Module 4.0
Communication Fundamentals

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Electronic Circuits & Communication Fundamentals
ECCF (CSC 304) for S.E. (Computer) – Semester III

What is Modulation ?

- Low frequency baseband signal needs shift in frequencies for transmission over channel
- Achieved by superimposing low frequency baseband signal on high frequency carrier
- Shift of low frequency baseband signal to a high frequency carrier signal is modulation
- Done such that one of the characteristics of the carrier varies with the baseband signal
- This is done to retain the characteristics of the baseband signal over the carrier signal

Advantages of Modulation ?

Modulating high frequency carrier signal with low frequency baseband signal :-

- Increases range of communication
- Reduction in the antenna height
- Allows multiplexing of signals
- Avoids mixing of signals
- Improves quality of reception

Advantages of Modulation ?

1. Increases Range of Communication :-

- Frequency of baseband signal is very low & cannot travel long distances by themselves
- When such signals are transmitted, they get heavily attenuated resulting in losses
- Attenuation of signal reduces with increase in frequency, hence can travel long distance
- Thus modulation process increases range of communication by upward frequency shift

Advantages of Modulation ?

2. Reduction in the Height of the Antenna :-

- For efficient transmission of signal, height of antenna should be at least kept at $\lambda/4$
- Here $\lambda = c/f$ where ‘c’ is the velocity of light & ‘f’ is frequency of transmitted signal
- For f = 15 kHz we get λ = 20 km giving height of h = 5 km which is impossible to construct
- For f = 10 MHz we get λ = 30 m giving height of h = 7.5 m which can be easily constructed

Advantages of Modulation ?

3. Allows Multiplexing of Signals :-

- Multiplexing is simultaneous transmission of 2 or more signals over the same carrier
- This is only possible with modulation where carriers can have different high frequencies
- Various baseband signals in audio range of 20 Hz – 20 kHz can easily be transmitted
- For wireless transmission like AM/FM this is helpful in broadcasting several frequencies

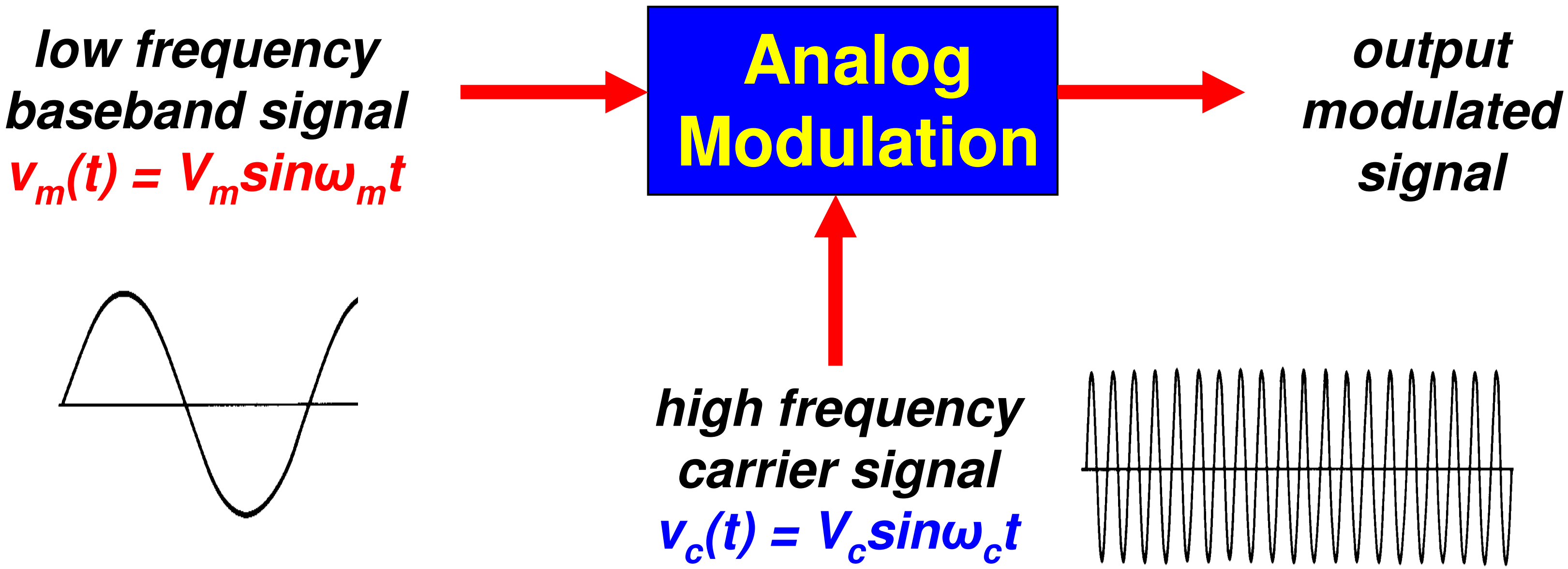
Advantages of Modulation ?

4. Avoids Mixing of Signals :-

- Baseband signals are in the audio frequency range between 20 Hz – 20 kHz
- If they are transmitted directly without any modulation, they will get all mixed up
- Due to this the receiver will not be able to separate out these baseband signals
- Modulation gives them all different carrier frequencies for the transmission process

Definition of Modulation

- Low frequency baseband signal represented by $v_m(t) = V_m \sin\omega_m t$ OR $v_m(t) = V_m \cos\omega_m t$
- High frequency carrier signal is represented by $v_c(t) = V_c \sin\omega_c t$ OR $v_c(t) = V_c \cos\omega_c t$



Definition of Modulation

Analog Modulation is defined as a process in which **one** of the **parameters** (characteristics) of a high frequency **carrier** signal **(amplitude, frequency or phase)** is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping other parameters constant.

- Carrier Amplitude – $V_c \alpha v_m(t)$
- Carrier Frequency – $f_c \alpha v_m(t)$
- Carrier Phase – $\theta_c \alpha v_m(t)$

one of them varies while two others remain constant

Types of Analog Modulation

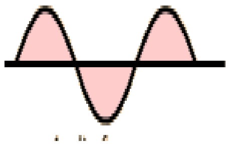
- **Amplitude Modulation (AM) where the carrier amplitude (V_c) varies**
- **Frequency Modulation (FM) where the carrier frequency (ω_c or f_c) varies**
- **Phase Modulation (PM) where phase of the carrier (θ_c) varies**

Amplitude Modulation (AM)

Amplitude Modulation is defined as process in which the **amplitude** of high frequency **carrier** signal is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping **phase** & **frequency** of the **carrier** signal **constant.**

low frequency baseband signal

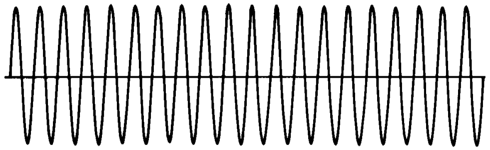
$v_m(t) = V_m \sin\omega_m t$



Amplitude Modulation

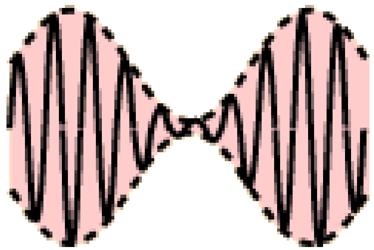
high frequency carrier signal

$v_c(t) = V_c \sin\omega_c t$



output amplitude modulated signal

$v_{AM}(t)$



Modulation Index (m_a)

Modulation Index also called as the modulation factor or modulation coefficient (m_a) refers to the depth of modulation of the carrier signal amplitude (V_c) by the instantaneous amplitude of modulating signal $v_m(t)$ & is the simply ratio of peak values of modulating signal amplitude (V_m) to the carrier signal amplitude (V_c)

$$\boxed{m_a = \frac{V_m}{V_c}}$$

mathematical definition of modulation index (m_a)

Modulation Index (m_a)

Modulation Index (m_a) for AM waveform takes one of the following values :-

- When $V_m = 0$ then $m_a = 0$ indicating case of zero (no) modulation being performed
- When $V_m < V_c$ then $m_a < 1$ indicating case of under modulation being performed
- When $V_m = V_c$ then $m_a = 1$ indicating case of critical modulation being performed
- When $V_m > V_c$ then $m_a > 1$ indicating case of over (excess) modulation being performed

Optimum Modulation Index (m_a)

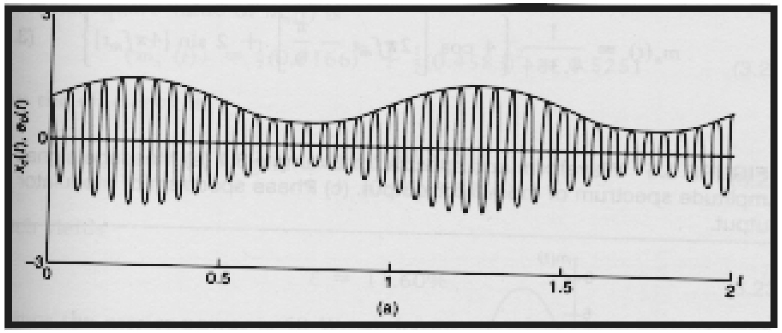
Modulation Index (m_a) for AM waveform simply is a parameter indicating depth of modulation

<u>After modulation it is also important to recover modulating signal from amplitude modulated (AM) waveform by removing the carrier signal</u>

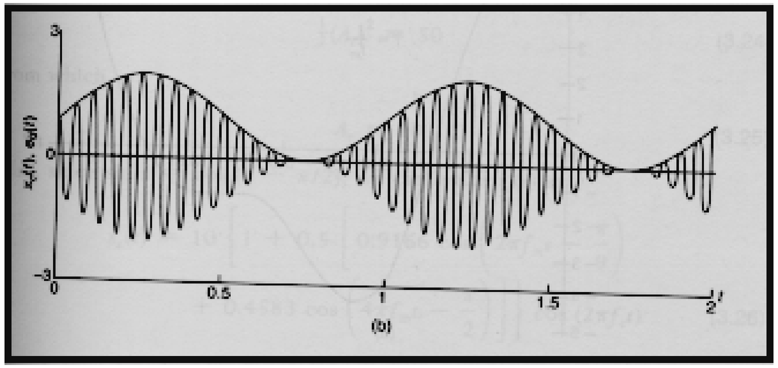
Hence the optimum modulation index (m_a) for an AM waveform should be maintained at :-

$\boxed{m_a \leq 1}$ → *important so that modulating signal can be easily recovered*

