DSIP – Lecturer 08

Image Enhancement in the Spatial Domain

Image Enhancement – Review

Types of image enhancement operations

Point/pixel operations	Output value at specific coordinates (x,y) is dependent only on the input value at (x,y)
Local operations	The output value at (x,y) is dependent on the input values in the *neighborhood* of (x,y)
Global operations	The output value at (x,y) is dependent on all the values in the input image

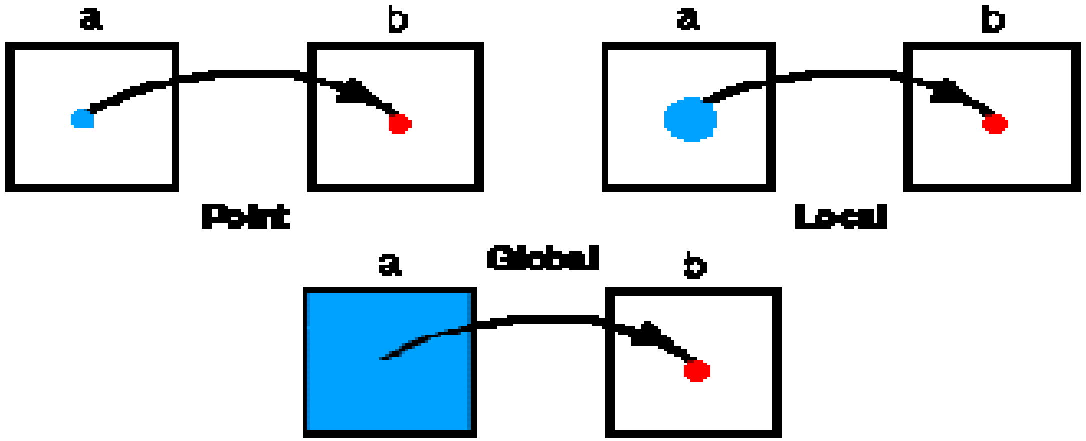


Image Enhancement

Process an image to make the result more suitable than the original image for a specific application

–*Image enhancement is subjective (problem /application oriented)*

Image enhancement methods:

Spatial domain: Direct manipulation of pixel in an image (on the image plane)

Frequency domain: Processing the image based on modifying the Fourier transform of an image

Many techniques are based on various combinations of methods from these two categories

Histograms

Histogram of an image with gray level (0 to L-1):

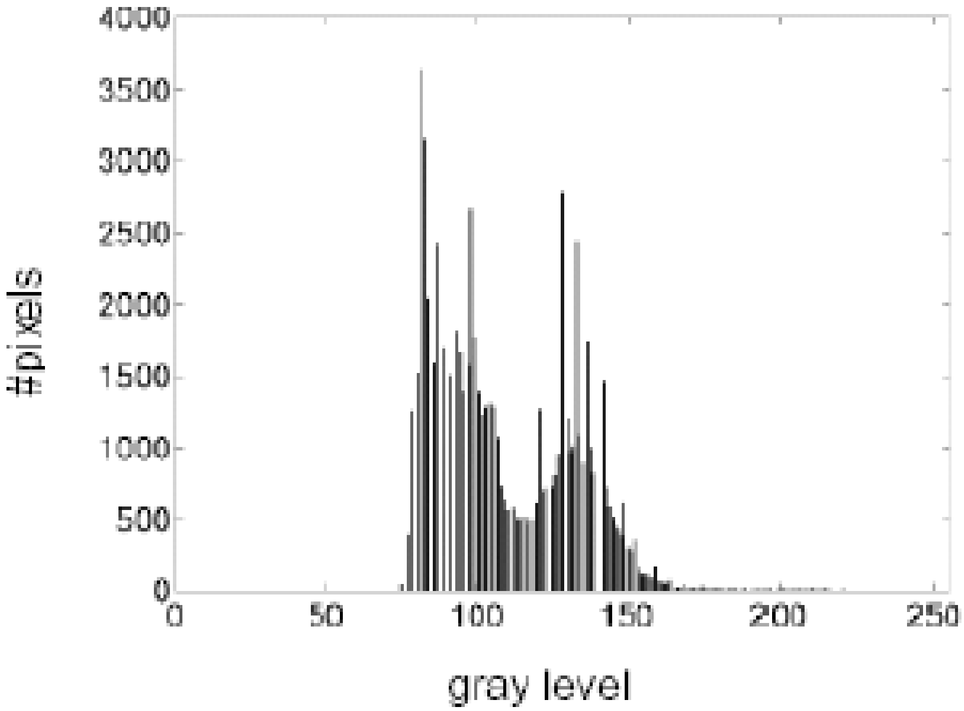
A discrete function $h(r_k) = n_k$, where r_k is the k^{th} gray level and n_k is the number of pixels in the image having gray level r_k .

How a histogram is obtained?

