Fractal Geometry:

- Natural objects such as cloud, moutains, trees, snowflakes, coastlines etc. are not easily described by "traditional geometry". The geometry of such objects in a nature is represented by fractal geometry.
- A fractal is a geometric figure whose parts contain the same statistical characters as a whole.
- There are mainly two properties of fractals.

1) Self-Similarity
2) Non-integer dimensions.

Self-Similarity

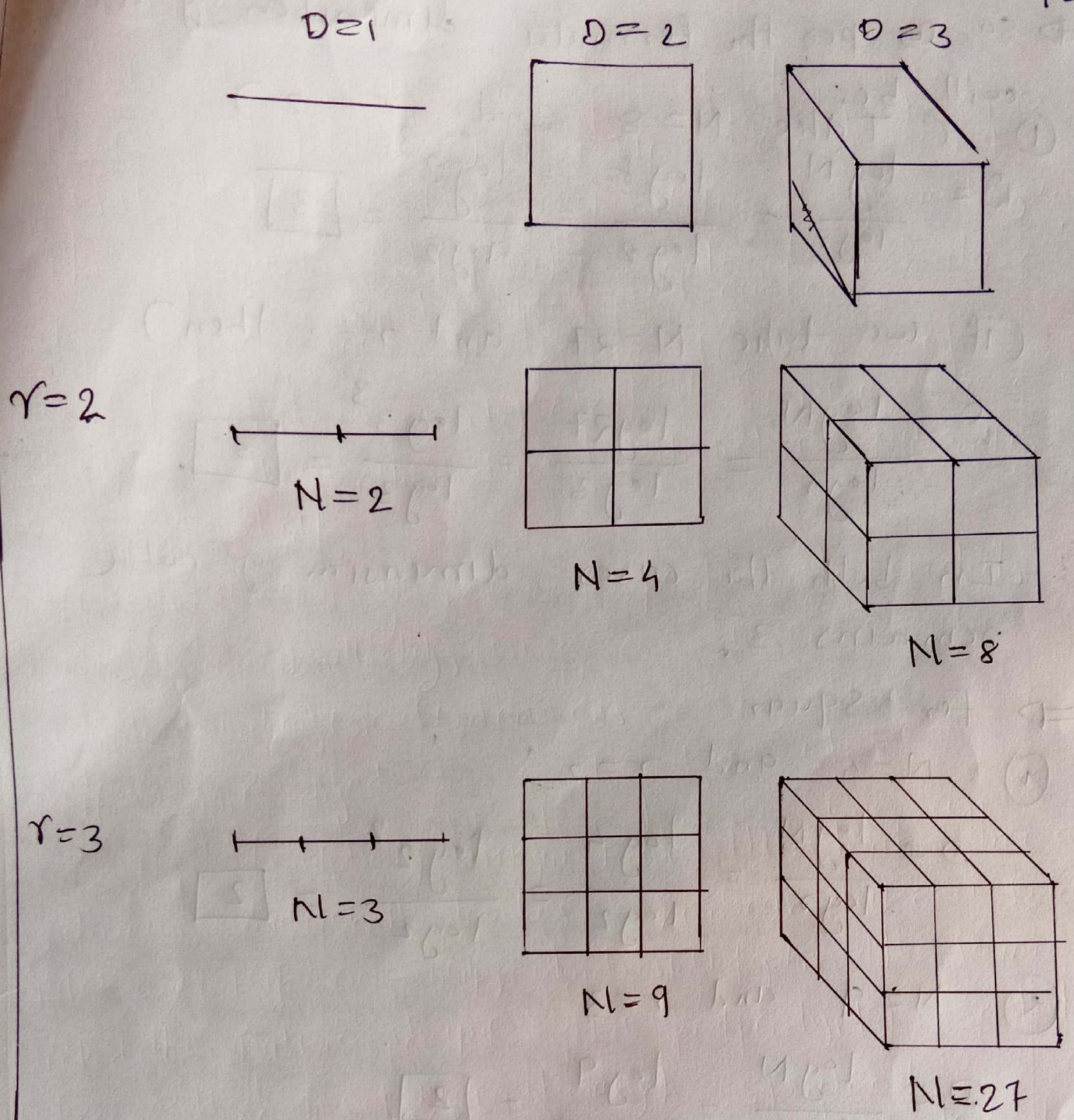
- Self-similarity means, fractals look same at various different scales - You can take a small extract of a shape and it looks the same as the entire shape. This property of fractals is called 'self-similarity'.

