DUAL SIMPLEX METHOD

Steps :

(1) Convert the problem into maximisation type

(2) All constraints are of the type $\leq$

(3) Add slack variables to convert inequalities into equalities.

(4) Compute $c_j - E_j$ for all columns.

Feasibility conditions :

(a) if all $c_j - E_j \leq 0$ and all $b_i \geq 0$, optimal basic feasible solution is optimal basic feasible.

(b) if all $c_j - E_j \leq 0$ and at least one $b_i < 0$, goto step (5)

(c) if any $c_j - E_j > 0$, method fails.

(5) In the 'b' column, select minimum among negatives. Corresponding variable is outgoing variable and the row is key row.

(6) In the key row,

(a) if all elements are ≥ 0, problem does not have a feasible solution.

(b) otherwise, compute 'θ' row, which consists of ratios of $c_j - E_j$ row to key row.

(6) contd.

Corresponding column is key column and the variable in that is the in coming variable. Intersection of key row and key column is a pivotal element.

(7) Make the pivotal entry as '1'. Carry out row operations and repeat iterations until either an optimal feasible solution is reached or there is an indication of non-existance of a feasible solution.

Problem:

minimise $Z = 20x_1 + 16x_2$

subject to

$$x_1 + x_2 \geq 12$$
$$2x_1 + x_2 \geq 17$$
$$x_1 \geq 2.5$$
$$x_2 \geq 6$$
$$x_1, x_2 \geq 0$$

Solution: Performing steps 1, 2 and 3, we have the following:

Maximise $W = -Z = -20x_1 - 16x_2$

subject to
$$-x_1 - x_2 + s_1 = -12$$
$$-2x_1 - x_2 + s_2 = -17$$
$$-x_1 + s_3 = -2.5$$
$$-x_2 + s_4 = -6$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

Dual Simplex Table :

	c_j	-20	-16	0	0	0	0		
e_i	CSV	x_1	x_2	s_1	s_2	s_3	s_4	b	
0	s_1	-1	-1	1	0	0	0	-12	
0	s_2	-2	-1	0	1	0	0	-17	$\rightarrow$
0	s_3	-1	0	0	0	1	0	-2.5	
0	s_4	0	-1	0	0	0	1	-6	
$E_j = \sum_{i=1}^{4} a_{ij} e_i$		0	0	0	0	0	0		
$c_j - E_j$		-20	-16	0	0	0	0		
θ		10 $\uparrow$	16						

<u>NOTE</u> : (1) -17 is most $-$ve in 'b' col^n. s_2 is outgoing. selecting that row,

(2) we compute 'θ' row entries only for $-$ve entries of key row.

(3) Select least $+$ve θ, x_1 is an incoming variable.

-2 is pivotal entry.

(4) $R_2 \rightarrow R_2/(-2)$, $R_1 \rightarrow R_1 + R_2$, $R_3 \rightarrow R_3 + R_2$ [Row Operations]

e_i	CSV	x_1	x_2	s_1	s_2	s_3	s_4	b	
	C_j	−20	−16	0	0	0	0		
0	s_1	0	−1/2	1	−1/2	0	0	−7/2	
−20	x_1	1	1/2	0	−1/2	0	0	17/2	
0	s_3	0	1/2	0	−1/2	1	0	6	
0	s_4	0	−1	0	0	0	1	−6	→
E_j		−20	−10	0	10	0	0		
$C_j - E_j$		0	−6	0	−10	0	0		
θ			12						
			↑						

$\begin{cases} R_4 \to R_4(-1) \\ R_1 \to R_1 + \frac{1}{2}R_4, \quad R_2 \to R_2 - \frac{1}{2}R_4, \quad R_3 \to R_3 - \frac{1}{2}R_4 \end{cases}$

e_i	CSV	x_1	x_2	s_1	s_2	s_3	s_4	b	
	C_j	−20	−16	0	0	0	0		
0	s_1	0	0	1	−1/2	0	−1/2	−1/2	→
−20	x_1	1	0	0	−1/2	0	1/2	11/2	
0	s_3	0	0	0	−1/2	1	1/2	3	
−16	x_2	0	1	0	0	0	−1	6	
E_j		−20	−16	0	10	0	6		
$C_j - E_j$		0	0	0	−10	0	−6		
θ					20		12		
							↑		

$R_1 \to R_1(-2), \quad R_2 \to R_2 - \frac{1}{2}R_1, \quad R_3 \to R_3 - \frac{1}{2}R_1,$

$R_4 \to R_4 + R_1$

	c_j	-20	-16	0	0	0	0	
e_i	CSV	x_1	x_2	s_1	s_2	s_3	s_4	b
0	s_4	0	0	-2	1	0	1	1
-20	x_1	1	0	1	-1	0	0	5
0	s_3	0	0	1	-1	1	0	5/2
-16	x_2	0	1	-2	1	0	0	7
E_j		-20	-16	12	4	0	0	
$c_j - E_j$		0	0	-12	-4	0	0	

All $c_j - E_j$ are ≤ 0 & all b_i are ≥ 0

$\therefore$ Optimal (& feasible) sol^n is given by

$x_1 = 5$, $x_2 = 7$, $W_{max} = -20(5) - 16(7)$
$= -212$

$\therefore Z_{min} = 212$.

<u>NOTE</u> : In dual simplex method, $c_j - E_j$ are ≤ 0 in every table. Continue iterations till all b_i are ≥ 0.

— x — x — x —