- *For B-bit image, initialize 2^B counters with 0*
- *Loop over all pixels x,y*
- *When encountering gray level f(x,y)=i, increment counter # i*

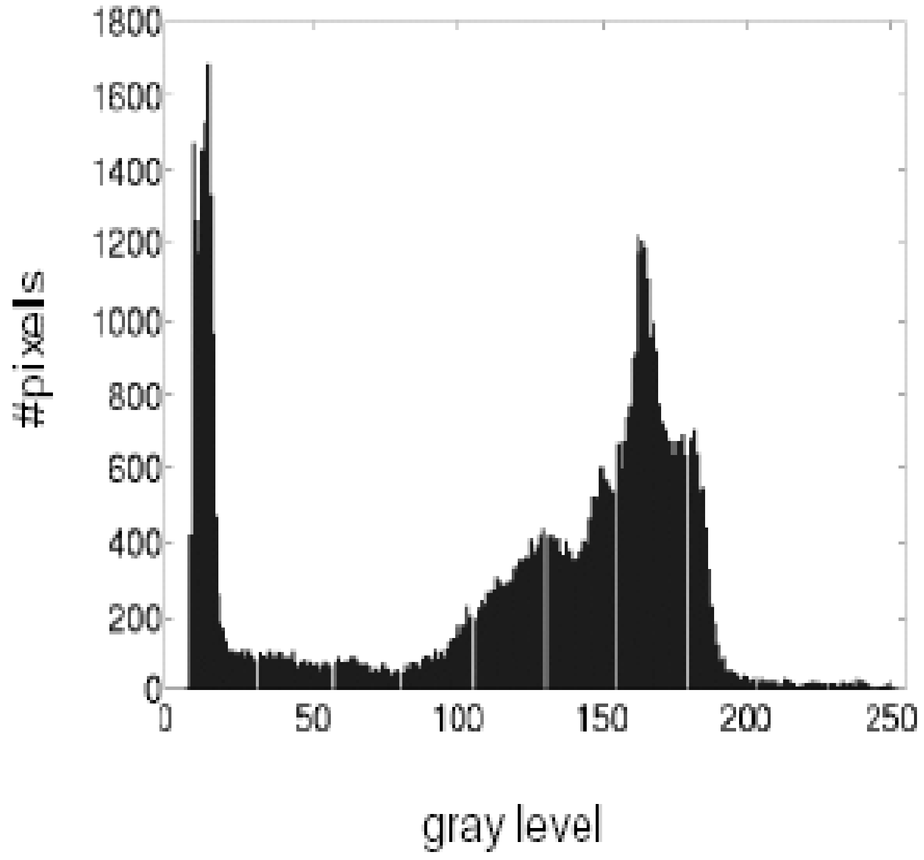
Normalized histogram: A discrete function $p(r_k) = n_k/n$, where n is the total number of pixels in the image. $p(r_k)$ estimates probability of occurrence of gray-level r_k

