

✓ Tree-Insert (T, p)

{

$q = T \rightarrow \text{root};$

$\text{prev} = \text{NULL};$ // parent of q .

while ($q \neq \text{NULL}$)

{

$\text{prev} = q;$

if ($p \rightarrow \text{key} < q \rightarrow \text{key}$)

$q = q \rightarrow \text{left};$

else

$q = q \rightarrow \text{right};$

}

$p \rightarrow \text{parent} = \text{prev};$

if $\text{prev} == \text{NULL}$

$T \rightarrow \text{root} = p;$

else if $p \rightarrow \text{key} < \text{prev} \rightarrow \text{key}$

$\text{prev} \rightarrow \text{left} = p;$

else

$\text{prev} \rightarrow \text{right} = p;$



RB-Tree-fixup (T, p);

// new node added
recoloring or
restructuring of T

}

RB-Tree-fixup (T, x) ← node to be added ②
 node to be adjusted
 while ($x \neq \text{Root}$ and $x \rightarrow \text{parent} \rightarrow \text{color} == \text{Red}$)

{

$p = x \rightarrow \text{parent};$

$gp = p \rightarrow \text{parent};$

if $gp \rightarrow \text{left} == p$ then

RB-~~left~~-correct (T, x, p, gp);

else

RB-~~right~~-correct (T, x, p, gp);

}

$x \rightarrow \text{color} = \text{black};$

}

RB-~~left~~-correct (T, x, p, gp)

{

$u = gp \rightarrow \text{right};$

if $u \rightarrow \text{color} == \text{red}$ then

{

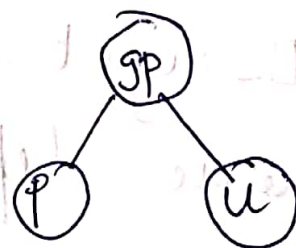
$p \rightarrow \text{color} = \text{black};$

$u \rightarrow \text{color} = \text{black};$

$gp \rightarrow \text{color} = \text{red};$

$u = gp;$

else // if $u \rightarrow \text{color} == \text{black}$



case 1

uncle is red

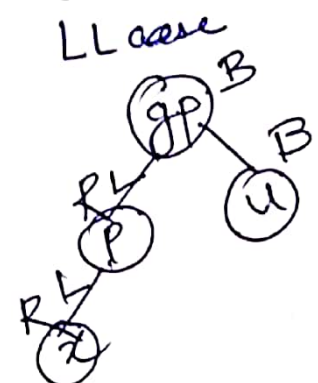
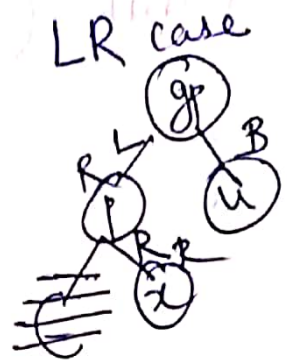
if (p → right == x) // x is right son of p.

~~p → right~~
rotateleft(p);
x = ~~p~~ → left

}
else

{ rotate right(gp);
gp → color = red;
p → color = black;
}

}



RB-right-correct is symmetric case where left \Rightarrow right.

Let $z \rightarrow$ search a node containing
desired data.

if ($z \rightarrow \text{left} == \text{null}$ or $z \rightarrow \text{right} == \text{null}$)
// case 1 or case 2 of
BST delete
 $y = z;$

else

{
 $y = \text{successor}(z);$
 $z \rightarrow \text{data} = y \rightarrow \text{data};$

}
if ($y \rightarrow \text{left} == \text{null}$) // In case 1 x is NULL
 $x = y \rightarrow \text{right};$

else $x = y \rightarrow \text{left};$

if ($x \neq \text{null}$) // if not case 1
 $x \rightarrow \text{parent} = y \rightarrow \text{parent};$

