Minimization of DFA

— Find equivalent DFA with minimum number of states.

Steps – ① Find and remove redundant states.

② Find equivalent states (using square optimization or visualization)

③ merge equivalent states into single state.

④ Redraw equivalent DFA transition diagram. & write final answer.

Step I – Remove redundant / non reachable states.

→ These states are not reachable from initial state on any possible sequence.

→ The presence or absense of these states does not affect the language accepted by automata. Let $S = \{q_0\}$ is a set of reachable states

Q \ Σ	0	1
→ q_0	q_1	q_5
q_1	q_6	q_2
* q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

Given DFA $M = (Q, \Sigma, \delta, q_0, F)$ $\quad q_0 = q_0$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$ $\quad F = \{q_2\}$

① $\delta(q_0, 0) = q_1$, $\boxed{q_1 \notin S}$, $\therefore S = \{q_0, q_1\}$

$\delta(q_0, 1) = q_5$, $\boxed{q_5 \notin S}$ $\therefore S = \{q_0, q_1, q_5\}$

② $\delta(q_1, 0) = q_6$ $\boxed{q_6 \notin S}$ $\therefore S = \{q_0, q_1, q_5, q_6\}$

$\delta(q_1, 1) = q_2$ $\boxed{q_2 \notin S}$ $\therefore S = \{q_0, q_1, q_5, q_6, q_2\}$

③ $\delta(q_5, 0) = q_2$ $\boxed{q_2 \in S}$ $\therefore$ No change in S

$\delta(q_5, 1) = q_6$ $\boxed{q_6 \in S}$ $\therefore$ — " —

④ $\delta(q_6, 0) = q_6$ $\boxed{q_6 \in S}$ $\therefore$ S is not changed.

$\delta(q_6, 1) = q_4$ $\boxed{q_4 \notin S}$ $\therefore S = \{q_0, q_1, q_5, q_6, q_2, q_4\}$

⑤ $\delta(q_2, 0) = q_0$ $\boxed{q_0 \in S}$ $\therefore$ No change in S

$\delta(q_2, 1) = q_2$ $\boxed{q_2 \in S}$ $\therefore$ No change in S.

⑥ $\delta(q_4, 0) = q_7$ $\boxed{q_7 \notin S}$ $\therefore S = \{q_0, q_1, q_5, q_6, q_2, q_4, q_7\}$

$\delta(q_4, 1) = q_5$ $\boxed{q_5 \in S}$ $\therefore$ No change in S.

⑦ $\delta(q_7, 0) = q_6$ $\boxed{q_6 \in S}$ } $\therefore$ No change in S.

$\delta(q_7, 1) = q_2$ $\boxed{q_2 \in S}$

$S = \{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}$

state q_3 is not reachable — remove

Step II – find equivalent states.

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

q_1	X I					
q_2	X I	X I				
* q_4	? ✓ II	X I	X I			
q_5	X-I	X I	X I	X I		
q_6	? X II	X I	X I	X I	X I	
= q_7	X I	✓ I	X I	X I	X I	X I
	q_0 *	q_1 =	q_2	q_4	q_5	q_6

Pass I – A final state can never be equivalent with non final state.

so q_2 is not equivalent with $q_0, q_4, q_5, q_6, q_7, q_1$

① $\delta(q_0, q_1) \Rightarrow (q_1, q_6)\ (q_5\ q_2)$ — $(q_5 + q_2)$ not equi so $q_0\ q_1$ also not equi

depends on — 0 transition — 1 transition — equivalence

② $\delta(q_0, q_4) \Rightarrow (q_1\ q_7)\ (q_5\ q_5)$ —— eq. ① can't say.

