

Theoretical Computer Science: Tutorial I

3) Design DFA for the following language over alphabet $\Sigma = \{a, b\}$

a) String that contains 3 consecutive b's

Sol:

i) DFA is Deterministic Finite Automata where machine (M) is defined as 5 tuple where
 $M = (Q, \Sigma, \delta, q_0, F)$
 where

$Q \rightarrow$ Finite set of states

$\Sigma \rightarrow$ Input alphabet

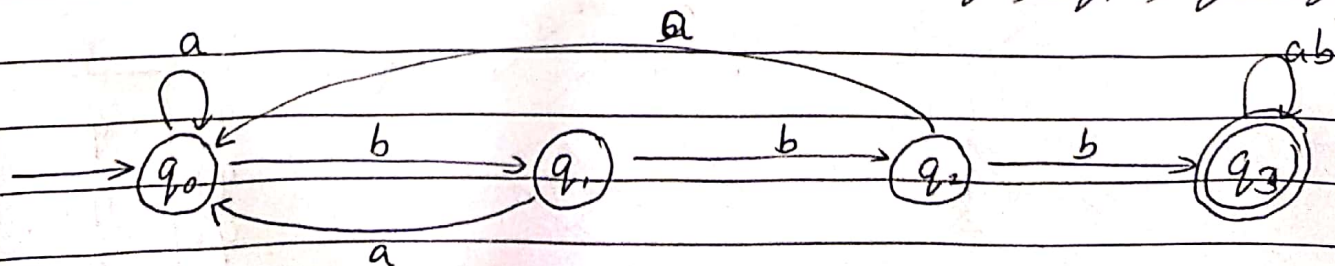
$\delta \rightarrow$ State transition function

$q_0 \rightarrow$ Initial state

$F \rightarrow$ set of Final State

ii) $q_0 = q_0$
 $\Sigma = \{a, b\} \quad \therefore Q = \{q_0, q_1, q_2, q_3\}$

iii) The string should contain 3 consecutive b's
 so there are 4 states i.e. q_0, q_1, q_2, q_3



iv) Transition Table:

q	Σ	a	b
q_0		q_0	q_1
q_1		q_0	q_2
q_2		q_0	q_3
q_3		q_3	q_3

$$\mathcal{Q} = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{ \cancel{a}, a, b \}$$

S: [Refer Transition Table]

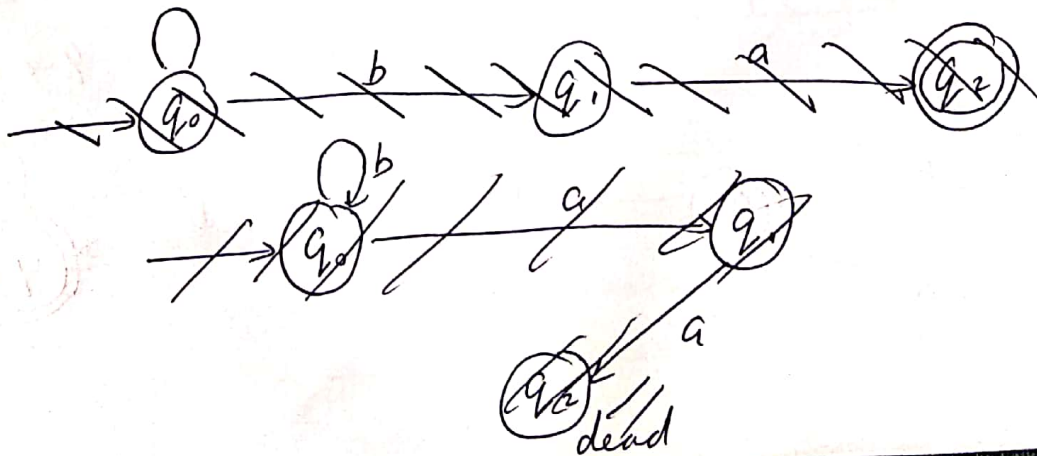
$$F = \{q_3\}$$

b) String in which occurrence of every 'a' is preceded by the occurrence of b.

