Summary, References, and Further Reading

The material in this chapter is representative of current techniques used for intensity transformations and spatial filtering. The topics were selected for their value as fundamental material that would serve as a foundation in an evolving field. Although most of the examples used in this chapter deal with image enhancement, the techniques presented are perfectly general, and you will encounter many of them again throughout the remaining chapters in contexts unrelated to enhancement.

The material in Section 3.1 is from Gonzalez [1986]. For additional reading on the material in Section 3.2, see Schowengerdt [2006] and Poyton [1996]. Early references on histogram processing (Section 3.3) are Gonzalez and Fittes [1977], and Woods and Gonzalez [1981]. Stark [2000] gives some interesting generalizations of histogram equalization for adaptive contrast enhancement.

For complementary reading on linear spatial filtering (Sections 3.4-3.7), see Jain [1989], Rosenfeld and Kak [1982], Schowengerdt [2006], Castleman [1996], and Umbaugh [2010]. For an interesting approach for generating Gaussian kernels with integer coefficients see Padfield [2011]. The book by Pitas and Venetsanopoulos [1990] is a good source for additional reading on median and other nonlinear spatial filters.

For details on the software aspects of many of the examples in this chapter, see Gonzalez, Woods, and Eddins [2009].

Problems

Solutions to the problems marked with an asterisk (*) are in the DIP4E Student Support Package (consult the book website: www.ImageProcessingPlace.com).

3.1 Give a single intensity transformation function for spreading the intensities of an image so the lowest intensity is 0 and the highest is $L-1$.

3.2 Do the following:

(a)* Give a continuous function for implementing the contrast stretching transformation in Fig. 3.2(a). In addition to m, your function must include a parameter, E, for controlling the slope of the function as it transitions from low to high intensity values. Your function should be normalized so that its minimum and maximum values are 0 and 1, respectively.

(b) Sketch a family of transformations as a function of parameter E, for a fixed value $m = L/2$, where L is the number of intensity levels in the image..

3.3 Do the following:

(a)* Propose a *set* of intensity-slicing transformation functions capable of producing all the individual bit planes of an 8-bit monochrome image. For example, applying to an image a transformation function with the property $T(r) = 0$ if r is 0 or even, and $T(r) = 1$ if r is odd, produces an image of the least significant bit plane (see Fig. 3.13). (*Hint:* Use an 8-bit truth table to determine the form of each transformation function.)

(b) How many intensity transformation functions would there be for 16-bit images?

(c) Is the basic approach in (a) limited to images in which the number of intensity levels is an integer power of 2, or is the method general for any number of *integer* intensity levels?

(d) If the method is general, how would it be different from your solution in (a)?

3.4 Do the following:

(a) Propose a method for extracting the bit planes of an image based on converting the value of its pixels to binary.

(b) Find all the bit planes of the following 4-bit image:

$$\begin{matrix} 0 & 1 & 8 & 6 \\ 2 & 2 & 1 & 1 \\ 1 & 15 & 14 & 12 \\ 3 & 6 & 9 & 10 \end{matrix}$$

3.5 In general:

(a)* What effect would setting to zero the lower-order bit planes have on the histogram of an image?

(b) What would be the effect on the histogram if we set to zero the higher-order bit planes instead?

3.6 Explain why the discrete histogram equalization technique does not yield a flat histogram in general.

3.7 Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass.

3.8 Assuming continuous values, show by an example that it is possible to have a case in which the transformation function given in Eq. (3-11) satisfies conditions (a) and (b) discussed in Section 3.3, but its inverse may fail condition (a′).

3.9 Do the following:

(a) Show that the discrete transformation function given in Eq. (3-15) for histogram equalization satisfies conditions (a) and (b) stated at the beginning of Section 3.3.

(b)* Show that the inverse discrete transformation in Eq. (3-16) satisfies conditions (a′) and (b) in Section 3.3 *only if* none of the intensity levels r_k, $k = 0,1,2,\ldots,L-1$, are missing in the original image.

3.10 Two images, $f(x,y)$ and $g(x,y)$ have unnormalized histograms h_f and h_g. Give the conditions (on the values of the pixels in f and g) under which you can determine the histograms of images formed as follows:

(a)* $f(x,y) + g(x,y)$

(b) $f(x,y) - g(x,y)$

(c) $f(x,y) \times g(x,y)$

(d) $f(x,y) \div g(x,y)$

Show how the histograms would be formed in each case. The arithmetic operations are elementwise operations, as defined in Section 2.6.

