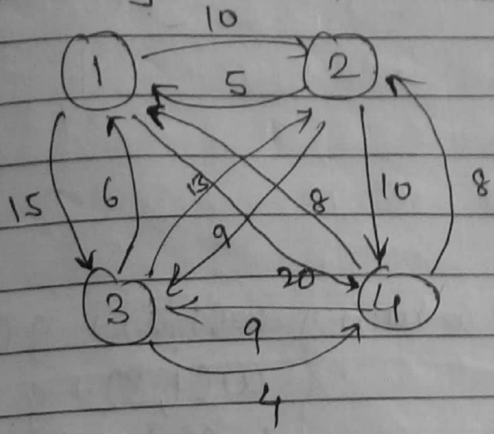
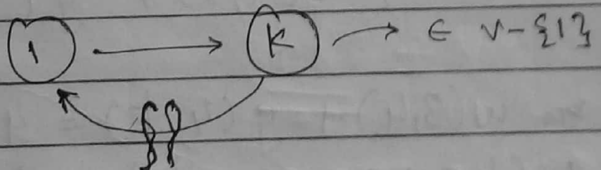
Travelling Salesperson Problem. (TSP)



Tour ← directed cycle having all vertices of G

salesperson starts from city 1 travels all city once & comes back to same city
Hence tour is directed cycle
To find tour with min cost
Imp → The tour starts & ends at vertex 1 by visiting intermediate vertices only once
Every tour consists of edge 1 → k, where k is $\in V - \{1\}$ & a path from k back to 1



If the tour is optimal then path from k to 1 is optimal (must be min cost path)
Let g(1, S) be a f^n describes the length of optimal tour where 1 is starting city & 'S' is set of intermediate nodes.
i.e. the tour started at 1 going through all vertices other than 1 & coming back to source vertex 1

$$g(i, S) = \min\left(w(i, k) + g(k, S - \{k\}) \right)$$

$S \downarrow V - \{1\}$

$k \uparrow$ set of vertices adj to node i

$g(i, \phi) = w(i, 1)$ ← base case

↑ no intermediate nodes.

For the given Q.

→ $g(1, \{2,3,4\}) = \min \begin{cases} w(1,2) + g(2, \{3,4\}) \\ w(1,3) + g(3, \{2,4\}) \\ w(1,4) + g(4, \{2,3\}) \end{cases}$

$g(2, \{3,4\}) = \min \begin{cases} w(2,3) + g(3, \{4\}) \\ w(2,4) + g(4, \{3\}) \end{cases}$

$g(3, \{2,4\}) = \min \begin{cases} w(3,2) + g(2, \{4\}) \\ w(3,4) + g(4, \{2\}) \end{cases}$

$g(4, \{2,3\}) = \min \begin{cases} w(4,2) + g(2, \{3\}) \\ w(4,3) + g(3, \{2\}) \end{cases}$

$g(3, \{4\}) = w(3,4) + g(4, \phi) = 4 + 8 = 12$

$g(4, \{2\}) = w(4,2) + g(2, \phi) = 8 + 5 = 13$

$g(4, \{3\}) = w(4,3) + g(3, \phi) = 9 + 6 = 15$

$\therefore g(2, \{4\}) = 18$

$g(2, \{3\}) = 15$

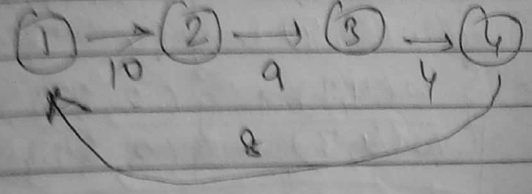
$g(3, \{2\}) = 18$

$\therefore g(4, \{2,3\}) = \begin{cases} 8 + 15 \to 23 & \leftarrow 2 \\ 9 + 18 \to 27 \end{cases}$

