Lab -1
Set 1 - Global Dynamics of GDP and Trade

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CS302, Modeling and Simulation

In this lab, we modeled the logistic equation to observe the growth of GDP and Trade for six countries, namely the United States of America, China, Japan, Germany, the United Kingdom, and India. We also tried to predict when India will become a $4 trillion and $5 trillion economy.

I. EQUATIONS

The first-order non-linear autonomous system is expressed by:

$$\dot{x} \equiv \frac{dx}{dt} = f(x)$$

where

$$f(x) = ax - bx^2 \tag{1}$$

Here, a and b are fixed parameters. The solution of the above logistic equation is given by:

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

where $x(0) = x_0$ is the initial condition and $k = \frac{a}{b}$
Eq. (2) gives the non-linear time-scale as

$$t_{nl} = \frac{1}{a}\ln(\frac{k}{x_0} - 1) \tag{3}$$

we posit the logistic equation to model the GDP and trade of various economies. If GDP is measured in US dollars and t in years, then the logistic model for the GDP is as follows,

$$\dot{G} \equiv \frac{dG}{dt} = \mathcal{G}(G) = \gamma_1 G - \gamma_2 G^2 \tag{4}$$

Here, a and b translate to γ_1 and γ_2 respectively, with $k_G = \gamma_1/\gamma_2$.
Equation of time(in years) to predict the GDP is as follows,

$$t = 1960 + \frac{1}{a}\ln\frac{x(k - x_0)}{x_0(k - x)}$$

Similarly, modeling the Trade as follows,

$$\dot{T} \equiv \frac{dT}{dt} = \mathcal{T}(T) = \tau_1 T - \tau_2 T^2 \tag{5}$$

where $T = T(t)$ is the Trade percentage expresses in US dollars, t is time measured in years. Also, a and b translate to τ_1 and τ_2 respectively, with $k_T = \tau_1/\tau_2$.

To explore the correlated growth of GDP (G) and Trade (T), we establish a linked autonomous dynamical system denoted by $\dot{G} = \mathcal{G}(T, G)$ and $\dot{T} = \mathcal{T}(T, G)$. The T-G phase solutions are determined by integrating,

$$\frac{dG}{dT} = \frac{\dot{G}}{\dot{T}} = \frac{\mathcal{G}(T, G)}{\mathcal{T}(T, G)} \tag{6}$$

In the linear regime, we get a scaling formula that goes as,

$$G(T) \sim T^{\alpha} \tag{7}$$

where $\alpha = \gamma_1/\tau_1$.

II. GDP AND TRADE MODELING GRAPHS

A. USA

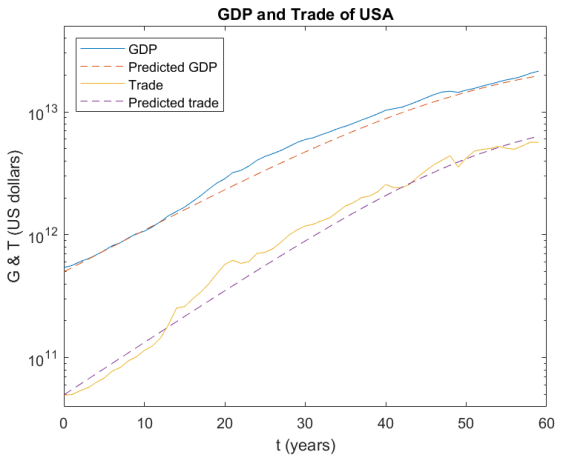

Fig. 1 The dotted curves follow the logistic equation with Table I and II's parameter values. The zero year of both plots is 1960.

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B. China

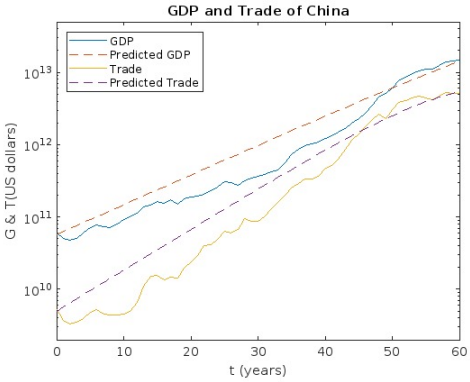

Fig. 2 The dotted curves follow the logistic equation with Table I and II's parameter values. The zero year of both plots is 1960, and both end in 2020.

