Number Systems Tips by CRACKU.IN

- Number Systems is the most important topic in the quantitative section.

- It is a very vast topic and a significant number of questions appear in CAT every year from this section.

- Learning simple tricks like divisibility rules, HCF and LCM, prime number and remainder theorems can help improve the score drastically.

- This document presents best short cuts which makes this topic easy and helps you perform better.

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Cracku Tip 1 - Number systems

HCF and LCM

- HCF * LCM of two numbers = Product of two numbers

- The greatest number dividing a, b and c leaving remainders of x_1, x_2 and x_3 is the HCF of $(a-x_1)$, $(b-x_2)$ and $(c-x_3)$.

- The greatest number dividing a, b and c (a<b<c) leaving the same remainder each time is the HCF of (c-b), (c-a), (b-a).

- If a number, N, is divisible by X and Y and HCF(X,Y) = 1. Then, N is divisible by X*Y

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Cracku Tip 2 - Number systems

Prime and Composite Numbers

▪ Prime numbers are numbers with only two factors, 1 and the number itself.

▪ Composite numbers are numbers with more than 2 factors. Examples are 4, 6, 8, 9.

▪ 0 and 1 are neither composite nor prime.

▪There are 25 prime numbers less than 100.

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Cracku Tip 3 - Number systems

Properties of Prime numbers

- To check if n is a prime number, list all prime factors less than or equal to √n. If none of the prime factors can divide n then n is a prime number.

- For any integer a and prime number p, $a^p - a$ is always divisible by p

- All prime numbers greater than 2 and 3 can be written in the form of 6k+1 or 6k-1

- If a and b are co-prime then $a^{(b-1)}$ mod b = 1.

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Cracku Tip 4 - Number systems

Theorems on Prime numbers

Fermat's Theorem:

Remainder of a^(p-1) when divided by p is 1, where p is a prime

Wilson's Theorem:

Remainder when (p-1)! is divided by p is (p-1) where p is a prime

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Cracku Tip 5 - Number systems

Theorems on Prime numbers

Remainder Theorem

- If a, b, c are the prime factors of N such that N= $a^p * b^q * c^r$

Then the number of numbers less than N and co-prime to N is

$\phi(N)$= N (1-1/a) (1 - 1/b) (1 - 1/c).

This function is known as the Euler's totient function.

Euler's theorem

- If M and N are co-prime to each other then remainder when $M^{\phi(N)}$ is divided by N is 1.

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Cracku Tip 6 - Number systems

- Highest power of n in m! is $[m/n]+[m/n^2]+[m/n^3]+\ldots$

Ex: Highest power of 7 in 100! = $[100/7] +[100/49]$ = 16

- To find the number of zeroes in n! find the highest power of 5 in n!

- If all possible permutations of n distinct digits are added together the sum = (n-1)! * (sum of n digits) * (11111... n times)

Cracku Tip 7 - Number systems

- If the number can be represented as N = $a^p * b^q * c^r$. . . then number of factors the is (p+1) * (q+1) * (r+1)

- Sum of the factors = $\frac{a^{p+1} - 1}{a - 1} * \frac{b^{q+1} - 1}{b - 1} * \frac{c^{r+1} - 1}{c - 1}$

- If the number of factors are odd then N is a perfect square.

- If there are n factors, then the number of pairs of factors would be n/2. If N is a perfect square then number of pairs (including the square root) is (n+1)/2

Cracku Tip 8 - Number systems

If the number can be expressed as $N = 2^p * a^q * b^r$ . . . where the power of 2 is p and a, b are prime numbers

- Then the number of even factors of $N = p(1+q)(1+r)$ . . .

- The number of odd factors of $N = (1+q)(1+r)$…

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Cracku Tip 9 - Number systems

Number of positive integral solutions of the equation $x^2 - y^2 = k$ is given by

- $\dfrac{\text{Total number of factors of k}}{2}$ (If k is odd but not a perfect square)

- $\dfrac{(\text{Total number of factors of k}) - 1}{2}$ (If k is odd and a perfect square)

- $\dfrac{\text{Total number of factors of } \frac{k}{4}}{2}$ (If k is even and not a perfect square)

- $\dfrac{(\text{Total number of factors of } \frac{k}{4}) - 1}{2}$ (If it is even and a perfect square)

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Cracku Tip 10 - Number systems

- Number of digits in a^b = [b $\log_m(a)$] + 1 ; where m is the base of the number and [.] denotes greatest integer function

- Even number which is not a multiple of 4, can never be expressed as a difference of 2 perfect squares.

Cracku Tip 11 - Number systems

- Sum of first n odd numbers is n^2

- Sum of first n even numbers is $n(n+1)$

- The product of the factors of N is given by $N^{a/2}$, where a is the number of factors

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Cracku Tip 12 - Number systems

- The last two digits of a^2, $(50 - a)^2$, $(50+a)^2$, $(100 - a)^2$ . . .. are same.

- If the number is written as 2^{10n}

When n is odd, the last 2 digits are 24.

When n is even, the last 2 digits are 76.

Cracku Tip 13 - Number systems

Divisibility

- Divisibility by 2: Last digit divisible by 2

- Divisibility by 4: Last two digits divisible by 4

- Divisibility by 8: Last three digits divisible by 8

- Divisibility by 16: Last four digit divisible by 16

Cracku Tip 14 - Number systems

Divisibility

- Divisibility by 3: Sum of digits divisible by 3

- Divisibility by 9: Sum of digits divisible by 9

- Divisibility by 27: Sum of blocks of 3 (taken right to left) divisible by 27

- Divisibility by 7: Remove the last digit, double it and subtract it from the truncated original number. Check if number is divisible by 7

- Divisibility by 11: (sum of odd digits) - (sum of even digits) should be 0 or divisible by 11

Cracku Tip 15 - Number systems

Divisibility properties

▪ For composite divisors, check if the number is divisible by the factors individually. Hence to check if a number is divisible by 6 it must be divisible by 2 and 3.

▪ The equation $a^n - b^n$ is always divisible by a-b. If n is even it is divisible by a+b. If n is odd it is not divisible by a+b.

▪ The equation $a^n + b^n$, is divisible by a+b if n is odd. If n is even it is not divisible by a+b.

Cracku Tip 16 - Number systems

- Converting from decimal to base b. Let R_1, R_2 . . . be the remainders left after repeatedly dividing the number with b. Hence, the number in base b is given by ... R_2R_1.

- Converting from base b to decimal - multiply each digit of the number with a power of b starting with the rightmost digit and b^0.

- A decimal number is divisible by b-1 only if the sum of the digits of the number when written in base b are divisible by b-1.

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Cracku Tip 17 - Number systems

Cyclicity

‣ To find the last digit of a^n find the cyclicity of a. For Ex. if a=2, we see that
‣ $2^1 = 2$
‣ $2^2 = 4$
‣ $2^3 = 8$
‣ $2^4 = 16$
‣ $2^5 = 32$

Hence, the last digit of 2 repeats after every 4th power. Hence cyclicity of 2 = 4. Hence if we have to find the last digit of a^n,
The steps are:
1. Find the cyclicity of a, say it is x
2. Find the remainder when n is divided by x, say remainder r
3. Find a^r if r>0 and a^x when r=0

Cracku Tip 18 - Number systems

- $(a + b)(a - b) = (a^2 - b^2)$

- $(a + b)^2 = (a^2 + b^2 + 2ab)$

- $(a - b)^2 = (a^2 + b^2 - 2ab)$

- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

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Cracku Tip 19 - Number systems

- $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

- $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

- $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

- When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

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