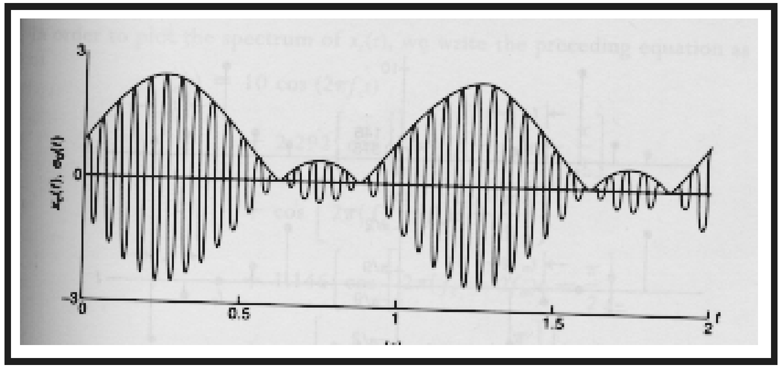
Modulation Index (m_a) Values



$m_a < 1$ hence AM wave is undermodulated



$m_a = 1$ hence AM wave is critically modulated

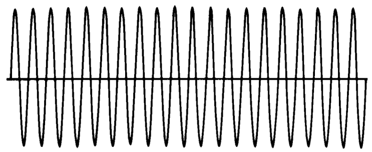


$m_a > 1$ hence AM wave is over / excess modulated

Mathematical Equation of AM wave

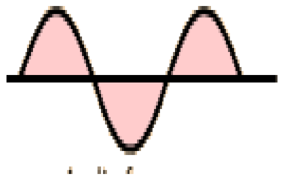
Carrier signal is mathematically represented by the following equation :-

$$v_c(t) = V_c \sin\omega_c t \text{ OR } v_c(t) = V_c \cos\omega_c t$$



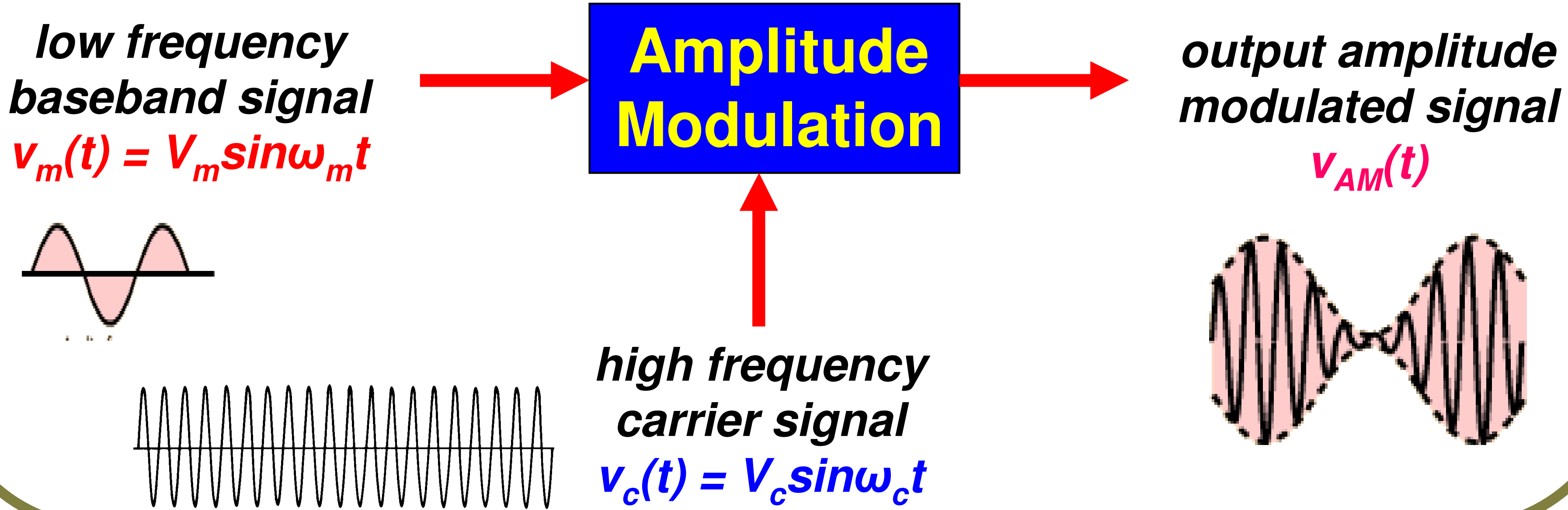
Modulating signal is mathematically denoted by the following equation :-

$$v_m(t) = V_m \sin\omega_m t \text{ OR } v_m(t) = V_m \cos\omega_m t$$



Mathematical Equation of AM wave

Amplitude Modulation is defined as process in which the **amplitude** of high frequency **carrier** signal is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping **phase** & **frequency** of the **carrier** signal **constant.**



Mathematical Equation of AM wave

Amplitude modulated (AM) waveform is mathematically represented by :-

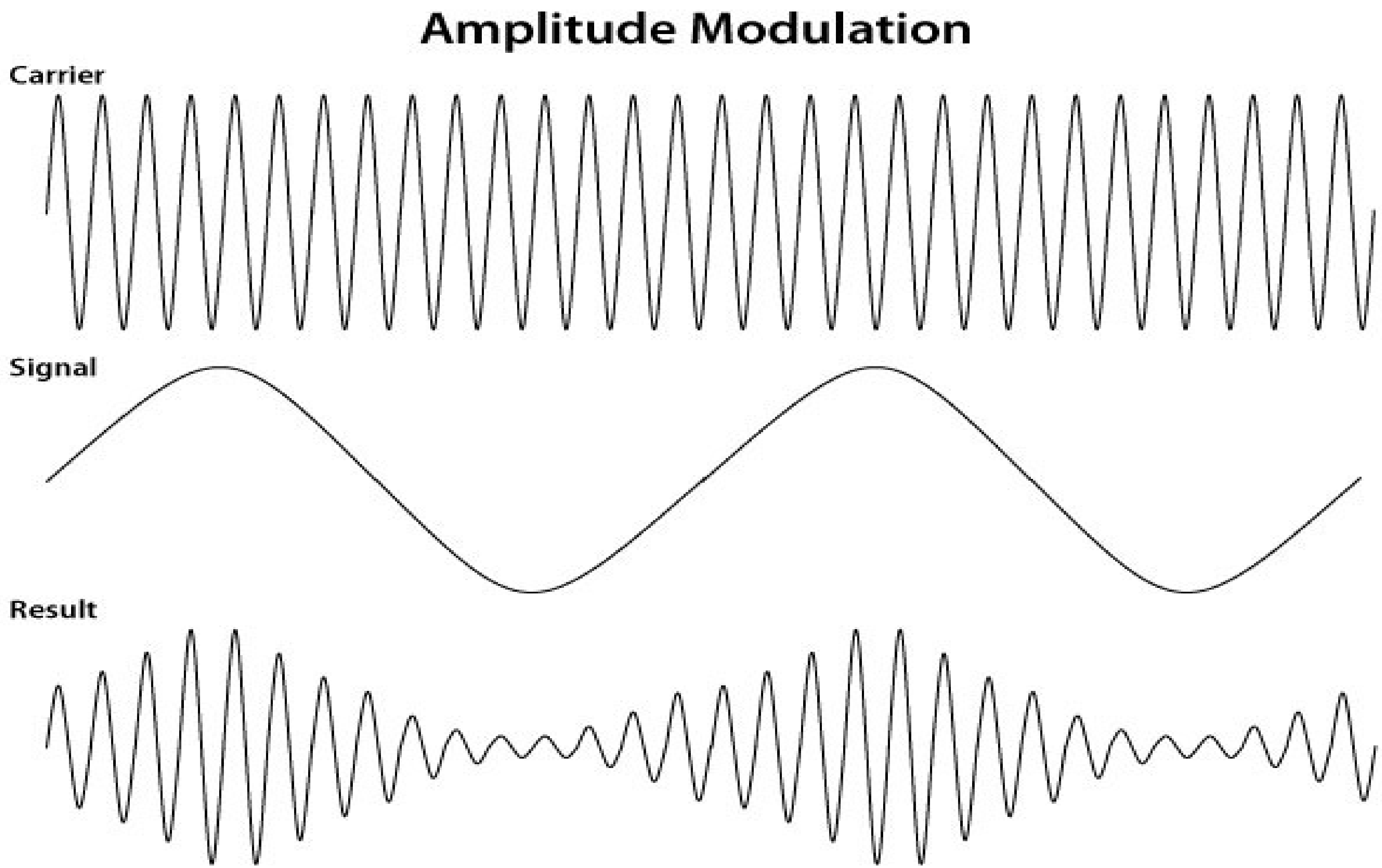
$$v_{AM}(t) = V_c\,[\,1 + m_a \sin\omega_m t\,]\,\sin\omega_c t$$

OR

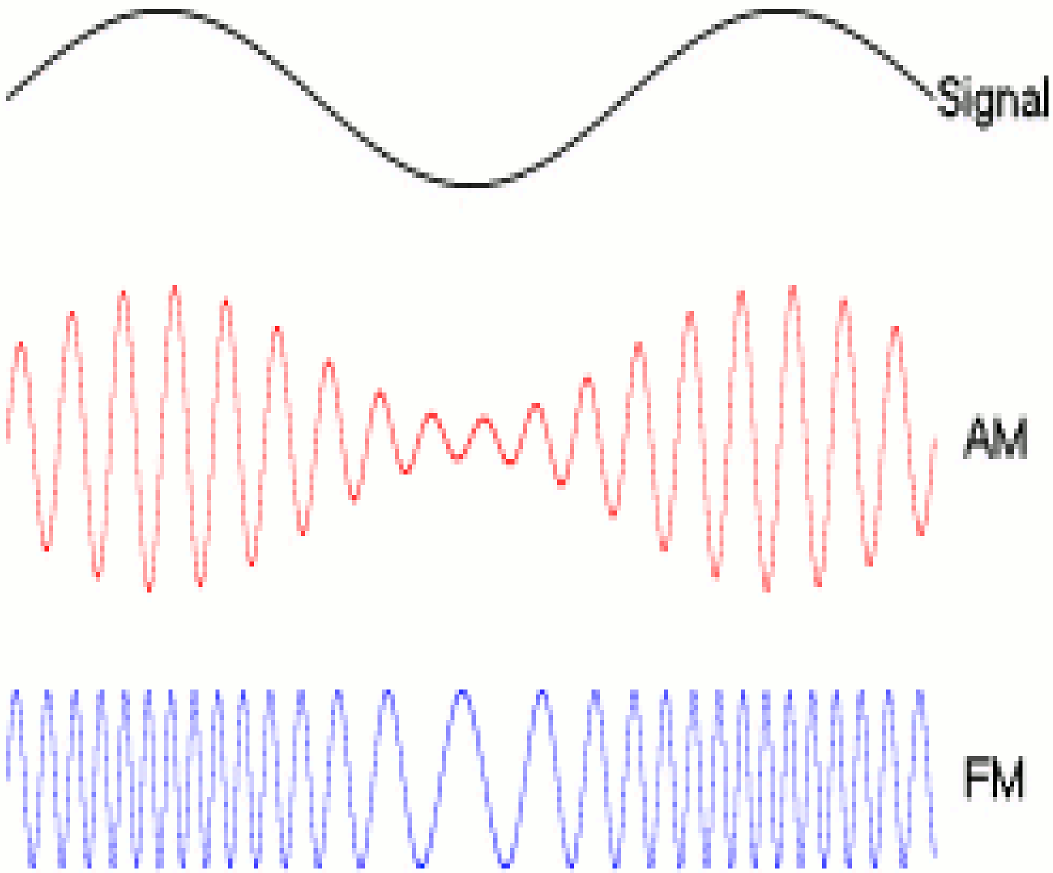
$$v_{AM}(t) = V_c\,[\,1 + m_a \cos\omega_m t\,]\,\cos\omega_c t$$

