

A Novel Implementation of 4-bit Carry Look-ahead Adder

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Abstract

Carry look-ahead adder (CLA) is a fast adder. This paper is dedicated to accelerate the 4-bit CLA circuit. In proposed circuit structure, XOR gate is replaced by NOR gate. Moreover, logic gates have less fan-in and fan-out and signal throughs one less MOS transistor in critical path. All designs are built in dynamic CMOS logic gates. Simulation results show that the proposed architecture has advantages over conventional circuits in speed, area and power consumption.

Key words: CLA, CMOS, HSpice and logic gate.

I. Introduction

Addition is the basic mathematical operation and an essential operating in Algorithm Logic Units (ALU) [1]. Currently, addition have many logic designs, such as ripple adder, carry select adder and so on [2]. In these circuits, the carry-sum of higher bits depend on the carry-out of lower significant bits and this causes long propagation delay. Carry look-ahead adder (CLA) solves this problem. This method makes the carry-sum of each bit to be calculated by inputs directly. Some articles implement the logic gate in different technologies, such as BiCMOS [3], in order to speed up the CLA operation. And some fabricate the logic gate only in NMOS transistors [4]. Circuits in [5]-[7] are accomplished in CMOS logic gate. They are constructed by XOR AND and OR gates. All of them have the same formulas. However, the design [8] uses NAND gate to replace the AND and NOT gates. This can efficiently decrease the cost and increase the speed. Therefore [8] is the fastest CLA design at present, and it has lower cost in area and power.

Our work is to accelerate the CLA circuit established by dynamic CMOS logic gates. This simplification scheme realizes this acceleration and reduces power consumption.

In this paper, section I introduces the background about CLA. In section II, the principle of 4-bit CLA circuit is described in part A. Part B gives the new derived functions and analysis. Comparisons of the simulation results are presented in section III. At last, a conclusion is given.

II. Implementation of Carry look-ahead Adder

A. The principle of 4-bit CLA

Function of the CLA is to calculate the carry-in of each bit from inputs directly. For addition with two inputs, the external carry-in is denoted as C_0 . Both inputs have 4 bits and they are represented as $A_3A_2A_1A_0$ and $B_3B_2B_1B_0$ respectively. Then sum of this addition $S_3S_2S_1S_0$ can be calculated by Full adders. Thus $S_3S_2S_1S_0 = A_3A_2A_1A_0 \oplus B_3B_2B_1B_0 \oplus C_3C_2C_1C_0$, where C_3 C_2 and C_1 are the carry-outs of previous bits.

The logic relationships between the carry-outs and inputs are expressed by equations (1)-(3). Among them, signals

Carry-Propagation (abbreviated P_i) and Carry-Generate (abbreviated G_i) are obtained $P_i = A_i \oplus B_i$ and $G_i = A_i \& B_i = A_i B_i$, in which i is the value 0 1 2 or 3. The symbol ' $\&$ ' represents logic AND and it can be omitted in function sometimes. Besides, the symbol '+' denotes the logic OR.

$$C_1 = G_0 + P_0 C_0 \quad (1)$$

$$C_2 = G_1 + P_1 G_0 + P_1 P_0 C_0 \quad (2)$$

$$C_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0 \quad (3)$$

$$G = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 \quad (4)$$

$$P = P_3 P_2 P_1 P_0 \quad (5)$$

In equations (4) and (5), G represents the carry-propagation signal of the 4-bit CLA adder and P is denoted as the carry-generate signal. Based on them, carry-out of this addition C_4 is given by formula $C_4 = G + P C_0$.

B. Proposed CLA Design

The basic circuit of 4-bit CLA can be accomplished according to the above equations (1)-(5). Our proposed structure is implemented as the following new derived functions.

On the one hand, substituting G_0 and P_0 into equation (1) we get $C_1 = A_0 B_0 + (A_0 \oplus B_0) C_0$. According to Table I, C_1 is simplified to formula $C_1 = A_0 B_0 + (A_0 + B_0) C_0$. From Table I, we know that the values of $A_0 + B_0$ and $A_0 \oplus B_0$ are different only when $A_0 B_0 = 11$. In this situation, the final value of C_1 equals to '1' because $A_0 \& B_0 = 1$.

On the other hand, the complementary logic gate is inverting for CMOS technology. Based on the DeMorgan's Theorems, C_1 is implemented as $C_1 = \overline{A_0 B_0} \overline{A_0 + B_0 + C_0}$. Replacing $\overline{A_0 B_0}$ and $\overline{A_0 + B_0}$ by G'_0 and P'_0 . In other words, $G'_0 = \overline{A_0 \& B_0} = \overline{A_0 B_0}$ and $P'_0 = \overline{A_0 + B_0}$. Thus the simplified formula of C_1 is finally written as equation (6).