Let $Q = \{q_0, q_1, q_2, q_3\}$

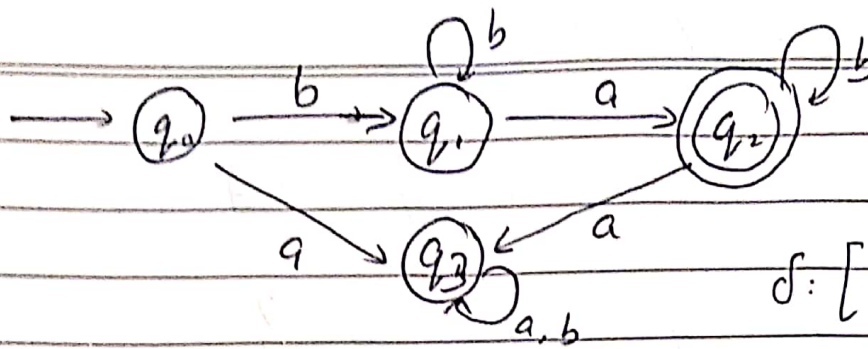
$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$



Transition Table -

$Q \backslash \Sigma$	a	b
q_0	q_3	q_1
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_3	q_3



$$F = \{q_2\}$$

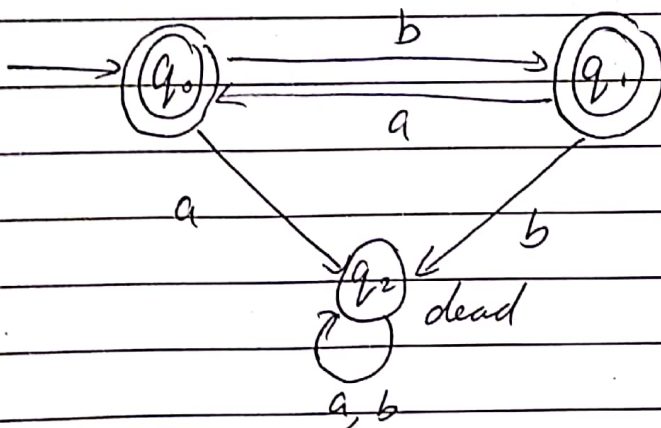
δ : [Refer Transition Table]

c) String which contain 'a' at every even position in the string.

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$



$$F = \{q_0, q_1\}$$

$Q \backslash \Sigma$	a	b
q_0	q_2	q_1
q_1	q_0	q_2
q_2	q_2	q_2

δ : [Refer Transition Table]

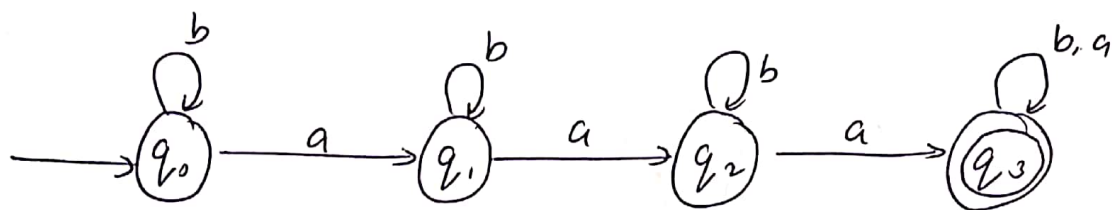
d) String with atleast 3 a's

Let $M = \{Q, \Sigma, \delta, q_0, F\}$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$



$$F = \{q_3\}$$

Q \ Σ	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_3	q_3

$$\delta(q_0, a) = q_1$$

$$\delta(q_1, a) = q_2$$

$$\delta(q_2, a) = q_3$$

$$\delta(q_3, a) = q_3$$

$$\delta(q_0, b) = q_0$$

$$\delta(q_1, b) = q_1$$

$$\delta(q_2, b) = q_2$$

$$\delta(q_3, b) = q_3$$

e) Even numbers of 'a' & odd numbers of 'b'