Example Histogram



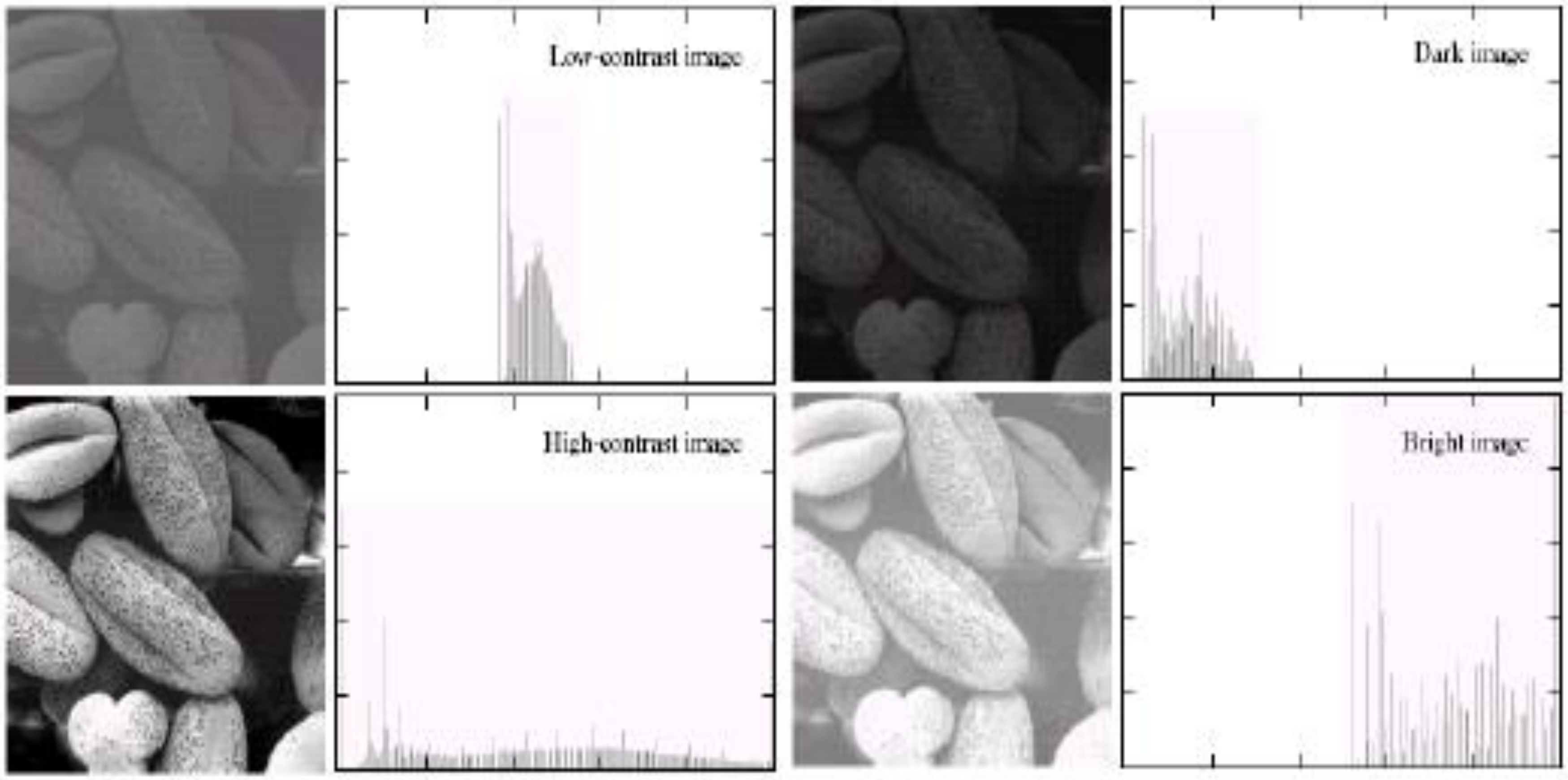
Pout image

Example Histogram



Cameraman image

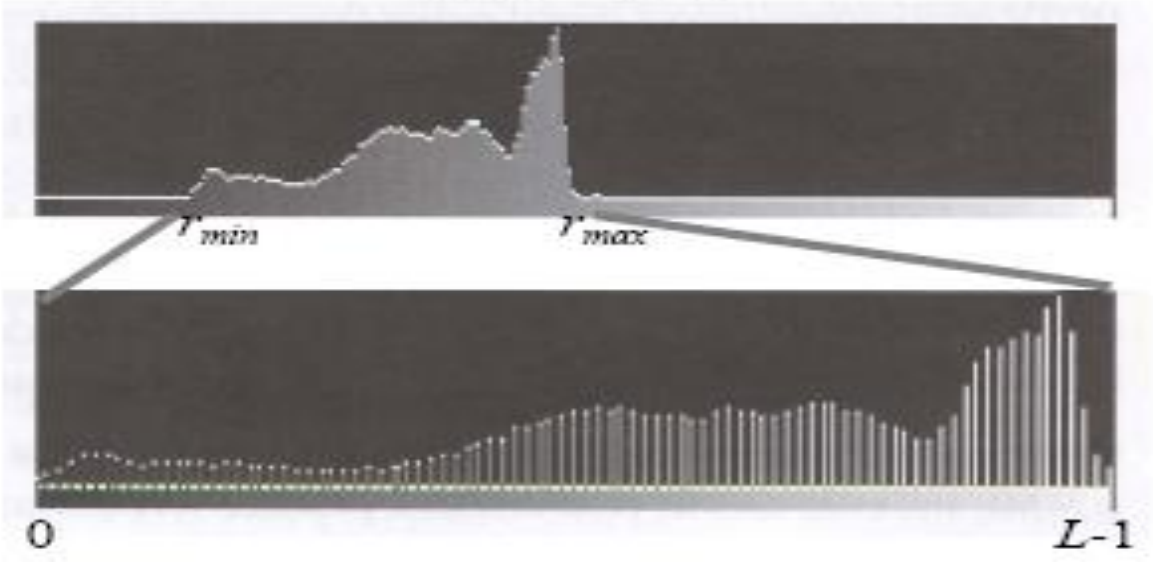
Histogram Examples



Contrast Stretching through Histogram

If r_{max} and r_{min} are the maximum and minimum gray level of the input image and L is the total gray levels of output image The transformation function for contrast stretching will be

$$s = T(r) = (r - r_{\min})\left(\frac{L}{r_{\max} - r_{\min}}\right)$$



Histogram Equalization

- Idea: To find a non-linear transformation

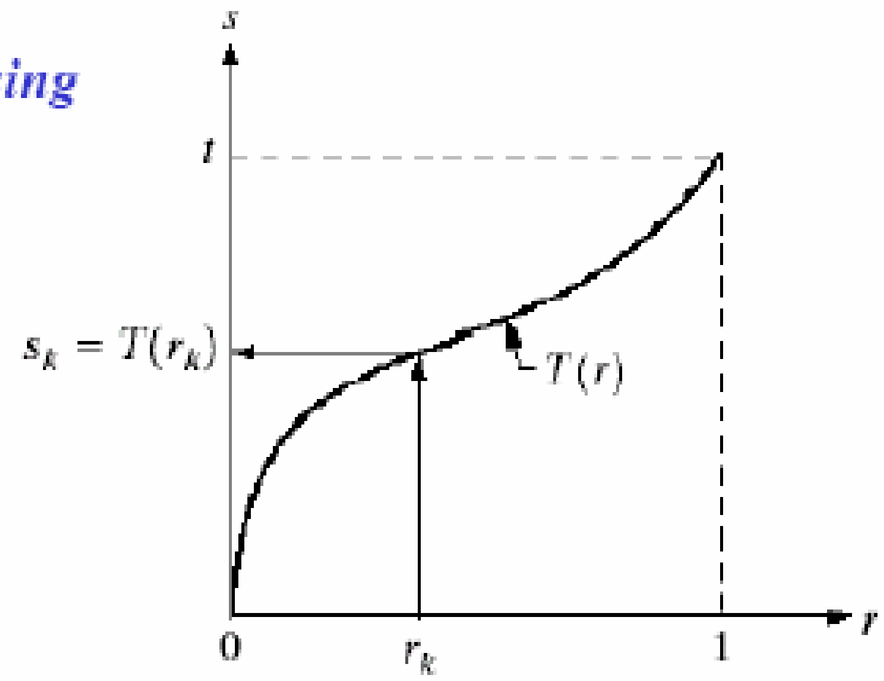
$$s = T(r)$$

to be applied to each pixel of the input image $f(x,y)$, such that a uniform distribution of gray levels in the entire range results for the output image $g(x,y)$.