if ($y \rightarrow \text{parent} == \text{null}$)
 $\text{root} = x;$

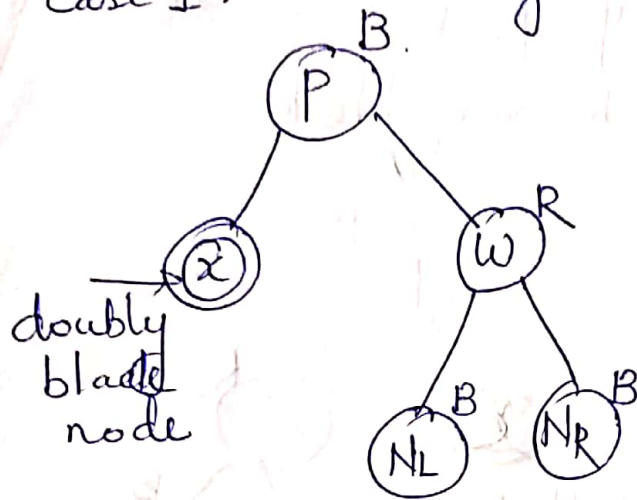
else $y \rightarrow \text{parent} \rightarrow \text{left} = x;$

else $y \rightarrow \text{parent} \rightarrow \text{right} = x;$

if ($y \rightarrow \text{color} == \text{black}$)
 $\text{RB-Delete-fixup}(T, x);$

$\text{free}(y);$

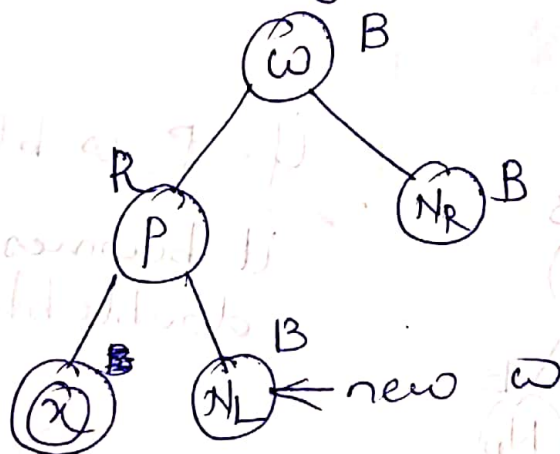
x is nonroot doubly black node
 if it is a left child \rightarrow 4 cases
 if it is a right child \rightarrow 4 cases (symmetric)
 case 1: x sibling is R.



Soln

- 1) swap colors of pkw
- 2) perform left rotation at parent

so resulting tree

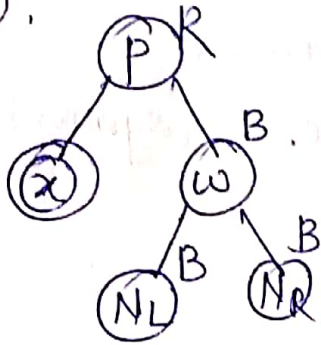


case 1 \rightarrow doesn't give answer
 its gets converted to
 case 2, case 3, case 4
 where sibling w is of black
 color.

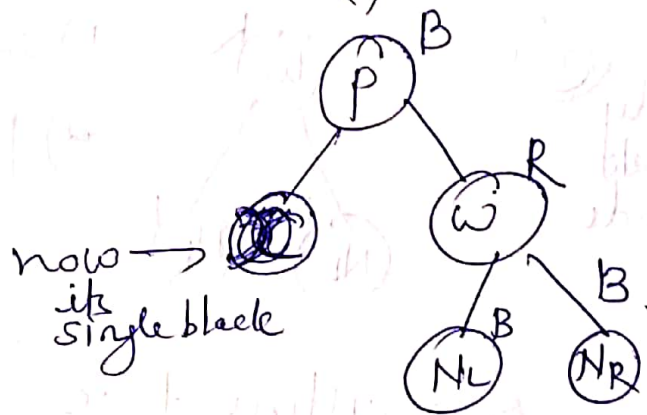
These cases are distinguished using
 by color's of w 's children.

case 2 : w black and NL & NR black

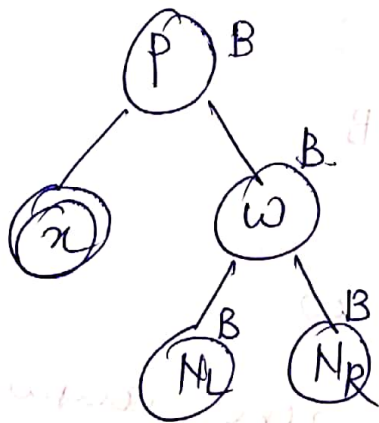
(a)