③ $\delta(q_0, q_5) \Rightarrow (q_1, q_2)\ (q_5, q_6)$ — not equivalent

④ $\delta(q_0, q_6) \Rightarrow (q_1, q_6)\ (q_5, q_4)$ —— eq. ②

⑤ $\delta(q_0, q_7) \Rightarrow (q_1, q_6)\ (q_5, q_2)$ —— not equivalent

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⑥ $\delta(q_1, q_4) \Rightarrow (q_6, q_7)\ (q_2, q_5)$ — not equivalent

⑦ $\delta(q_1, q_5) \Rightarrow (q_6, q_2)\ (q_2, q_6)$ — not equivalent

⑧ $\delta(q_1\ q_6) \Rightarrow (q_6, q_6)\ (q_2, q_4)$ — not equivalent

⑨ $\delta(q_1, q_7) \Rightarrow (q_6, q_6)\ (q_2, q_2)$ ← equivalent

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⑩ $\delta(q_4, q_5) \Rightarrow (q_7, q_2)\ (q_5, q_6)$ — not equivalent

⑪ $\delta(q_4, q_6) \Rightarrow (q_7, q_6)\ (q_5, q_4)$ — not equivalent

⑫ $\delta(q_4, q_7) \Rightarrow (q_7, q_6)\ (q_5, q_2)$ — not equivalent

(13) $(q_5, q_6) \Longrightarrow (q_2, q_6)\ (q_6, q_4)$ not ~~equivalent~~ equivalent

$(q_5, q_7) \Longrightarrow (q_2, q_6)\ (q_6, q_2)$ not equivalent

(14) $(q_6, q_7) \Longrightarrow (q_6, q_6)\ (q_4, q_2)$ not equivalent

Pass 2

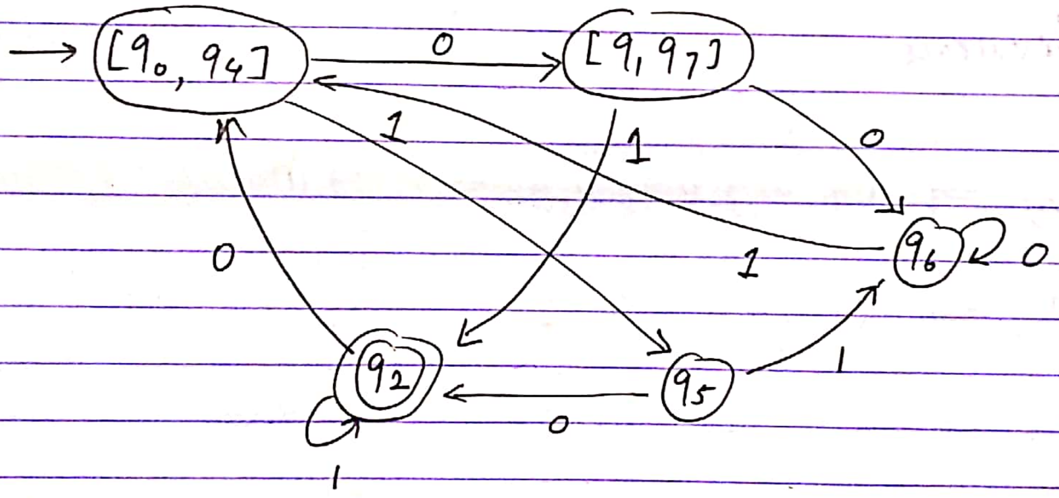
$(q_0\ q_4) \Longrightarrow (q_1\ q_7)\ (q_5\ q_5)$ — equivalent

$(q_0, q_6) \Longrightarrow (q_1\ q_6)\ (q_5\ q_4)$ — not equival.

Step III equivalent states are

(1) q_0 and q_4 and (2) q_1 and q_7

merge them & redraw TD.



Final answer - equivalent DFA

$$M' = (Q', \Sigma', \delta', q_0', F')$$

$Q' = \{[q_0, q_4], [q_1, q_7], q_2, q_5, q_6\}$

$\Sigma' = \{0, 1\}$ δ' as per diagram

$q_0' = [q_0, q_4]$

$F' = \{q_2\}$

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