please refer to your notebook for the complete derivation

Graphical Representation of AM wave



Graphical Representation of AM wave

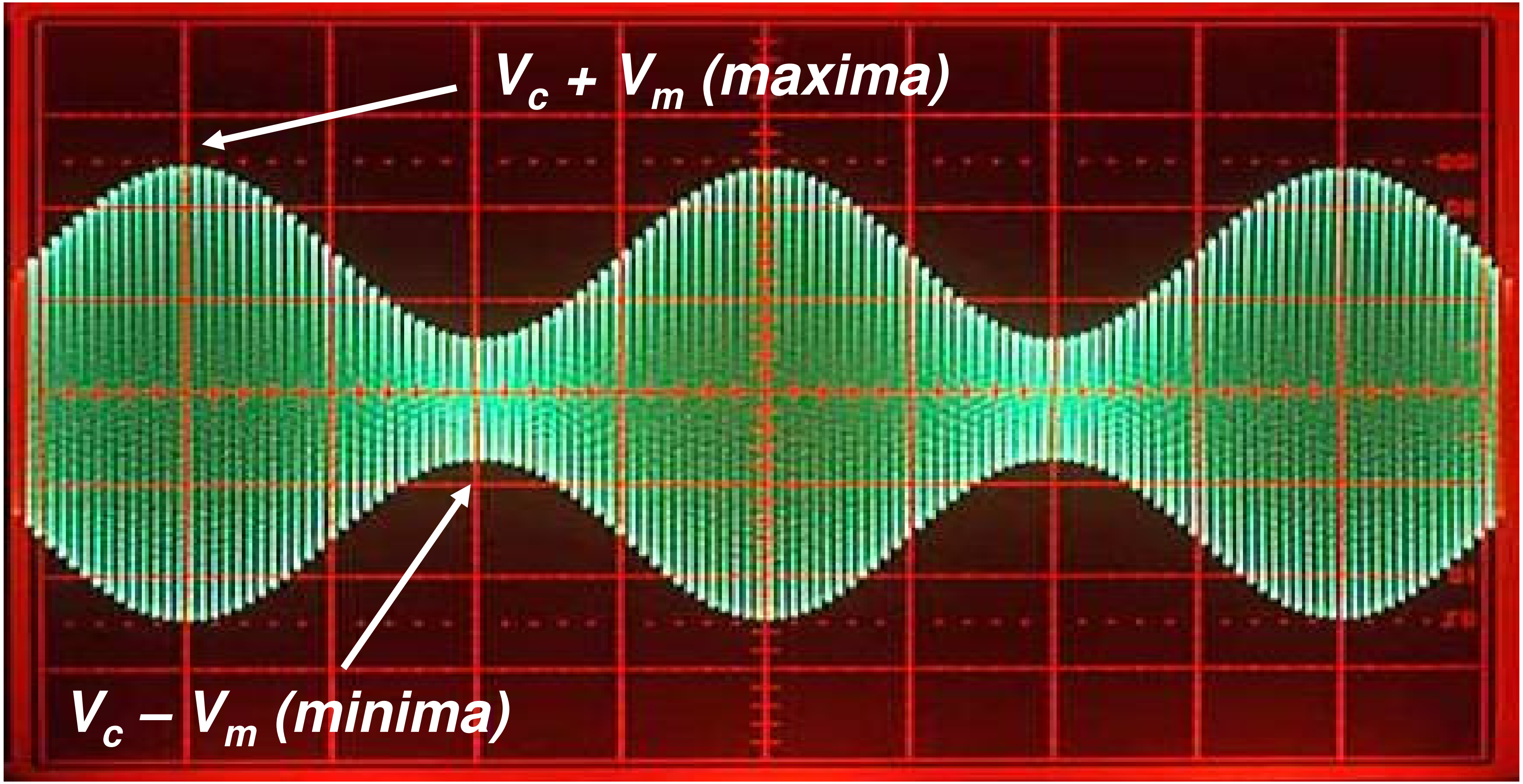


modulating or baseband signal

amplitude variations in V_c due to $v_m(t)$

frequency variations in ω_c due to $v_m(t)$

Graphical Representation of AM wave



Amplitude Modulated (AM) wave on CRO

Frequency Spectrum of AM wave

Amplitude modulated (AM) waveform is thus mathematically represented by :-

$$v_{AM}(t) = V_c\,[\,1 + m_a \sin\omega_m t\,]\,\sin\omega_c t$$

OR

$$v_{AM}(t) = V_c\,[\,1 + m_a \cos\omega_m t\,]\,\cos\omega_c t$$

please refer to your notebook for the complete derivation

Frequency Spectrum of AM wave

Amplitude modulated (AM) waveform is thus mathematically represented by :-

$v_{AM}(t) = V_c \cos \omega_c t$ → *carrier frequency component*

$+$

$\dfrac{m_a V_c}{2} \cos(\omega_c + \omega_m)t$ → *upper side band (USB)*

$+$

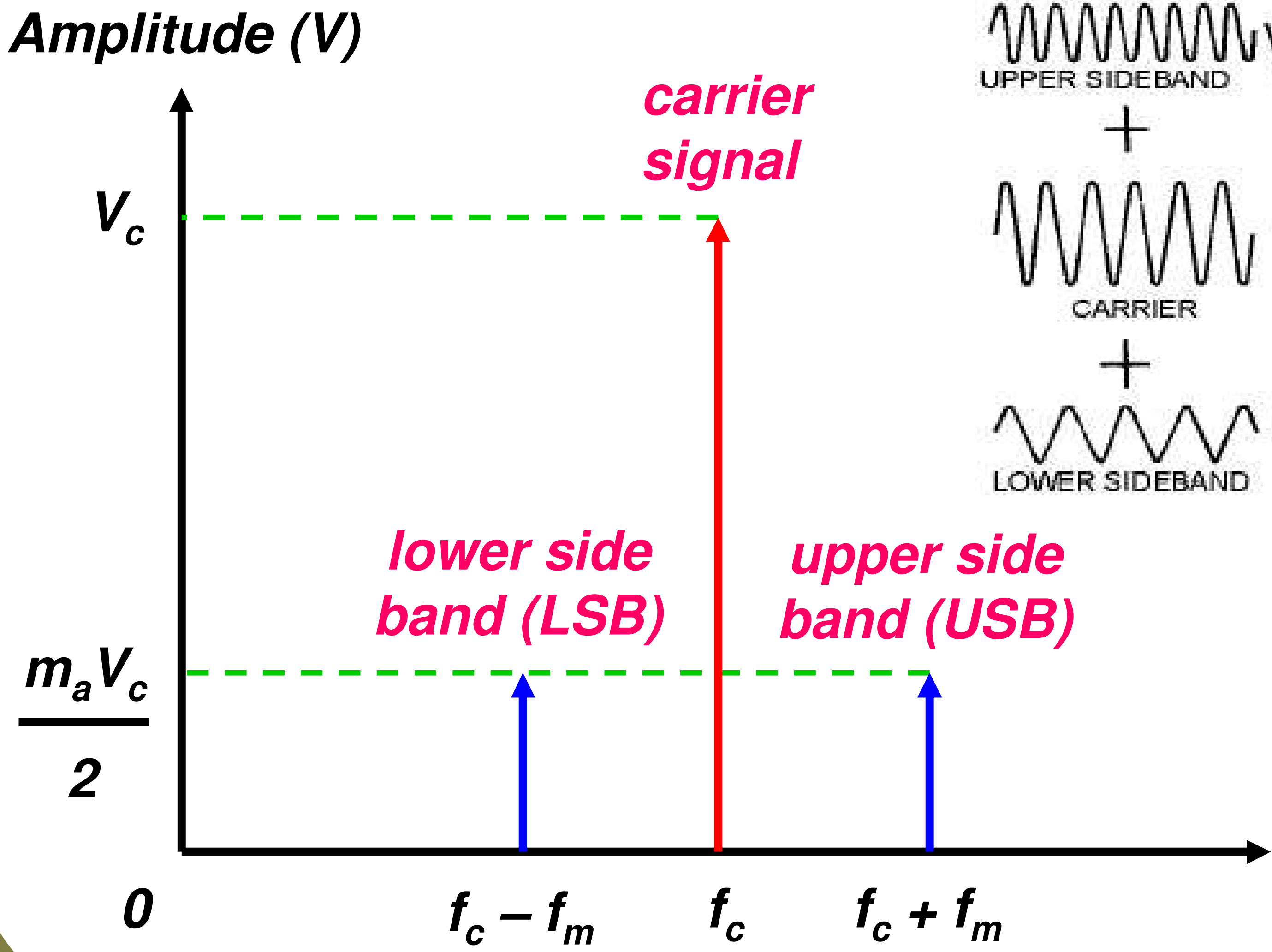
$\dfrac{m_a V_c}{2} \cos(\omega_c - \omega_m)t$ → *lower side band (LSB)*

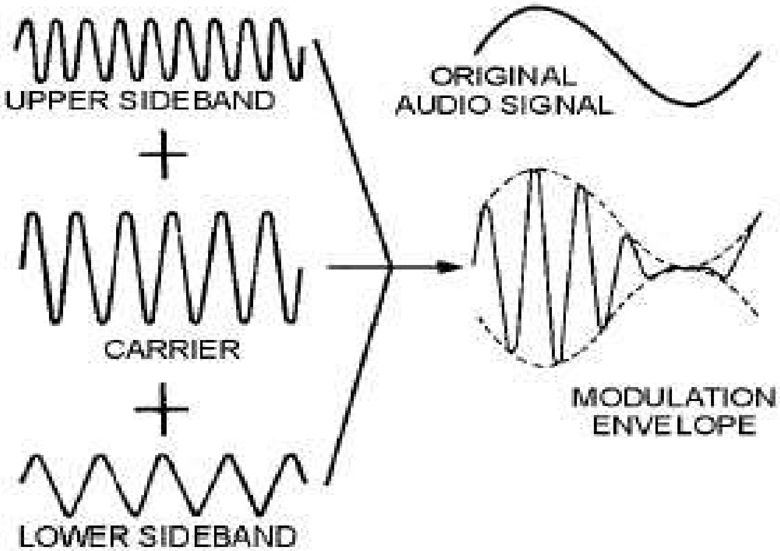
Frequency Spectrum of AM wave

Amplitude modulated (AM) waveform is thus composed of the following components :-

- **Carrier Signal itself, having an amplitude of V_c & frequency of f_c Hz**
- **Upper Sideband (USB) having amplitude of $m_a V_c/2$ & a frequency of $f_c + f_m$ Hz**
- **Lower Sideband (LSB) having amplitude of $m_a V_c/2$ & a frequency of $f_c - f_m$ Hz**