C. Japan

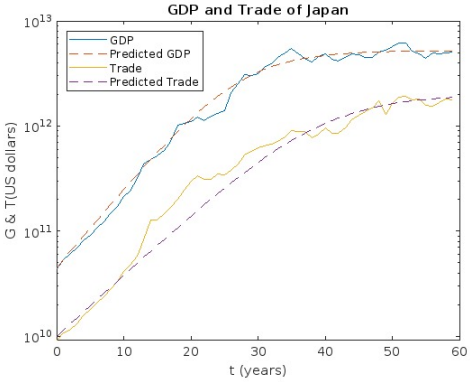

Fig. 3 The dotted curves follow the logistic equation with the parameter values in Table I and Table II. The zero year of both plots is 1960, and both end in 2019.

D. Germany

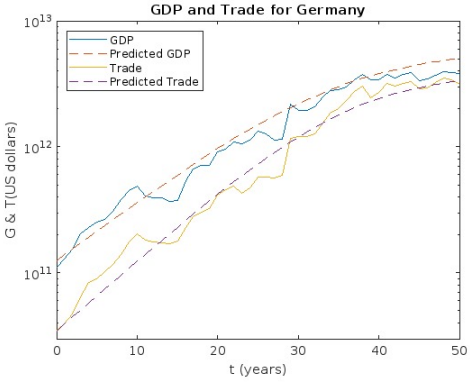

Fig. 4 The dotted curves follow the logistic equation with the parameter values in Table I and Table II. The zero year of both plots is 1970, and both end in 2020.

E. UK

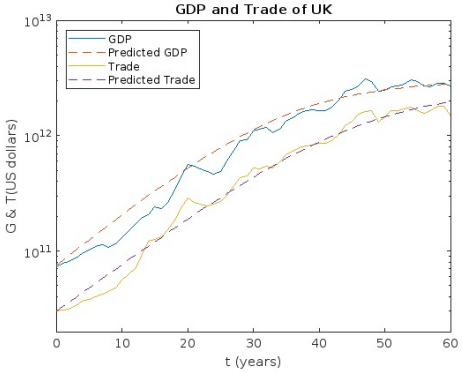

Fig. 5 The dotted curves follow the logistic equation with the parameter values in Table I and Table II. The zero year of both plots is 1960, and both end in 2020.

F. India

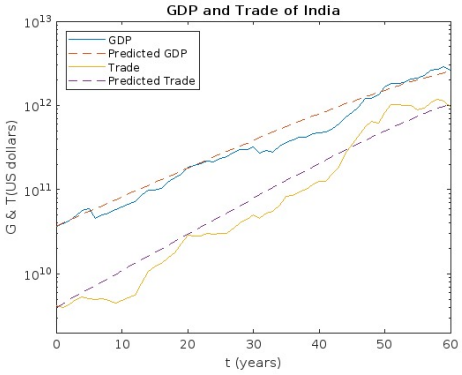

Fig. 6 The dotted curves follow the logistic equation with the parameter values in Table I and Table II. The zero year of both plots is 1960, and both end in 2020.

III. GDP VS TRADE MODELING GRAPHS

A. USA

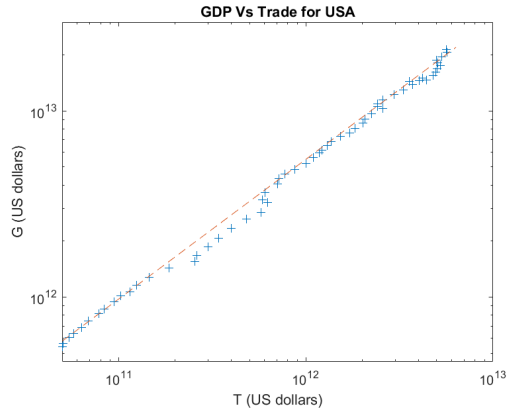

Fig. 7 The dotted line follows Eq. (7) with $\alpha = 0.75$ (see Table VI). The plot begins in 1960 and ends in 2019.