- Assuming ideal, continuous case, with normalized histograms
  - *that* $0 \le r \le 1$ *and* $0 \le s \le 1$
  - $T(r)$ *is single valued i.e., there exists* $r = T^{-1}(r)$
  - $T(r)$ *is monotonically increasing*

Histogram Equalization

A function $T(r)$ is ***monotonically increasing***
if $T(r_1) < T(r_2)$ for $r_1 < r_2$,
and ***monotonically decreasing***
if $T(r_1) > T(r_2)$ for $r_1 < r_2$.



Example of a transformation function which is both single valued and monotonically increasing

Histogram Equalization

The discrete approximation of the transformation function for histogram equalization is:

$$\boxed{s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) \quad \text{for} \quad 0 \le k \le L-1}$$

where $p_r(r_j) = \dfrac{n_j}{n}, \quad j = 0, \ldots\ldots, L-1 \quad and \quad n = \sum_{j=0}^{L-1} n_j$

n_j : number of pixels with gray level r_j

n : total number of pixels

Note: For digital images, gray-level pdf cannot be exactly uniform after histogram equalization

Histogram Equalization

- Spreading out the frequencies in an image (or equalising the image) is a simple way to improve dark or washed out images
- The formula for histogram equalisation is given where
  - r_k: input intensity
  - s_k: processed intensity
  - k: the intensity range (e.g 0.0 – 1.0)
  - n_j: the frequency of intensity j
  - n: the sum of all frequencies

$$\boxed{\begin{aligned} s_k &= T(r_k) \\ &= \sum_{j=1}^{k} p_r(r_j) \\ &= \sum_{j=1}^{k} \frac{n_j}{n} \end{aligned}}$$

Histogram Equalization: Example

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Histogram Equalization: Example

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 12 & 53 & 93 & 146 & 53 & 73 & 166 \\ 65 & 32 & 12 & 215 & 235 & 202 & 130 & 158 \\ 57 & 32 & 117 & 239 & 251 & 227 & 93 & 166 \\ 65 & 20 & 154 & 243 & 255 & 231 & 146 & 130 \\ 97 & 53 & 117 & 227 & 247 & 210 & 117 & 146 \\ 190 & 85 & 36 & 146 & 178 & 117 & 20 & 170 \\ 202 & 154 & 73 & 32 & 12 & 53 & 85 & 194 \\ 206 & 190 & 130 & 117 & 85 & 174 & 182 & 219 \end{bmatrix}$$

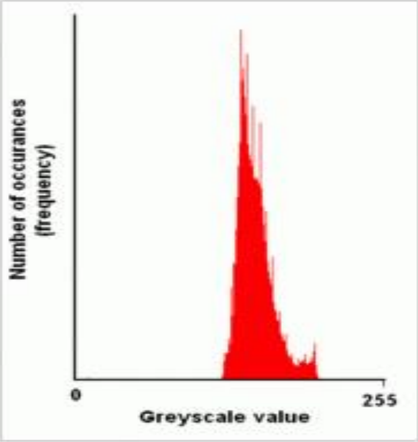
Initial Image

Image After Equalization

Notice that the minimum value (52) is now 0 and the maximum value (154) is now 255.