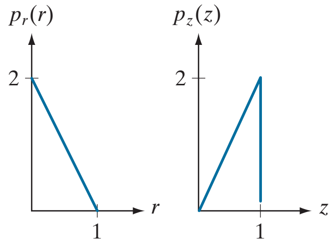
3.11 Assume continuous intensity values, and suppose that the intensity values of an image have the PDF $p_r(r) = 2r/(L-1)^2$ for $0 \le r \le L-1$, and $p_r(r) = 0$ for other values of r.

(a)* Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(b)* Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity PDF is $p_z(z) = 3z^2/(L-1)^3$ for $0 \le z \le L-1$ and $p_z(z) = 0$ for other values of z.

(c) Express the transformation function from (b) directly in terms of r, the intensities of the input image.

3.12 An image with intensities in the range [0,1] has the PDF, $p_r(r)$, shown in the following figure. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown in the figure. Assume continuous quantities, and find the transformation (expressed in terms of r and z) that will accomplish this.



3.13* In Fig. 3.25(b), the transformation function labeled (2) [$G^{-1}(s_k)$ from Eq. (3-23)] is the mirror image of (1) [$G(z_q)$ in Eq. (3-21)] about a line joining the two end points. Does this property always hold for these two transformation functions? Explain.

3.14* The local histogram processing method discussed in Section 3.3 requires that a histogram be computed at each neighborhood location. Propose a method for updating the histogram from one neighborhood to the next, rather than computing a new histogram each time.

3.15 What is the behavior of Eq. (3-35) when $a = b = 0$? Explain.

3.16 You are given a computer chip that is capable of performing linear filtering in real time, but you are not told whether the chip performs correlation or convolution. Give the details of a test you would perform to determine which of the two operations the chip performs.

3.17* We mentioned in Section 3.4 that to perform convolution we rotate the kernel by 180°. The rotation is "built" into Eq. (3-35). Figure 3.28 corresponds to correlation. Draw the part of the figure enclosed by the large ellipse, but with w rotated 180°. Expand Eq. (3-35) for a general 3×3 kernel and show that the result of your expansion corresponds to your figure. This shows graphically that convolution and correlation differ by the rotation of the kernel.

3.18 You are given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a)* Give a sketch of the area encircled by the large ellipse in Fig. 3.28 when the kernel is centered at point (2,3) (2nd row, 3rd col) of the image shown above. Show specific values of w and f.

(b)* Compute the convolution $w \star f$ using the *minimum* zero padding needed. Show the details of your computations when the kernel is centered on point (2,3) of f; and then show the final full convolution result.

(c) Repeat (b), but for correlation, $w \,☆\, f$.

3.19* Prove the validity of Eqs. (3-36) and (3-37).

3.20 The kernel, w, in Problem 3.18 is separable.

(a)* By inspection, find two kernels, w_1 and w_2 so that $w = w_1 \star w_2$.

(b) Using the image in Problem 3.18, compute $w_1 \star f$ using the *minimum* zero padding (see Fig. 3.30). Show the details of your computation when the kernel is centered at point (2,3) (2nd row, 3rd col) of f and then show the full convolution.

(c) Compute the convolution of w_2 with the result from (b). Show the details of your computation when the kernel is centered at point (3,3) of the result from (b), and then show the full convolution. Compare with the result in Problem 3.18(b).

3.21 Given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Give the convolution of the two.

(b) Does your result have a bias?

3.22 Answer the following:

(a)* If $\mathbf{v} = [1 \;\; 2 \;\; 1]^T$ and $\mathbf{w}^T = [2 \;\; 1 \;\; 1 \;\; 3]$, is the kernel formed by $\mathbf{v}\mathbf{w}^T$ separable?

(b) The following kernel is separable. Find w_1 and w_2 such that $w = w_1 \star w_2$.

$$w = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{bmatrix}$$

3.23 Do the following:

(a)* Show that the Gaussian kernel, $G(s,t)$, in Eq. (3-45) is separable. (*Hint:* Read the first paragraph in the discussion of separable filter kernels in Section 3.4.)

