Greibach Normal Form
(GNF)

A CFG $G = (V T P S)$ is said to be in GNF if every production is of the form $A \rightarrow a\alpha$ $\alpha \in V$ [string of zero or more no. of variables]

① Lang. should be without ϵ production rules. & simplified.

② RHS should start with one terminal symbol followed by zero or more number of variables.

Steps - ① simplify Grammar.

② Bring every production to form $A \rightarrow a\alpha$ or $A \rightarrow \alpha$ where $\alpha \in V$

③ Rename variables $A_1, A_2, \ldots$

④ $A_i \rightarrow A_j \alpha$ rule for every $i > j$ convert it to $i \leq j$ by substitution.

⑤ Remove left recursion.

⑥ get GNF productions by substituting already GNF production.

Q- Find a GNF equivalent of following CFG.

$$\boxed{\begin{aligned} S &\rightarrow BA \mid ab \\ B &\rightarrow AB \mid a \\ A &\rightarrow Bb \mid BB \end{aligned}}$$

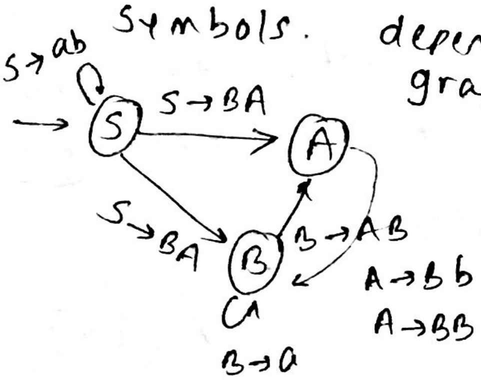
(1) step 1 - Explain what is GNF.

(2) step 2 - simplify the Grammar.

Grammar does not contain ϵ production and unit productions.

Generating symbols are (a, b)
↓ $B \rightarrow a$, $S \rightarrow ab$
(a, b, S, B) are gener.
↓ $A \rightarrow Bb$
(a, b, S, B, A) all gener.

To find Non reachable symbols. dependancy graph



all the symbols are reachable from S.

No useless symbol is there.

(3) step 3 - ~~Rename~~ ~~var~~ Bring every production to form $A \rightarrow a\alpha$ or $A \rightarrow \alpha$

where $|\alpha| \geq 2$ then α should contain all variables.

$S \rightarrow BA \mid aD$ —

$B \rightarrow AB \mid a$

$A \rightarrow BD \mid BB$

$D \rightarrow b$

$A \rightarrow Bb \Rightarrow$ convert to $A \rightarrow \alpha$ form by adding

$D \rightarrow b$

④ <u>step 4</u> - Rename all the variables. $A_1 \cdots A_4$

$A_1 \rightarrow A_2 A_3 \mid a A_4$

$A_2 \rightarrow A_3 A_2 \mid a$

$A_3 \rightarrow A_2 A_4 \mid A_2 A_2$

$A_4 \rightarrow b$

⑤ <u>step 5</u> - convert every prod. $A_i \rightarrow A_j$ where $i > j$ to $i \leq j$ format

$A_1 \rightarrow A_2 A_3 \mid a A_4$

$A_2 \rightarrow A_3 A_2 \mid a$

$A_3 \rightarrow A_3 A_2 A_4 \mid A_3 A_2 A_2$
$\mid a A_4 \mid a A_2$

$A_4 \rightarrow b$

Need to modify

① $A_3 \rightarrow A_2 A_4$ $i > j$

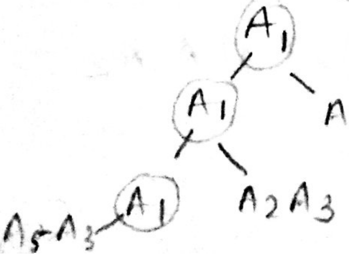
② $A_3 \rightarrow A_2 A_2$ substitute

Value of A_2

⑥ <u>step 6</u> - Remove left recursion -

*** if $A_1 \rightarrow A_1 A_2 A_3 \mid A_5 A_3$ ⟹ rewrite

left recursion present



strings generated are ⟹ $\underline{A_5 A_3} (\underbrace{A_2 A_3}_{B_1})^n, n \geq 0$

