DSIP – Lecturer 05

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Basic Relationships Between Pixels

- Neighborhood
- Adjacency
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Neighborhood Operations in Images

Neighbors of a Pixel

- A pixel *p* at coordinates (*x*,*y*) has four *horizontal* and *vertical* neighbors whose coordinates are given by:

  (x+1,y), (x-1, y), (x, y+1), (x,y-1)

	(x, y-1)	
(x-1, y)	*P (x,y)*	(x+1, y)
	(x, y+1)	

This set of pixels, called the 4-*neighbors* or *p*, is denoted by $N_4(p)$. Each pixel is one unit distance from (x,y) and some of the neighbors of p lie outside the digital image if (x,y) is on the border of the image.

Basic Connectivity of Pixels:

	p(x-1, y)	
p(x, y-1)	p(x,y)	p(x, y+1)
	p(x+1, y)	

$$N_a(P) = N_4(p) + N_o(P)$$

p(x-1,y-1)		p(x-1,y+1)
	p(x,y)	
p(x+1,y-1)		p(x+1,y+1)

Neighbors of a Pixel

- The four *diagonal* neighbors of *p* have coordinates:

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

(x-1, y+1)		(x+1, y-1)
	P (x,y)	
(x-1, y-1)		(x+1, y+1)

and are denoted by $N_D(p)$.

These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$.

(x-1, y+1)	(x, y-1)	(x+1, y-1)
(x-1, y)	*P (x,y)*	(x+1, y)
(x-1, y-1)	(x, y+1)	(x+1, y+1)

As before, some of the points in $N_D(p)$ and $N_8(p)$ fall outside the image if (x,y) is on the border of the image.

Adjacency and Connectivity

- Let *V*: a set of intensity values used to define adjacency and connectivity.
- In a binary image, $V = \{1\}$, if we are referring to adjacency of pixels with value 1.
- In a gray-scale image, the idea is the same, but *V* typically contains more elements, for example, $V = \{180, 181, 182, \ldots, 200\}$
- If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values.

Types of Adjacency

1. **4-adjacency:** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
2. **8-adjacency:** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
3. **m-adjacency =(mixed)**

Types of Adjacency

- **m-adjacency:**

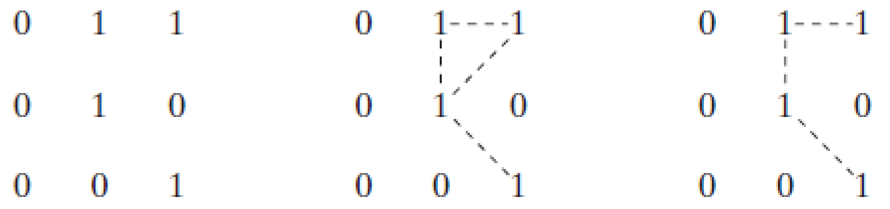
  Two pixels *p* and *q* with values from *V* are m-adjacent if :

  - q is in $N_4(p)$ or
  - q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixel whose values are from V (no intersection)

- **Important Note:** the type of adjacency used must be specified

Types of Adjacency

- Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.
- For example:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Types of Adjacency

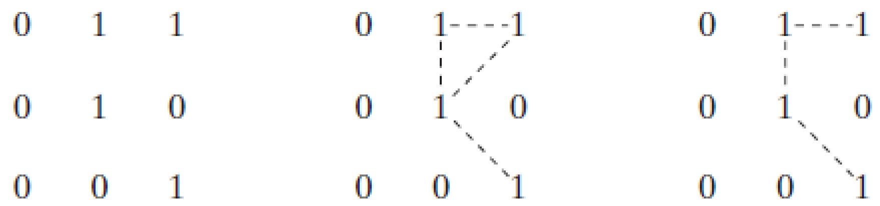
- In this example, we can note that to connect between two pixels (finding a path between two pixels):
  - In 8-adjacency way, you can find multiple paths between two pixels
  - While, in m-adjacency, you can find only one path between two pixels
- So, m-adjacency has eliminated the multiple path connection that has been generated by the 8-adjacency.
- Two subsets *S1* and *S2* are adjacent, if some pixel in *S1* is adjacent to some pixel in *S2*. Adjacent means, either 4-, 8- or m-adjacency.

A Digital Path

- A digital path (or curve) from pixel *p* with coordinate (x,y) to pixel *q* with coordinate (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), …, (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$
- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

A Digital Path

- Return to the previous example:



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

In figure (b) the paths between the top right and bottom right pixels are 8-paths. And the path between the same 2 pixels in figure (c) is m-path

Connectivity

- Let *S* represent a subset of pixels in an image, two pixels *p* and *q* are said to be connected in *S* if there exists a path between them consisting entirely of pixels in *S*.
- For any pixel *p* in *S*, the set of pixels that are connected to it in *S* is called a *connected component* of *S*. If it only has one connected component, then set *S* is called a *connected set*.