(b) Because G is separable and circularly symmetric, it can be expressed in the form $G = \mathbf{v}\mathbf{v}^T$. Assume that the kernel form in Eq. (3-46) is used, and that the function is sampled to yield an $m \times m$ kernel. What is $\mathbf{v}$ in this case?

3.24* Show that the product of a column vector with a row vector is equivalent to the 2-D convolution of the two vectors. The vectors do not have to be of the same length. You may use a graphical approach (as in Fig. 3.30) to support the explanation of your proof.

3.25 Given K, 1-D Gaussian kernels, $g_1, g_2, \ldots, g_K$, with arbitrary means and standard deviations:

(a)* Determine what the entries in the third column of Table 3.6 would be for the product $g_1 \times g_2 \times \cdots \times g_K$.

(b) What would the fourth column look like for the convolution $g_1 \star g_2 \star \cdots \star g_K$?

(*Hint:* It is easier to work with the variance; the standard deviation is just the square root of your result.)

3.26 The two images shown in the following figure are quite different, but their histograms are the same. Suppose that each image is blurred using a 3×3 box kernel.

(a)* Would the histograms of the blurred images still be equal? Explain.

(b) If your answer is no, either sketch the two histograms or give two tables detailing the histogram components.

3.27 An image is filtered four times using a Gaussian kernel of size 3×3 with a standard deviation of 1.0. Because of the associative property of convolution, we know that equivalent results can be obtained using a single Gaussian kernel formed by convolving the individual kernels.

(a)* What is the size of the single Gaussian kernel?

(b) What is its standard deviation?

3.28 An image is filtered with three Gaussian lowpass kernels of sizes 3×3, 5×5, and 7×7, and standard deviations 1.5, 2, and 4, respectively. A composite filter, w, is formed as the convolution of these three filters.

(a)* Is the resulting filter Gaussian? Explain.

(b) What is its standard deviation?

(c) What is its size?

3.29* Discuss the limiting effect of repeatedly filtering an image with a 3×3 lowpass filter kernel. You may ignore border effects.

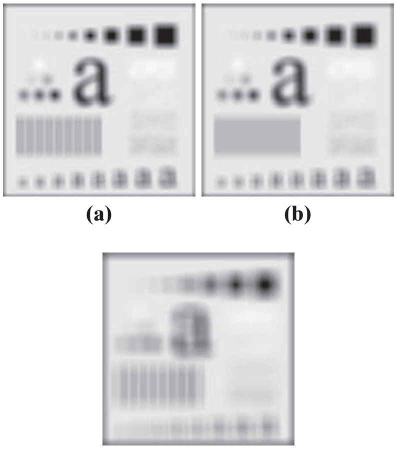
3.30 In Fig. 3.42(b) the corners of the estimated shading pattern appear darker or lighter than their surrounding areas. Explain the reason for this.

3.31* An image is filtered with a kernel whose coefficients sum to 1. Show that the sum of the pixel values in the original and filtered images is the same.

3.32 An image is filtered with a kernel whose coefficients sum to 0. Show that the sum of the pixel values in the filtered image also is 0.

3.33 A single point of light can be modeled by a digital image consisting of all 0's, with a 1 in the location of the point of light. If you view a single point of light through a defocused lens, it will appear as a fuzzy blob whose size depends on the amount by which the lens is defocused. We mentioned in Section 3.5 that filtering an image with a box kernel is a poor model for a defocused lens, and that a better approximation is obtained with a Gaussian kernel. Using the single-point-of-light analogy, explain why this is so.

3.34 In the original image used to generate the three blurred images shown, the vertical bars are 5 pixels wide, 100 pixels high, and their separation is 20 pixels. The image was blurred using square box kernels of sizes 23, 25, and 45 elements on the side, respectively. The vertical bars on the left, lower part of (a) and (c) are blurred, but a clear separation exists between them.



(a) **(b)**

(c)

However, the bars have merged in image (b), despite the fact that the kernel used to generate this image is much smaller than the kernel that produced image (c). Explain the reason for this.

3.35 Consider an application such as in Fig. 3.41, in which it is desired to eliminate objects smaller than those enclosed by a square of size $q \times q$ pixels. Suppose that we want to reduce the average intensity of those objects to one-tenth of their original average value. In this way, their intensity will be closer to the intensity of the background and they can be eliminated by thresholding. Give the (odd) size of the smallest box kernel that will yield the desired reduction in average intensity in only one pass of the kernel over the image.

