**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

Ans. Normal distribution at 45 and 8, let x = time take to transmission of customer’s car, customer requirement is time x<=50mins, P(x>50)=Service manager can not meet his commitment. P(x>50) = 1- P(x<=50), Z = (X- 45)/8.0, P(x<=50) = p(Z <= (50-45)/8.0=P(0<=0.625)=73.4%, P(X>50) = 100-73.4% = 0.2676.

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

Ans. False. P(X>44) = 1 – P(X<=44), P(X<=44) = P(Z<=(44-38)/6) = P(Z<=1)=84.1345%

P(X>44) = 100 – 84.1345 = 15.86%.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. True. P(X<=30) = 9.12%, So the no. of employees under age of 30 with P=0.0912\*400

=36.48.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans. If X1 and X2 are independent variables, Similarly if Z = aX + bY,

In given, 2X1~ N(2 u, 4 SD^2) & X1 +X2 ~ N( u + u,SD^2 + SD^2) ~ N( 2 u, 2SD^2)

2X1 – (X1 + X2) = N( 4u, 6 SD^2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

Ans. The P of getting value between a and b should be 0.99, the P outside = 1 – 0.99,

The P towards left from a = -0.005 (i.e. 0.01/2), P towards right from b = 0.005

The standard normal variable Z, Z = ( X –u ) / SD, For P(0.005) the Z value is -2.57,

Z \* SD + u = X, Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4, Z(+0.005)\*20+100 = 48.6.

The option D is correct.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
8. Specify the 5th percentile of profit (in Rupees) for the company
9. Which of the two divisions has a larger probability of making a loss in a given year?