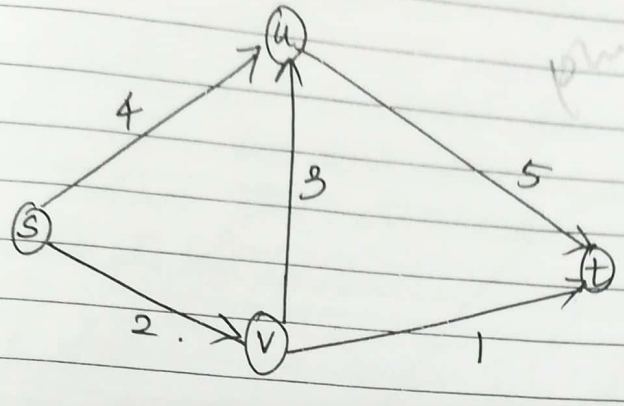
Q:



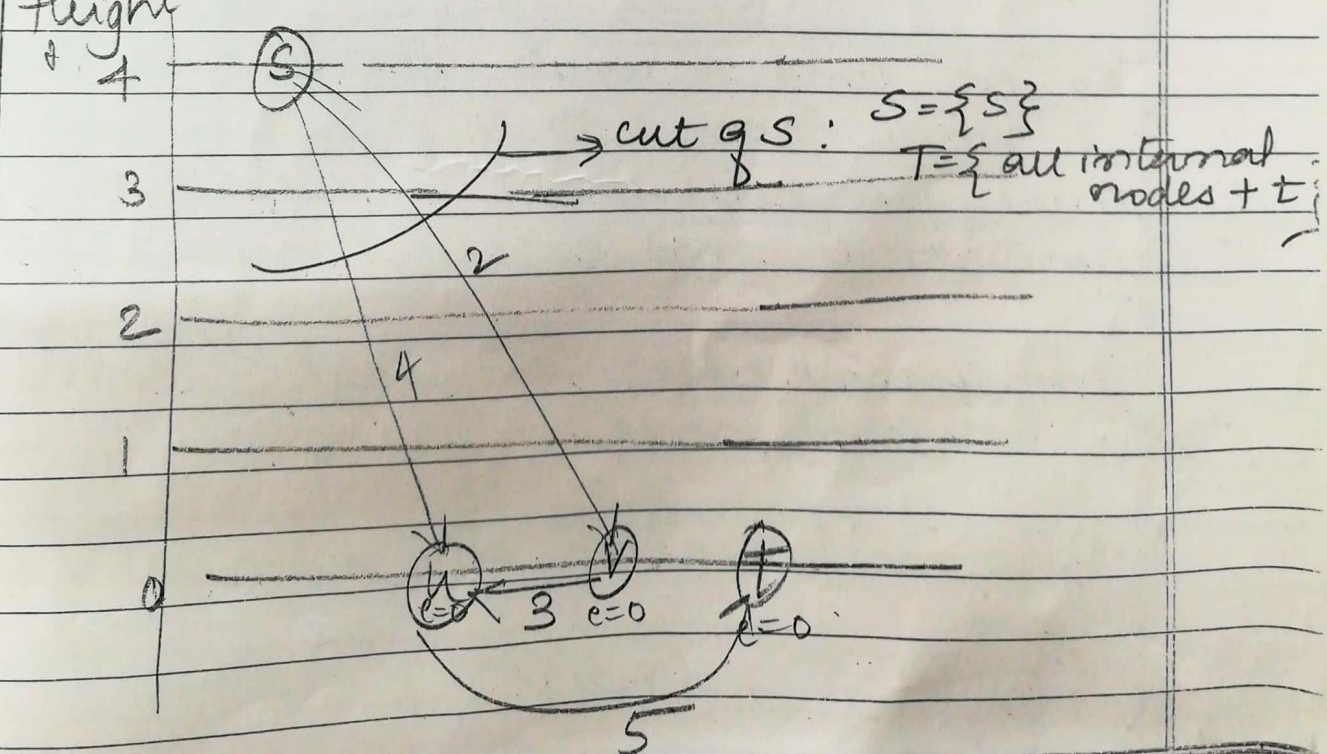
Sol: Step 1:

