r}{R_c})^2 + 2\frac{\vec{R}_c.\vec{r}}{R_c^2})^{-3/2} \\
&\approx R_c^{-3}(1 - 3\frac{\vec{R}_c.\vec{r}}{R_c^2})
\end{aligned}$$

Hence the torque can be written as

$$\begin{aligned}
N_G &= \frac{\mu\vec{R}_c}{R_c^3} \times \int_B \vec{r}(1 - 3\frac{\vec{R}_c.\vec{r}}{R_c^2})dm \\
&= \frac{\mu\vec{R}_c}{R_c^3} \times \int_B \vec{r}dm - 3\frac{\mu\vec{R}_c}{R_c^5} \times \int_B (\vec{R}_c.\vec{r})\vec{r}dm
\end{aligned}$$

The first term above involving the integral will be zero since value of the integral is simply position vector of centre of mass of satellite in Satellite Body Reference Frame which is zero. The second term involving integral can be simplified by using the triple product identity (CAB-BAC Rule).

$$\vec{r} \times (\vec{R}_c \times \vec{r}) = r^2\vec{R}_c - (\vec{R}_c.\vec{r})\vec{r}$$

Using the above equation in torque equation one gets

$$N_G = 3\frac{\mu\vec{R}_c}{R_c^5} \times \int_B (\vec{r} \times (\vec{R}_c \times \vec{r} - r^2\vec{R}_c)dm$$

Now the second term in the integral is zero because

$$-(\vec{R}_c \times \vec{R}_c)\frac{3\mu}{R_c^5}\int_B r^2 dm = 0$$

Now $\vec{r}$ is expressed in body frame. The first term can be simplified by using a result in rigid body dynamics which states

$$\int_B (\vec{r} \times (\vec{v} \times \vec{r}))dm = [I]\vec{v}$$

where is $\vec{v}$ is any vector and [I] is the moment of inertia tensor of the body.

Therefore the expression for gravity gradient becomes

$$\boxed{N_G = \frac{3\mu}{|\vec{R}_c|^5}(\vec{R}_c \times [I]\vec{R}_c)} \tag{1.1}$$

1.2 Aerodynamic Drag Torque

For a satellite in low earth orbit, the main disturbance torque is exerted by aerodynamic drag. Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid. As mentioned, the drag equation with a constant drag coefficient gives the force experienced by an object moving through a fluid at relatively large velocity (i.e. high Reynolds number, Re ¿ 1000). This is also called quadratic drag. The calculations for torque will be done for a cuboid satellite. The drag force $d\vec{f}_{drag}$ acting on an infinitesimal area element dA with normal vector $\hat{N}$, is given by the drag equation:

$$d\vec{f}_{drag} = -\frac{1}{2}\rho C_d |V|^2 (\hat{N}.\hat{V})\hat{V} dA$$

where $\hat{V}$ is a unit vector in the direction of satellites translation with respect to the atmosphere, ρ is the atmospheric density and C_d is the drag coefficient. Since there is no measured value available for C_d, it can be set to 2. [*Wertz Page 573-574*] The total torque about the centre of mass will then be

$$\begin{aligned} N &= \int_B (\vec{r} - \vec{r}_{com}) \times d\vec{f}_{drag} \\ &= \int_B \vec{r} \times d\vec{f}_{drag} + \int_B d\vec{f}_{drag} \times \vec{r}_{com} \\ &= \int_B \vec{r} \times d\vec{f}_{drag} + \vec{F}_{total} \times \vec{r}_{com} \end{aligned}$$

where $\vec{r}_{com}$ is vector from geometric centre of satellite to centre of mass of satellite. For evaluating the term in the integral, break up the integration over 3 surfaces because the maximum number of faces getting hit by air molecules opposing motion at a time is 3.

$$N_1 = -\frac{\rho C_d V^2}{2} \int_B \vec{r} \times (\hat{V}.\hat{N})\hat{V} dA$$

