PDA INTRODUCTION

PDA
(Push Down Automata)

- PDA is machine that is equivalent to a machine with FA + stack memory that increases the capacity of FA.

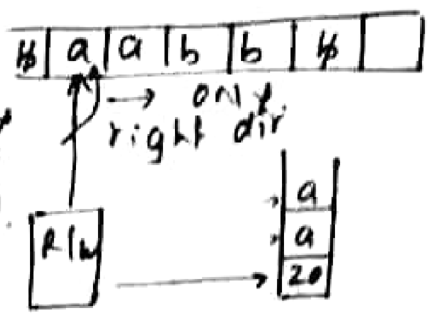
- Power of PDA is

FA < PDA < post machine < TM.

- lang	Reg. Lang	CFL	CSL / NDTM/LBA	Recursively Enumerable Language.
-	type 3.	type 2	type 1	type 0
-	RG	CFG	CSG	UNRestricted Grammar.

CFL has ↑ every PDA ↓ every CFL

every PDA



(1) ∞ input tape - length

(2) only read symbol.

only right dir

(3) stack. [Push/POP/NOP]

(4) check empty condition of stack thru Initial stack symbol.

(5) read top of stack thru POP.

(6) $q_0 \rightarrow q_1$
state change.

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Formal definition of PDA

PDA is a machine

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

Q = set of states

Σ = input alphabet

Γ = stack symbols.

δ = transition function

q_0 = Initial state $q_0 \subseteq Q$

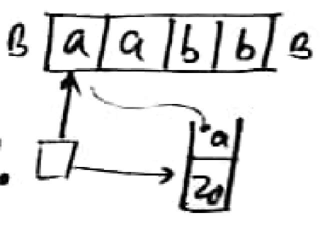
z_0 = stack initial symbol.

F = set of final state $F \subseteq Q$

$\delta: Q \times (\Sigma \cup \epsilon) \times \Gamma \longrightarrow Q \times \Gamma^*$

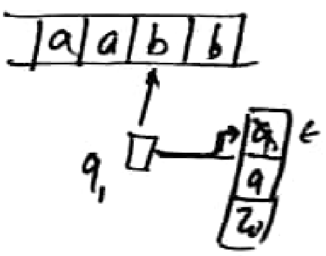
- Q → current state
- $(\Sigma \cup \epsilon)$ → input symbol on tape
- Γ → topmost stack symbol.
- Q → new state
- Γ^* → operation on stack: Push, Pop, NOP.

Push



$\delta(q_0, a, z_0) = (q_1, az_0)$ topmost replaced with a

POP



$\delta(q_1, b, a) = (q_2, \epsilon)$ topmost replaced with ϵ

NOP

$\delta(q_1, b, a) = (q, a)$ a replaced with a.

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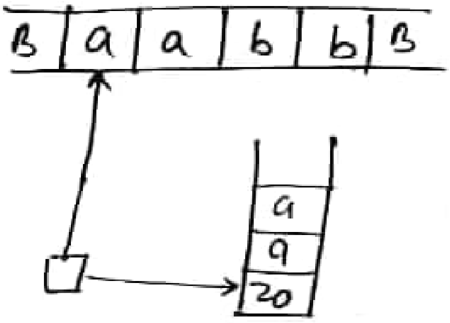
DESIGN PDA - SAMPLE

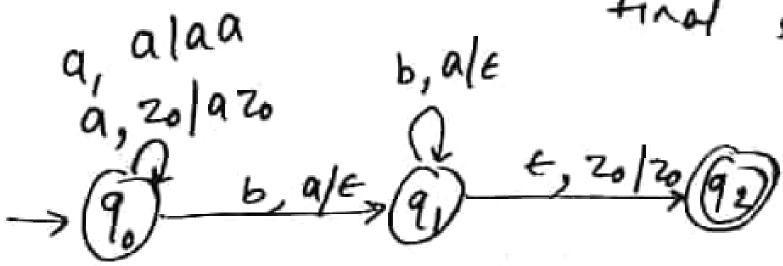
Q Design PDA for $a^n b^n$

steps
1. design transition diagram with logic explanation
2. find out transition rules.
3. show Instantaneous description
4. final statement machine.