no. of levels = n+1 (0, 1, 2 ..., n) where n is the no. of nodes

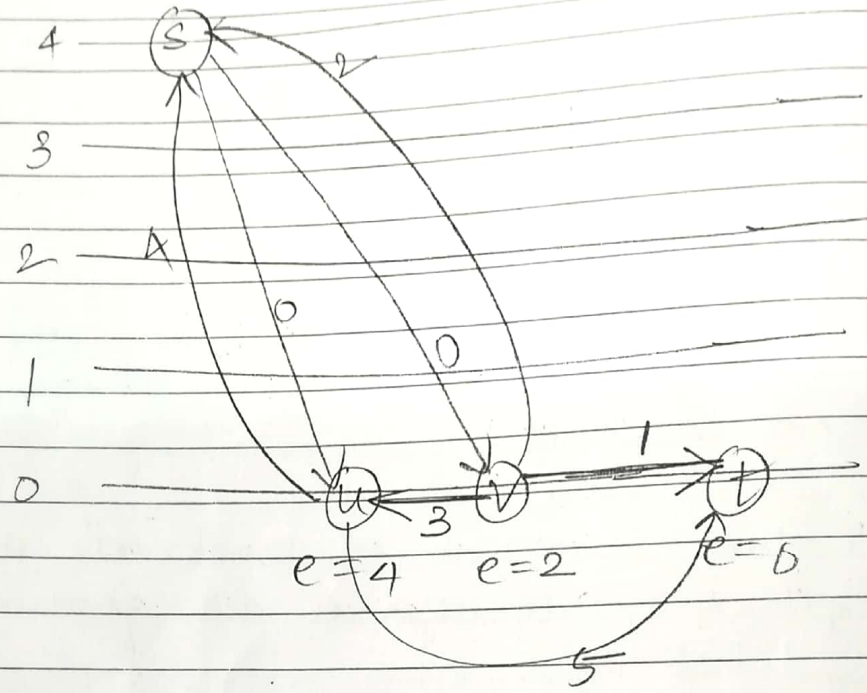
$h(s) = n$

Initialise source to level 'n' and all the other vertices are at ground level i.e level 0

Excess flow for all ~~sources~~ nodes except source is 0 ~~is~~ at the start



Step 2: Along the cut of S: send maximum flow.



The nodes which have excess flow greater than zero, are called active nodes.

So we send 4 units from s to u and 2 units from s to v.

Thus we will have excess flow of 4 units at u and excess flow of 2 at v.

Admissable edge: an edge with some capacity from any internal node to sink.

Thus we have ~~one~~ two admissable edges here

$(u,t)=5$, $(v,t)=1$

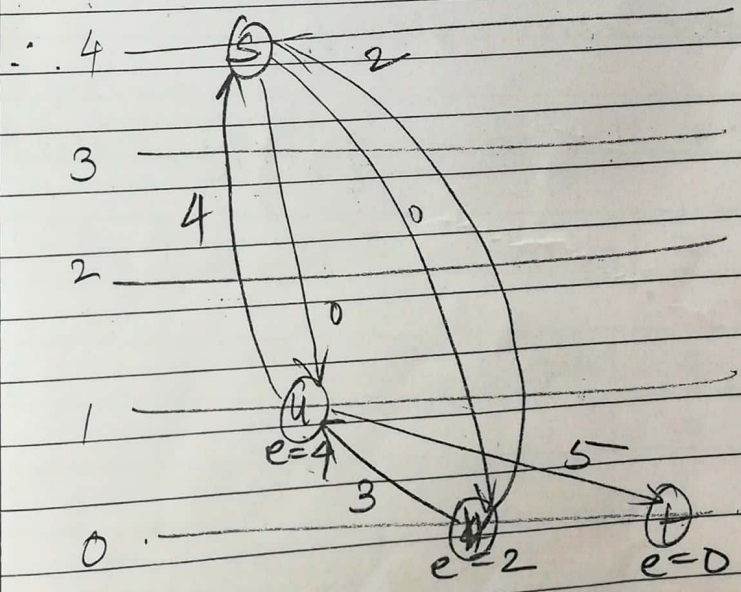
Suppose we choose (u,t) as the admisable edge.
∴ But we cannot send from u → t because they are at the same level.
∴ we have to increase the height of u.
∴ relabel(u):

$$\text{relabel}(u) = 1 + \min\{h(v) \mid c_f(u,v) > 0\}$$

h(v): height of all adjacent nodes such that capacity of the edges (u,v) is strictly greater than zero.