Let $M = (Q, \Sigma, \delta, q_0, F)$

There are four states in this problem i.e. Let

$q_0 \rightarrow$ Even 'a's and Even 'b's

$q_1 \rightarrow$ Odd 'a's and Even 'b's

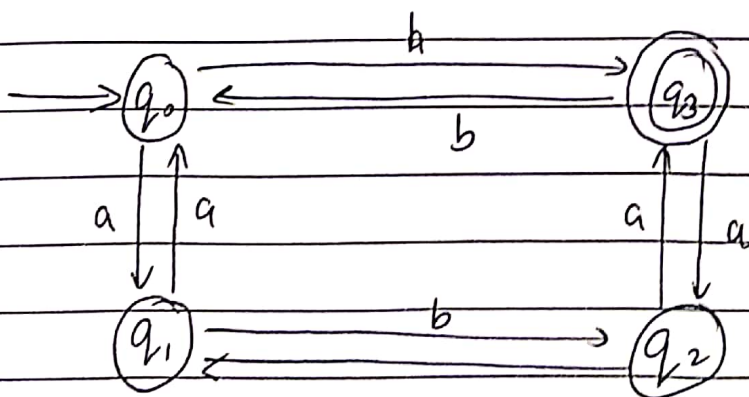
$q_2 \rightarrow$ Odd 'a's and Odd 'b's

$q_3 \rightarrow$ Even 'a's and Odd 'b's

$$\therefore Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = a, b$$

$$F = \{q_3\}$$



Q \ Σ	a		b
	a	b	
q_0	q_1	q_3	
q_1	q_0	q_2	
q_2	q_3	q_1	
q_3	q_2	q_0	

$$\therefore \delta(q_0, a) = q_1$$

$$\delta(q_0, b) = q_3$$

$$\delta(q_1, a) = q_0$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_3$$

$$\delta(q_2, b) = q_1$$

$$\delta(q_3, a) = q_2$$

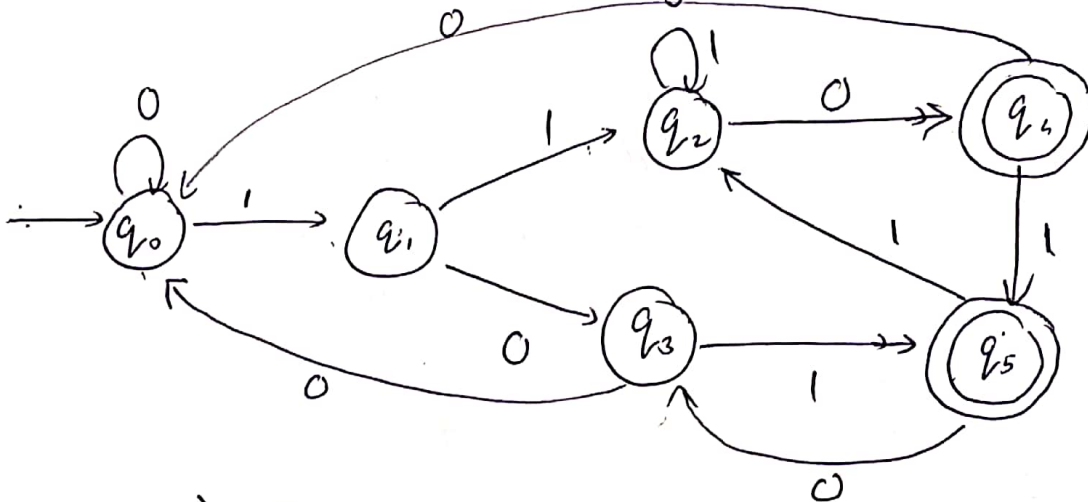
$$\delta(q_3, b) = q_0$$

f) string that ends with either "110" or "101"

Let $M = (Q, \Sigma, \delta, q_0, F)$

Let $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$q_0 = q_0$
 $\Sigma = \{1, 0\}$



Q \ Σ	0	1
q_0	q_0	q_1
q_1	q_3	q_2
q_2	q_4	q_2
q_3	q_0	q_5
q_4	q_0	q_5
q_5	q_3	q_2

$F = \{q_4, q_5\}$

$\delta \Rightarrow$ [As per the transition Diagram]

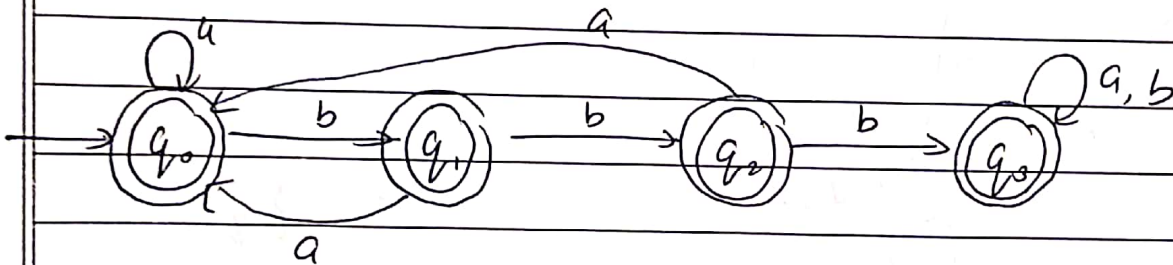
g) String does not contain any occurrence of three consecutive b's

