# TUTORIAL 8

1) Convert to CNF (Context free grammar to Chomsky Normal form)

$S \to ABC$

$A \to a \mid b$

$B \to Bb \mid bb$

$C \to aC \mid cc \mid ba$

<u>Steps:</u> <u>Step 1:</u>

Ⓐ Remove $\epsilon$ production

- No $\epsilon$ production present

Ⓑ Remove unit production

- No unit production present

Ⓒ Remove useless symbols / non-generating symbols:

$G = (a, b)$

$A \to a \mid b$

$B \to bb$

$C \to ba$

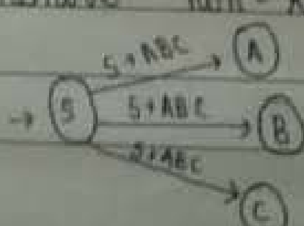
$\therefore G = (a, b, A, B, C)$

$S \to ABC$

$\therefore G = (a, b, A, B, C, S)$

No non-generating symbols

Ⓓ Remove non-reachable symbols




all symbols reachable

no change in grammar

Chomsky Normal Form of CFG is conversion of produ[ctions in]
such a way that either $\boxed{A \to BC}$ or $\boxed{A \to a}$

Step 2: Add Chomsky variables

$C_a \to a$ , $C_b \to b$

All rules must be in the form

$A \to BC$ $A \to a$

① $S \to ABC$

② $A \to a$

③ $A \to b$

④ $B \to Bb$

⑤ $B \to bb$

⑥ $C \to aC$

⑦ $C \to CC$

⑧ $C \to ba$

$C_a \to a$

$C_b \to b$

$S \to AC_1$

$C_1 \to BC$

$S \to C_2C$

$C_2 \to AB$

$A \to a$

$A \to b$

$B \to BC_b$

$B \to C_bC_b$

$C \to C_aC$

$C \to CC$

$C \to C_bC_a$

CNF is $(V, T, P, S)$

P as shown above

$V = \{S, A, B, C, C_a, C_b, C_1, C_2\}$

$T = \{a, b\}$

$S = S$

5) Convert the grammar to GNF (Greibach Normal Form of Context Free Grammar)

$S \to ABA$

$A \to aA \mid \epsilon$

$B \to Bb \mid \epsilon$

~~Nb~~ A CFG, $G = (V, T, P, S)$ is said to be in GNF if every production is of the form

$A \to a\alpha$ $\alpha \in V$ (string of 0 or more no of variables

Step 1 (A) Remove ~~next~~ $\epsilon$ - production

$S \to ABA \mid AB \mid B \mid AA \mid BA \mid A$

$A \to aA \mid a$ | $A \to \epsilon$

$B \to Bb \mid b$ | $B \to \epsilon$

(B) Remove unit production $S \to A$, $S \to B$

- ~~No unit production~~

$\therefore S \to ABA \mid AB \mid aA \mid a \mid AA \mid BA \mid Bb \mid b$

(C) Remove useless / non-generating symbols

$G = \{a, b\}$

$A \to a$

$B \to b$

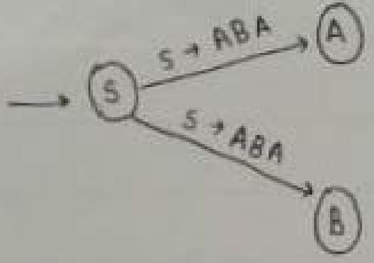
$G = \{a, b, A, B\}$

$S \to ABA$

$G = \{a, b, A, B, S\}$

All symbols are generating

(ii) Remove non-reachable symbols




All symbols are reachable

<u>Step 2</u>: Bring every production to the form $A \to a\alpha$ or $A \to a$
where $|\alpha| \geq 2$ then $\alpha$ should contain all variables

$S \to ABA \mid AB \mid AA \mid BA \mid a \mid aA \mid Bb \mid b$

$A \to aA \mid a$

$B \to Bb \mid b$ left recursion

(i) Convert $B \to Bb$ into form $A \to \alpha$

$B \to b$ $\qquad$ $B \to bb$

(ii) ~~Convert~~ ~~S → ABA~~

(1) Rename all variables

$A_1 \to A_2 A_3 A_2 \mid A_2 A_3 \mid A_2 A_2 \mid A_3 A_2 \mid a \mid a A_2 \mid \underline{A_3 b} \mid b$ $\quad$ $A_4$ $\quad$ $b \to A_4$

$A_2 \to a A_2 \mid a$

$A_3 \to \underline{A_3 b} \mid b$ $\quad$ $A_4 \to b$

(2) Convert every production $A_i \to A_j$ where $i > j$ to $i \leq j$

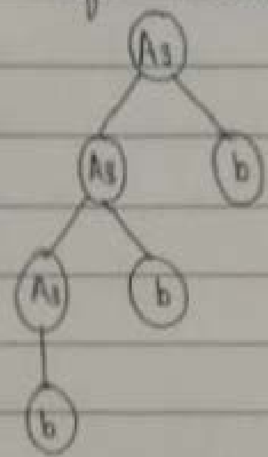
$A_1 \to A_2 A_3 A_2 \mid A_2 A_3 \mid A_2 A_2 \mid A_3 A_2 \mid a \mid a A_2 \mid A_3 b \mid b$