$A_1 \rightarrow A_5 A_3 B_1$

$A_1 \rightarrow A_5 A_3$

$B_1 \rightarrow A_2 A_3 B_1 \mid A_2 A_3$

... next page

(6) step 6 - Remove left recursion

$A_1 \rightarrow A_2 A_3 \mid a A_4$

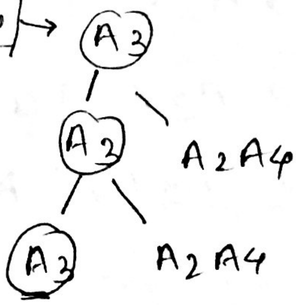
$A_2 \rightarrow A_3 A_2 \mid a$

$A_3 \rightarrow A_3 A_2 A_4 \mid a A_4 \mid A_3 A_2 A_2 \mid a A_2$

$A_4 \rightarrow b$

A_3 production has left recursion. so remove it by adding new variable B_3.

$\boxed{\text{for } A_3 \rightarrow A_3 A_2 A_4}$ →

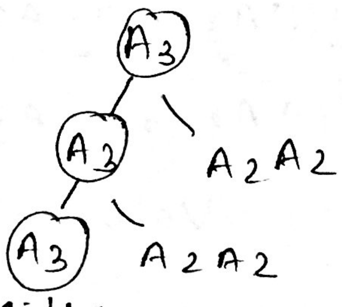


either aA_4 or aA_2

$a A_4 \underbrace{(A_2 A_4)^n}_{B_3}$ or $a A_2 \underbrace{(A_2 A_4)^n}_{B_3}$

$A_3 \rightarrow a A_4 \mid a A_2 \mid$

$\boxed{\text{for } A_3 \rightarrow A_3 A_2 A_2}$



either aA_4 or aA_2

→ strings of type $a A_4 (A_2 A_2)^n$ or $a A_2 \underbrace{(A_2 A_2)^n}_{B_{\cancel{3}4}}$

$A_3 \rightarrow a A_4 \mid a A_2 \mid a A_4 B_3 \mid a A_2 B_3$ & b_4 & B_4

$B_3 \rightarrow A_2 A_4 B_3 \mid A_2 A_4 \mid A_2 A_2 B_{\cancel{3}\,4} \mid A_2 A_2$

Step 7 - convert all productions to GNF.

A_4, A_3 is already in GNF.

$A_1\ A_2\ B_3$ can be converted to GNF.

$$\left.\begin{array}{l} A_4 \rightarrow b \\ A_3 \rightarrow aA_4 \mid aA_2 \mid aA_4B_3 \mid aA_2B_3 \end{array}\right\} \text{already in GNF.}$$

$B_3 \rightarrow A_2A_4B_3 \mid A_2A_2B_3 \mid A_2A_4 \mid A_2A_2.$

$A_2 \rightarrow aA_4A_2 \mid aA_2A_2 \mid aA_4B_3A_2 \mid aA_2B_3A_2 \mid a$

$A_1 \rightarrow A_2A_3 \mid aA_4$

rewrite it as

$A_1 \rightarrow aA_4A_2A_3 \mid aA_2A_2A_3 \mid aA_4B_3A_2A_3 \mid$
$aA_2B_3A_2A_3 \mid aA_3 \mid aA_4$

$B_3 \rightarrow A_2A_4B_3 \mid A_2A_2B_3 \mid A_2A_4 \mid A_2A_2$

rewrite B_3 by substituting A_2

$B_3 \rightarrow aA_4A_2A_4B_3 \mid aA_2A_2A_4B_3 \mid aA_4B_3A_2B_3 \mid$
$aA_2B_3A_2A_4B_3 \mid aA_4B_3 \mid aA_4A_2A_2B_3 \mid$
$aA_2A_2A_2B_3 \mid aA_4B_3A_2A_2B_3 \mid aA_2B_3A_2A_2$
$\mid aA_2B_3 \mid aA_4A_2A_2 \mid aA_2A_2A_2 \mid aA_4B_3A_2A_2$
$aA_2B_3A_2A_2 \mid aA_2 \mid aA_4A_2A_2 \mid aA_2A_2A_2 \mid$
$aA_4B_3A_2A_2 \mid aA_2B_3A_2A_2 \mid aA_2$

20 terms

Step 8 - write GNF Grammar rules.

$A_4 \rightarrow b$

A_3, A_2, A_1 and B_3.