3.36 With reference to order-statistic filters (see Section 3.5):

(a)* We mentioned that isolated clusters of dark or light (with respect to the background) pixels whose area is less than one-half the area of a median filter are forced to the median value of the neighbors by the filter. Assume a filter of size $n \times n$ (n odd) and explain why this is so.

(b) Consider an image having various sets of pixel clusters. Assume that all points in a cluster are lighter or darker than the background (but not both simultaneously in the same cluster), and that the area of each cluster is less than or equal to $n^2/2$. In terms of n, under what condition would one or more of these clusters cease to be isolated in the sense described in part (a)?

3.37 Do the following:

(a)* Develop a procedure for computing the median of an $n \times n$ neighborhood.

(b) Propose a technique for updating the median as the center of the neighborhood is moved from pixel to pixel.

3.38 In a given application, a smoothing kernel is applied to input images to reduce noise, then a Laplacian kernel is applied to enhance fine details. Would the result be the same if the order of these operations is reversed?

3.39* Show that the Laplacian defined in Eq. (3-50) is isotropic (invariant to rotation). Assume continuous quantities. From Table 2.3, coordinate rotation by an angle θ is given by

$$x' = x\cos\theta - y\sin\theta \quad \text{and} \quad y' = x\sin\theta + y\cos\theta$$

where (x, y) and (x', y') are the unrotated and rotated coordinates, respectively.

3.40* You saw in Fig. 3.46 that the Laplacian with a −8 in the center yields sharper results than the one with a −4 in the center. Explain the reason why.

3.41* Give a 3×3 kernel for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using a box filter of size 3×3.

3.42 Show that subtracting the Laplacian from an image gives a result that is proportional to the unsharp mask in Eq. (3-55). Use the definition for the Laplacian given in Eq. (3-53).

3.43 Do the following:

(a)* Show that the magnitude of the gradient given in Eq. (3-58) is an isotropic operation (see the statement of Problem 3.39).

(b) Show that the isotropic property is lost in general if the gradient is computed using Eq. (3-59).

3.44 Are any of the following highpass (sharpening) kernels separable? For those that are, find vectors $\mathbf{v}$ and $\mathbf{w}$ such that $\mathbf{v}\mathbf{w}^T$ equals the kernel(s).

(a) The Laplacian kernels in Figs. 3.45(a) and (b).

(b) The Roberts cross-gradient kernels shown in Figs. 3.50(b) and (c).

(c)* The Sobel kernels in Figs. 3.50(d) and (e).

3.45 In a character recognition application, text pages are reduced to binary using a thresholding transformation function of the form in Fig. 3.2(b). This is followed by a procedure that thins the characters until they become strings of binary 1's on a background of 0's. Due to noise, binarization and thinning result in broken strings of characters with gaps ranging from 1 to 3 pixels. One way to "repair" the gaps is to run a smoothing kernel over the binary image to blur it, and thus create bridges of nonzero pixels between gaps.

(a)* Give the (odd) size of the smallest box kernel capable of performing this task.

(b) After bridging the gaps, the image is thresholded to convert it back to binary form. For your answer in (a), what is the minimum value of the threshold required to accomplish this, without causing the segments to break up again?

3.46 A manufacturing company purchased an imaging system whose function is to either smooth or sharpen images. The results of using the system on the manufacturing floor have been poor, and the plant manager suspects that the system is not smoothing and sharpening images the way it should. You are hired as a consultant to determine if the system is performing these functions properly. How would you determine if the system is working correctly? (*Hint:* Study the statements of Problems 3.31 and 3.32).

3.47 A CCD TV camera is used to perform a long-term study by observing the same area 24 hours a day, for 30 days. Digital images are captured and transmitted to a central location every 5 minutes. The illumination of the scene changes from natural daylight to artificial lighting. At no time is the scene without illumination, so it is always possible to obtain an acceptable image. Because the range of illumination is such that it is always in the linear operating range of the camera, it is decided not to employ any compensating mechanisms on the camera itself. Rather, it is decided to use image processing techniques to post-process, and thus normalize, the images to the equivalent of constant illumination. Propose a method to do this. You are at liberty to use any method you wish, but state clearly all the assumptions you made in arriving at your design.