Frequency Spectrum of AM wave





Power Distribution in AM Wave

Amplitude modulated (AM) waveform is thus mathematically represented by :-

$$v_{AM}(t) = V_c \cos\omega_c t$$ → *carrier frequency component*

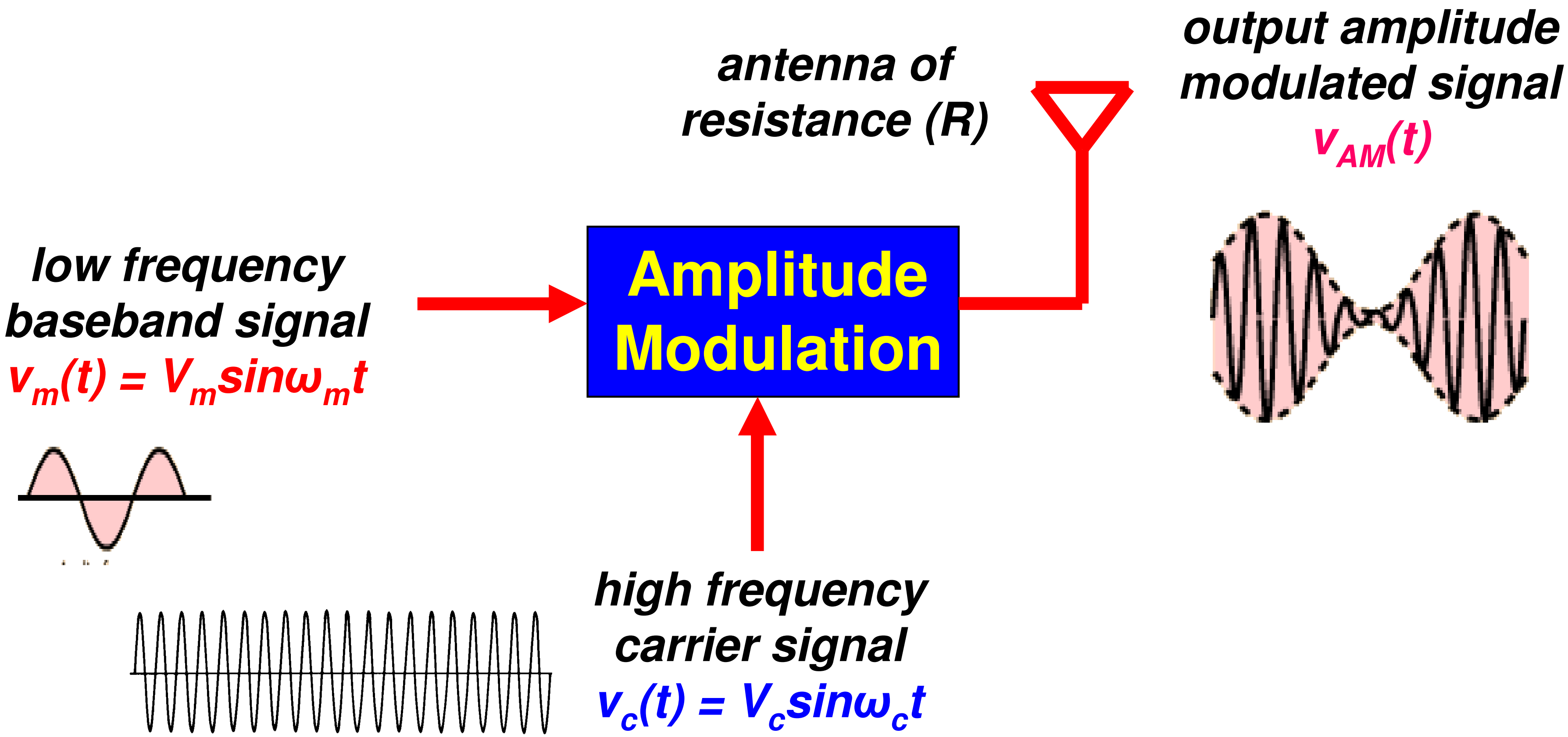
$+$

$$\frac{m_a V_c \cos(\omega_c + \omega_m)t}{2}$$ → *upper side band (USB)*

$+$

$$\frac{m_a V_c \cos(\omega_c - \omega_m)t}{2}$$ → *lower side band (LSB)*

Power Distribution in AM Wave



Power Distribution in AM Wave

Power contained (P_T) in amplitude modulated (AM) waveform is the total sum of :-

1. **Power in the carrier signal (P_C)**
2. **Power in the upper sideband (P_{USB})**
3. **Power in the lower sideband (P_{LSB})**

$$\boxed{P_T = P_C + P_{USB} + P_{LSB}}$$

Power Distribution in AM Wave

Carrier Power (P_C) :- $$P_C = \frac{V_c^2}{2R}$$ → 2

USB Power (P_{USB}) :- $$P_{USB} = \frac{m_a^2 V_c^2}{8R}$$ → 3

LSB Power (P_{LSB}) :- $$P_{LSB} = \frac{m_a^2 V_c^2}{8R}$$ → 4

Power Distribution in AM Wave

Power in sidebands (P_{USB} & P_{LSB}) can easily be expressed in terms of carrier power (P_C) :-

$$\boxed{P_{LSB} = P_{USB} = \left(\frac{m_a^2}{4}\right)\left(\frac{V_c^2}{2R}\right) = \left(\frac{m_a^2}{4}\right)P_C}$$

please refer to your class notes for the derivation

Power Distribution in AM Wave

The total power (P_T) can now be expressed by the following equation as shown below :-

$$P_T = P_C + P_{LSB} + P_{USB} \longrightarrow 6$$

$$P_T = P_C + \left(\frac{m_a^2}{4}\right)P_C + \left(\frac{m_a^2}{4}\right)P_C \longrightarrow 7$$

$$P_T = P_C + \left(\frac{m_a^2}{2}\right)P_C \longrightarrow 8$$

Power Distribution in AM Wave

The total power (P_T) can now be expressed by the following equation as shown below :-

$$\underbrace{\boxed{P_T = P_C\left(1+\frac{m_a^2}{2}\right)}}$$

please refer to your class notes for the derivation

Power Distribution in AM Wave

$$\frac{P_T}{P_C} = 1 + \frac{m_a^2}{2}$$

$$\frac{m_a^2}{2} = \frac{P_T}{P_C} - 1$$

$$m_a^2 = 2\left(\frac{P_T}{P_C} - 1\right)$$

Representation of modulation index (m_a) in terms of the total power (P_T) & carrier power (P_C)

Modulation Index (m_a) calculation

Modulation Index (m_a) can be calculated from the power equation as shown below :-

$$\boxed{m_a = \sqrt{2\left(\frac{P_T}{P_C} - 1\right)}}$$

please refer to your class notes for the derivation

Transmission Efficiency (η) in AM

- **Transmission efficiency (η) is defined as the ratio of the useful power to the total power**
- **In AM, frequency spectrum consists of both sidebands (USB & LSB) with carrier signal**
- **Information (modulating signal) is present in only both sidebands & NOT in carrier signal**
- **Hence useful transmitted power is only that of the sidebands (P_{USB} & P_{LSB})**
- **Carrier transmission results only in wastage of power (P_C) since NONE of it is useful**

Transmission Efficiency (η) in AM

Transmission Efficiency (η) is expressed in the context of definition as :-

$$\eta = \frac{\text{Useful Power in AM}}{\text{Total Power in AM}}$$

$$\boxed{\eta = \frac{P_{USB} + P_{LSB}}{P_T}}$$

basically defined as ratio of the sidebands power to total power

Transmission Efficiency (η) in AM

$$\eta = \frac{P_{USB} + P_{LSB}}{P_T} \quad \Rightarrow \quad \eta = \frac{\left(\frac{m_a^2}{4}\right)P_C + \left(\frac{m_a^2}{4}\right)P_C}{P_C\left(1+\frac{m_a^2}{2}\right)}$$

please refer to your class notes

$$\eta = \frac{\left(\frac{m_a^2}{2}\right)P_C}{P_C\left(1+\frac{m_a^2}{2}\right)} \quad \Rightarrow \quad \eta = \frac{\left(\frac{m_a^2}{2}\right)}{\left(1+\frac{m_a^2}{2}\right)}$$

Transmission Efficiency (η) in AM

Transmission Efficiency (η) is expressed in the terms of modulation index (m_a) as :-

$$\boxed{\eta = \frac{m_a^2}{2 + m_a^2}}$$

please refer to your class notes for the derivation

Transmission Efficiency (η) in AM

Transmission Efficiency (η) is expressed in the terms of modulation index (m_a) as :-

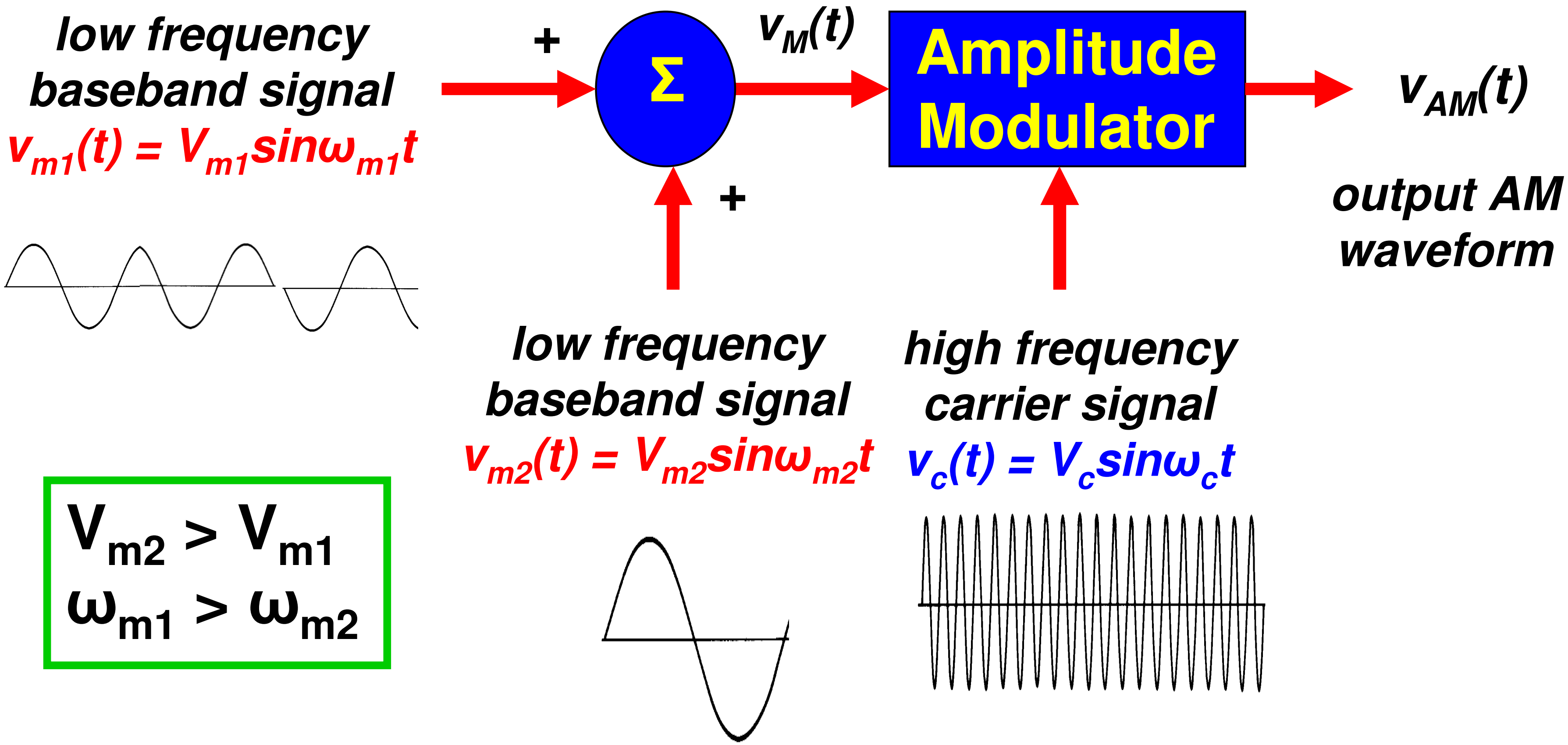
$$\eta = \frac{m_a^2}{2 + m_a^2}$$

assume m_a = 1 for critical modulation

$$\eta = \frac{(1)^2}{2+(1)^2}$$

$$\boxed{\eta = \frac{1}{3} = 33.33\%}$$

AM of Multiple Baseband Signals



refer to your class notes for complete description, derivation & analysis of the frequency spectrum

AM of Multiple Baseband Signals

Overall equation of the AM waveform is given by the following as shown below :-

$$\boxed{v_{AM}(t) = V_c[1 + m_{a1} cos\,\omega_{m1} t + m_{a2} cos\,\omega_{m2} t] cos\,\omega_c t}$$

refer to your class notes for complete description, derivation & analysis of the frequency spectrum