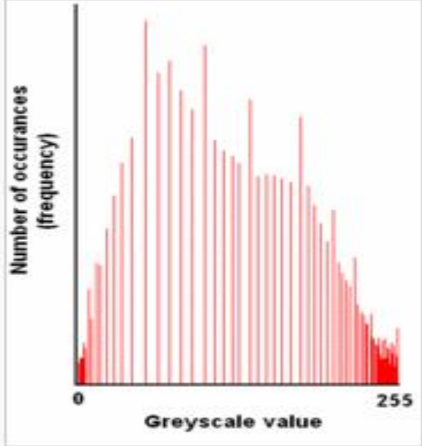
Histogram Equalization: Example

An unequalized image



Corresponding histogram

The same image after histogram equalization



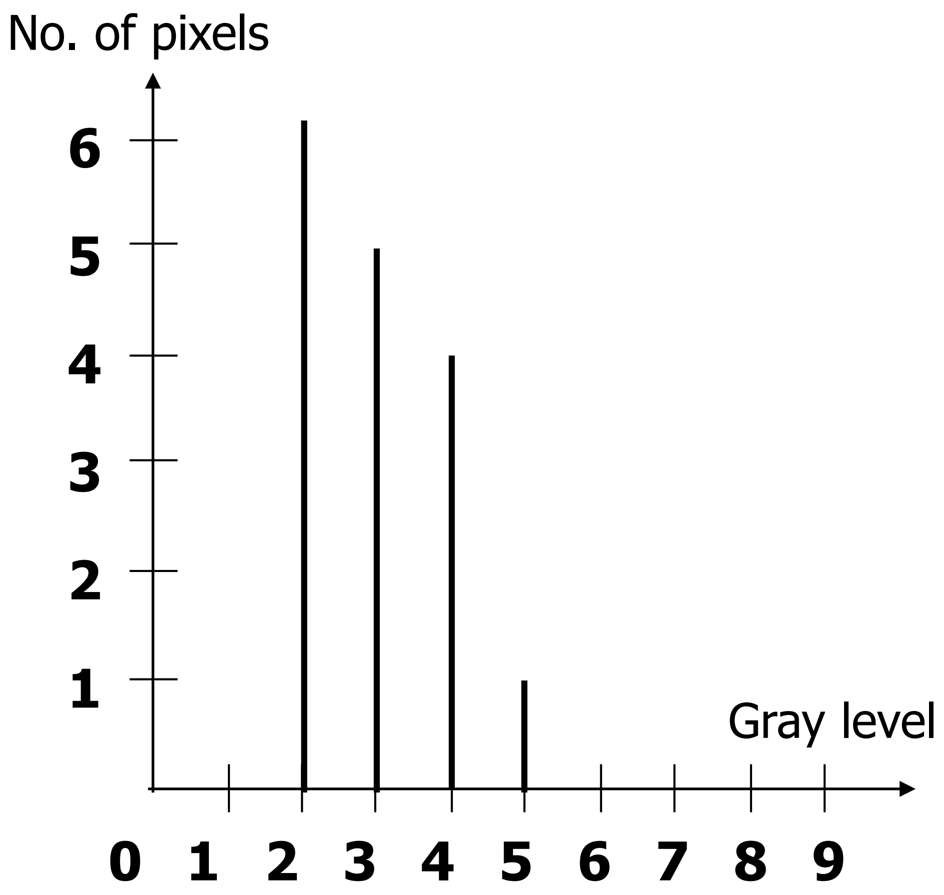
Corresponding histogram

Histogram Equalization: Example

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



histogram

Histogram Equalization: Example

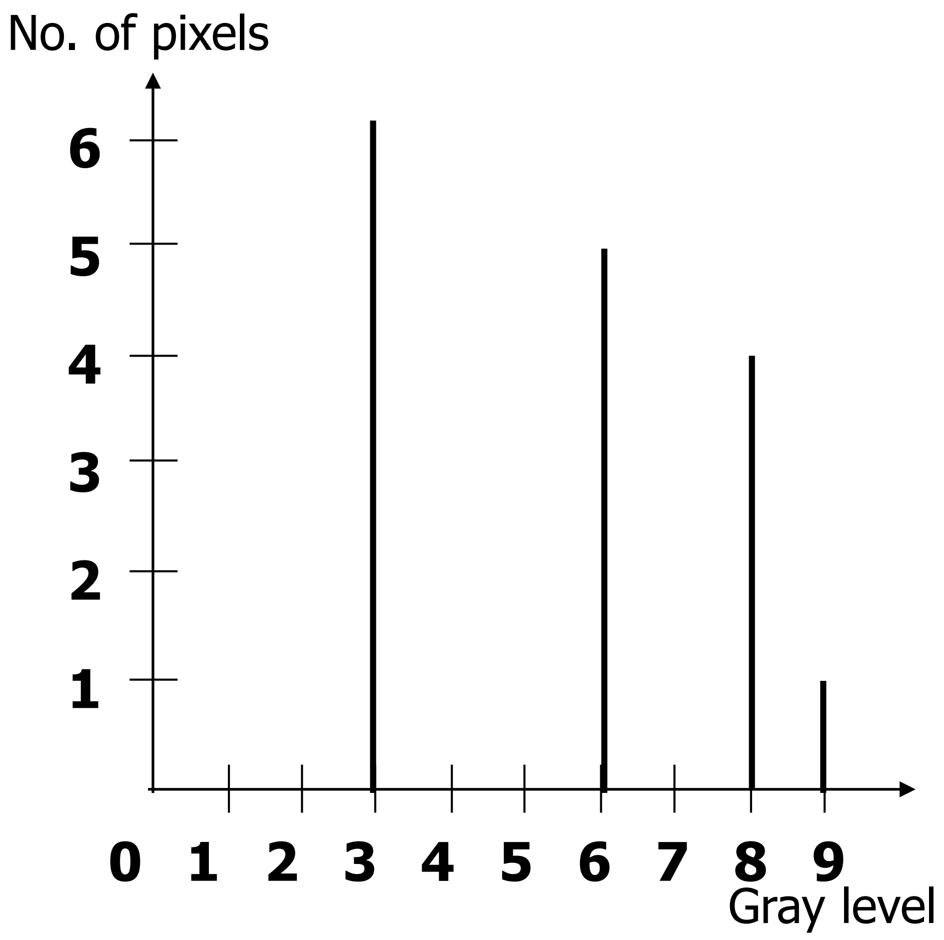
Gray Level(j)	0	1	2	3	4	5	6	7	8	9
No. of pixels	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^{k} n_j$	0	0	6	11	15	16	16	16	16	16
$s = \sum_{j=0}^{k} \frac{n_j}{n}$	0	0	6 / 16	11 / 16	15 / 16	16 / 16	16 / 16	16 / 16	16 / 16	16 / 16
s x 9	0	0	3.3 ≈3	6.1 ≈6	8.4 ≈8	9	9	9	9	9

Histogram Equalization: Example

3	6	6	3
8	3	8	6
6	3	6	9
3	8	3	8

Output image

Gray scale = [0,9]



Histogram equalization

Histogram Matching

Histogram matching (histogram specification)

—A processed image has a specified histogram

Let $p_r(r)$ and $p_z(z)$ denote the continous probability density functions of the variables r and z. $p_z(z)$ is the specified probability density function.

