January 19, 2021

PHY432 (Quantum Mechanics-II)

Problem set-1

(WKB approximation)

1. Assume the wavefunction for the Schrödinger equation be given by

$$\psi(x) = A\ e^{iS(x)/\hbar}$$

where $S(x)$ is a real function and A is a constant. Show that you can derive the WKB wave function by expanding $S(x)$ in the power series of $\hbar$ and keeping terms upto first order in $\hbar$.

2. Using the WKB approximation, calculate the transition coefficient of a potential barrier for a particle with energy $E < V_0$:

$$V(x) = V_0(1 - \frac{x^2}{a^2}), \quad for \quad -a \leq x \leq a,$$

$$V(x) = 0, \quad otherwise.$$

3. Use the WKB approximation to find the allowed energies (E_n) of an infinite well with a self of height V_0 extending half-way across:

$$V(x) = \begin{cases} V_0, & \text{if } 0 < x < a/2 \\ 0, & \text{if } a/2 < x < a \\ \infty. & \text{otherwise} \end{cases}$$

4. Apply WKB method to the one-dimensional motion of a particle of mass m in a potential that equals to $-V_0$ at $x = 0$, changes linearly with x until it vanishes at $x = \pm a$, and is zero for $|x| > a$. Find all the bound energy levels obtained in this approximation if $mV_0a^2/\hbar^2 = 40$.

5. Calculate the bound state energies of hydrogen atom using the WKB approximation

$$\int_{x_1}^{x_2} k(x)dx = (n - 1/2)\pi.$$

Don't omit the centrifugal potential. Comment about the $l = 0$ and $l \neq 0$ results.