B. China

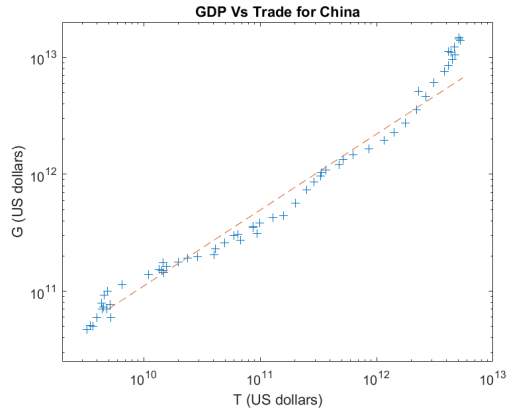

Fig. 8 The dotted line follows Eq. (7) with $\alpha = 0.65$ (see Table VI). The plot begins in 1960 and ends in 2020.

C. Japan

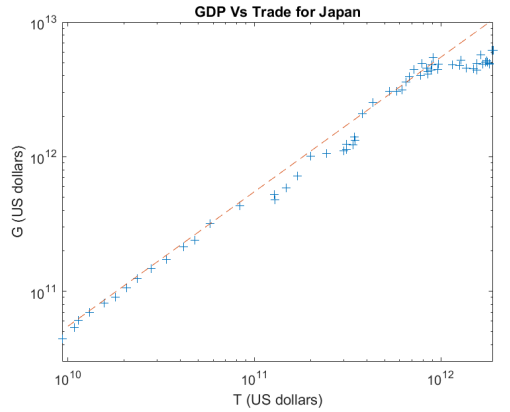

Fig. 9 The dotted line follows Eq. (7) with $\alpha = 1.00$ (see Table VI). The plot begins in 1960 and ends in 2019.

D. Germany

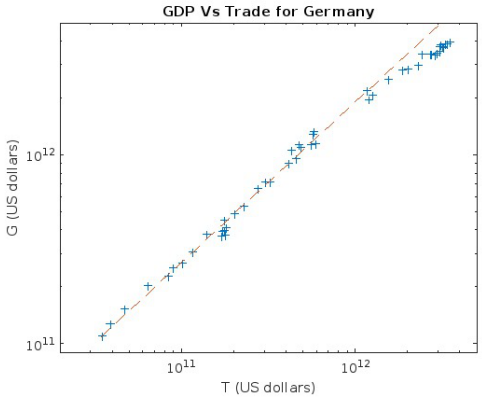

Fig. 10 The dotted line follows Eq. (7) with $\alpha = 0.85$ (see Table VI). The plot begins in 1970 and ends in 2020.

E. UK

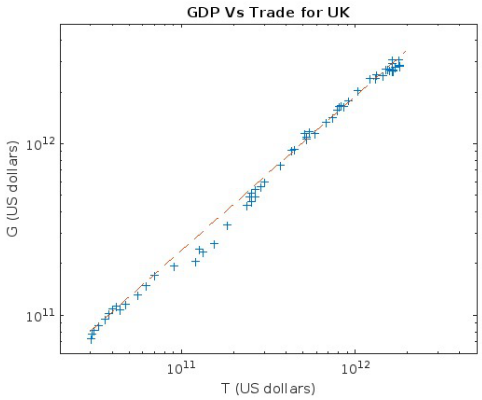

Fig. 11 The dotted line follows Eq. (7) with $\alpha = 0.90$ (see Table VI). The plot begins in 1960 and ends in 2020.

