

Recommender Systems: Content-based Systems & Collaborative Filtering

Mining of Massive Datasets

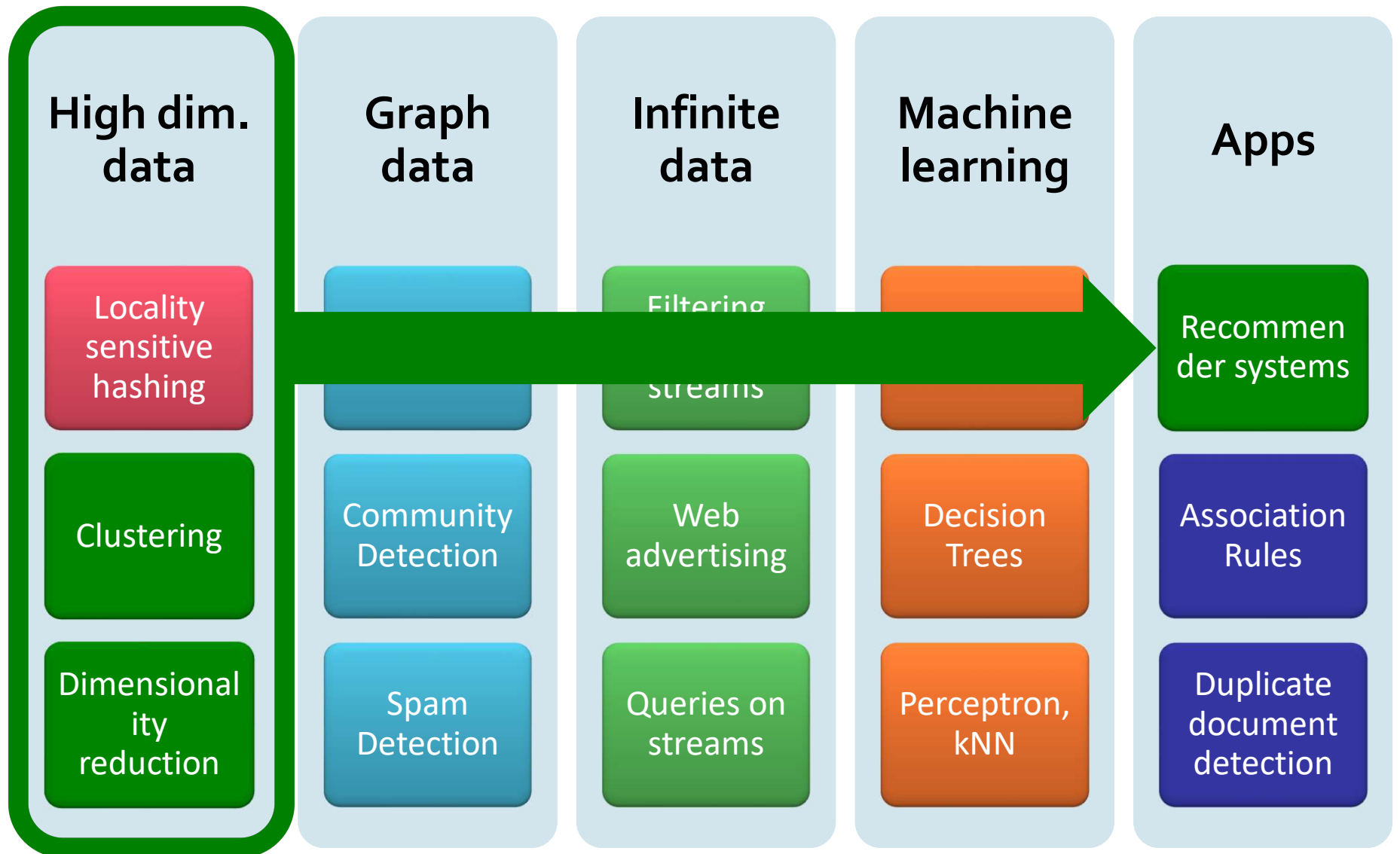
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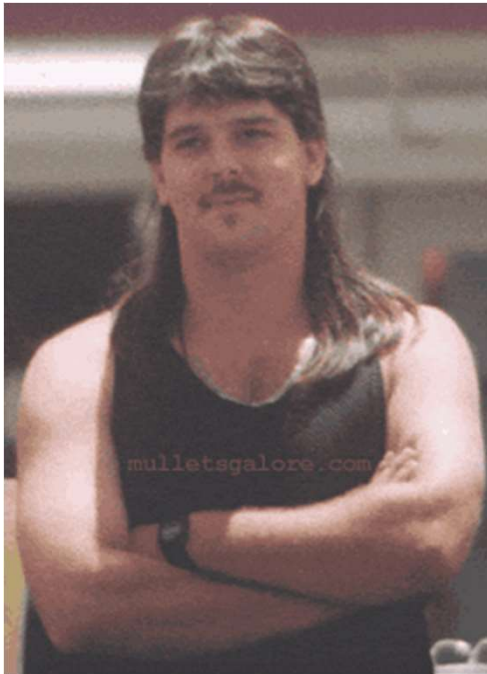
<http://www.mmds.org>