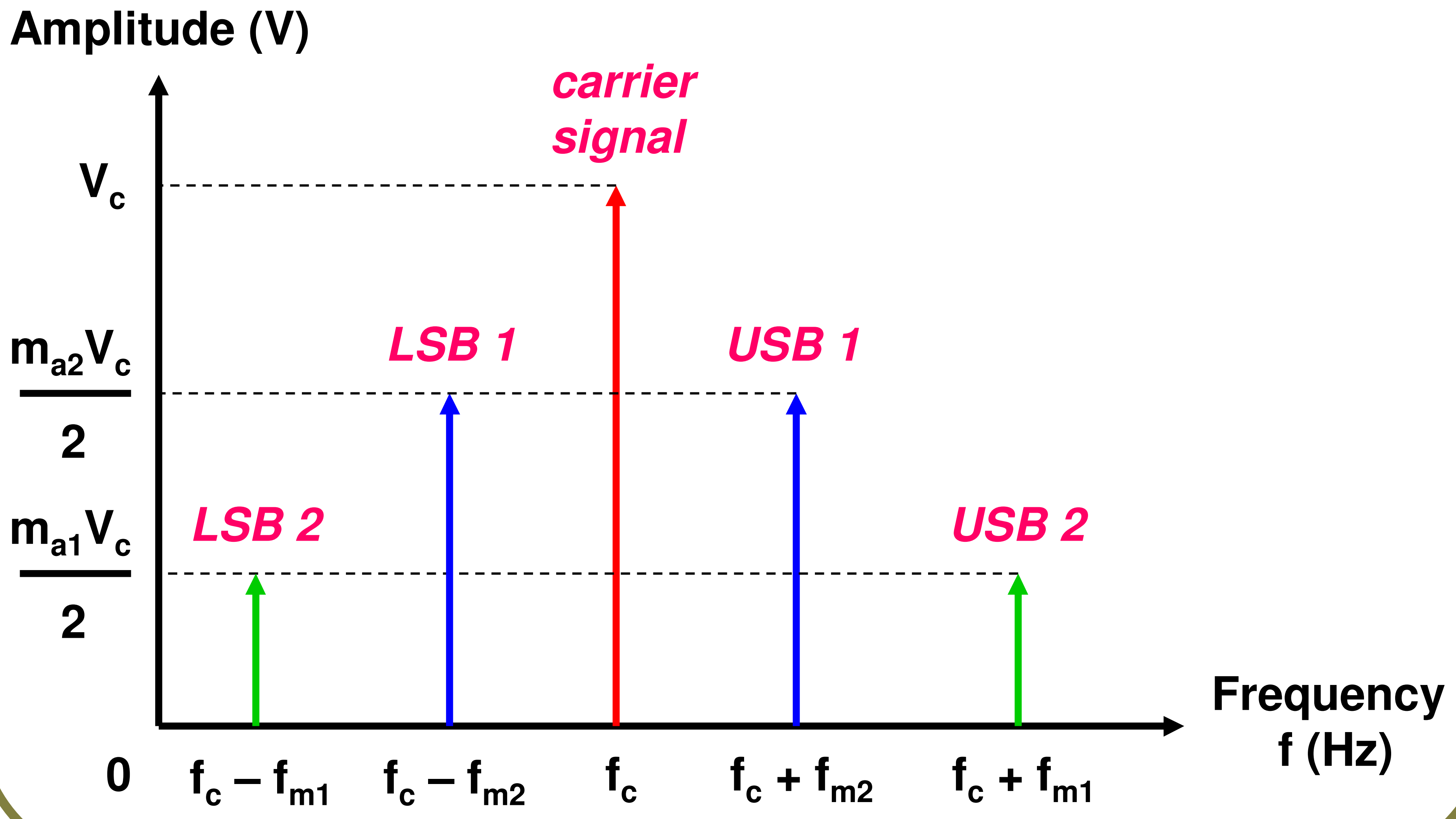
AM of Multiple Baseband Signals

Frequency spectrum representation of the AM waveform is given by following equation :-

$$v_{AM}(t) = V_c \cos\omega_c t + \frac{m_{a1}V_c}{2}\left[\cos(\omega_c + \omega_{m1})t + \cos(\omega_c - \omega_{m1})t\right]$$
$$+ \frac{m_{a2}V_c}{2}\left[\cos(\omega_c + \omega_{m2})t + \cos(\omega_c - \omega_{m2})t\right]$$

refer to your class notes for complete description, derivation & analysis of the frequency spectrum

Frequency Spectrum of AM wave



AM of Multiple Baseband Signals

Overall modulation index (m_T) is expressed in terms of individual modulation index values :-

$$\boxed{m_T = \sqrt{m_{a1}^2 + m_{a2}^2 + m_{a3}^2 + \ldots}}$$

please refer to your class notes for the derivation

AM of Multiple Baseband Signals

The total power (P_T) can now be expressed by the following equation as shown below :-

$$\boxed{P_T = P_C\left(1+\frac{m_T^2}{2}\right)}$$

please refer to your class notes for the derivation

Types of Amplitude Modulation (AM)

- Dual Sideband Full Carrier (DSBFC)
- Dual Sideband Suppressed Carrier (DSBSC)
- Single Sideband Modulation (SSB)

Please refer to your class notebook for mathematical analysis & derivations

How is DSBSC better than DSBFC ?

ADVANTAGES OF DSBSC OVER DSBFC :-

1. Power is wasted as the carrier contains no information and each of sideband carries the same information independently
2. The double sideband suppressed carrier (DSBSC) is introduced to eliminate the carrier & hence improve the power efficiency
3. It is a technique in which it is transmitting both the sidebands without the carrier (the carrier is being suppressed or removed)

Total Power Saved in DSBSC

Total power saved in DSBSC is expressed in the context of definition as :-

$$PS = \frac{\text{Power Saved in AM}}{\text{Total Power in AM}}$$

$$\boxed{PS = \frac{P_C}{P_T}}$$

basically defined as ratio of the carrier power to total power

Total Power Saved in DSBSC

$$PS = \frac{P_C}{P_T} \Rightarrow PS = \frac{P_C}{P_C\left[1 + \frac{m_a^2}{2}\right]}$$

please refer to your class notes

$$PS = \frac{1}{\left[1 + \frac{m_a^2}{2}\right]} \Rightarrow PS = \frac{2}{2 + m_a^2}$$

Total Power Saved in DSBSC

Total Power Saved (PS) is expressed in terms of the modulation index (m_a) as :-

$$\boxed{PS = \frac{2}{2 + m_a^2}}$$

please refer to your class notes for the derivation

Total Power Saved in DSBSC

Total Power Saved (PS) is expressed in terms of the modulation index (m_a) as :-

$$PS = \frac{2}{2 + m_a^2}$$

assume m_a = 1 for critical modulation

$$PS = \frac{2}{2+(1)^2}$$

$$\boxed{PS = \frac{2}{3} = 66.67\%}$$

How is SSB better than DSBSC ?

ADVANTAGES OF SSB OVER DSBSC :-

1. Power saving is higher in SSB as compared to DSBSC, only one sideband is transmitted
2. Bandwidth reduces to half, hence compared to DSBSC, bandwidth of SSB is only f_m Hz
3. As bandwidth reduces to half, more number of channels can now be easily transmitted
4. This gives an advantage in saving power & bandwidth compared to DSBSC & DSBFC

Total Power Saved in SSB (SC)

Total power saved in SSB (SC) is expressed in the context of definition as :-

$$PS = \frac{Power\ Saved\ in\ AM}{Total\ Power\ in\ AM}$$

$$\boxed{PS = \frac{P_C + P_{SB}}{P_T}}$$

defined as ratio of carrier + sideband power to total power

Total Power Saved in SSB (SC)

$$PS = \frac{P_C + P_{SB}}{P_T}$$

$$PS = \frac{P_C + \left[\frac{m_a^2}{4}\right] P_C}{P_C\left[1 + \frac{m_a^2}{2}\right]}$$

please refer to your class notes

$$PS = \frac{1 + \frac{m_a^2}{4}}{1 + \frac{m_a^2}{2}}$$

$$PS = \frac{4 + m_a^2}{2(2 + m_a^2)}$$

Total Power Saved in SSB (SC)

Total Power Saved (PS) is expressed in terms of the modulation index (m_a) as :-

$$\boxed{PS = \frac{4 + m_a^2}{2(2 + m_a^2)}}$$

please refer to your class notes for the derivation

Total Power Saved in SSB (SC)