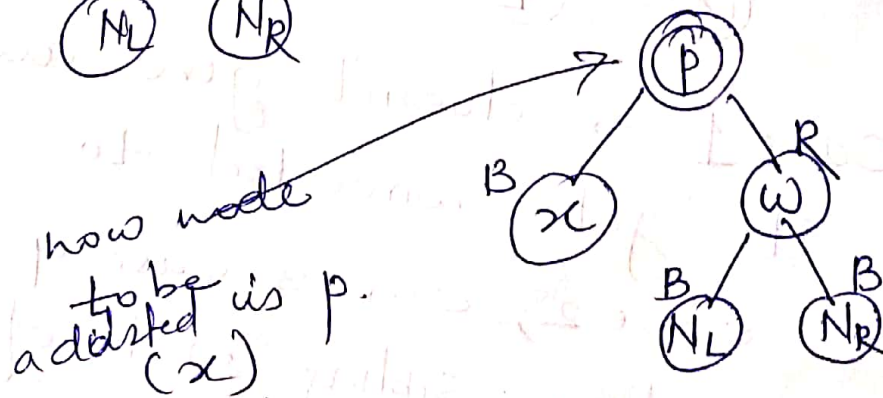
Give blackness of x & w to P.
if P is Red, its becomes black.



(b)

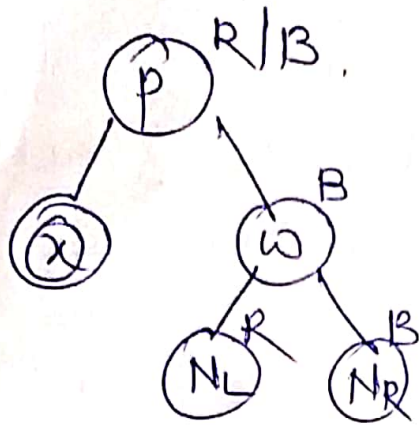


if P is black, it becomes double black.



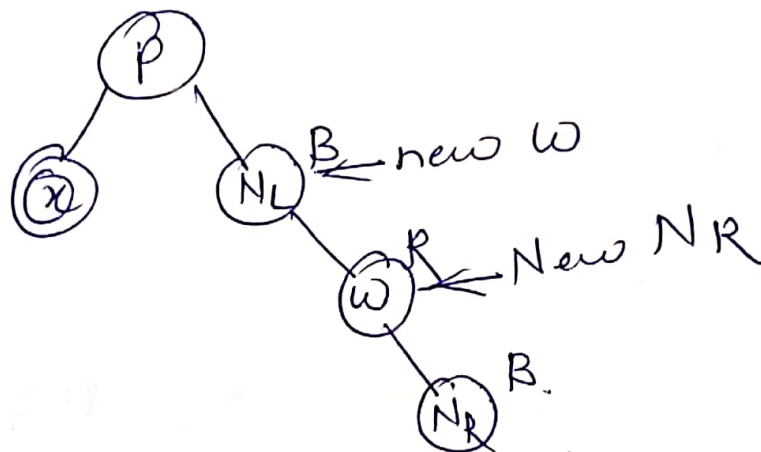
Violation of property moves one level up.

case 3: to black, $N_L \rightarrow R$ & $N_R \rightarrow B$.



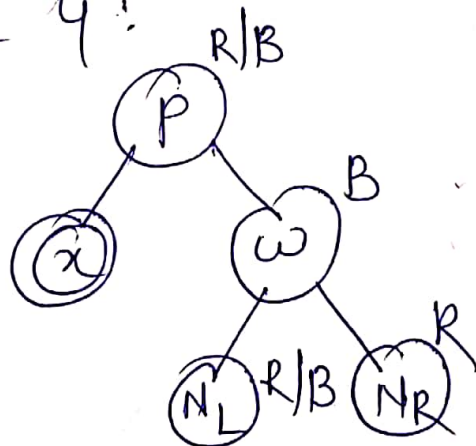
1) swap colors of N_L & w .

