Find L^{-1} of the following functions

(I) Type - I: Using Partial Fractions

(1) $$\frac{1}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

(2) $$\frac{1}{s(s^2-a^2)} = \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s-a}$$

(3) $$\frac{1}{(s-a)(s+a)^2} = \frac{A}{s-a} + \frac{B}{s+a} + \frac{C}{(s+a)^2}$$

(4) $$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$$

(5) $$\frac{-3s^2+20s-24}{(s-2)\underbrace{(s^2-3s+2)}_{(s-2)(s-1)}} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

(6) $$\frac{2\left[s^4+3s^3+s^2+s+2\right]}{s^3\underbrace{(s^2+3s+2)}_{(s+1)(s+2)}} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1} + \frac{E}{s+2}$$

(7) $$\frac{s^3+s^2+4s+44}{(s^2+4)\underbrace{(s^2-s-2)}_{(s+1)(s-2)}} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{Cs+D}{s^2+4}$$

(8) $$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{As+B}{s^2+2s+2} + \frac{Cs+D}{s^2+2s+5}$$

(or)

Let $u = s^2+2s \Rightarrow F(s) = \dfrac{u+3}{(u+2)(u+5)} = \dfrac{A}{u+2} + \dfrac{B}{u+5}$

*: Find A & B and replace u by s^2+2s & then find L^{-1}

Using Convolution Theorem

(1) $\dfrac{1}{(s+3)(s-1)}$ (2) $\dfrac{1}{s(s^2-a^2)}$ (3) $\dfrac{s}{(s^2+a^2)^2}$

(4) $\dfrac{1}{s(s^2+4)}$ (5) $\dfrac{1}{(s-a)(s+a)^2}$ (6) $\dfrac{1}{(s-2)^4(s+3)}$

(7) $\dfrac{s^2}{(s^2+9)^2}$ (8) $\dfrac{s^2}{(s^2+4)^2}$ (9) $\dfrac{1}{s(s^2-2s+5)}$

~~(10)~~ (p.o) $\cancel{\dfrac{1}{(s-2)^4(s+3)}}$ (10) $\dfrac{s+1}{(s^2+2s+2)^2}$ (12) $\dfrac{(s+2)^2}{(s^2+4s+8)^2}$

Hint: $L^{-1}[\downarrow]$
$= e^{-2t} \underbrace{L^{-1}\left[\dfrac{s^2}{(s^2+4)^2}\right]}_{\text{Use (8)}}$

(13) $\dfrac{s^2}{(s^2+4)^3} = \left(\dfrac{1}{s^2+4}\right)\underbrace{\left(\dfrac{s^2}{(s^2+4)^2}\right)}_{\text{Use: (8)}}$ (14) $\dfrac{s}{(s^2+1)(s^2+4)}$

II) Using the formula: $L^{-1}[F(s)] = -\left(\frac{1}{t}\right) L^{-1}[F'(s)]$

$\left[F'(s) = \frac{d}{ds} F(s)\right]$

(1) $\log\left[\dfrac{s+a}{s+b}\right]$ (2) $\log\left[\dfrac{s^2-4}{(s-3)^2}\right]$ (3) $\frac{1}{2}\log\left[\dfrac{s^2-a^2}{s^2}\right]$

(4) $\tan^{-1}(2/s)$ (5) $\tan^{-1}(2/s^2)$ (6) $\cot^{-1}(s+1)$

I) Solve: (Using L.T)

1) $(D^3 - 2D^2 + 5D)y = 0$; $y(0)=0$; $y'(0)=0$; $y''(0)=1$

2) $(D^2 - 3D + 2)y = 4e^{2t}$; $y(0) = -3$; $y'(0) = 5$

3) $(D^2 + 2D + 5)y = e^{-t}\sin t$; $y(0)=0$; $y'(0)=1$

4) $(D^2 - D - 2)y = 20\sin(2t)$; $y(0)=1$; $y'(0)=2$

5) $(D^2 + 3D + 2)y = 2(t^2+t+1)$; $y(0)=2$; $y'(0)=0$

6) $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$; $y(0)=1$, $y'(0)=0$; $y''(0)=-2$

Extra Problems

Find L^{-1} of $F(s)$:

① $= \dfrac{1-5s}{s^2+2s-3}$

② $\dfrac{1}{s^3+1}$ [Hint : $F(s) = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2-s+1}$]

Ans: $f(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{t/2}\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)$

③ $\dfrac{2s}{s^4+4}$ Ans: $\sin t \sinh(t)$

④ $\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$

Pb ④ $\frac{1}{3}e^{-t}[\sin(2t)+\sin t]$

Ans: $\frac{1}{5}e^{-2t}[\cos(2t)-\cos(3t)]$ * Ans for Pb ⑦

⑤ $\dfrac{s+2}{(s+3)(s+1)^3}$ Ans: $\frac{1}{8}\left[e^{-3t} + e^{-t}\{2t^2+2t-1\}\right]$

⑥ $\dfrac{s^3+6s^2+14s}{(s+2)^4}$ Ans: $e^{-2t}[1+t^2-2t^3]$

⑦ $\dfrac{s+2}{(s^2+4s+8)(s^2+4s+13)}$ Ans: *

⑧ $\dfrac{s^3-3s^2+6s-4}{(s^2-2s+2)^2}$ Ans: $e^t[\cos t + t\sin t]$

$s^2-s-s+2$

$s(s-1)-(s-2)$

$[(s-1)^2+1]^2$

⑨ $\dfrac{1}{s^2(s+3)^2}$ Ans: $\frac{1}{27}\left[-2+3t+2e^{-3t}+3t^2e^{-3t}\right]$

⑩ $\dfrac{s^2+8s+27}{(s+1)(s^2+4s+13)}$ Ans. $2e^{-t}+e^{-2t}\{\sin(3t)-\cos(3t)\}$