Region and Boundary

- **Region**

  Let *R* be a subset of pixels in an image, we call *R* a region of the image if *R* is a connected set.

- **Boundary**

  The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

Region and Boundary

If *R* happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image.

This extra definition is required because an image has no neighbors beyond its borders

Normally, when we refer to a region, we are referring to subset of an image, and any pixels in the boundary of the region that happen to coincide with the border of the image are included implicitly as part of the region boundary.

Distance Measures

- For pixels p, q and z, with coordinates (x,y), (s,t) and (v,w), respectively, D is a distance function if:

  (a) $D(p,q) \geq 0$ ($D(p,q) = 0$ iff $p = q$),

  (b) $D(p,q) = D(q,p)$, and

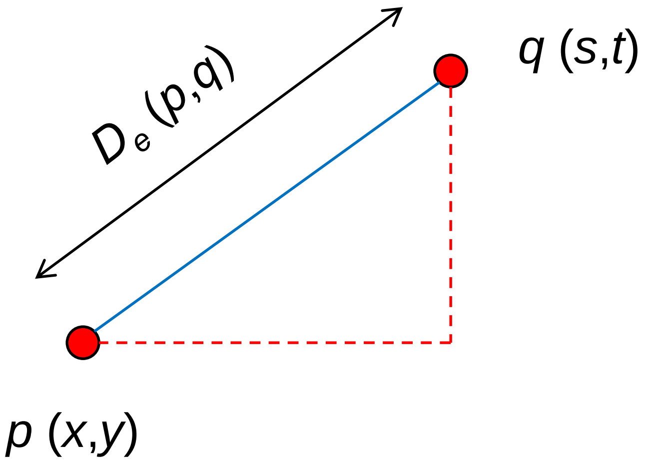
  (c) $D(p,z) \leq D(p,q) + D(q,z)$.

Distance Measures

- The *Euclidean Distance* between *p* and *q* is defined as:

$$D_e(p,q) = [(x - s)^2 + (y - t)^2]^{1/2}$$

Pixels having a distance less than or equal to some value r from (x,y) are the points contained in a disk of radius r centered at (x,y)

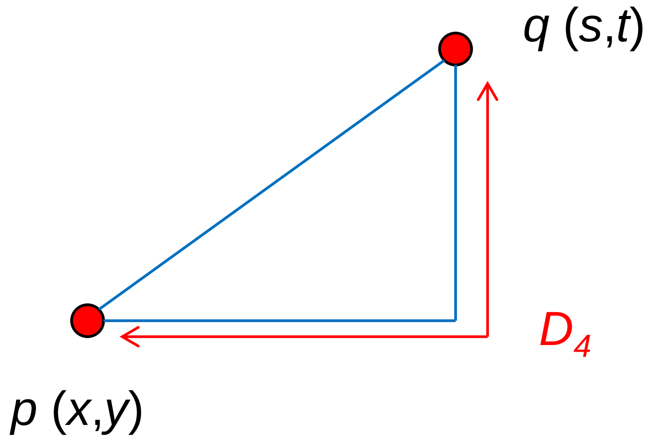


Distance Measures

- The D_4 *distance* (also called *city-block distance*) between *p* and *q* is defined as:

  $$D_4(p,q) = |x - s| + |y - t|$$

Pixels having a D_4 distance from (x,y), less than or equal to some value r form a Diamond centered at (x,y)



Distance Measures

Example:

The pixels with distance $D_4 \leq 2$ from (x,y) form the following contours of constant distance.

The pixels with $D_4 = 1$ are

the 4-neighbors of (x,y)

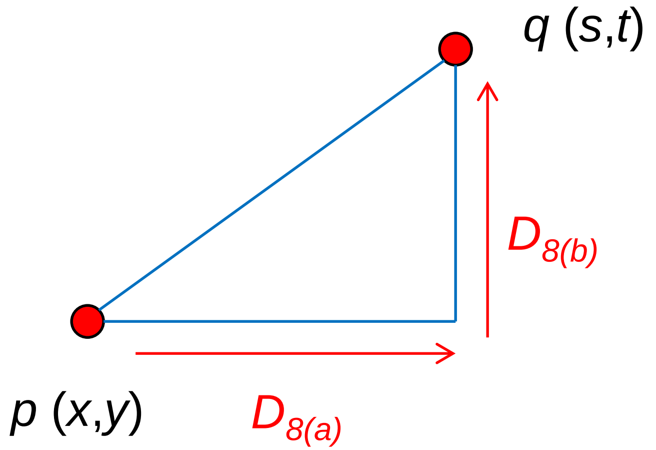
		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measures

- The D_8 *distance* (also called *chessboard distance*) between *p* and *q* is defined as:

  $$D_8(p,q) = \max(|x - s|, |y - t|)$$

Pixels having a D_8 distance from (x,y), less than or equal to some value r form a square Centered at (x,y)



$D_8 = \max(D_{8(a)}, D_{8(b)})$

Distance Measures

Example:

D_8 distance ≤ 2 from (x,y) form the following contours of constant distance.