For eg; if you look carefully at a fern leaf, you will notice that every little leaf - part of the bigger one - has the same shape as the whole fern leaf. The same is with fractals: You can magnify them many times and after every step you will see the same shape.

2) Non-integer Dimensions:

- classical geometry deals with objects of integer dimensions:
  - zero dimensional points
  - One dimensional lines and curves
  - two dimensional planes figures such as square, circle etc.
  - Three dimensional solids such as cube, spheres etc
- However, many natural phenomena are better described by using a dimension that are non-integers. That is using dimensions between 2 and 3 or 1 and 2 etc.
- So, while straight line has a dimension one, a fractal may have dimension between one and two.
- If we take an object residing in Euclidean dimension D and reduce its linear size by $1/r$ in each spatial direction. then its measure (ie length, area or volume) would increase to $N = r^D$ times the original.
- This is explained in the following fig.

Scanned by CamScanner



$$\boxed{N = r^D}$$

Here, N is the number of self-similar pieces and r is the magnification factor.
Then the dimension D can be found as,

$$\text{dimension } D = \frac{\log N}{\log r} = \frac{\log(\text{No. of self-similar pieces})}{\log(\text{magnification factor})}$$

⇒ So, as per the formula, dimension of a cube will be:

① (Take $N=8$ and $r=2$)

$$D = \frac{\log N}{\log r} = \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2} = \boxed{3}$$

(if we take $N=27$ and $r=3$ then)

$$D = \frac{\log N}{\log r} = \frac{\log 27}{\log 3} = \frac{\log 3^3}{\log 3} = \boxed{3}.$$

In both the cases, dimension of cube remains 3.

⇒ For square.

① $N=4$ and $r=2$

$$D = \frac{\log N}{\log r} = \frac{\log 4}{\log 2} = \frac{\log 2^2}{\log 2} = \boxed{2}$$

② $N=9$ and $r=3$

$$D = \frac{\log N}{\log r} = \frac{\log 9}{\log 3} = \boxed{2}.$$

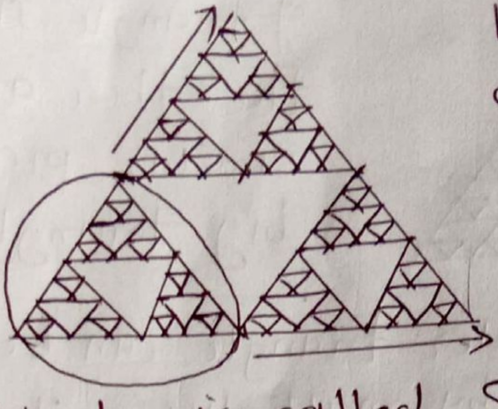
In both the cases, dimension of ~~cut~~ square remain 2.

Thus, we can define the fractal dimension of a self-similar object as,

$$\text{fractal dimension} = \frac{\log(\text{no. of self-similar pieces})}{\log(\text{magnification factor})}$$

Scanned by CamScanner

• Now, consider the following triangle.



Here $N=3$
and $r=2$

The above triangle is called Sierpinski triangle. The above pattern is generated by finding the midpoints of the line segments of the largest triangle. Then, by connecting these midpoints smaller triangles have been created. This pattern is then repeated for the smaller triangles, infinitely.