$A_2 \to a A_2 \mid a$

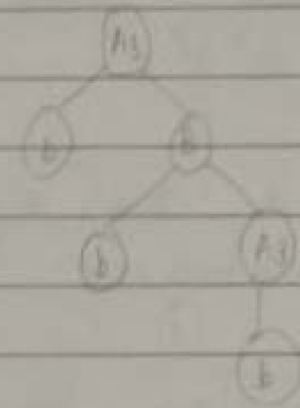
$A_3 \to A_3 b \mid b$

$A_4 \to b$

③ Remove left recursion.



$A_3 \to \cancel{A_3 (b)^n \mid b}$



$A_3 \to b \mid B \quad b$

$B \to bA_3 \mid b$

$\therefore$ S → ABA $\quad A_1 \to A_2A_3A_2 \mid A_2A_3 \mid A_2A_2 \mid A_3A_2 \mid a \mid aA_2 \mid A_3A_4 \mid b$

$A_2 \to aA_2 \mid a$

$A_3 \to bB \mid b$

$B \to bA_3 \mid b$

$A_4 \to b$

④ Convert all production to GNF

$A_2$, $A_3$, $B$ are already in GNF

rewrite $A_1 \to A_2A_3A_2$ in GNF

$A_1 \to aA_2A_3A_2 \mid aA_3A_2 \mid aA_2A_3 \mid aA_3 \mid aA_2A_2 \mid aA_2 \mid bBA_2 \mid bA_2 \mid a \mid aA_2 \mid bBA_4 \mid b$

⑤ Final Grammar rules

$A_1 \to aA_2A_3A_2 \mid aA_3A_2 \mid aA_2A_3 \mid aA_3 \mid aA_2A_2 \mid aA_2 \mid bBA_2 \mid bA_2 \mid a \mid aA_2 \mid bBA_4 \mid b$

$A_2 \to aA_2 \mid a$

$A_3 \to bB \mid b$

$B \to bA_3 \mid b$

$A_4 \to b$

2) $L = \{a^n b^m c^n \mid m \le n \le 2m\}$

Assume L is CFL

Let pumping lemma be p

∴ test case where $m = n$

$w = a^p b^p c^p$

If $m = 3$, then $n = 3$

$w =$ aaabbb ccc

Case i: $uv^i x y^i z$ for $i \ge 0$

Now dividing into 2 cases

Case (i): v, y contain 3 symbols a, b, c

$s = \underbrace{aa}_{u}\ \underbrace{ab}_{v}\ \underbrace{b}_{x}\underbrace{bc}_{y}\ \underbrace{cc}_{z}$

Taking $i = 2$

$uv^2 x y^2 z = aa\,(ab)^2 b\,(bc)^2 cc$

$=$ aaaa bbbbb cccc

$= a^4 b^5 c^4$

The abow language doesnt belong to language

Case (ii) v, y doesnt contain all 3 symbols a, b, c

$s = \underbrace{aa}_{u}\ \underbrace{a}_{v}\ \underbrace{b}_{x}\ \underbrace{bb}_{y}\ \underbrace{ccc}_{z}$

if $uvxyz \in L$ then $uv^i x y^i z$ should also be

$uv^2 x y^2 z = aa(a)^2\, b\, (bb)^2$ ccc

$= a^4\, b^5$ ccc

m cannot be 4 & 5

This given language is not a CFL by condition

2) $L = \{a^m b^n \mid n <= m <= 2n\}$

→ Assume L is CFL

Let the pumping lemma be p

∴ Consider $m = n$

$w = a^p b^p$

If $m = 3$ then $n = 3$

$w = aaabbb$

<u>Case 1</u> $uv^i xy^i z$ for L $i \geq 0$

Now dividing into 2 cases

Case (1): v, y contains all symbols a, b

$s = \underset{u}{a}\,\underset{v}{a}\,\underset{x}{a}\;\underset{y}{bb}\,\underset{z}{b}$

Taking $i = 2$

$uv^2xy^2z = a(aa)a(bbbb)b$

$= a^4 b^5$

The above language doesn't belong to L

Case (ii): v, y doesn't contain all symbols

$s = \underset{u}{aaaaa}\;\underset{v}{b}\,\underset{x}{b}\,\underset{y}{bb}\,\underset{z}{b}$

Taking $i = 2$

$uv^2xy^2z = (aaaaa)\ bb\ b\ bbbb\ b$

$= a^5 b^8$

The above language doesn't belong to L