F. India

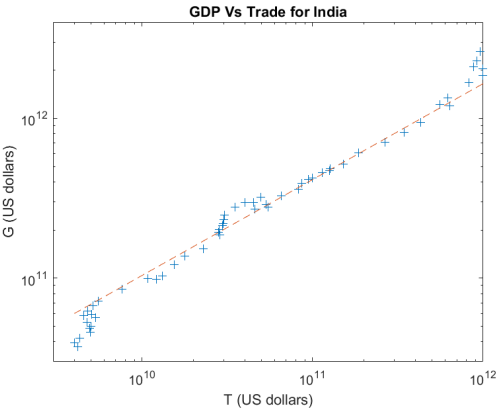

Fig. 12 The dotted line follows Eq. (7) with $\alpha = 0.60$ (see Table VI). The plot begins in 1960 and ends in 2020.

IV. GDP GROWTH COMPARISON GRAPHS

A. USA and China

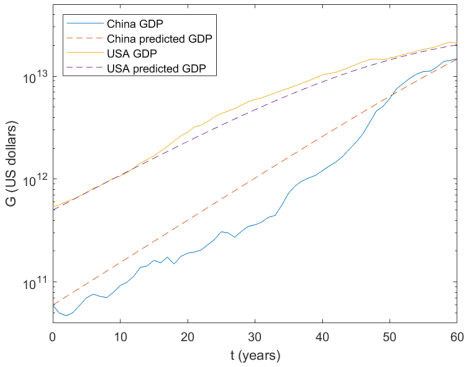

Fig. 13 The smooth dotted curves model the GDP growth of both countries, with the values of γ and k in Table IV. The World Bank data of the annual GDP is from 1960 (t = 0) to 2020.

B. Japan and India

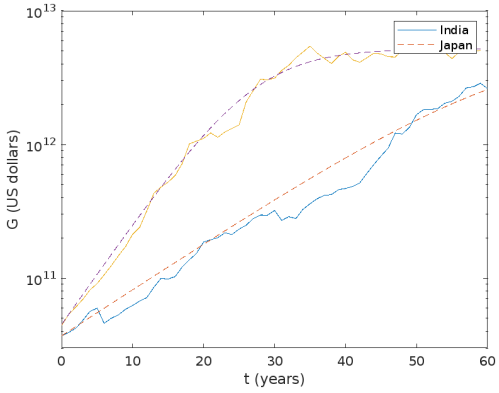

Fig. 14 The smooth dotted curves model the GDP growth of both countries, with the values of γ and k in Table IV. The World Bank data of the annual GDP starts from 1960 (t=0) and ends in the year 2019 for Japan, and the year 2020 for India.

C. Germany and UK

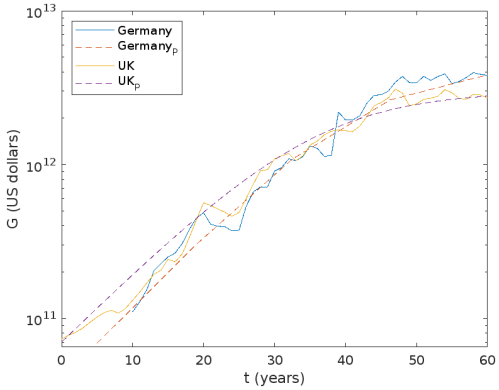

Fig. 15 The smooth dotted curves model the GDP growth of both countries, with the values of γ and k in Table IV. The World Bank data of the annual GDP of the UK start in 1960 (t = 0) and end in 2020. However, for Germany, the GDP data start 10 years later e.g. from 1970 (t = 10 years).