High Dimensional Data



Example: Recommender Systems



■ Customer X

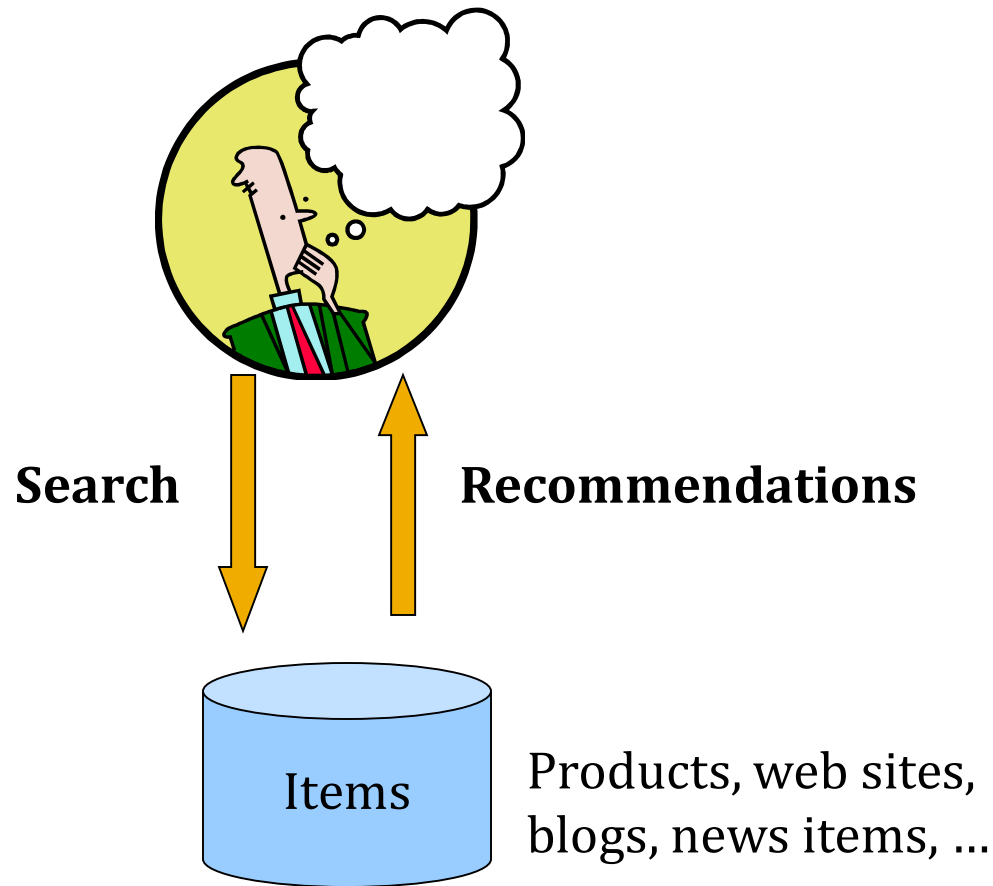
- Buys Metallica CD
- Buys Megadeth CD



■ Customer Y

- Does search on Metallica
- Recommender system suggests Megadeth from data collected about customer X

Recommendations



Examples:

amazon.com.