Let s be the random variable with the probability

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Obtain a transformation function G

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

Histogram Matching

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

Histogram Matching: Procedure

- Obtain $p_r(r)$ from the input image and then obtain the values of s

$$s = (L-1)\int_0^r p_r(w)dw$$

- Use the specified PDF and obtain the transformation function G(z)

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

- Mapping from s to z

$$z = G^{-1}(s)$$

Histogram Matching: Example

Assuming continuous intensity values, suppose that an image has the intensity PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L\text{-}1 \\ 0\ , & \text{otherwise} \end{cases}$$

Find the transformation function that will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \dfrac{3z^2}{(L-1)^3}, & \text{for } 0 \le z \le (L\text{-}1) \\ 0, & \text{otherwise} \end{cases}$$

Histogram Matching: Example

Find the histogram equalization transformation for the input image

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = (L-1)\int_0^r \frac{2w}{(L-1)^2} dw = \frac{r^2}{L-1}$$

Find the histogram equalization transformation for the specified histogram

$$G(z) = (L-1)\int_0^z p_z(t)dt = (L-1)\int_0^z \frac{3t^2}{(L-1)^3} dt = \frac{z^3}{(L-1)^2} = s$$

The transformation function

$$z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)^2 \frac{r^2}{L-1}\right]^{1/3} = \left[(L-1)r^2\right]^{1/3}$$

Histogram Matching: Discrete Cases

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k, round the value to the integer range [0, L-1].

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range [0, L-1].

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

- Mapping from s_k to z_q

$$z_q = G^{-1}(s_k)$$

Example: Histogram Matching

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

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Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

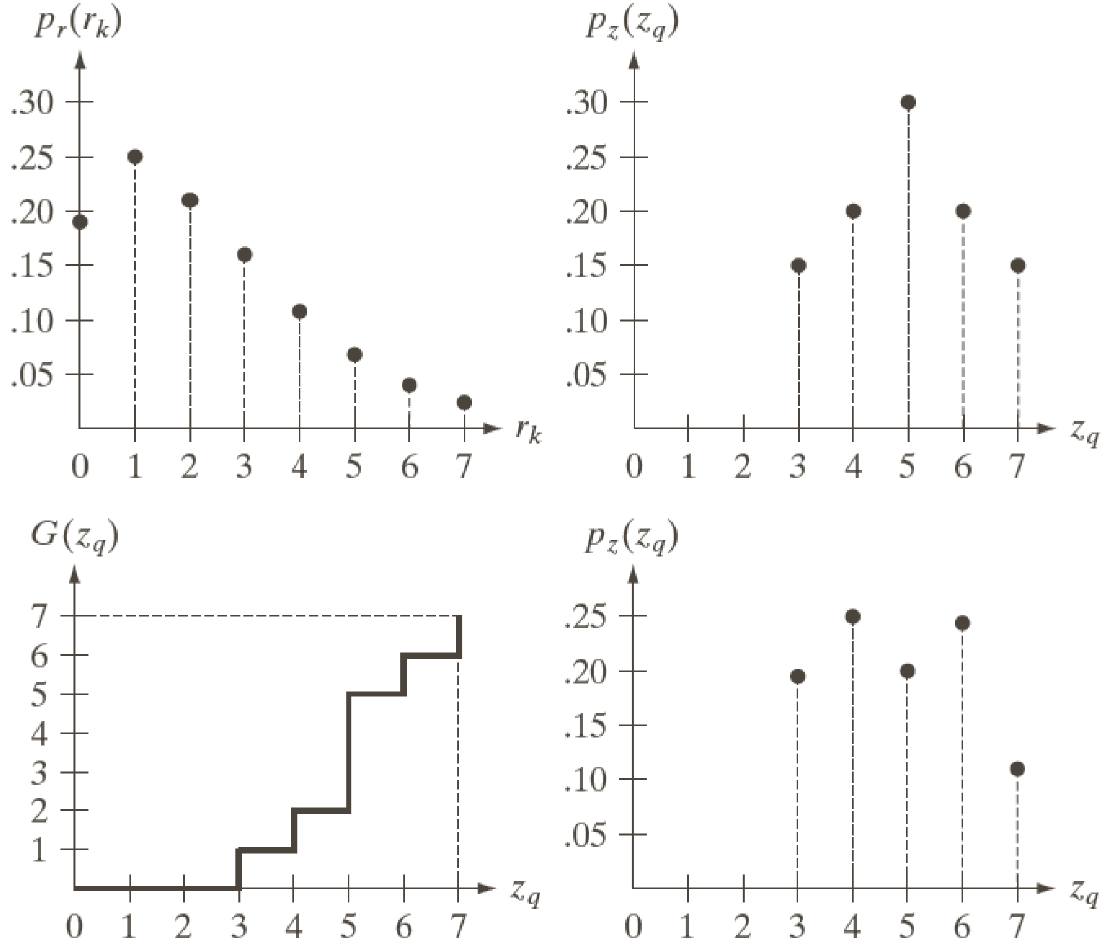
$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$
$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_5) = 4.55 \rightarrow 5 \qquad G(z_6) = 5.95 \rightarrow 6$$
$$G(z_7) = 7.00 \rightarrow 7$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Example: Histogram Matching



a	b
c	d

FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

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Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$G(z_1) = 0.00 \rightarrow 0$ $\quad$ $G(z_2) = 0.00 \rightarrow 0$

$G(z_3) = 1.05 \rightarrow 1$ **s_0** $\quad$ $G(z_4) = 2.45 \rightarrow 2$ **s_1**

$G(z_5) = 4.55 \rightarrow 5$ **s_2** $\quad$ $G(z_6) = 5.95 \rightarrow 6$ **s_3**

$G(z_7) = 7.00 \rightarrow 7$ **s_4 s_5 s_6 s_7**

Example: Histogram Matching

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

r_k

0
1
2
3
4
5
6
7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Example: Histogram Matching