2) Right rotate at w

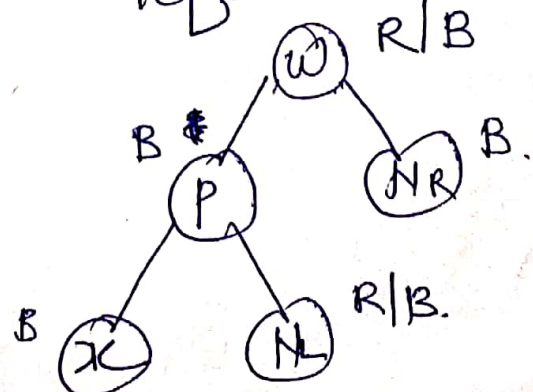


This is transformed to case 4.
where w is black, N_R is Red,
 $N_L \rightarrow R/B$.

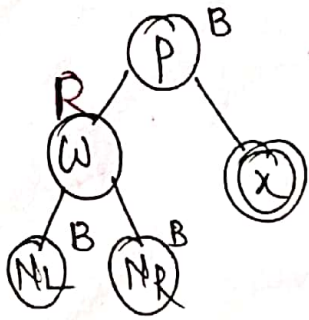
- case 4:



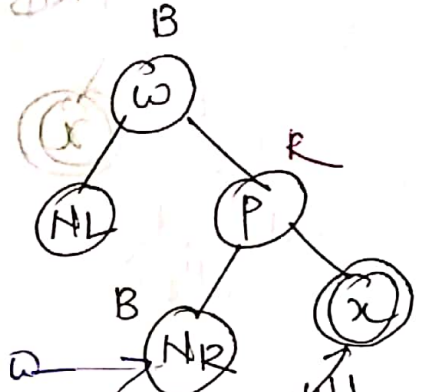
$N_R \rightarrow \text{color} = w \rightarrow \text{color}$
 $w \rightarrow \text{color} = p \rightarrow \text{color}$
 $p \rightarrow \text{color} = \text{black}$
 left rotate at p



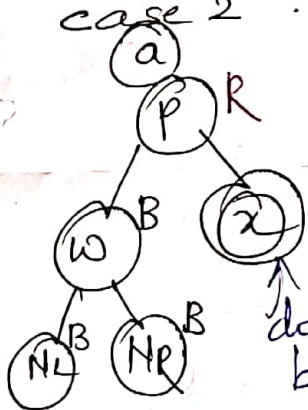
x is nonroot doubly black node & it is a right child.
 case 1': x sibling with R



- 1) swap colors of p & w
- 2) perform right rotation at parent.

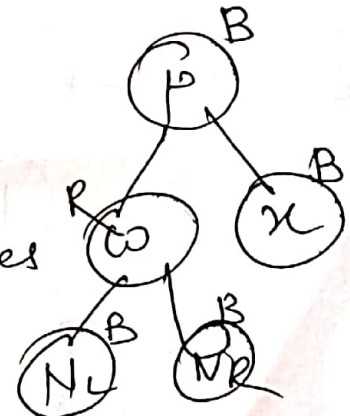


case 1' doesn't give answer, its get converted to case 2', case 3' or case 4' where sibling w is of black color. These cases are distinguished using the colors of w 's children.
 case 2': w black, NL & NR black

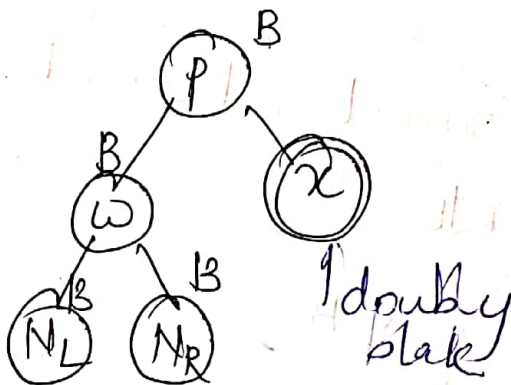


Give blackness of x & w to p .