Total Power Saved (PS) is expressed in terms of the modulation index (m_a) as :-

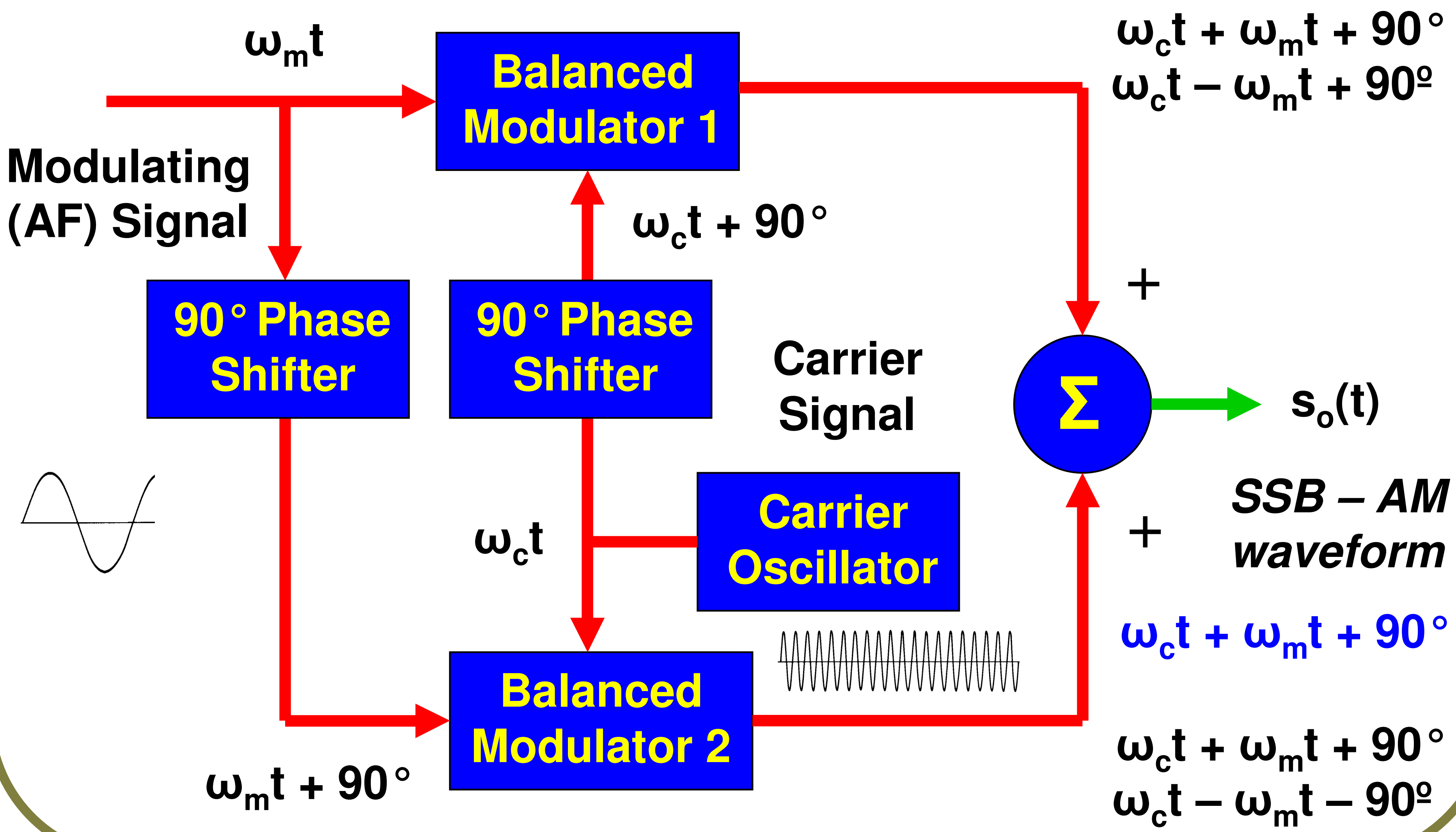
$$PS = \frac{4 + m_a^2}{2(2 + m_a^2)}$$

assume $m_a = 1$ for critical modulation

$$PS = \frac{4 + (1)^2}{2[2 + (1)^2]}$$

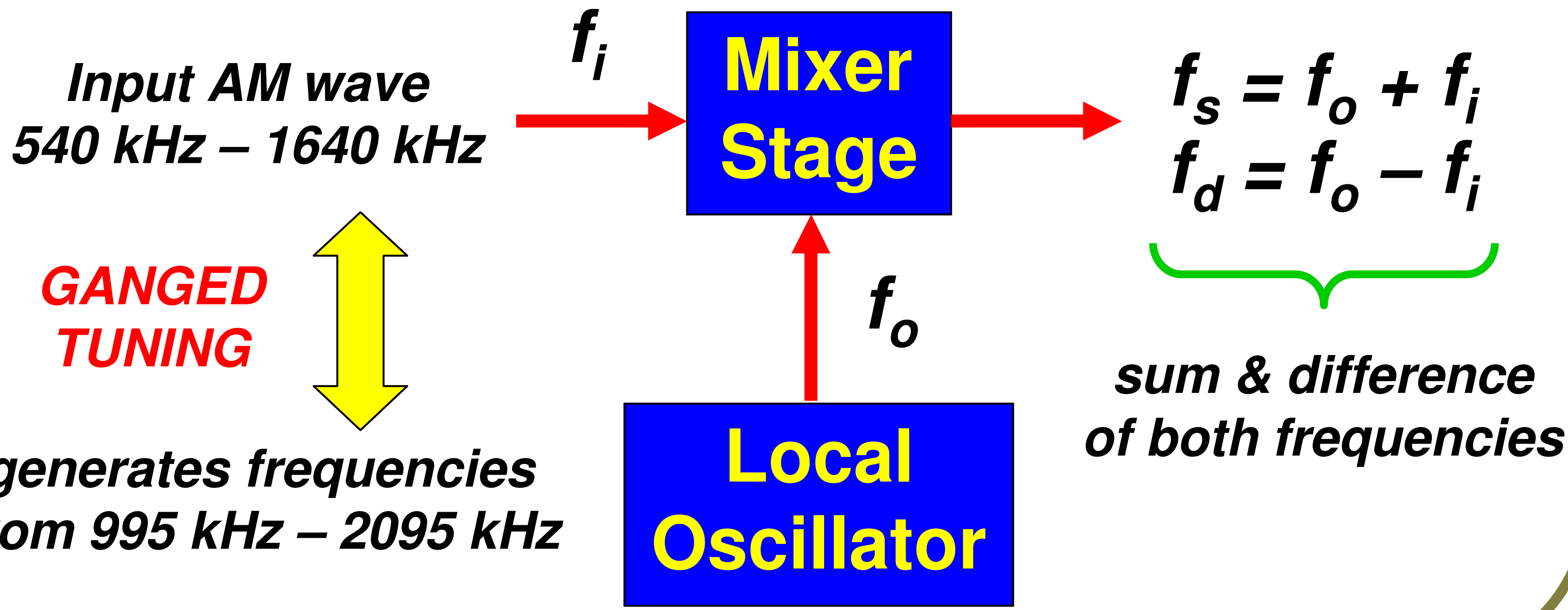
$$\boxed{PS = \frac{5}{6} = 83.33\%}$$

Single Sideband (SSB) Generation – The Phase Shift Method

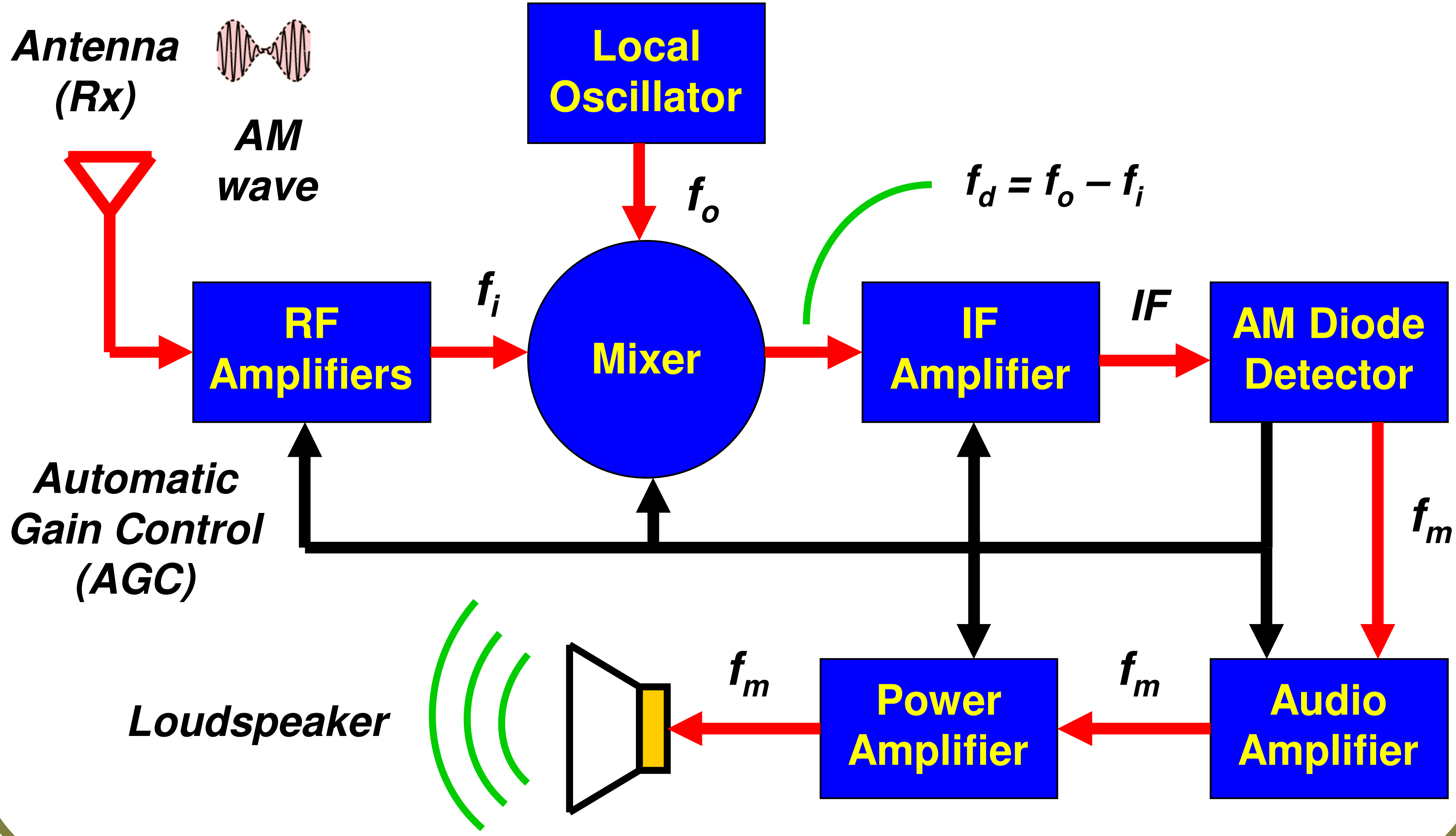


The Superhetrodyne Principle

The superhetrodyne process involves mixing of two different frequency signals such that the output is a sum & difference of both the input signal frequencies



The AM Superhetrodyne Receiver



The AM Superhetrodyne Receiver

Block Diagram Description :-

- Input RF amplifier stages, all tuned together used to select & amplify the input frequency
- AM diode detector used to demodulate AM wave to recover modulating signal $v_m(t)$
- Audio amplifier amplifies the modulating received signal (increases the amplitude)
- Power amplifier raises the power level to a sufficient stage to drive the loudspeakers

The AM Superhetrodyne Receiver

Block Diagram Description :-

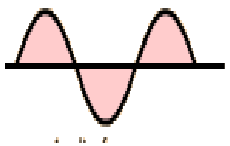
- **RF amplifier stages designed for frequency selection between 540 kHz to 1640 kHz**
- **Local oscillator tuning mechanically linked with RF amplifier from 995 kHz to 2095 kHz**
- **Mixer produces a single constant frequency (IF) of 455 kHz over entire AM tuning range**
- **IF amplifier is narrow-band amplifier having high selectivity to select only IF frequency**

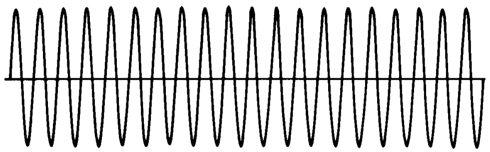
Frequency Modulation (FM)

Frequency Modulation is defined as process in which the **frequency** of high frequency **carrier** signal is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping **phase** & **amplitude** of the **carrier** signal **constant.**

low frequency baseband signal

$v_m(t) = V_m \sin\omega_m t$





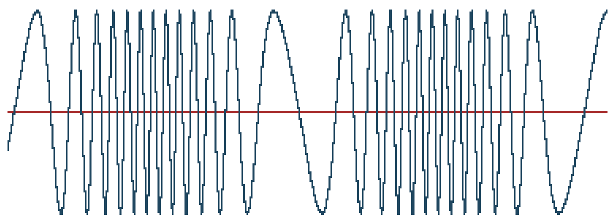
Frequency Modulation

high frequency carrier signal

$v_c(t) = V_c \sin\omega_c t$

output frequency modulated signal

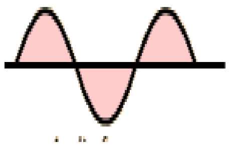
$v_{FM}(t)$



Frequency Modulation (FM)

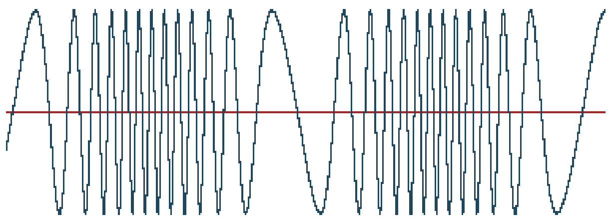
- **AF modulating (baseband) signal given by** $v_m(t) = V_m \sin\omega_m t$ **OR** $v_m(t) = V_m \cos\omega_m t$
- **HF carrier signal is given by the equation of** $v_c(t) = V_c \sin\omega_c t$ **OR** $v_c(t) = V_c \cos\omega_c t$