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

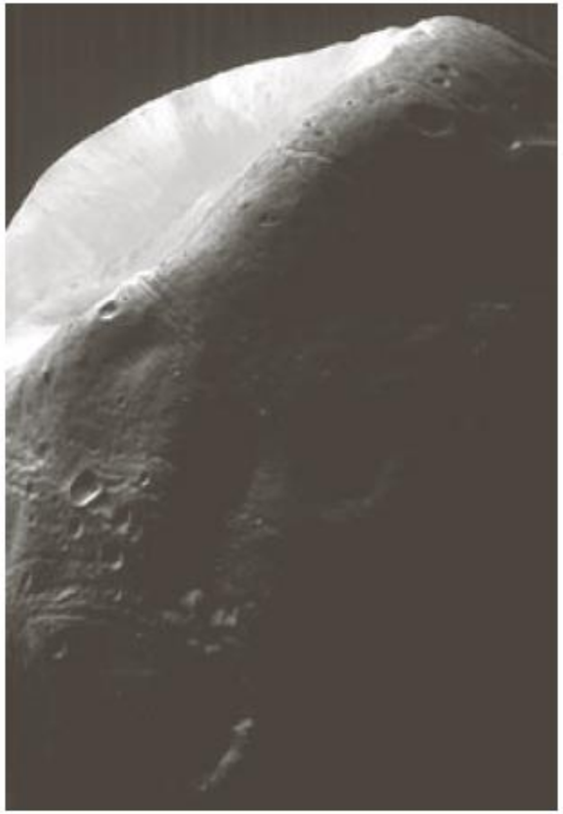
$$4 \rightarrow 7$$

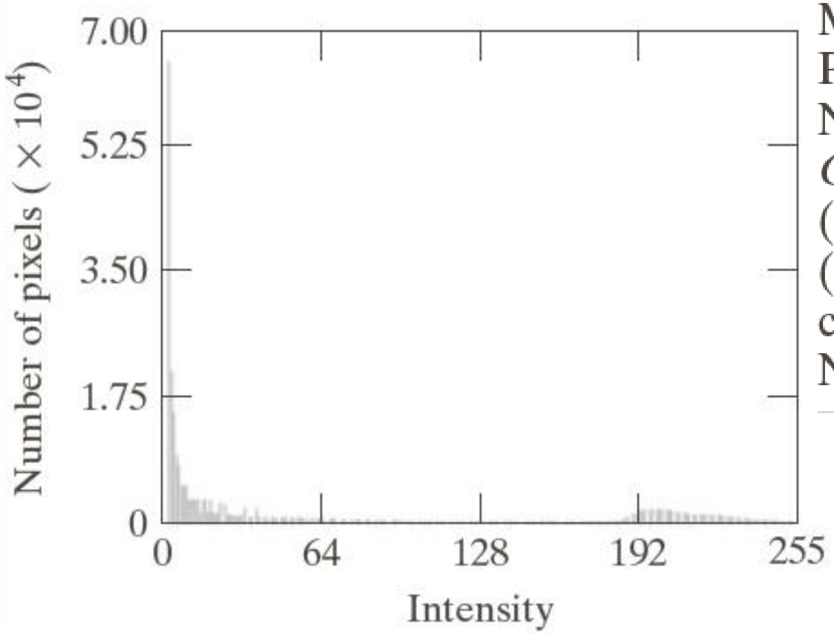
$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

