Advanced Algorithms

Assignment 1

Given date: 3th August 2019

Date of Submission: 16th August 2019 (Before 2.00pm)

Note: Do not copy. Assessment is done purely on submission time and oral performance.

1.	If the set of stack operations included a MULTIPUSH operation, which pushes k items onto the stack, would the O(1) bound on the amortized cost of stack operations continue to hold?
2.	Suppose we perform a sequence of n operations on a data structure in which the i^{th} operation costs i if i is an exact power of 2, and 1 otherwise. Use accounting and potential method to determine the amortized cost per operation.
4.	Suppose we wish not only to increment a counter but also to reset it to zero (i.e., make all bits in it 0). Counting the time to examine or modify a bit as theta(1), show how to implement a counter as an array of bits so that any sequence of n INCREMENT and RESET operations takes time O(n) on an initially zero counter.
5.	Analyze the hiring problem using indicator random variables.
6	Use indicator random variables to solve the following problem, which is known as the *hat-check problem*. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers who get back their own hat?
7.	Let A[1….n] be an array of n distinct numbers. If i < j and A[i] > A[j], then the pair (i,j) is called an inversion of A. Suppose that the elements of A form a uniform random permutation of <1,2, …, n>. Use indicator random variables to compute the expected number of inversions.
8.	How many people must there be in a room before there is a 50% chance that two of them were born on the same day of the year? (Birthday paradox)
9	Discuss balls and bins problem and its uses in hashing.