

# Bi-linear Transformation (B.T.)

Q1

I Find the fixed points of the foll. B.T. - [Solve:  $w = f(z) = z$ ]  
Ans

①  $w = \frac{2z+6}{z+7}$

①  $\{1, -6\}$

②  $w = \frac{3z-5i}{iz-1}$

②  $\{i, -5i\}$

③  $w = \frac{z-1-i}{z+2}$

③  $\{-i, -1+i\}$

④  $w = \frac{2z+4i}{iz+1}$ ; Also show that

④  $\{4i, -i\}$

the 2 fixed points together with any point  $z$  and its image  $w$  form a set of four points having constant cross-ratio

[i.e. S.T.  $(z_1, z_2, z_3, z_4) = \frac{5}{2}$  a const]

$\rightarrow \begin{cases} z_1 = z; \\ z_2 = w = \frac{2z+4i}{iz+1} \\ z_3 = 4i \\ z_4 = -i \end{cases}$

⑤  $w = \frac{2z-2+iz}{i+z}$  ⑥  $w = \frac{2z-5}{z+4}$

⑤  $1 \pm i$  ⑥  $-1 \pm 2i$

II Find the B.T. that maps the points

①  $z = \{-1, 1, \infty\}$  onto  $w = \{-i, -1, i\}$

②  $z = \{2, 1, 0\}$  onto  $w = \{1, 0, i\}$

③  $z = \{-1, 0, 1\}$  onto  $w = \{-1, -i, 1\}$

④  $z = \{-i, 0, i\}$  onto  $w = \{-1, i, 1\}$

⑤  $z = \{i, -1, 1\}$  onto  $w = \{0, 1, \infty\}$

⑥  $z = \{0, 1, \infty\}$  onto  $w = \{-5, -1, 3\}$

⑦  $z = \{\infty, i, 0\}$  onto  $w = \{0, i, \infty\}$

⑧  $z = \{1+i, -i, 2-i\}$  onto  $w = \{0, 1, i\}$

Ans:-

$w = \frac{iz+i-2}{z+1-2i}$

$w = \frac{2i(z-1)}{z(1+i)-2}$

$w = \frac{z-i}{1-iz}$

$w = \frac{i(1-z)}{1+z}$

$w = \frac{2(z-i)}{(1+i)(z-1)}$

$w = \frac{3z-5}{z+1}$

$w = -\frac{1}{z}$

$w = \frac{2z-2-2i}{(i-1)z-3-5i}$

⑨  $z = \{0, -i, -1\}$  onto  $w = \{i, 1, 0\}$

⑩  $z = \{0, -i, 2i\}$  onto  $w = \{5i, \infty, -i/3\}$

⑪  $z = \{0, -1, \infty\}$  onto  $w = \{-1, -2-i, i\}$

$w = -i \left[ \frac{z+1}{z-1} \right]$  (02)

$w = \frac{3z-5i}{iz-1}$

$w = \frac{iz-2}{z+2}$

III Find the images under B.T.:-

① Find the image of the real axis of  $z$ -plane under  $w = \frac{1}{z+i}$   
 Ans:  $u^2 + v^2 + v = 0$ ; Centre:  $(0, -1/2)$ ,  $ra = 1/2$

② S.T.  $w = \frac{i-z}{i+z}$  maps the circle  $|z|=1$  onto the imaginary axis of the  $w$ -plane.

③ S.T.  $w = \frac{2z+3}{z-4}$  maps the circle  $x^2 + y^2 - 4x = 0$  in  $z$ -Plane into the st. line  $4u+3=0$  in  $w$ -plane.

④ S.T.  $w = \frac{iz+2}{4z+i}$  maps the real axis in  $z$ -plane into a circle  $u^2 + v^2 + \frac{7}{4}v - \frac{1}{2} = 0$ ; Find the pre-image of the centre of this circle  
 Ans: Centre:  $(0, -7/8)$ ,  $ra = 9/8$   
 $z = i/4$

⑤ S.T.  $w = \frac{1+iz}{1-iz}$  maps the interior of the unit circle  $|z| < 1$  in  $z$ -Plane into the upper left half of the  $w$ -plane  $u < 0$

⑥ S.T.  $w = \frac{z}{1-z}$  maps the upper half of  $z$ -plane onto the upper half of  $w$ -plane. Find the image of the circle  $|z|=1$  under this transn.  
 Ans:  $u = -1/2$

⑦ S.T.  $w = \frac{z-i}{iz-1}$  maps  $\text{Im}(z) \geq 0$  onto  $|w| \leq 1$   
 $y \geq 0$

⑧ Find the image of the segment of the real axis between  $z=+1$  &  $z=-1$  under  $w = \frac{1+iz}{z-i}$   
 Ans: lower half of  $|w|=1$