The terms in the bracket can be broken into integration over 3 surfaces, each of which are in a direction of satellite velocity. Needless to say, $\hat{V}.\hat{N}$ will be postitive for such surfaces . Morever, the Satellite body reference frame has x,y and z axes lied along perpendiculars to surfaces. Therefore $\hat{V}.\hat{N}$ for a surface perpendicular to x axis will be $|\hat{V}.\hat{x}|$. Using this one can simplify the integral as a sum over faces:

$$\sum_{x,y,z} \vec{r} dA |\hat{V}.\hat{x}| \times \hat{V}$$

The term $\vec{r}$ dA for a face will be vector for centre of that face times area of the face. For example, for a face perpendicular to x direction the value will be $\frac{l_2 l_3 l_1}{2}(\frac{\hat{V}.\hat{x}}{|\hat{V}.\hat{x}|})\hat{x}$

The sum thus becomes

$$\begin{aligned}
&\sum_{x,y,z} l_2 l_3 \frac{l_1}{2}\left(\frac{\hat{V}.\hat{x}}{|\hat{V}.\hat{x}|}\right)\hat{x}|\hat{V}.\hat{x}| \times \hat{V} \\
=&\sum_{x,y,z} \frac{l_1 l_2 l_3}{2}(\hat{V}.\hat{x})\hat{x} \times \hat{V} \\
=&\frac{l_1 l_2 l_3}{2}\left(\sum_{x,y,z}(\hat{V}.\hat{x})\hat{x}\right) \times \hat{V} \\
=&\frac{l_1 l_2 l_3}{2} V \times \hat{V} \\
=&0
\end{aligned}$$

The integral term therefore turned out to be zero. So the total torque is simply $\vec{F}_{total} \times \vec{r}_{com}$ The physical meaning of drag equation is that the drag force on a surface will be some constant times the area projected in the direction of velocity. The projected area of satellite in direction of velocity is $\sum_{x,y,z} l_2 l_3 |\hat{V}.\hat{x}|$ The total aerodynamic drag torque will thus be

$$\boxed{\vec{r}_{com} \times \frac{\rho C_d V^2}{2}(l_2 l_3 |\hat{V}.\hat{x}| + l_1 l_3 |\hat{V}.\hat{y}| + l_2 l_1 |\hat{V}.\hat{z}|)\hat{V}} \tag{1.2}$$

where $\vec{V}\ is\ \vec{V}_{satellite} - \vec{V}_{atmosphere}$.

1.3 Solar Radiation Torque

Radiation hitting the surface of the satellite has momentum associated with it and thus generates a torque around the centre of mass. The majority solar radiation experienced by the satellite is due to radiation emitted from the sun. Force due to radiation is due to reflection and absorption of radiation are given by [*Satellite Orbits. see References*]

$$\vec{F}_{abs} = (1-e)PAcos(\theta)\hat{e}$$
$$\vec{F}_{ref} = -2ePAcos^2(\theta)\hat{n}$$

where e denotes coefficient of reflectivity of the surface of satellite, $\hat{e}$ denotes the unit vector in the direction of radiation which in our case is the same as negative of the unit sun vector $\hat{v}_{sun}$. θ is the angle between normal to the surface and incoming radiation. $\hat{n}$ is the unit outward normal vector to the surface. P is the solar radiation pressure due to sun which is constant throughout the satellite's orbit. A is the area of surface upon which radiation is incident. For torque calculation due to $\vec{F}_{abs}$ we can use the similarity between $\vec{F}_{abs}$ and Aerodynamic drag torque. Both force equations have projected area of surface in equation and force opposite to some fixed vector. Thus there would be a similar form of torque equation for torque due to $\vec{F}_{abs}$.

$$\vec{N}_{abs} = \vec{r}_{com} \times P(1-e)(l_2l_3|\hat{v}_{sun}.\hat{x}| + l_1l_3|\hat{v}_{sun}.\hat{y}| + l_2l_1|\hat{v}_{sun}.\hat{z}|)\hat{v}_{sun}$$