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

- **Dm distance:**

  is defined as the shortest m-path between the points.

  In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

Distance Measures

- Example:

  Consider the following arrangement of pixels and assume that p, p_2, and p_4 have value 1 and that p_1 and p_3 can have can have a value of 0 or 1

  Suppose that we consider

  the adjacency of pixels

  values 1 (i.e. $V = \{1\}$)

	p_3	p_4
p_1	p_2	
p		

Distance Measures

- Cont. Example:

  Now, to compute the D_m between points p and p_4

  Here we have 4 cases:

  Case1: If p_1 =0 and p_3 = 0

  The length of the shortest m-path

  (the D_m *distance) is 2 (p, p_2, p_4)*

	0	1
0	1	
1		

Distance Measures

- Cont. Example:

  Case2: If p_1 =1 and p_3 = 0

  now, p_1 *and p will no longer be adjacent (see m-adjacency definition)*

  then, the length of the shortest

  path will be 3 (p, p_1, p_2, p_4)

	0	1
1	1	
1		

Distance Measures

- Cont. Example:

  Case3: If p_1 =0 and p_3 = 1

  The same applies here, and the shortest –m-path will be 3 (p, p_2, p_3, p_4)

	1	1
0	1	
1		

Distance Measures

- Cont. Example:

  Case4: If p_1 =1 and p_3 = 1

  The length of the shortest m-path will be 4 (p, p_1 , p_2, p_3, p_4)

	1	1
1	1	
1		

HW

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

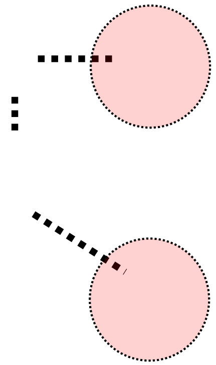
8-adjacent

m-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$



8-adjacent **m-adjacent**

The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

(1,3), (1,2), (2,2), (3,3)

Question 1

- **In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 2

- **In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Part 1

Part 2

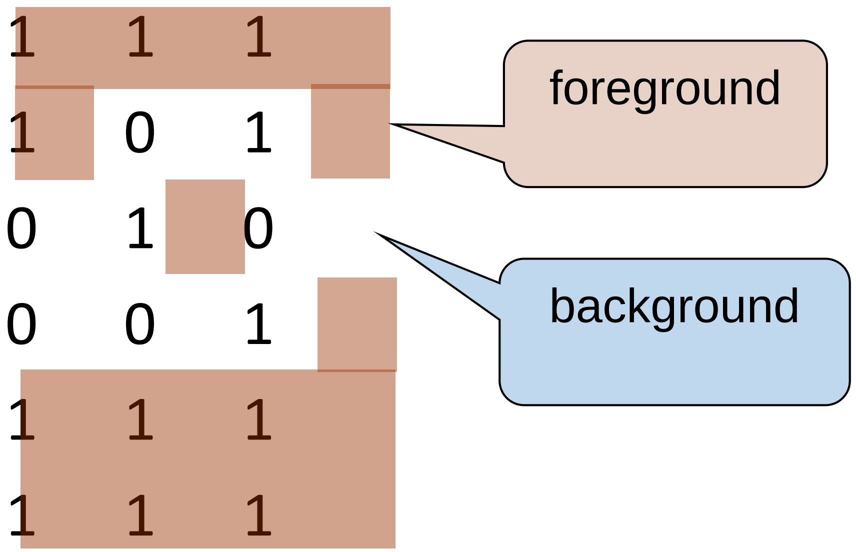
- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**



Question 3

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 5

- **In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

- **In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Syllabus

- **4.0 Digital Image Fundamentals 08**
- **4.1** Introduction to Digital Image, Digital Image Processing System, Sampling and Quantization
- **4.2** Representation of Digital Image, Connectivity
- **4.3** Image File Formats: BMP, TIFF and JPEG.
- **5.0 Image Enhancement in Spatial domain 10**
- **5.1** Gray Level Transformations, Zero Memory Point Operations,
- **5.2** Histogram Processing, Histogram equalization.
- **5.3** Neighborhood Processing, Spatial Filtering, Smoothing and Sharpening Filters, Median Filter.
- **6.0 Image Segmentation 06**
- **6.1** Segmentation based on Discontinuities (point, Line, Edge),
- **6.2** Image Edge detection using Robert, Sobel, Previtt masks, Image Edge detection using Laplacian Mask.

- **Total 24**