$g(3, \{2,4\}) = \begin{cases} 13 + 18 \\ 4 + 13 \to 17 & \leftarrow 4 \end{cases}$

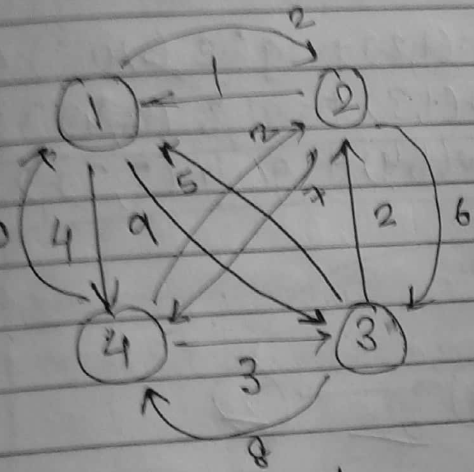
$g(2, \{3,4\}) = \begin{cases} 9 + 18 \quad 27 \\ 10 + 15 \quad 25 & \leftarrow 4 \end{cases}$

$g(1,\{2,3,4\}) = \begin{cases} 10 + 23 & 33 \quad \cancel{\leftarrow 2} \\ 15 + 47 & 32 \quad \leftarrow 3. \\ 13 + 23 & 36 \end{cases}$



cost = 31.

(8



soln. The optimal tour cost is $g(1, \{2,3,4\})$.

(no intermediates)

$|S| = 0$

$g(2, \phi) = 1$

$g(3, \phi) = 5$

$g(4, \phi) = 10$

$|S| = 1$

$g(2, \{3\}) = 6 + 5 = 11$

$g(2, \{4\}) = 7 + 10 = 17$

$g(3, \{2\}) = 2 + 1 = 3$

$g(3, \{4\}) = 8 + 10 = 18$

$g(4, \{2\}) = 2 + 1 = 3$

$g(4, \{3\}) = 3 + 5 = 8$

For $|S| = 2$

$$g(2, \{3,4\}) = \begin{cases} w(2,3) + g(3,\{4\}) = 6 + 18 \to 24 \\ w(2,4) + g(4,\{3\}) = 7 + 8 \to 15 \to 4 \end{cases}$$

$$g(3, \{2,4\}) = \begin{cases} w(3,2) + g(2,\{4\}) = 2 + 17 \\ w(3,4) + g(4,\{2\}) = 8 + 3 \to 11 \to 4 \end{cases}$$

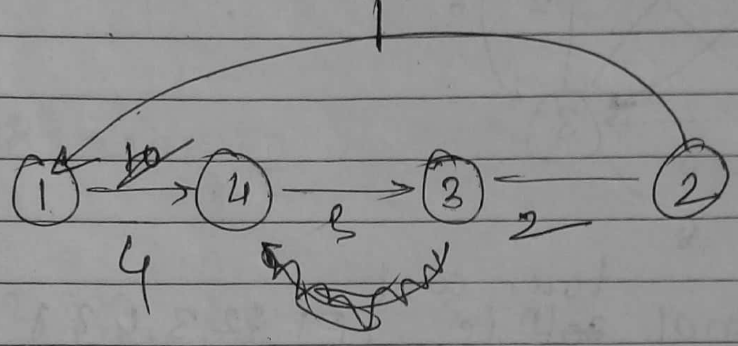
$$g(4, \{2,3\}) = \begin{cases} w(4,2) + g(2,\{3\}) = 2 + 11 \\ \boxed{w(4,3) + g(3,\{2\}) = 3 + 3 \to 6} \to 3 \end{cases}$$

For $|S| = 3$

$$g(1, \{2,3,4\}) = w(1,2) + g(2,(3,4)) = 2 + 15$$

$$w(1,3) + g(3\,(2,4)) = 9 + 11$$

$$\boxed{w(1,4) + g(4\,(2,3)) = 4 + 6 \to 10} \to 4$$



$= \underline{\underline{10}}$