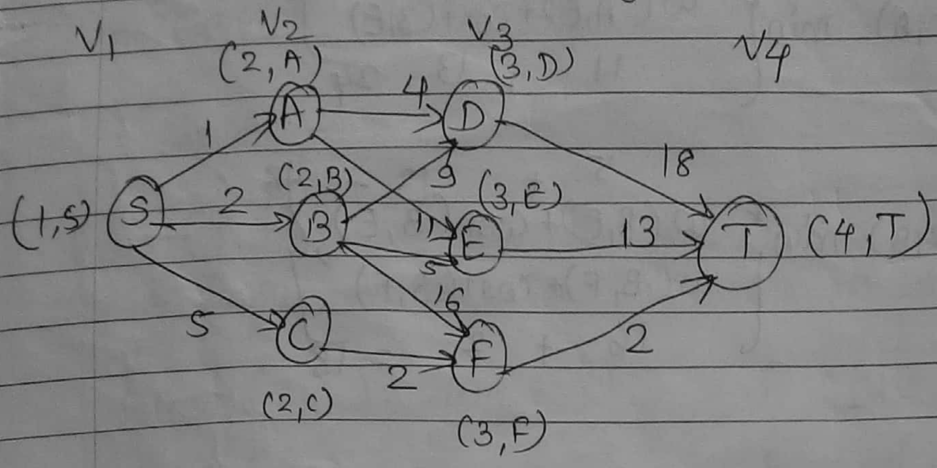
Multistage graph.



dynamic programming is used to solve this problem because sometimes greedy approach fails.

forward approach (for outgoing edges)

1) Cost(i, j) - cost of path from node j in stage i to sink node in k^{th} (last) stage.

Cost(i, j) is given by formula

$\min \{ w(j,l) + cost(i+1, l) \}$ where l is set of vertices adjacent to node j

$Cost(k-1, j) = w(j, T)$ if $(j,T) \in E$

$Cost(k-1, j) = \infty$ if $(j,T) \notin E$

$$Cost(1,S) = \min \begin{cases} \overset{1 + 22}{w(S,A) + Cost(2,A)}, & = 23 \\ \overset{2 + 18}{w(S,B) + Cost(2,B)}, & = 20 \\ \underset{5 + 4}{w(S,C) + Cost(2, C)} & = 9 ✓ \end{cases}$$

2)

$$4 + 18 = 22 \checkmark$$

$$Cost(2,A) = \min\begin{cases} w(A,D) + cost(3,D), \\ w(A,E) + Cost(3,E) \end{cases}$$

$$11 + 13 \quad 24$$

$$5 + 13 = 18$$

$$Cost(2,B) = \min\begin{cases} w(B,E) + Cost(3,E), \\ w(B,F) + Cost(3,F) \end{cases}$$

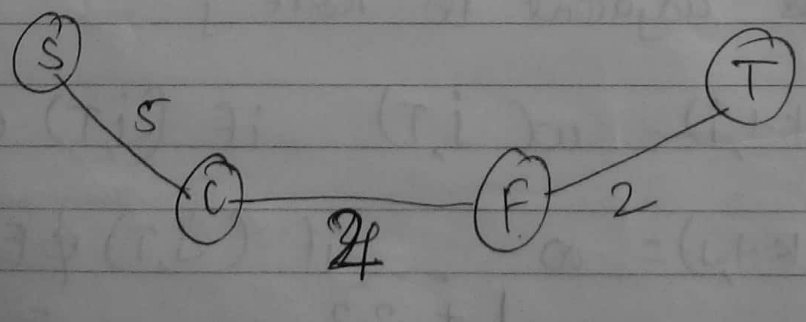
$$16 + 2 = 18$$

$$Cost(2,C) = \min\begin{cases} w(C,F) + Cost(3,F) \\ 2 + 2 \end{cases} = 4$$

$$Cost(3,D) = w(D,T)$$
$$= 18$$

$$Cost(3,E) = w(E,T)$$
$$= 13$$

$$Cost(3,F) = w(F,T)$$
$$= 2$$



Backward approach

sink ⟶ source

BCost(i, j) = Cost of the path from source node s from stage 1 to j^{th} node in stage i

$BCost(2, j) = w(s, j)$ if $s, j \in E$

$BCost(2, j) = \infty$ if $s, j \notin E$

$BCost(i, j) = \min \{ w(l, j) + BCost(i-1, l) \}$
from i-1 stage

l is set of vertices ~~adjacent to j~~ to which j is adjacent.

$$BCost(4, T) = \min \begin{cases} \overset{18}{w(D, T)} + \overset{5}{BCost(3, D)} & = 23 \\ \overset{13}{w(E, T)} + \overset{7}{BCost(3, E)} & = 20 \\ \underset{2}{w(F, T)} + \underset{7}{BCost(3, F)} & = 9 \end{cases}$$

$$BCost(3, D) = \min \begin{cases} \overset{4}{w(A, D)} + \overset{1}{BCost(2, A)} & = 5 \\ \underset{9}{w(B, D)} + \underset{2}{BCost(2, B)} & = 11 \end{cases}$$

$$BCost(3, E) = \min \begin{cases} \overset{5}{w(B, E)} + \overset{2}{BCost(2, B)}, & = 7 ✓ \\ \underset{11}{w(A, E)} + \underset{1}{BCost(2, A)} & = 12 \end{cases}$$

$$BCost(3, F) = \min \begin{cases} \overset{16}{w(B, F)} + \overset{2}{BCost(2, B)}, & = 18 \\ \underset{2}{w(C, F)} + \underset{5}{BCost(2, C)} & = 7 ✓ \end{cases}$$

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$BCost(2, A) = w(S, A) = 1$

$BCost(2, B) = w(S, B) = 2$

$BCost(2, C) = w(S, C) = 5$

$$S \xrightarrow{5} C \xrightarrow{2} F \xrightarrow{2} T = \underline{9}$$