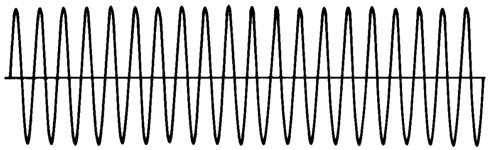
low frequency baseband signal
$v_m(t) = V_m \sin\omega_m t$



Frequency Modulation

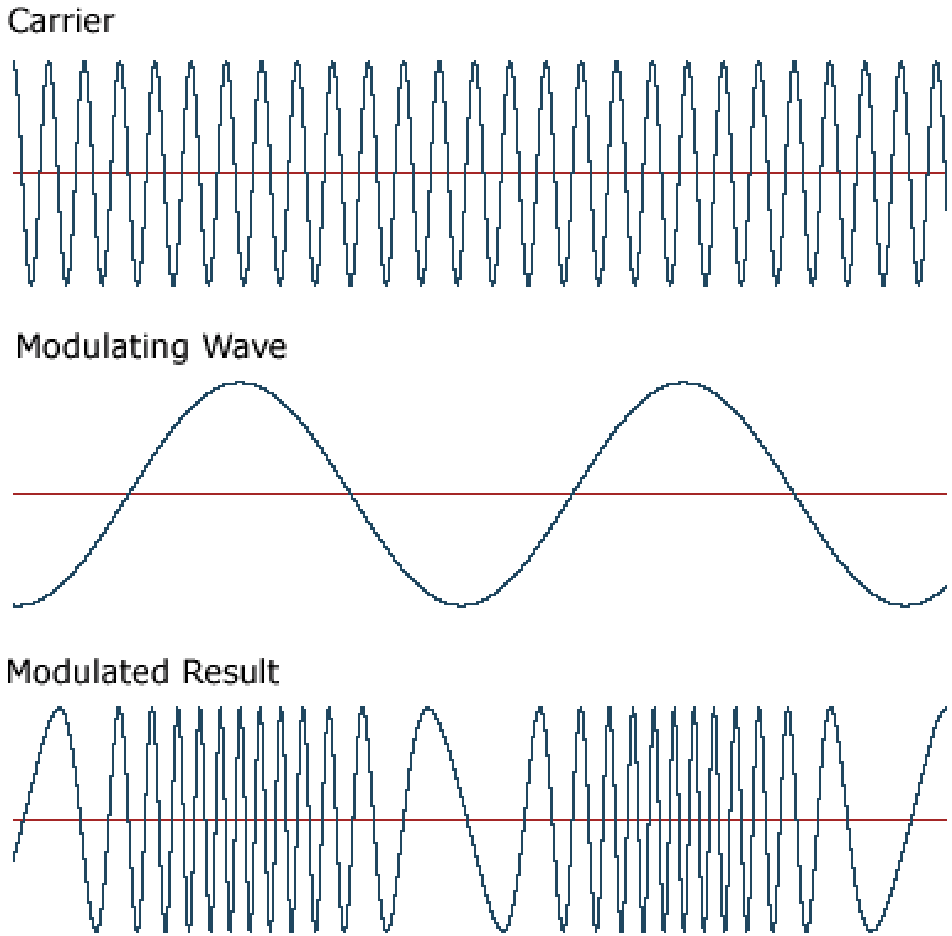
output frequency modulated signal
$v_{FM}(t)$





high frequency carrier signal
$v_c(t) = V_c \sin\omega_c t$

Frequency Modulation (FM)



HF carrier signal

$v_c(t) = V_c \cos\omega_c t$

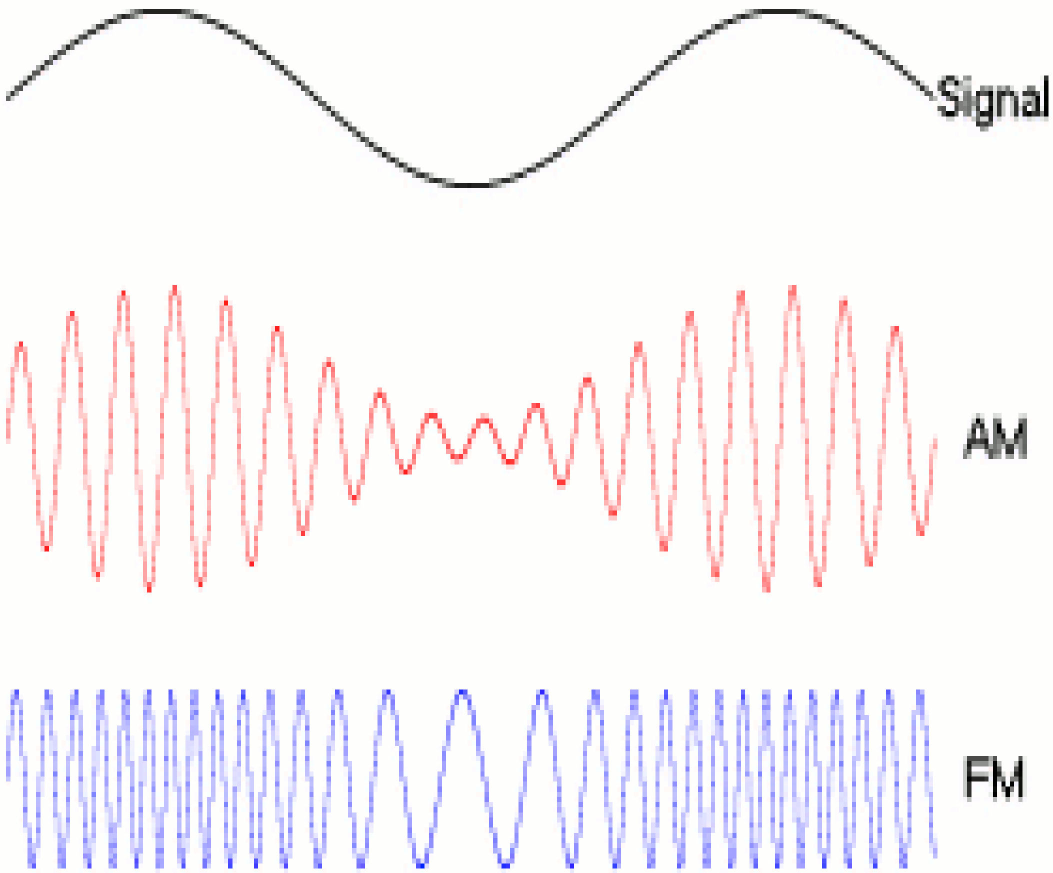
modulating signal

$v_m(t) = V_m \cos\omega_m t$

output signal

$FM = v_{FM}(t)$

AM & FM Waveforms



modulating or baseband signal

amplitude variations in V_c due to $v_m(t)$

frequency variations in ω_c due to $v_m(t)$

Concept of Frequency Deviation

- **Frequency Deviation** is change in carrier frequency (f_c) with time due to the input modulating baseband signal & expressed mathematically as :-

$$\boxed{\delta(t) = k_F v_m(t)}$$

k_F is constant (Hz/V) frequency sensitivity

- **Maximum Frequency Deviation** refers to the highest change in the carrier signal frequency (fc) due to the input modulating baseband signal given by :-

$$\boxed{\delta_{max} = k_F V_m}$$

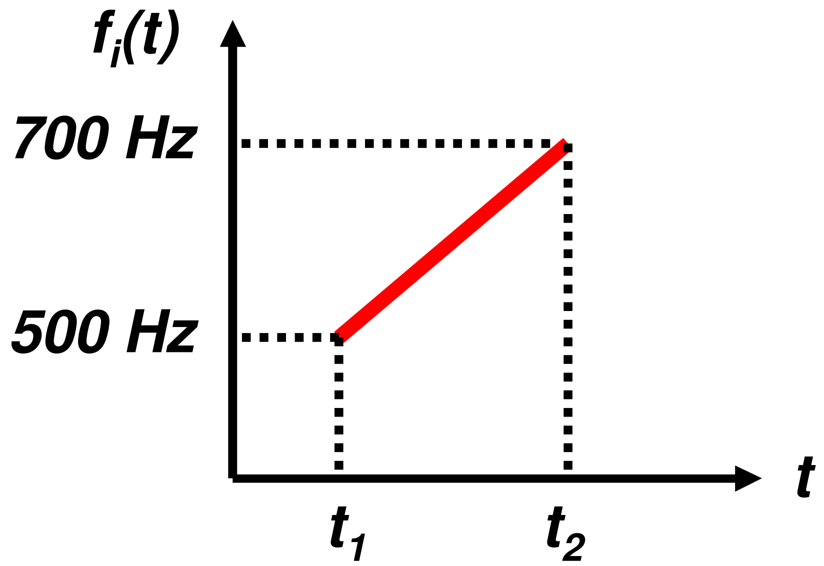
V_m is maximum OR peak signal amplitude

maximum frequency deviation is proportional to the maximum or peak amplitude of input modulating (baseband) signal

Concept of Instantaneous Frequency

Instantaneous Frequency refers to variation in output frequency of FM wave, defined at all the points of time (t) expressed as follows :-

$$\boxed{f_i(t) = f_c + \delta(t)}$$



based on frequency deviation, instantaneous frequency $f_i(t)$ is either above (more) or below (less) the carrier frequency f_c

Modulation Index (m_f)

- **Modulation Index (m_f)** of FM wave is defined as ratio of the maximum frequency deviation (δ_{max}) to modulating signal frequency (f_m) :-

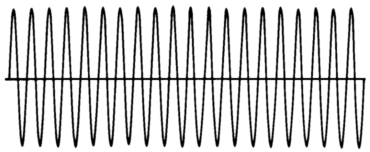
$$\boxed{m_f = \frac{\delta_{max}}{f_m}}$$

since frequency deviation & modulating signal frequency both carry the unit in Hz, it is a dimensionless quantity very much like the modulation index (m_a) of an AM wave

Concept of Angle Modulation

Carrier signal is mathematically represented by the following equation :-

$$v_c(t) = V_c \sin\omega_c t \text{ OR } v_c(t) = V_c \cos\omega_c t$$



In terms of vector carrier signal mathematically is also represented by :-

$$v_c(t) = V_c \sin[\theta(t)]$$

where $\theta(t)$ is the angular component incorporating frequency & phase shift

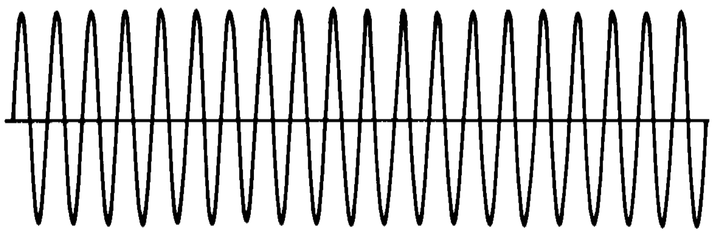
since $\theta(t) = \omega_c t + \Phi(t)$