logic



1. for all a's push on stack
2. for all b's pop stack.
3. for B symbol on tape if stack is empty final state.



$$M = (\underbrace{\{q_0, q_1, q_2\}}_{Q}, \underbrace{\{a, b\}}_{\Sigma}, \{a, b, z_0\}, \delta, q_0, z_0, \{q_2\})$$

1. $\delta(q_0, a, z_0) = (q_0, az_0)$
2. $\delta(q_0, a, a) = \delta(q_0, aa)$
3. $\delta(q_0, b, a) = (q_1, \epsilon)$
4. $\delta(q_1, b, a) = (q_1, \epsilon)$
5. $\delta(q_1, \epsilon, z_0) = (q_2, z_0)$ or $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

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Instantaneous Description -

$q_0, aabb, z_0 \vdash q_0, abb, az_0 \vdash q_0, bb, aaz_0$

$\vdash q_1, b, az_0 \vdash q_1, \epsilon, z_0 \vdash q_2, \epsilon, z_0$ final state.

Final statement -

PDA is $M = (Q, \Sigma, \delta, \Gamma, q_0, z_0, F)$

$Q = \{q_0, q_1, q_2\}$ $\Sigma = \{a, b\}$ $z_0 =$ Initial (top of) to stack

$\delta : Q \times \{\Sigma \cup \epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$ $F = \{q_2\}$

as per diagram.

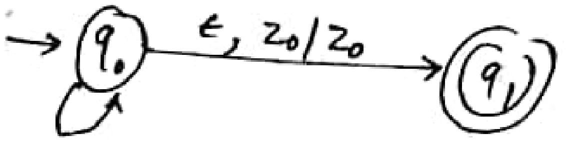
Q - Design PDA for CFL that checks wellformed-ness of paranthesis. $\Sigma = \{ (, [,],) \}$

① logic : Push [, (on stack.

Pop for) + (on stack +] + [on stack

② Transition diagram

[	(	(	)	)	(	(	)	]	[	]	B



$[, z_0 / [z_0$
$[, [/ [[$
$[, (/ [($ push
$[,) / [)$

$[,] / \epsilon$ - pop

$(, z_0 / (z_0$
$(, [/ ([$ push

$(,) / \epsilon$ - pop.

$(,] / []$ - push

③ δ function for all transitions.

~~$\delta(q_0, z_0, \epsilon$~~

$\delta(q_0, [, z_0) = (q_0, [z_0)$

$\vdots$

④ I Description.

$q_0, (()[]), z_0 \vdash q_0, ()[]), (z_0$
$\vdash q_0,)[]), ((z_0 \vdash q_0, []), (z_0.$
$\vdash q_0,]), [(z_0 \vdash q_0,), (z_0 \vdash q_0 \epsilon z_0$
$\vdash q_1$ final state