D. long term outcome of the GDP

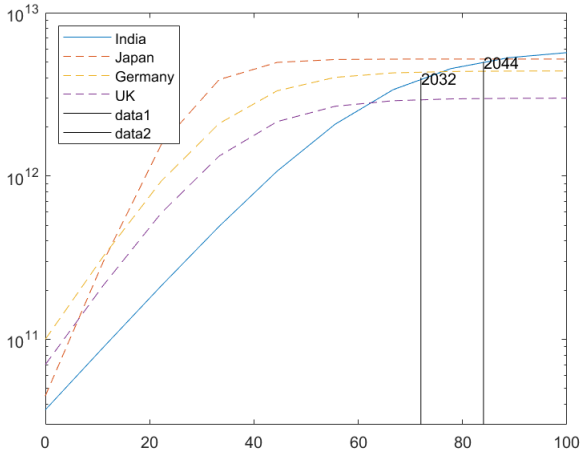

Fig. 16

The logistic functions for GDP of various countries were fitted using World Bank data up to 2020. Two intersections of these functions occurred before 2020: one in 1966, when Japan's GDP surpassed the UK's, and the other in 2000, when Germany's GDP surpassed the UK's. These intersections align with the actual GDP data. The remaining three intersections are predicted to occur after 2020. The first of these occurred in 2021 when India's GDP surpassed the UK's. Predictions suggest India's GDP will surpass Germany's in 2033 and Japan's in 2045.

V. STATISTICS

- Predicting years in which India's GDP will be $4 trillion and $5 trillion

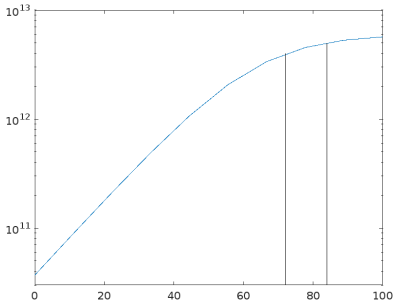

Fig. 15

From the above graph, it can be shown that India will be a $4 trillion economy by the year 2032 and a $5 trillion economy by the year 2044.

TABLE I: Statistical Analysis of GDP

Country	μ_G	σ_G
USA	0.0499	0.0872
China	-0.3629	0.2467
Japan	-0.0336	0.3125
Germany	0.0489	0.1744
UK	0.1101	0.1937
India	-0.1390	0.1777

TABLE II: Statistical Analysis of Trade

Country	μ_T	σ_T
USA	0.1239	0.2028
China	-0.3838	0.3292
Japan	0.2527	0.4016
Germany	0.0588	0.2356
UK	0.0036	0.1641
India	-0.1793	0.3483

VI. CONCLUSIONS

- According to the analysis, a mathematical equation can be used to model the economies of the US, Japan, and the UK. The statistics for Germany, India, and China are not entirely consistent. Long-term trends meet the equation, but some points don't match the equation's results.
- India's projected ascent to becoming the world's third-largest economy by surpassing Germany and Japan in 2035 and 2047 respectively reflects ongoing interest and varying predictions regarding India's economic trajectory. This is because while other nations' economies are stabilizing, India is continuously expanding.
- The model's findings showed a direct correlation between a nation's trade activity and GDP for almost every economy studied. It highlights how important trade is for promoting economic growth.

TABLE III: Countries, G-T Correlation, and Alpha Values

Country	c	α
USA	$10^{3.72}$	0.75
China	$10^{4.5}$	0.65
Japan	$10^{0.7}$	1.00
Germany	$10^{2.08}$	0.85
UK	$10^{1.48}$	0.90
India	$10^{5.005}$	0.60

TABLE IV: Countries and Parameters to Fit GDP (G)

Country	γ_1 (per annum)	k_G (trillion USD)	t_{nl} (years)
USA	0.080	30.0	50
China	0.095	80.0	76
Japan	0.175	5.2	26
Germany	0.110	4.4	32
UK	0.105	3.0	35
India	0.080	6.0	64

TABLE V: Countries and Parameters to Fit Trade (T)

Country	τ_1 (per annum)	k_T (trillion USD)	t_{nl} (years)
USA	0.099	10.0	53
China	0.130	10.0	58
Japan	0.135	2.0	39
Germany	0.130	3.9	36
UK	0.095	2.5	46
India	0.100	3.0	66

[1] Abhin Kakkad, Arnab K. Ray *Global dynamics of GDP and trade* (September 24, 2022).
[2] Arnab K. Ray *Logistic forecasting of GDP competitiveness* (August 15, 2023).