For the remaining carry-outs C_3 and C_2 , using the same way, we obtain the expressions (7)-(8). In particular, G and P are replaced by G'_{out} and P'_{out} which are shown in equations (9)-(10). In them, $G'_i = \overline{A_i \& B_i} = \overline{A_i B_i}$ and $P'_i = \overline{A_i + B_i}$. C_4 can be built as function $C_4 = G'_{out} (P'_{out} + \overline{C_0})$. Eventually, Fig.1 demonstrates the detailed circuit structure of the proposed 4-bit CLA.

$$C_1 = \overline{G'_0} (\overline{P'_0} + \overline{C_0}) \quad (6)$$

$$C_2 = \overline{G'_1} (\overline{P'_1} + \overline{G'_0}) + (\overline{P'_1} + \overline{P'_0}) C_0 \quad (7)$$

$$C_3 = \overline{G'_2} (\overline{P'_2} + \overline{G'_1}) + (\overline{P'_2} + \overline{P'_1}) \overline{G'_0} (\overline{P'_0} + \overline{C_0}) \quad (8)$$

$$G'_{out} = \overline{G'_3} (\overline{P'_3} + \overline{G'_2}) + (\overline{P'_3} + \overline{P'_2}) \overline{G'_1} (\overline{P'_1} + \overline{G'_0}) \quad (9)$$

$$P'_{out} = \overline{P'_3} + \overline{P'_2} \overline{P'_1} + \overline{P'_0} \quad (10)$$

Path from B_0 through C_3 to S_3 is the longest. To function $\overline{C_3}$ of our design and C_3 of [8], Fig.2 illustrates a dynamic CMOS structure for both of them. By construct, logic gates in Fig.2.(a) has less fan-in and signal throughs one less transistor than logic gates in Fig.2.(b). Moreover, the proposed logic gates G'_i and P'_i have less fan-out and logic gates C_i have less fan-in compared with corresponding gates in [8].

TABLE I
LOGIC RELATIONSHIP TRUTH TABLE OF C_1

C_0	A_0B_0	$A_0 \& B_0$	$A_0 \oplus B_0$	$A_0 + B_0$	C_1
	$A_0B_0=00$	0	0	0	0
$C_0=0$	$A_0B_0=01$	0	1	1	0
or 1	$A_0B_0=10$	0	1	1	0
	$A_0B_0=11$	1	0	1	1

III. Results and Comparsons

Based on the 0.25 μ m technology, circuits are implemented in dynamic logic gates which are fabricated in minimal size using typical model SPICE transistors of [9]. The inverter has the 0.5 μ m PMOS width and the 0.25 μ m NMOS width. The voltage is 2.5v and all outputs have no extra loads. Fig.3 shows the waves of the longest paths in HSpice.

Simulation results of circuits are listed in Table II. Seen from the data, delay of the 4-bit CLA is reduced from 1.008ns to 0.732ns. Besides, our design uses less transistors, so the area of transistor is less. Thus, power consumption during the latency of the proposed design is lower than [8].

IV. Conclusion

In this paper, we implement a novel 4-bit CLA circuit. Logic gates in the proposed design have less fan-in and fan-out. And XOR gate is replaced by NOR gate. Meanwhile, signal throughs one less MOS transistor in the critical paths. Power consumption and area have also decreased. Comparisons show that our design has better circuit performance than reported designs. This scheme is also applied to the other architecture.

TABLE II
RESULTS OF 4-BIT CLA DESIGNS

Design	Delay	Area	Power consumption
[8]	1.008ns	37.937 μ m ²	6.988 $\times 10^{-13}$ J
Our	0.732ns	30.125 μ m ²	5.075 $\times 10^{-13}$ J

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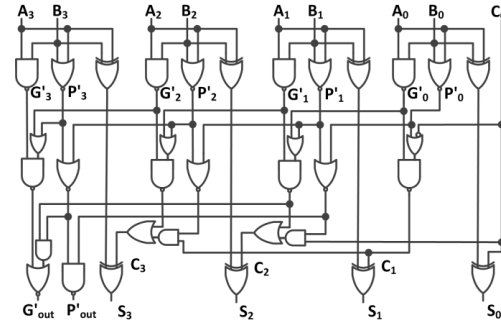


Fig. 1. Proposed circuit structure of 4-bit CLA.

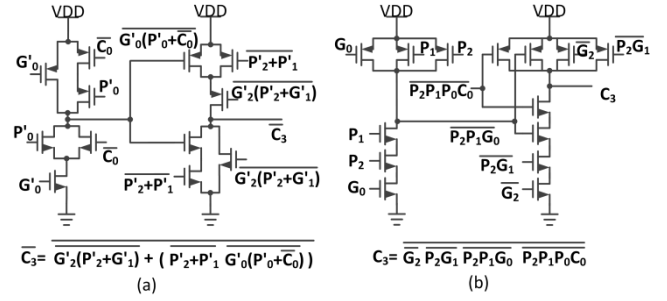


Fig. 2. (a) A dynamic CMOS logic structure of $\overline{C_3}$ in our design.

(b) A dynamic CMOS logic structure of C_3 in [8].

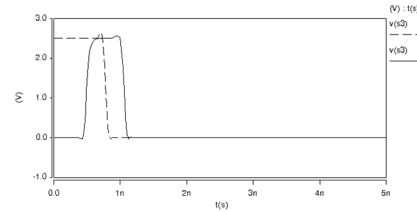


Fig. 3. The simulation waves of S_3 in HSpice.

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