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Non-deterministic PDA

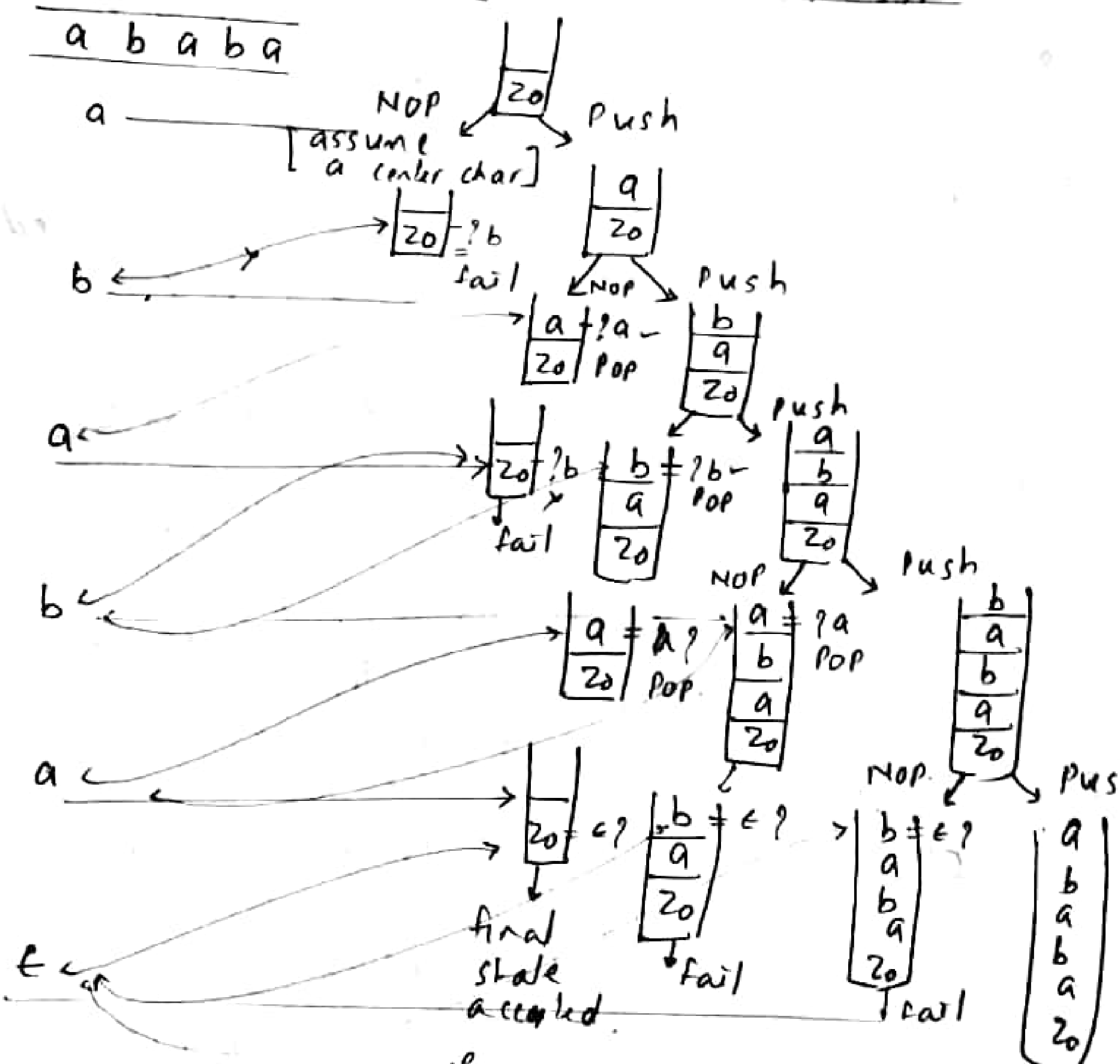
DPDA + non determinism = NPDA. more powerful than DPDA.

Q design PDA for odd and even palindromes.

(1) odd length PDA – waw^R or wbw^R

There is no way to identify centered char. and PDA moves to right side only.

(2) odd length string logic – $\not{b}$|a|b|a|b|a|$\not{b}$



(1) $\delta(q_0, a, \epsilon) = (q_1, \epsilon)$ NOP, (q_0, a) push
[any sym on stack]

(2) $\delta(q_0, b, \epsilon) = (q_1, \epsilon)$ NOP, (q_0, b) push

(3) $\delta(q_1, a, a) = (q_1, \epsilon)$ pop

(4) $\delta(q_1, b, b) = (q_1, \epsilon)$ pop

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$ accept

or $= (q_2, z_0)$ final st.

