



Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (West), Mumbai-400058-India
(Autonomous College Affiliated to University of Mumbai)

PRACTICAL NO. 3	DESIGN AND ANALYSIS OF ALGORITHMS
NAME	VEDANTI ANIL WADATKAR
UID	2021700072
BATCH	D4
PROBLEM STATEMENT	Let us consider two matrices X and Y. We want to calculate the resultant matrix Z by multiplying X and Y.
EXPLANATION	Following is simple Divide and Conquer method to multiply two square matrices. <ul style="list-style-type: none">● Divide matrices A and B in 4 sub-matrices of size N/2 x N/2 as shown in the below diagram.● Calculate following values recursively. $ae + bg$, $af + bh$, $ce + dg$ and $cf + dh$.
	$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$ <p style="text-align: center;">A B C</p> <p>A, B and C are square matrices of size N x N a, b, c and d are submatrices of A, of size N/2 x N/2 e, f, g and h are submatrices of B, of size N/2 x N/2</p>
TIME COMPLEXITY	Here, we assume that integer operations take O(1) time. There are three for loops in this algorithm and one is nested in other. Hence, the algorithm takes O(n ³) time to execute.

<p>ALGORITHM FOR NAIVE METHOD</p>	<p>Algorithm: Matrix-Multiplication (X, Y, Z)</p> <p>for i = 1 to p do</p> <p> for j = 1 to r do</p> <p> Z[i,j] := 0</p> <p> for k = 1 to q do</p> <p> Z[i,j] := Z[i,j] + X[i,k] × Y[k,j]</p>
<p><u>STRASSEN'S</u> <u>MATRIX</u> <u>MULTIPLICATION</u> <u>METHOD</u></p>	<div> <div> $\begin{matrix} p1 = a(f - h) \\ p3 = (c + d)e \\ p5 = (a + d)(e + h) \\ p7 = (a - c)(e + f) \end{matrix}$ </div> <div> $\begin{matrix} p2 = (a + b)h \\ p4 = d(g - e) \\ p6 = (b - d)(g + h) \end{matrix}$ </div> </div> <p>The A x B can be calculated using above seven multiplications. Following are values of four sub-matrices of result C</p> <div> <div> $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ <p>A</p> </div> <div> $\times \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ <p>B</p> </div> <div> $= \begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}$ <p>C</p> </div> </div> <p>A, B and C are square matrices of size N x N a, b, c and d are submatrices of A, of size N/2 x N/2 e, f, g and h are submatrices of B, of size N/2 x N/2 p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2</p> <p>$T(N) = 7T(N/2) + O(N^2)$</p> <p>From Master's Theorem, time complexity of above method is $O(N \log 7)$ which is approximately $O(N^{2.8074})$</p>