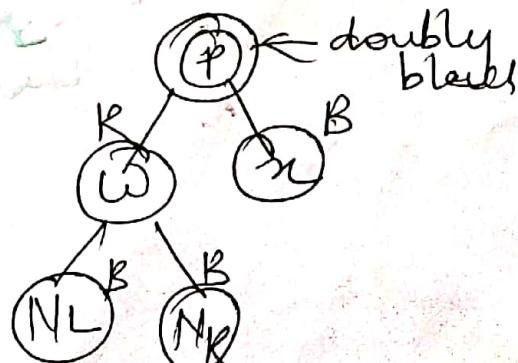
If p is red, it becomes black, stop.



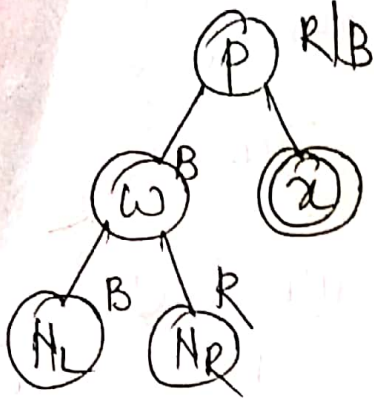
(B)



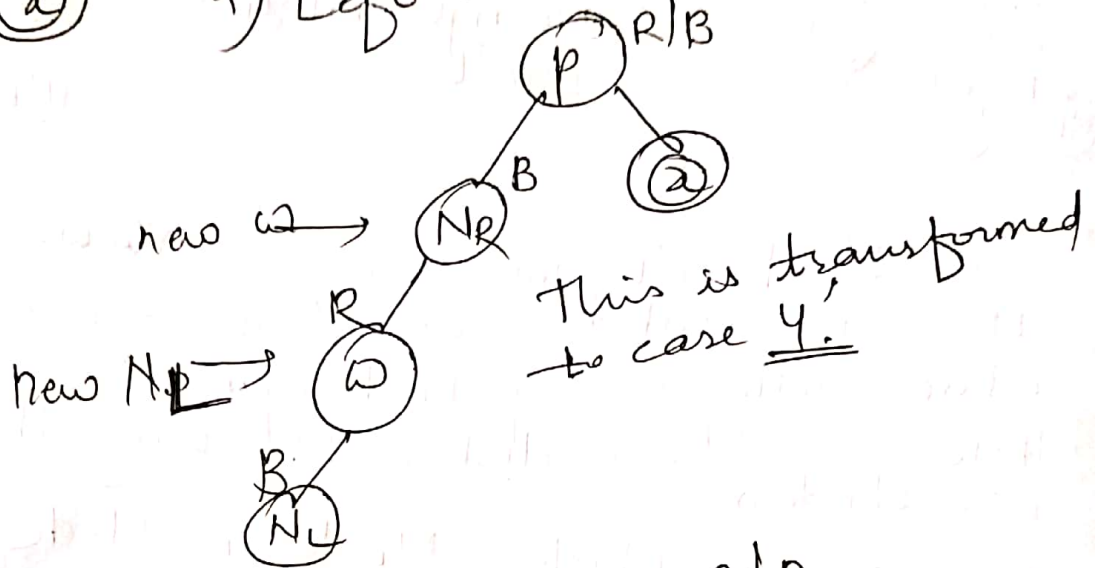
If p is black
 p becomes double black
 Violation of property moves one level up



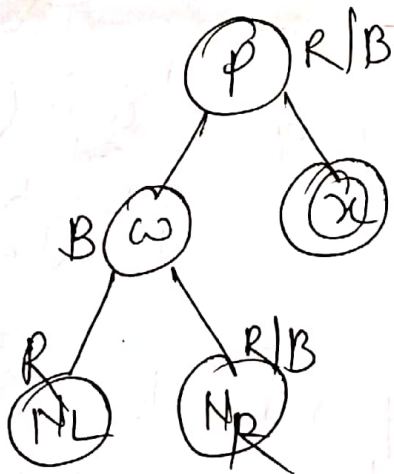
Case 3': w is black, N_L is B & N_R is R



- 1) Swap colors of N_R & w .
- 2) Left rotate at w .



Case 4' $\rightarrow w \rightarrow B, N_L \rightarrow R, N_R \rightarrow R/B$.



$N_L \rightarrow \text{color} = w \rightarrow \text{color}$
 $w \rightarrow \text{color} = P \rightarrow \text{color}$
 $P \rightarrow \text{color} = \text{black}$
 x becomes singly black

right rotate at P