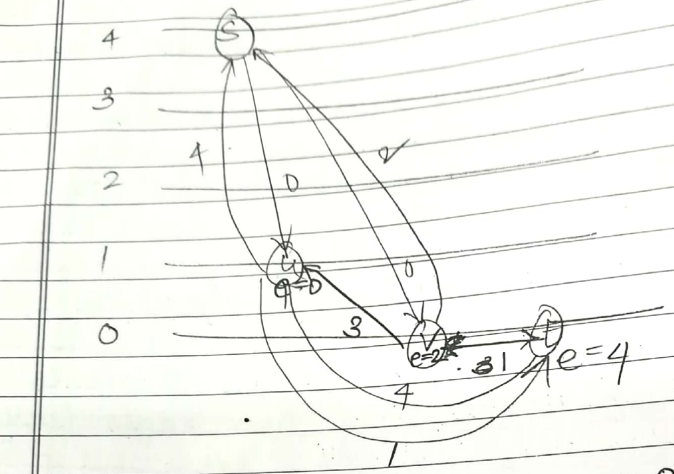
∴ for u:

$$= 1 + \min\{h(s), h(t)\}$$
$$= 1 + \min\{4, 0\}$$
$$= 1 + 0$$
$$= 1$$



Now it is at higher level.

∴ push (u,t).

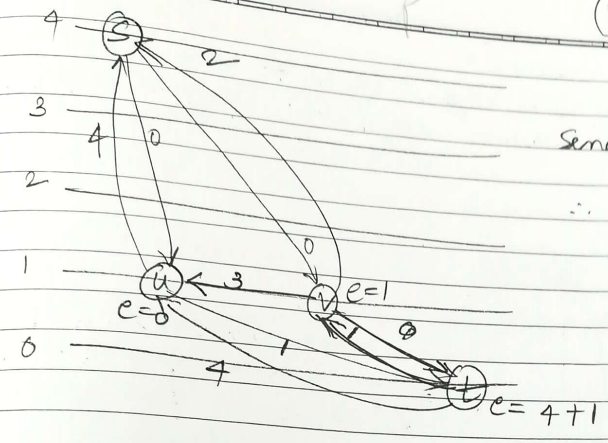


∴ Now excess flow at $u = 0$.
Now t has excess flow but it cannot be considered as active node because it is sink.

Now active vertices are v
and admissable edges are $v \to t$ but they are at the same level.

$$\therefore \text{relable}(v) = 1 + \min\{h(u), h(s), h(t)\}$$
$$= 1 + \min\{1, 4, 0\}$$
$$= 1 + 0$$
$$= 1.$$

∴ now $h(v) = 1$
and push (v, t)



Send 1 unit from v to t.
$\therefore e(v) = 2-1 = 1$
$e(t) = 4+1$

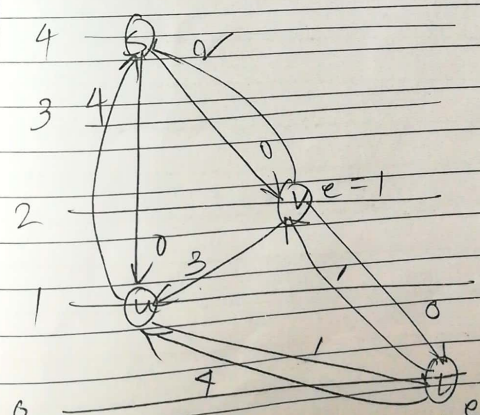
Now active nodes: v
but no admissable edges.
∴ increase height of v:

relabel(v) = ~~1+~~ min {h(s), h(u), h(t)}

= 1 + min {4, 1, ~~∞~~}

~~But~~ = 1 + 1
= 2.

t is not considered because (v,t) = 0 and condition is $C_f(u,v) > 0$

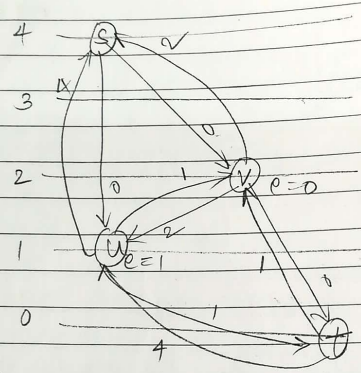


active nodes : v

admissable edge : (v, u) ✓

∴ Send excess capacity : 1 unit along v to u

Now



Now excess capacity of $u = 1$

∴

Active nodes = u

admissable edge : u, t

∴ Send 1 unit of excess along u, t