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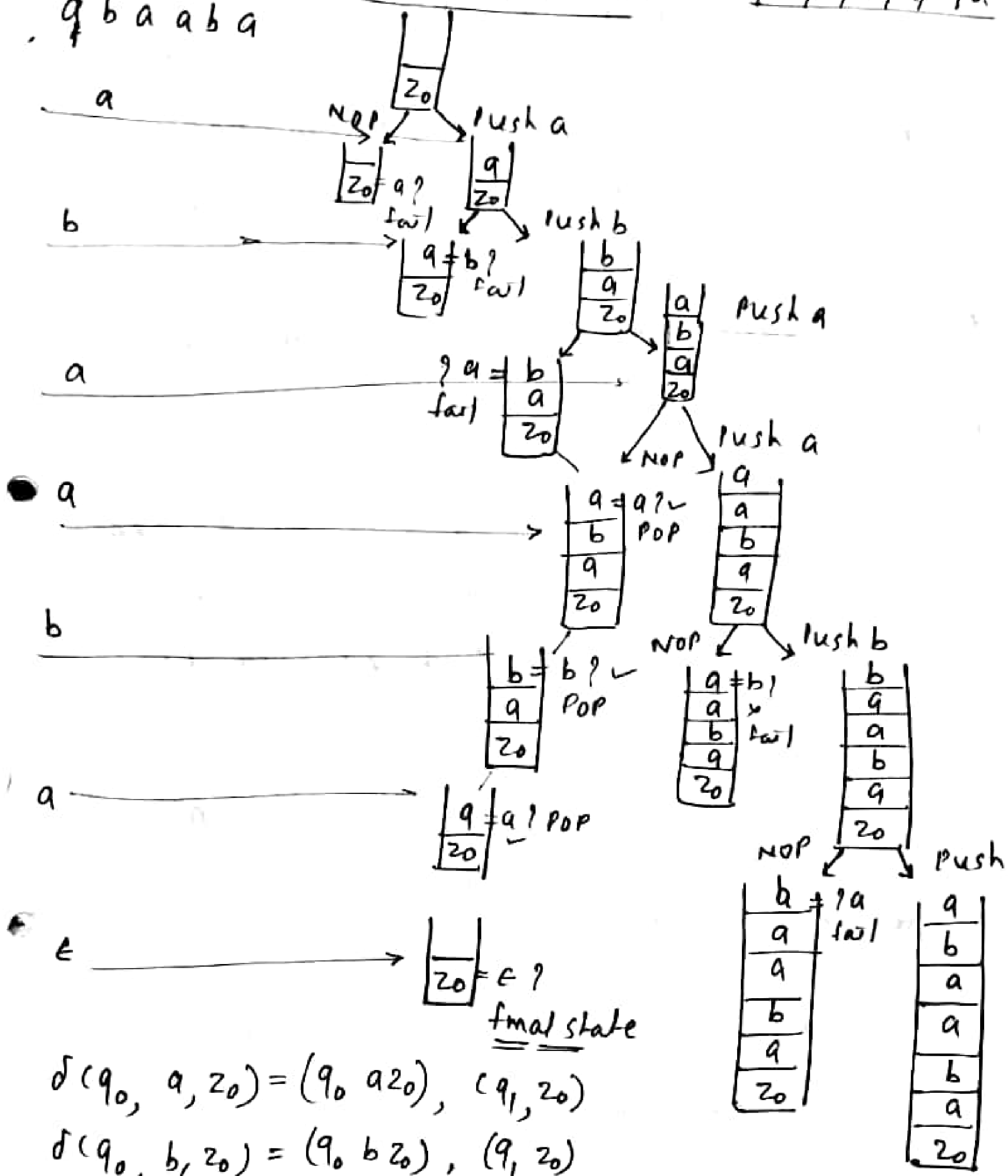
Even palindrome

a | b | a | a | b | a



$\delta(q_0, a, z_0) = (q_0\ a z_0),\ (q_1, z_0)$

$\delta(q_0, b, z_0) = (q_0\ b z_0),\ (q_1\ z_0)$

$\delta(q_0, a, a) = (q_0, aa)\ (q_1, a)\ (q_1, \epsilon)$

$\delta(q_0, a, b) = (q_0, ab)\ (q_1, b)$

$\delta(q_0, b, a) = (q_0, ba)\ (q_1, a)$

$\delta(q_0, b, b) = (q_0, bb)\ (q_1, b)\ (q_1, \epsilon)$

$\delta(q.\ \epsilon, z_0) = (q_1\ \epsilon)$ | empty stack or $(q_2\ z_0)$ final

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(1) step1 - convert grammer in either CNF/GNF CFL

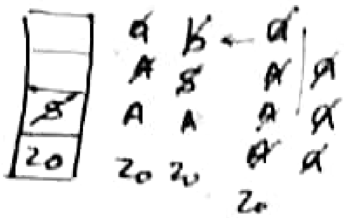
(2) step2 - convert it to PDA / design PDA.