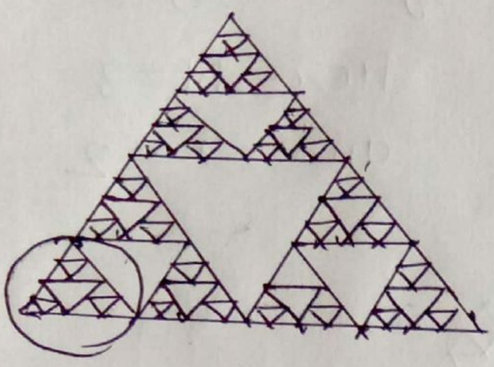
• Now, we can compute the dimension of Sierpinski triangle (S). Note that the triangle S consists of 3 self-similar pieces, each with magnification factor 2.

• So fractal dimensions of S is

$$= \frac{\log(\text{No. of self-similar pieces})}{\log(\text{magnification factor})}$$

$$= \frac{\log 3}{\log 2} = 1.58$$

• Note, that, S also ~~can contain~~ ~~a~~ consists of 9 self-similar pieces with magnification factor 4.



if you consider smaller piece of triangle shown by circle the there are 9 such similar pieces in whole big triangle $(N = 9)$

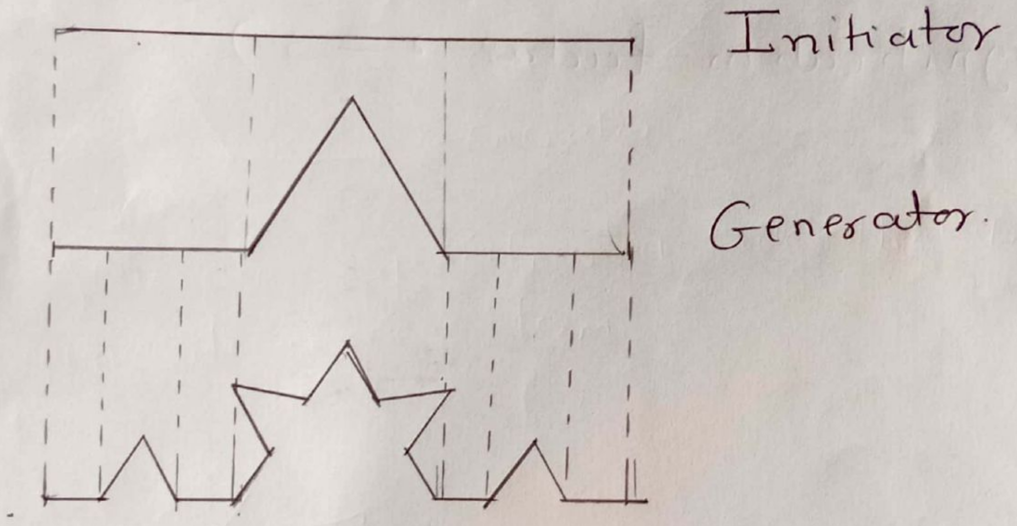
and each smaller triangle can be brough to original size by multiplying it by 4 in each direction, so magnification factor is $r = 4$

$$\therefore \frac{\log N}{\log r} = \frac{\log 9}{\log 4} = 1.58$$

Note, that ~~dim~~ dimension remain same.

The Koch Curve:

- The Koch curve begins with a straight line of length 1, called <u>initiator</u>.
- The divide into three equal segments and replace the middle segment by the two sides of an equilateral triangle of the same length as the segment being removed. This new form is called <u>generator</u>.



- Now repeat, taking each of the four resulting segments, dividing ~~into~~ each into three equal parts and replacing each of the middle segment by two sides of equilateral triangle.

Fractal dimension of Koch curve:

- The Koch curve is self-similar with 4 non-overlaping copies of itself, each scaled by the factor $r < 1$.

$\therefore$ dimension $d = \dfrac{\log(\text{No. of smaller pieces})}{\log(\text{magnification factor})}$

$= \dfrac{\log 4}{\log 3} = 1.26$

Note that, each line has been transformed into 4 identical lines (i.e $N=4$), ~~and~~ each one third the size of the original (ie magnification factor $r=3$).