Example: Histogram Matching

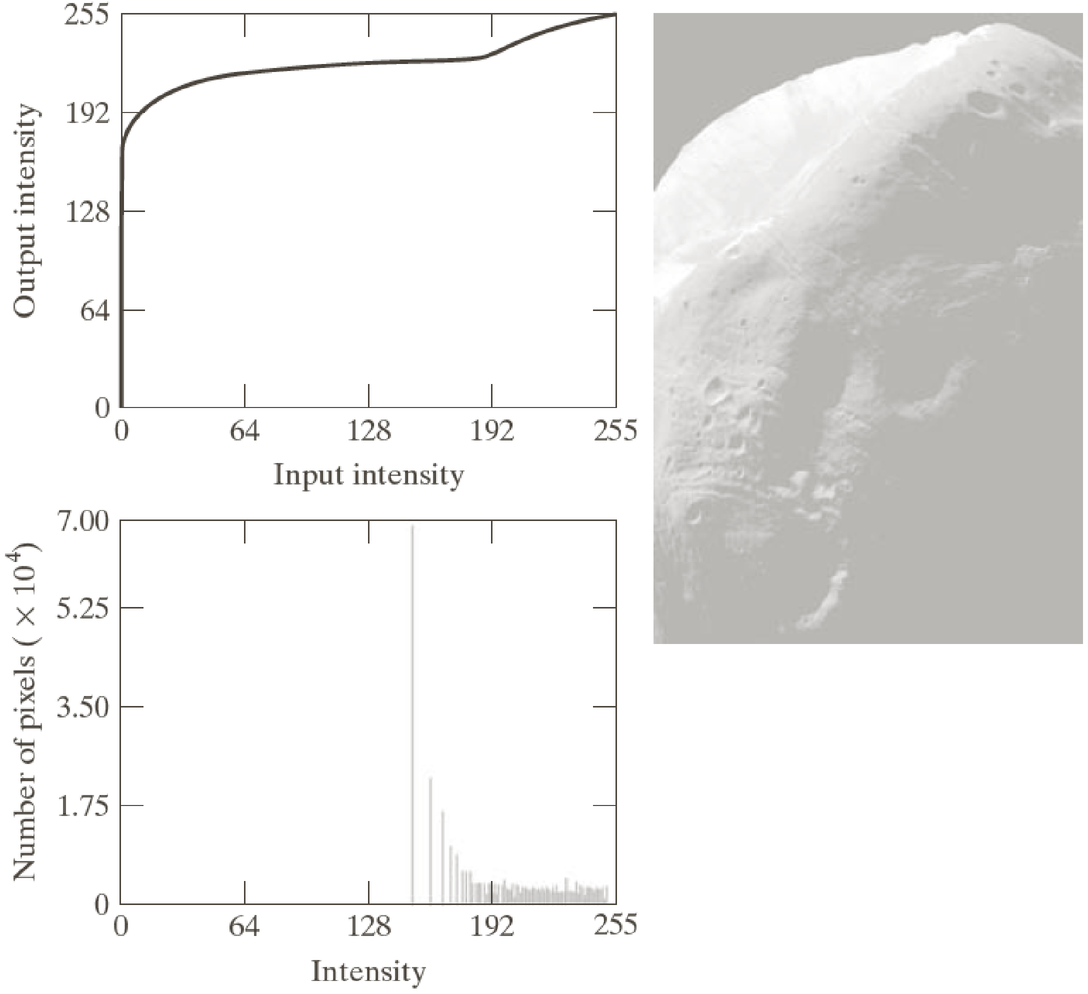




a b

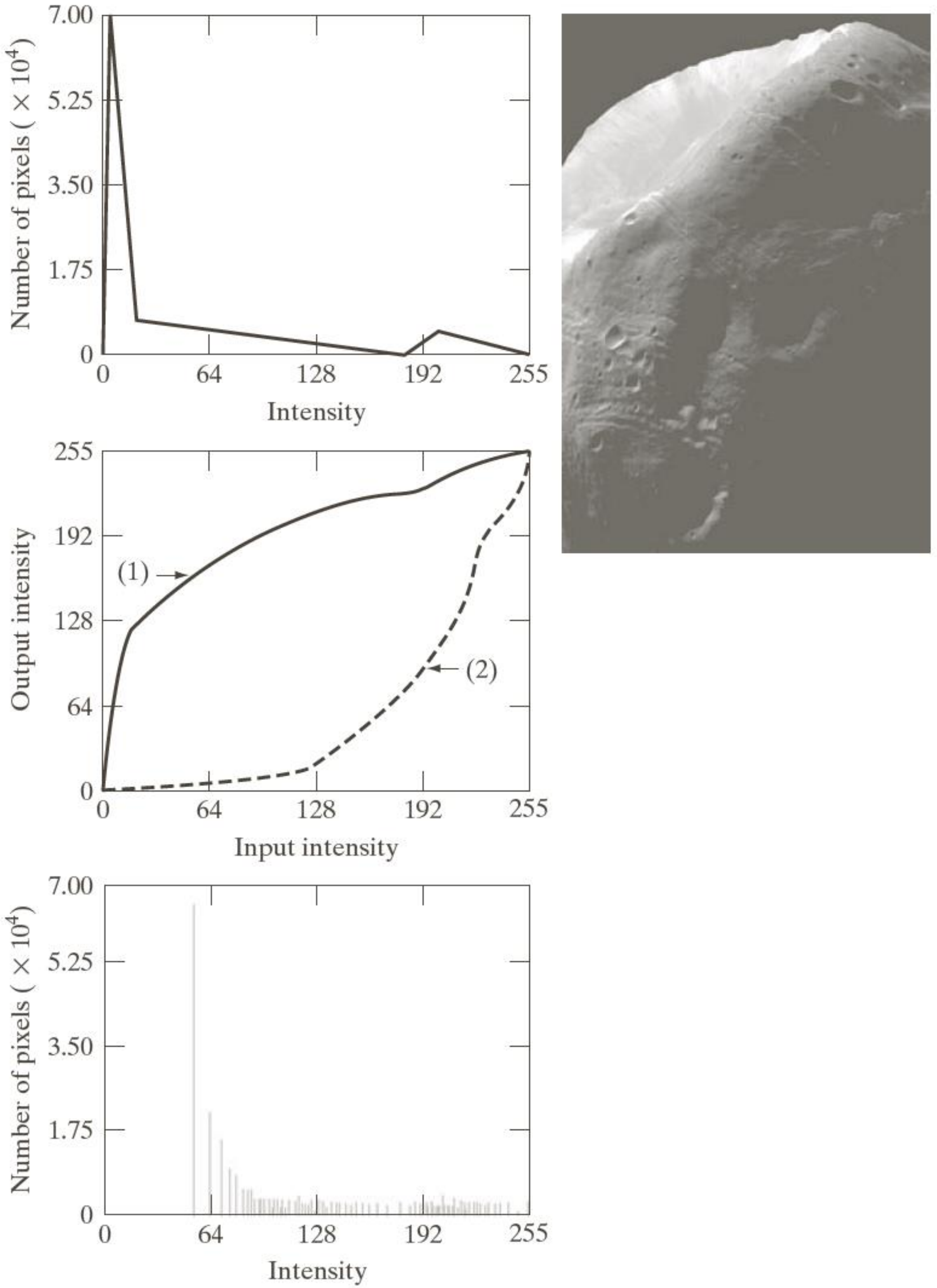
FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Example: Histogram Matching



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).