$S \rightarrow aAA$
$A \rightarrow bS \mid aS \mid a$

step1 - Grammer is already in GNF format

step2 - Identify string belongs to this grammar.

W/E	a	b	a	a	a	a	#



$\delta(q_0, \epsilon, z_0) = (q_1, S z_0)$

$\delta(q_1, a, S) = (q_1, aAA)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, A) = (q_1, bS)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, a, A) = \{(q_1, aS)\ (q_1, a)\}$

$\delta(q_1, \epsilon, z_0) = (q_2\ z_0)$ final state

Instaneous Description

$(q_1, abaaaa, Sz_0) \vdash (q_1, abaaaa, aAA)$

$\vdash (q_1, baaaa, AAz_0) \vdash (q_1, baaaa, bSAz_0)$

$\vdash (q_1, aaaa, SAz_0) \vdash (q_1, aaaa, aAAAz_0)$

$\vdash (q_1, aaa, AAAz_0) \vdash (q_1, aaa, aAAz_0)$

$\vdash (q_1, aa, AAz_0) \vdash (q_1, aa, aAz_0)$

$\vdash (q_1, a, A, z_0) \vdash (q_1, a, az_0)$

$\vdash (q_1\ \epsilon\ z_0) \vdash q_2$ final state.

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CONVERT PDA TO CFG

Convert ~~CFG to~~ PDA to CFG

① $S \rightarrow [q_0 \; z \; q_i]$ for each $q_i \in Q$, z start symbol.

② $\delta(q_i, a, B) \rightarrow (q_j, C)$

$[q_i \; B \; q] \rightarrow a[q_j \; C \; q]$ for each $q \in Q$.

③ $\delta(q_i, a, B) \rightarrow (q_j, C_1 C_2)$

$[q_i \; B \; p_1] \rightarrow a[q_j \; C_1 \; p_2][p_2 \; C_2 \; p_1]$ for each $p \in Q$.

