

Simplification of CFG

CFG - context Free Grammar

for lang. [context Free Language (CFL)]

= Type 2 lang.

= lang of Push Down Automata (PDA or PDM)

$$CFG = (V, T, P, S)$$

V = set of variables

T = set of terminals

P = set of rules

S = starting symbol.

Lang. $L = \{a^n b^n \mid n \geq 0\}$ is a CFL.

so strings $ab, aabb, aaabbb, \dots, aaaaaabbbbb \dots$ belong to this language.

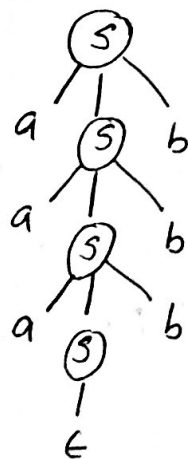
if we consider a string

aaabbb

start from symbol

the string is generated as

per parse tree



$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{ S \rightarrow aSb, S \rightarrow \epsilon \}$$

$$S = S$$

In a given CFG, there is a possibility of having useless production rules and useless symbols. so the grammar should be simplified before using.

Steps for simplification of CFG -

- ① Remove Null (ϵ) productions. like $A \rightarrow \epsilon$
- ② Remove unit productions. like $A \rightarrow B$
- ③ Remove Non generating symbols. ie Variables that are not deriving terminal strings.
- ④ Remove non reachable symbols. ie Symbols not appearing in any sentential form.

Q simplify the given CFG.

(1) $S \rightarrow OAO / |B| / BB$
*

$$A \rightarrow C_*$$

$$B \rightarrow S | A_+$$

$$C \rightarrow S | \epsilon$$

step I - Eliminate ϵ production rules.

if $c \rightarrow t$

nullable symbols = ${}^*(C, A, B)$
 $\neq S$

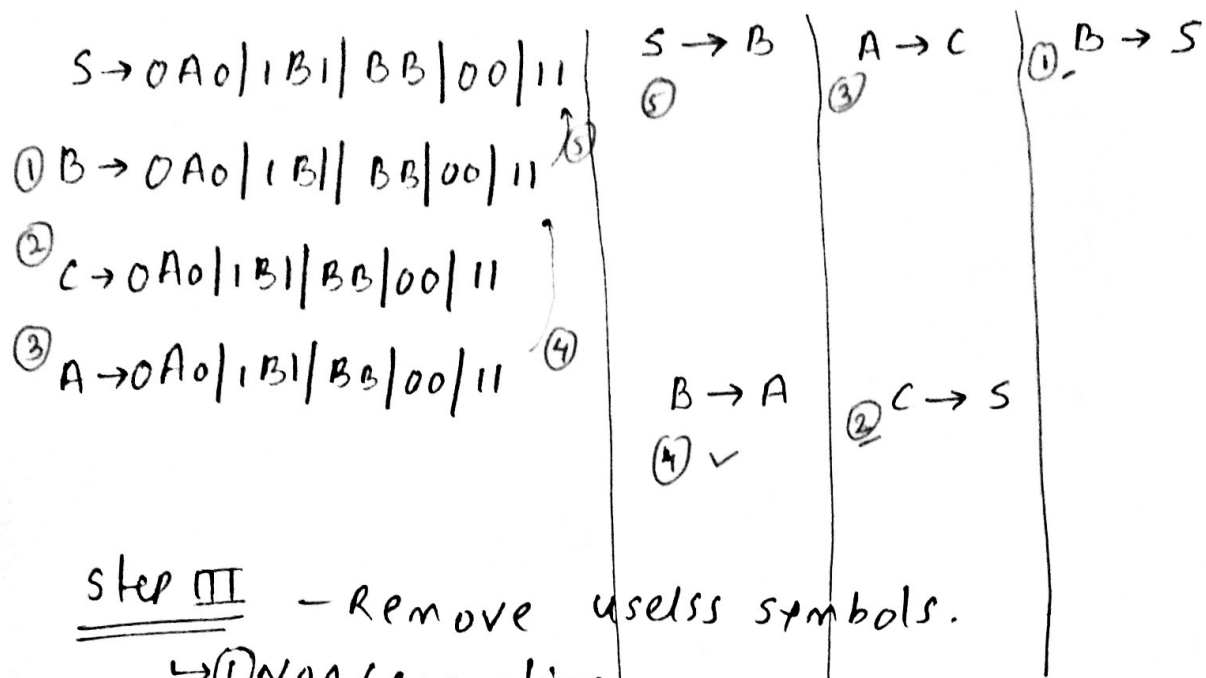
$S \rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11 \mid B$

$$A \rightarrow C$$

$$B \rightarrow S/A$$

$$C \rightarrow S$$

step II - Eliminate unit productions



step III - Remove useless symbols.

↳ (1) Non Generating

Assume generating symbols are
 $\{0, 1\}$

$S \rightarrow 00 \quad C \rightarrow 00$
 $B \rightarrow 11 \quad A \rightarrow 11$

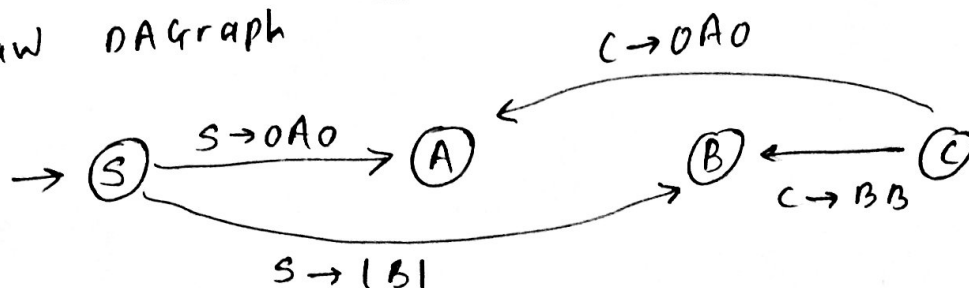
$\{0, 1, S, B, C, A\}$

All are generating symbols.

∴ CFG is unchanged.

↳ (2) Non reachable

draw DAG graph



C is unreachable symbol. remove C

Ans: simplified CFG

$S \rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$
$B \rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$
$A \rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11$