The torque due to $\vec{F}_{ref}$ is given by

$$\begin{aligned}\vec{N}_{ref} &= -\int_B (\vec{r}-\vec{r}_{com}) \times 2ePAcos^2(\theta)\hat{n} \\ &= -\int_B \vec{r} \times 2ePAcos^2(\theta)\hat{n} + \int_B \vec{r}_{com} \times 2ePAcos^2(\theta)\hat{n}\end{aligned}$$

The torque should turn out same irrespective of choice of origin as long as axes are aligned. So one can use origin to be geometric centre of the cuboid satellite. The first term in the integral is therefore torque about geometric centre of satellite. We can break integration as sum over three surfaces because at a time maximum 3 surfaces are exposed to radiation. From the equation,we see that force is perpendicular to the surface of incidence. Thus the equivalent force vector for a single surface passes though centre of the surface perpendicular it and thus produces no torque about geometric centre. Similarly for the other 2 surfaces. The first integral is hence zero. The second integral can be written as

$$\vec{N}_{ref} = \vec{r}_{com} \times \int_B 2ePAcos^2(\theta)\vec{n}$$

As the axes are perpendicular to the surfaces of satellite and at a time only one out of a pair of opposite surfaces can face the radiation one can write the normal vector in terms of sun vector. For example, for a face perpendicular to x axis the normal vector can be written as

$$\vec{n} = \frac{(\hat{v}_{sun}.\vec{x})\vec{x}}{|\hat{v}_{sun}.\vec{x}|}$$

The $cos^2(\theta)$ term for the same surface can be written as

$$cos^2(\theta) = |\hat{v}_{sun}.\vec{x}|^2$$

The torque therefore is

$$\vec{N}_{ref} = \vec{r}_{com} \times 2eP(l_2 l_3(|\hat{v}_{sun}.\hat{x}|\hat{v}_{sun}.\hat{x})\hat{x} + l_1 l_3(|\hat{v}_{sun}.\hat{y}|\hat{v}_{sun}.\hat{y})y + l_2 l_1(|\hat{v}_{sun}.\hat{z}|\hat{v}_{sun}.\hat{z})\hat{z})$$

The total solar radiation torque is therefore

$$\vec{N} = \vec{N}_{ref} + \vec{N}_{abs} \tag{1.3}$$

Note that this torque is only applicable when satellite isn't in the eclipse region.

Chapter 2

2.1 Conclusion

The final torques in the Satellite Body reference Frame are

- Gravity Gradient Torque

$$\vec{N}_G = \frac{3\mu}{|\vec{R}_c|^5}(\vec{R}_c \times [I]\vec{R}_c)$$

- Aerodynamic Drag Torque

$$\vec{N}_{AD} = \vec{r}_{com} \times \frac{\rho C_d V^2}{2}(l_2 l_3|\hat{V}.\hat{x}| + l_1 l_3|\hat{V}.\hat{y}| + l_2 l_1|\hat{V}.\hat{z}|)\hat{V}$$

  where $\vec{V}\ is\ \vec{V}_{satellite} - \vec{V}_{atmosphere}$.

- Solar Radiation Pressure Torque

$$\vec{N}_{SP} = \vec{r}_{com} \times P(1-e)(l_2 l_3|\hat{v}_{sun}.\hat{x}| + l_1 l_3|\hat{v}_{sun}.\hat{y}| + l_2 l_1|\hat{v}_{sun}.\hat{z}|)\hat{v}_{sun} + \\ \vec{r}_{com} \times 2eP(l_2 l_3(|\hat{v}_{sun}.\hat{x}|\hat{v}_{sun}.\hat{x})\hat{x} + l_1 l_3(|\hat{v}_{sun}.\hat{y}|\hat{v}_{sun}.\hat{y})y + l_2 l_1(|\hat{v}_{sun}.\hat{z}|\hat{v}_{sun}.\hat{z})\hat{z})$$

2.2 References

James R. Wertz. Spacecraft Attitude Determination and Control. Kluwer Academic Publishers, 1978.
Satellite Orbits by Oliver Montenbruck. Springer Publications.