Sol: Let $M = (Q, \Sigma, \delta, q_0, F)$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$q_0 = q_0$$

$$\Sigma = \{a, b\}$$



$Q \backslash \Sigma$	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

$$F = \{q_0, q_1, q_2\}$$

$$\delta(q_0, a) = q_0$$

$$\delta(q_0, b) = q_1$$

$$\delta(q_1, a) = q_0$$

$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_0$$

$$\delta(q_2, b) = q_3$$

$$\delta(q_3, a) = q_3$$

$$\delta(q_3, b) = q_3$$

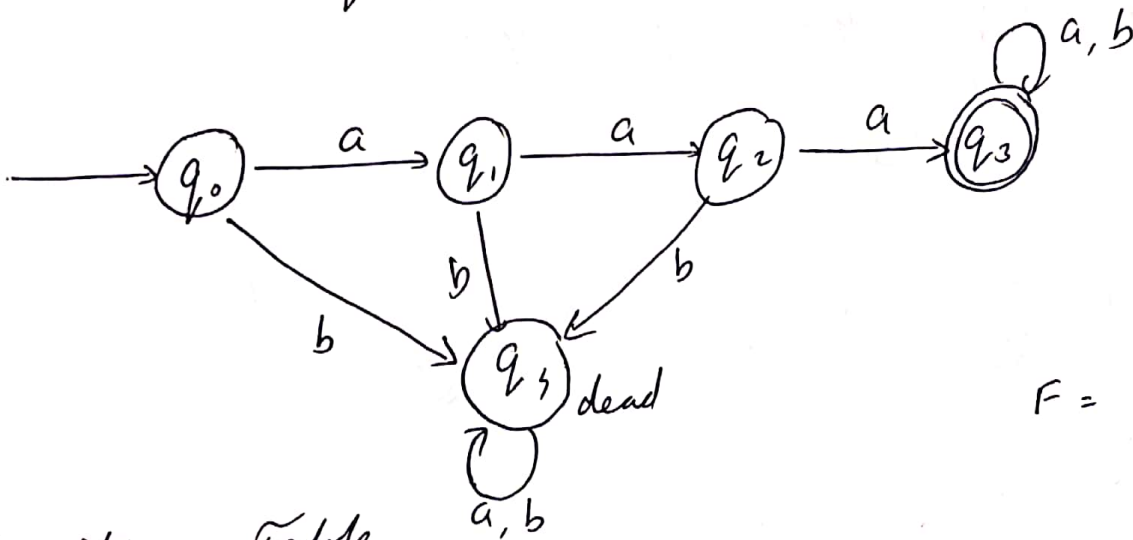
h) String starts with 3 consecutive 'a's

Sol: Let $M = (Q, \Sigma, \delta, q_0, A)$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$



$$F = \{q_3\}$$

Transition Table

Q	Σ		
		a	b
q_0		q_1	q_4
q_1		q_2	q_4
q_2		q_3	q_4
q_3		q_3	q_3
q_4		q_4	q_4

δ : [Refer Transition Table]

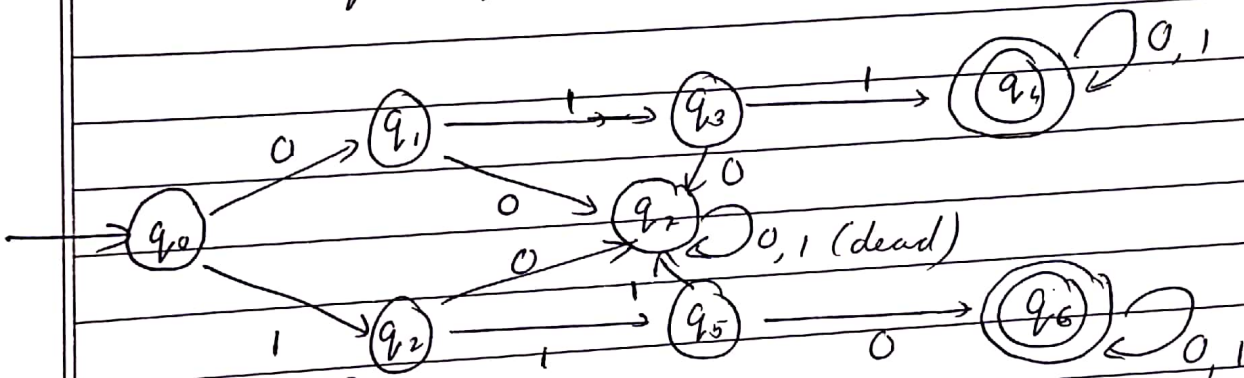
i) String starts with either '011' or '110' over $\Sigma = \{0, 1\}$

Sol: Let $M = (Q, \Sigma, \delta, q_0, F)$

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$

$$\Sigma = \{a, b\}$$

$$q_0 = q_0$$



Transition Table

$Q \backslash \Sigma$	0	1
q_0	q_1	q_2
q_1	q_7	q_3
q_2	q_7	q_5
q_3	q_7	q_4
q_4	q_4	q_4
q_5	q_6	q_7
q_6	q_6	q_6
q_7	q_7	q_7

$$F = \{q_4, q_6\}$$

δ : [Refer the Transition table]