Date: January 26, 2021

# PHY432 (Quantum Mechanics-II)
## Problem set-2

(Time-independent perturbation theory: non-degenerate cases)

1. Consider the infinite potential well of width $a$:

   $V(x) = 0$, for $0 \leq x \leq a$, $\quad V(x) = \infty$ otherwise.

   Suppose we put a delta-function bump in the center of the square well:

   $H' = aw\ \delta(x - a/2)$, where $w$ is a constant.

   (a) Find the first order correction to the energy levels. Explain why the energies are not perturbed for even $n$.

   (b) Find the first three nonzero terms in the expansion of the first order correction to the ground state wave function.

2. Consider a charged harmonic oscillator in one dimension is placed in a uniform electric field $\mathcal{E}$, so that the electrostatic potential $V(x) = -q\mathcal{E}x$, where $q$ is the charge of the oscillator.

   (a) Assuming the electric field is very weak, calculate the leading nonzero correction to the energy level $E_n$.

   (b) Calculate the first order correction to the eigenstate $|n\rangle$.

   (c) It is also possible to solve the above problem exactly. Compare the result in (a) with the exact solution for the energy.

3. The hamiltonian of the *anharmonic* oscillator is given by

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \sigma x^3 \qquad (\sigma << 1)$$

   Calculate the leading order correction to the harmonic oscillator energy due to the anharmonic term and show that the energy levels are no longer equidistant.

4. A rigid rotator is described by the Hamiltonian

$$H_0 = \frac{L^2}{2I}$$

   where $L$ denotes the rotator's angular momentum and $I$ its moment of inertia.

   (a) What are the eigenfunctions and eigenvalues of the above Hamiltonian?

(b) In presence of a uniform electric field $\mathcal{E}$ applied along z-axis, the potential is given by $H' = -\mathcal{E} d \cos\theta$ where $d$ is the electric dipole moment of the rotator.

(i) Though all the levels except for $l = 0$ are degenerate, argue that we can still use non-degenerate perturbation theory to calculate the Stark effect.

(ii) Calculate the lowest non-vanishing correction to the energy levels.

5. Consider the hydrogen atom and assume that the proton, instead of being a point particle, is a uniformly charged sphere of radius $R$. This means that the Coulomb potential is now modified to

$$
\begin{aligned}
V(r) &= -\frac{3e^2}{8\pi\epsilon_0 R^3}\left(R^2 - \frac{1}{3}r^2\right) \qquad r < R \quad (<< a_0) \\
&= -\frac{e^2}{4\pi\epsilon_0 r} \qquad r > R
\end{aligned}
$$

Calculate the energy shift for the $n = 1,\ l = 0$ state to lowest non-vanishing order.