Concept of Angle Modulation

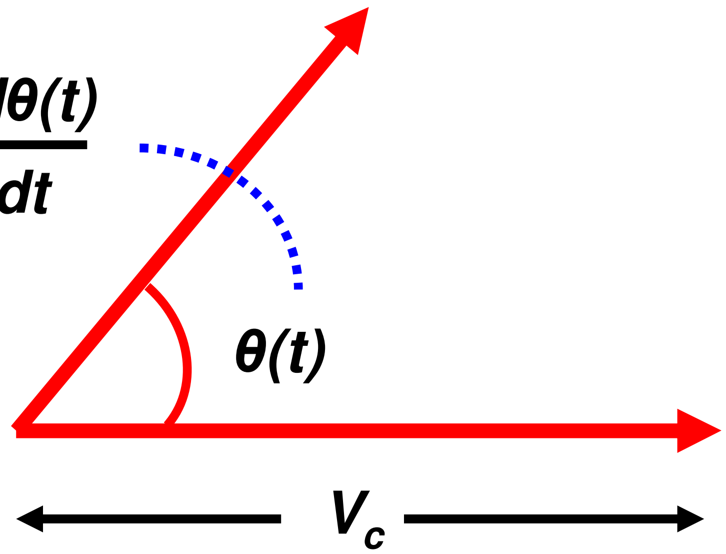
In terms of vector carrier signal mathematically is also represented by :-

$$\boxed{v_c(t) = V_c \sin[\theta(t)]}$$

where $\theta(t) = \omega_c t + \Phi(t)$



$$\omega(t) = \frac{d\theta(t)}{dt}$$



vector representation of carrier waveform

Concept of Angle Modulation

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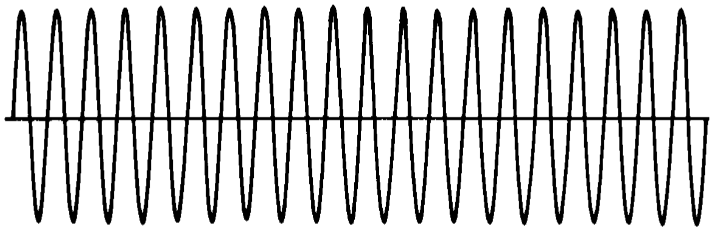

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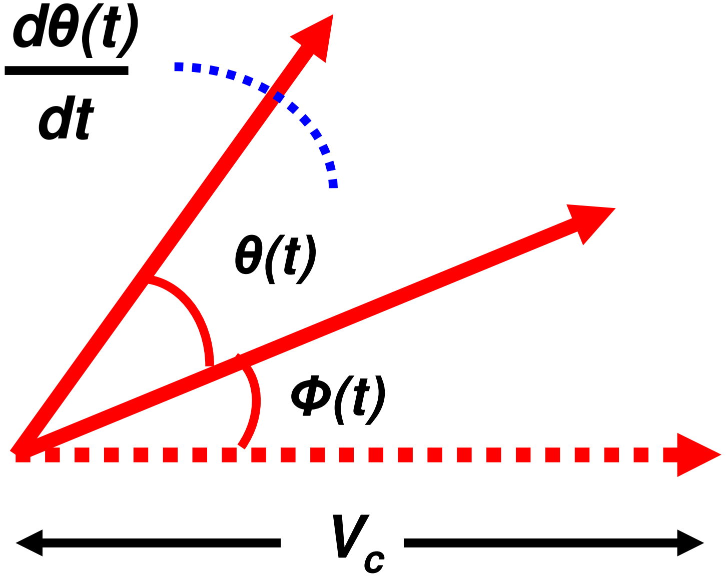

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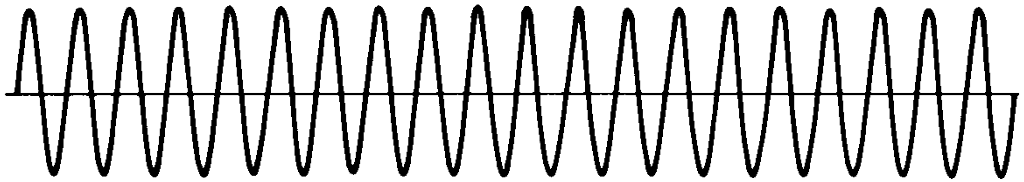

where $\theta(t) = \omega_c t + \Phi(t)$

frequency modulation (FM) make $\omega_c \; \alpha \; v_m(t)$

phase modulation (PM) make $\Phi(t) \; \alpha \; v_m(t)$

Mathematical Analysis of FM

- **AF modulating (baseband) signal given by** $v_m(t) = V_m \sin\omega_m t$ **OR** $v_m(t) = V_m \cos\omega_m t$
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low frequency baseband signal
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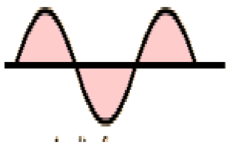
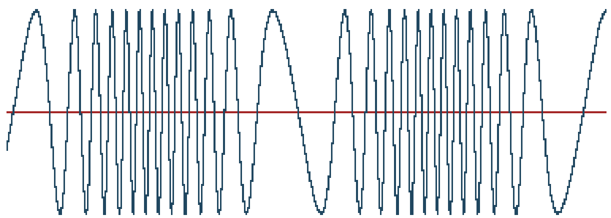
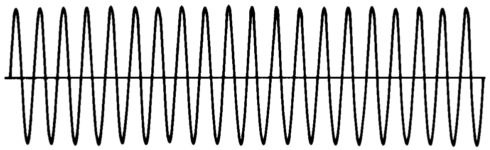


Frequency Modulation

output frequency modulated signal
$v_{FM}(t)$



high frequency carrier signal
$v_c(t) = V_c \sin\omega_c t$



Mathematical Analysis of FM

- **The equation of FM wave thus obtained :-**

$$v_{FM}(t) = V_c cos(\omega_c t + m_f sin\omega_m t)$$

- **This equation cannot be analyzed by using simple trigonometric functions but instead can be solved by using Bessel Functions**

thus only by using Bessel Functions the complete equation of FM wave is obtained in terms of carrier frequency & sidebands

Frequency Spectrum of FM

Expanding above equation by using Bessel Functions :-

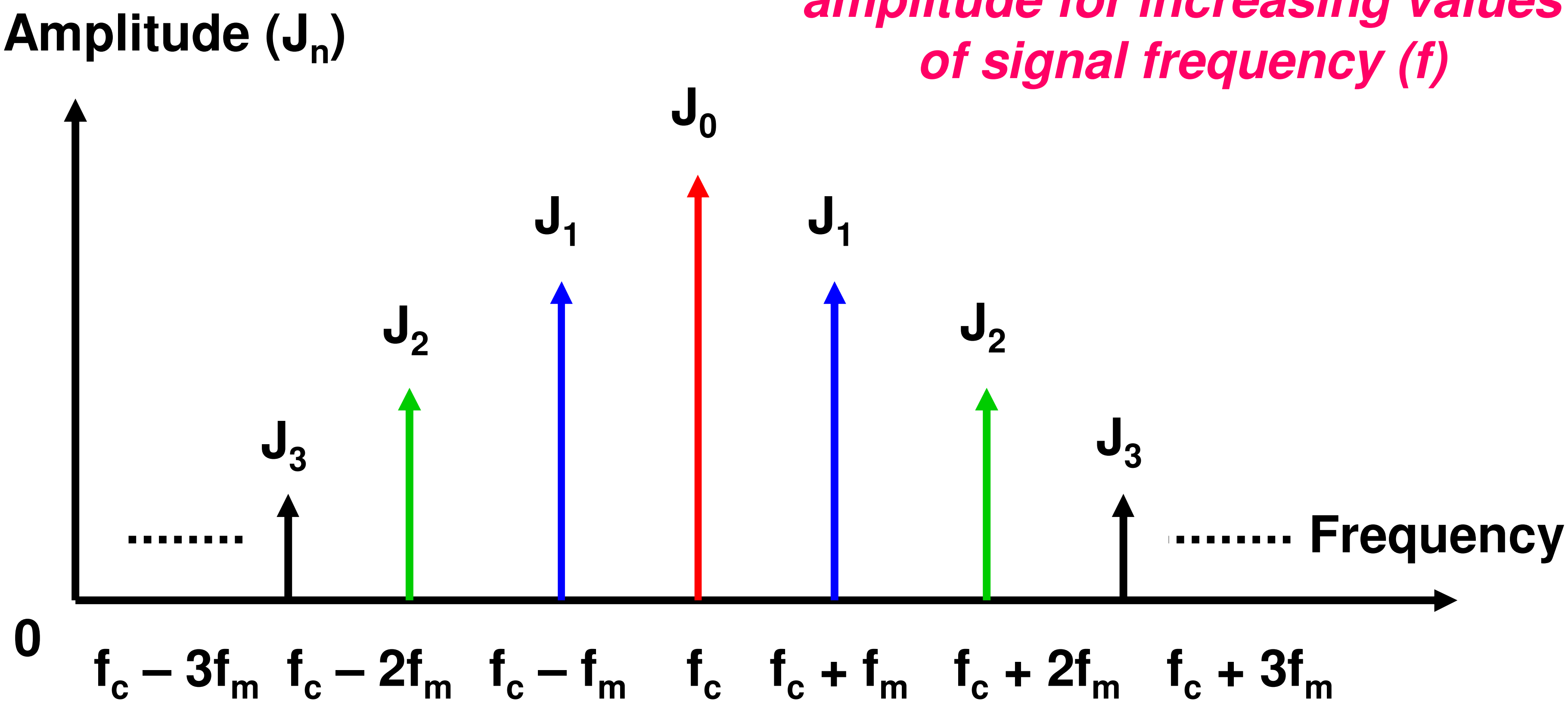
$$v_{FM}(t) = V_c\{\cos\omega_c t[J_0 + J_2\cos 2\omega_m t + J_4\cos 4\omega_m t.......]$$
$$-\sin\omega_c t[J_1\sin\omega_m t + J_3\sin 3\omega_m t.....]\}$$

$$v_{FM}(t) = V_c\{J_0\cos\omega_c t + J_1[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t]$$
$$+ J_2[\cos(\omega_c + 2\omega_m)t - \cos(\omega_c - 2\omega_m)t].....+ J_5....\}$$

- **where J_0 is Bessel Function of first type & n^{th} order**
- **J_0 – amplitude of the carrier signal**
- **J_n – amplitude of the sidebands, with frequency $\omega_c + n\omega_m$**

Frequency Spectrum of FM

FM spectrum contains infinite sidebands with a decreasing amplitude for increasing values of signal frequency (f)



Frequency Spectrum of FM

- **Some of the sidebands & carrier signal has negative amplitudes for some values of 'm_f'**
- **This indicates that the signal represented by that amplitude is only 180° out of phase**
- **FM spectrum varies considerably in terms of bandwidth based upon modulation index**
- **The higher the modulation index (m_f) more is the bandwidth of the FM wave**
- **As 'm_f' increases, carrier amplitude decrease & sideband amplitude will increase**

Bandwidth of FM wave

- Theoretically FM waves consists an <u>infinite</u> number of sidebands (both LSB & USB)
- So, ideally the frequency modulated (FM) should have an <u>infinite</u> bandwidth
- However Bessel Functions for some higher 'm_f' values become <u>relatively insignificant</u>
- Hence only those terms should taken into consideration whose amplitudes or Bessel Function coefficients are significant

Bandwidth of FM wave

From Bessel Function analysis, theoretical bandwidth of FM wave taking into account only the significant sidebands is given by :-

$$\boxed{BW = 2nf_m}$$

However a more practical approach uses 'approximation' of the FM bandwidth given by Carson's rule as :-

$$\boxed{BW = 2(\delta_{max} + f_m)}$$

Carson's rule approximates bandwidth reasonably well only for higher modulation index values (m_f)

Power Equations in FM wave

- In frequency modulation (FM) amplitude of FM wave is same as that of the carrier
- Amplitude of sidebands is relatively smaller compared to the carrier amplitude (V_c)
- Since they are relatively insignificant, power content of sidebands can be ignored too

$$\boxed{P_T = P_C = \frac{V_C^2}{2R}}$$

this equation indicates that total power in FM is equal to the carrier power itself where R = resistance of the transmitting (Tx) antenna