- $M = (\{q_0\; q_1\}, \{0, 1\}, \{z_0, x\}, \delta, q_0, z_0, \phi)$

① add productions for start symbol z_0

$S \rightarrow [q_0 \; z_0 \; q_0]$

$S \rightarrow [q_0 \; z_0 \; q_1]$

② $\delta(q_0, 1, z_0) = (q_0, x z_0)$

$[q_0 \; z_0 \; q_0] \rightarrow 1[q_0 \; x \; q_0][q_0 \; z_0 \; q_0]$

$[q_0 \; z_0 \; q_0] \rightarrow 1[q_0 \; x \; q_1][q_1 \; z_0 \; q_0]$

$[q_0 \; z_0 \; q_1] \rightarrow 1[q_0 \; x \; q_0][q_0 \; z_0 \; q_1]$

$[q_0 \; z_0 \; q_1] \rightarrow 1[q_0 \; x \; q_1][q_1 \; z_0 \; q_1]$

Q PDA to CFG -

① $\delta(q_0, 1, z_0) = \{(q_0, x z_0)\}$

② $\delta(q_0, 1, x) = \{(q_0, xx)\}$

③ $\delta(q_0, 0, x) = \{(q_1, x)\}$

④ $\delta(q, \epsilon, x) \; \{(q_1, \epsilon)\}$

⑤ $\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$

⑥ $\delta(q_1, 0, z_0) = \{(q_0, z_0)\}$

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$Q = \{q_0, q_1\}$

$\delta(q_0, 1, x) = (q_0\, xx)$

$[q_0\, x\, q_0] = 1[q_0\, x\, q_0]\,[q_0\, x\, q_0]$

$[q_0\, x\, q_0] = 1[q_0\, x\, q_1]\,[q_1\, x\, q_0]$

$[q_0\, x\, q_1] = 1[q_0\, x\, q_0]\,[q_0\, x\, q_1]$

$[q_0\, x\, q_1] = 1[q_0\, x\, q_1]\,[q_1\, x\, q_1]$

④ $\delta(q_0, 0, x) = (q_1, x)$

$[q_0\, x\, q_0] = 0[q_1\, x\, q_0]$

$[q_0\, x\, q_1] = 0[q_1\, x\, q_1]$

⑤ $\delta(q_0, \epsilon, z_0) = (q_1, \epsilon)$

$[q_0\, z_0\, q_1] \to \epsilon$

⑥ $\delta(q_1, 1, x) = (q_1, \epsilon)$

$[q_1\, x\, q_1] \Rightarrow 1$

⑦ $\delta(q_1, 0, z_0) = (q_0, z_0)$

$[q_1\, z_0\, q_0] \to 0[q_0\, z_0\, q_0]$

$[q_0\, z_0\, q_1] \to 0[q_0\, z_0\, q_1]$

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CFG TO PDA SAMPLE

Convert from CFG to PDA

① $S \rightarrow aSb$
$S \rightarrow a \mid b \mid \epsilon$

step 1 - convert it to CNF/GNF

(1) remove ϵ symbols

$S \rightarrow aSb \mid a \mid b \mid ab$

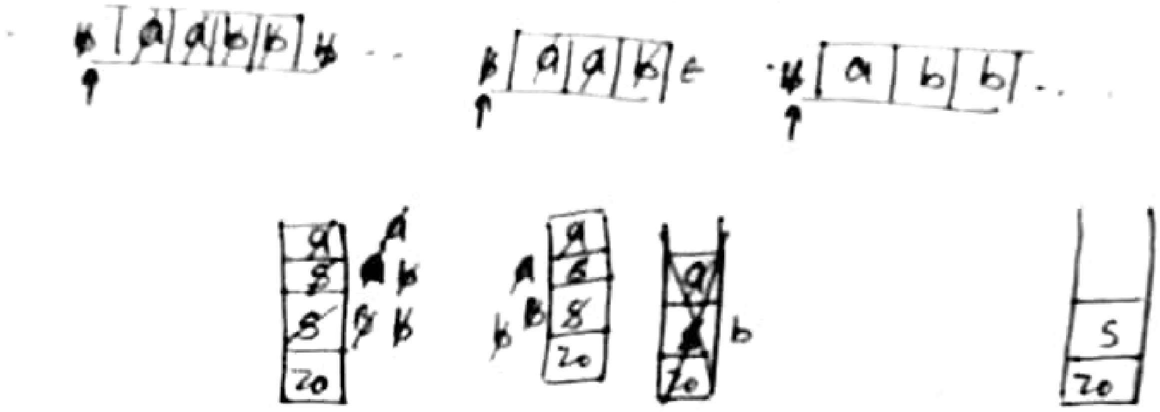
(2) convert it to CNF

G: $S \rightarrow aSB \mid a \mid b \mid ab$

$B \rightarrow b$

step 2 - $M = (Q\ \Sigma\ \delta\ \Gamma\ q_0\ z_0\ S)$

strings of Grammar G are



① $\delta(q_0, \epsilon, z_0) = (q_1, S)$

② $\delta(q_1, a, S) = \{(q_1, aSB), (q_1, a), (q_1, bab)\}$

③ $\delta(q_1, a, a) = (q_1, \epsilon)$

④ $\delta(q_1, b, b) = (q_1, \epsilon)$

⑤ $\delta(q_1, b, B) = (q_1, b)$

⑥ $\delta(q_1, \epsilon, z_0) = (q_2, z_0)$ final

ID $\delta(q_0, \epsilon\ aabb, z_0)$

$\vdash (q_1, aabb, Sz_0)$

$\vdash (q_1, aabb, aSBz_0)$

$\vdash (q_1, abb, SBz_0)$

$\vdash (q_1, abb, abBz_0)$

$\vdash (q_1, bb, bBz_0)$

$\vdash (q_1, b, Bz_0)$

$\vdash (q_1, b, bz_0)$

$\vdash (q_1, \epsilon, z_0) \vdash (q_2, z_0)$

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