Signal Processing

LAB REPORT:5

DFT and FFT

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1) DFT for frequency analysis of CT signals:

(a).

Given, $p[n] = cos(2\pi f_o n/fs) = cos(w_o n),\ where\ w_o = 2\pi f_o/fs$

Now,

$$P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} p[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} cos[w_o n]e^{-j\omega n}$$

$$\Rightarrow P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (e^{jw_o n} + e^{-jw_o n})e^{-j\omega n}/2 = \sum_{n=-\infty}^{\infty} (e^{jn(w_o - w)} + e^{-jn(w_o + w)})\ /2$$

$$\Rightarrow P(e^{j\omega}) = (\delta[w_o - w] + \delta[w_o + w]) * \pi$$

In general, $\Rightarrow P(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (\delta[w_o - w - 2\pi k] + \delta[w_o + w + 2\pi k])$

(b) From the above equation we can conclude that location of impulses are $[-w_o]$ & $[w_o]$ where $w_o = 2\pi f_o/fs$.

(c) Given, $x[n] = p[n] \times w[n]$. From Multiplication property of DTFT we know that, if $p[n] \leftrightarrow P(e^{j\omega})$ and $w[n] \leftrightarrow W(e^{j\omega})$, then

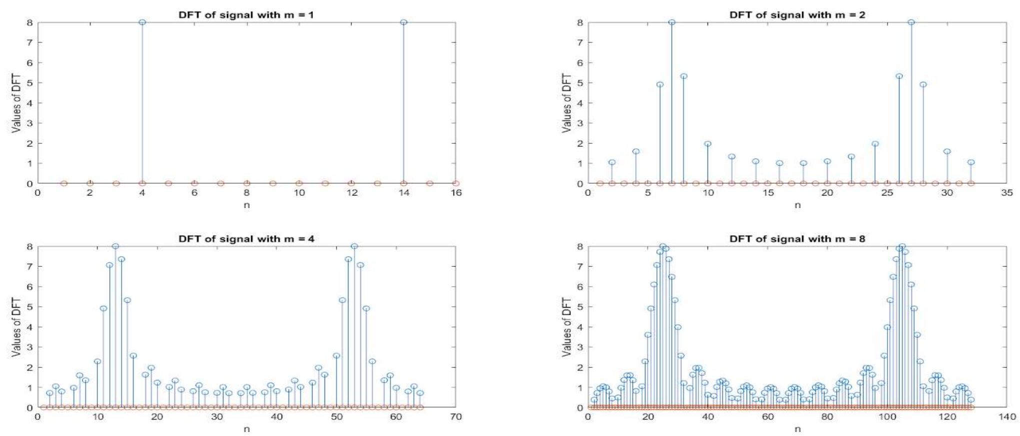
$$X(e^{j\omega}) = P(e^{j\omega}) * W(e^{j\omega}),\ where\ '*'\ is\ convolution\ operator.$$

In magnitude spectrum when we take the whole p[n], we got 2 impulse. But if we take a window then we will get 2 less sharp pulses and several pulses with less magnitude instead of zero.

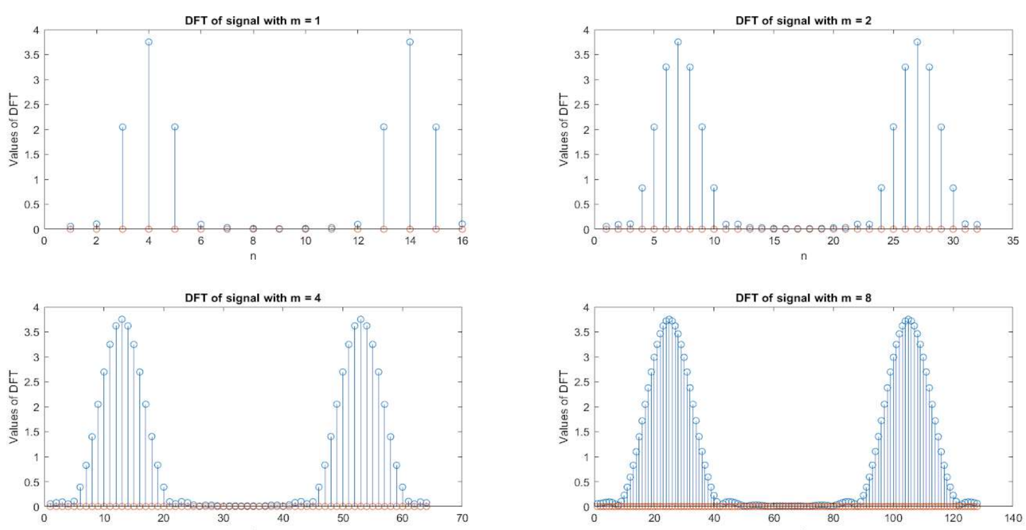
(d) The plots are not consistent with what derived in part c. A difference is observed in between the theoretical calculations and one that are obtained from Matlab plots.
This is due to the *spectral leakage.* FFT in matlab assumes that the signal repeats itself after a certain interval of time. This occurs when signal being measured is not periodic. So there is the difference between theoretical calculations and DFT calculated from FFT.

(e) As the length of the signal increases the value of N also increases so we get better resolution in FFT. Increasing L makes the output more denser.

(g) Without Hanning window



With Hanning window



These are the following observations when $w[n]$ is an L-length Hanning window:

- Main lobe: The main around the peaks has the values which decrease uniformly in DFT obtained using hanning window.
- Spectral leakage: It is less as compared to previous one.

So, we can conclude that Hanning window provides good frequency resolution and leakage protection with fair amplitude accuracy.

(i) The top 3 strongest frequencies (in Hz) present in the audio signal, " *Audio Files_0.wav"* are as follows:

Frequency	Power
31	2520.8
51	5128.5
131	1379.13

The top 3 strongest frequencies (in Hz) present in the audio signal, " *sample.wav"* are as follows:

Frequency	Power
1488	3280.19
2976	6472.35
6694	2857.13

2) Direct and DFT based convolutions:

(a).

(b)The expected length , when x1 and x2 are signals:

- Linear convolution: length(x1) + length(x2) - 1
- Circular Calculation: max(length(x1) , length(x2))

(c)From computation from DFT method:

- Linear convolution: length(x1) + length(x2) - 1
- Circular Calculation: max(length(x1) , length(x2))

(d) We observe that from both the methods of using formula and by using DFT and IDFT we got same plot for both linear and circular convolution.

3) DFT of some signals:

We can identify the high frequency and low frequency from the magnitude plot.

(a) .

- N = 4: max = 1; min = 2.
- N = 16: max = 1; min = 5.
- N = 64: max = 1; min = 17.

(b)Max: 4; Min = 1.

(c)Max: 4; Min = 1.

(d)Max: 4; Min = 1.

(e)Max: 1; Min = 11.

(f) Max: 11; Min = 1.