StumbleUpon



del.icio.us



movielens
helping you find the *right* movies

last.fm™
the social music revolution

Google
News

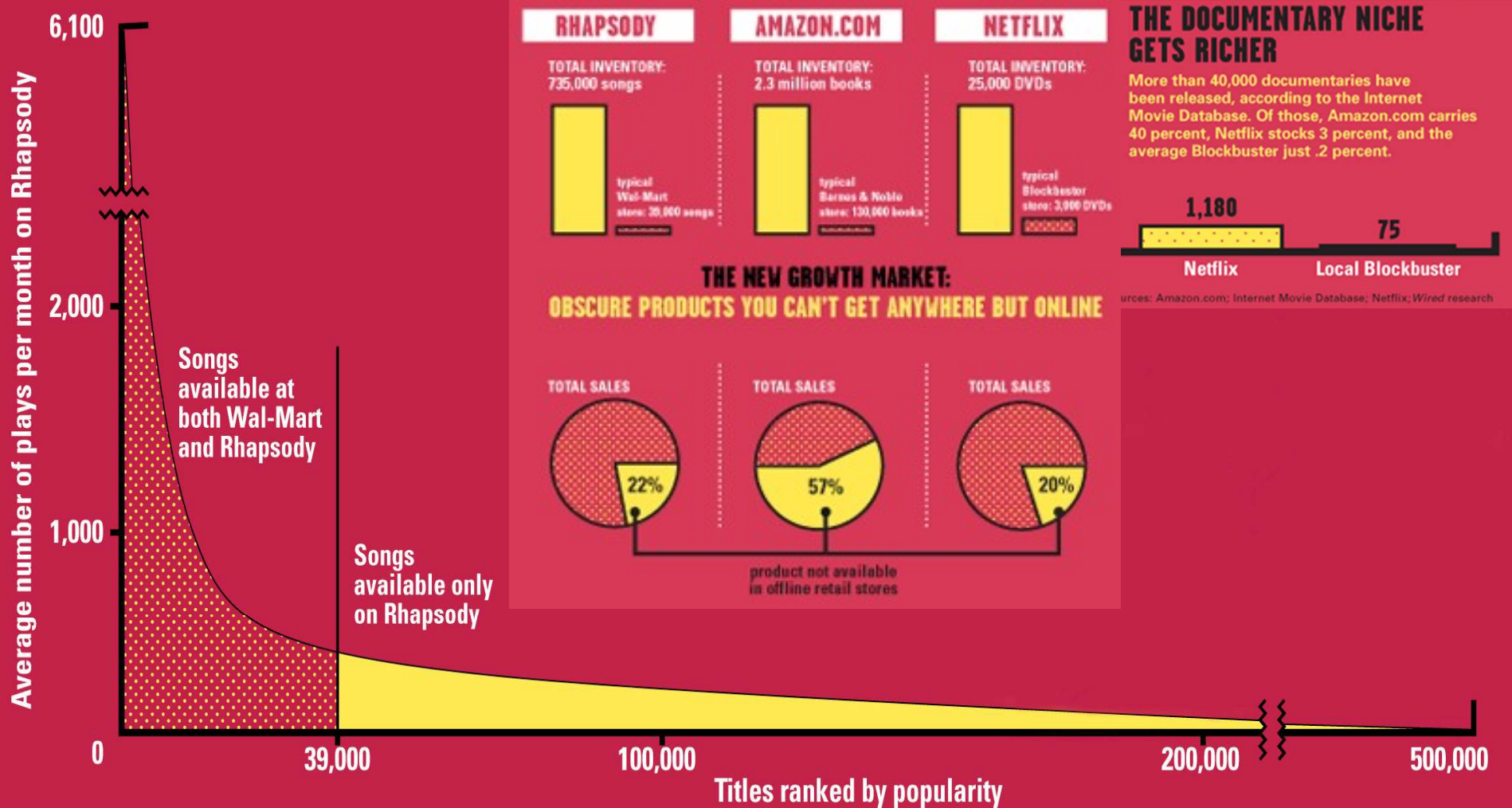
YouTube

XBOX
LIVE

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
 - Also: TV networks, movie theaters,...
- **Web enables near-zero-cost dissemination of information about products**
 - From scarcity to abundance
- **More choice necessitates better filters**
 - Recommendation engines
 - How **Into Thin Air** made **Touching the Void** a bestseller: <http://www.wired.com/wired/archive/12.10/tail.html>

Sidenote: The Long Tail



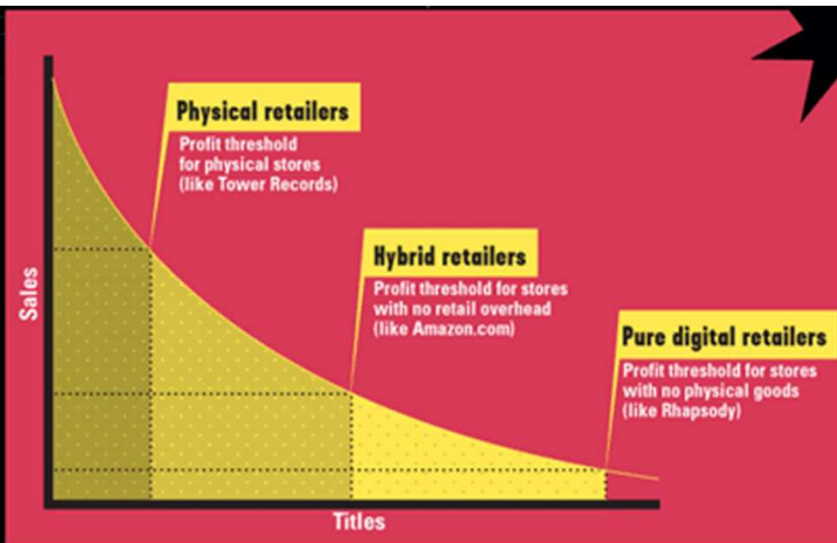
Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks
Source: Chris Anderson (2004)

Physical vs. Online

THE BIT PLAYER ADVANTAGE

Beyond bricks and mortar there are two main retail models – one that gets halfway down the Long Tail and another that goes all the way. The first is the familiar hybrid model of Amazon and Netflix, companies that sell physical goods online. Digital catalogs allow them to offer unlimited selection along with search, reviews, and recommendations, while the cost savings of massive warehouses and no walk-in customers greatly expands the number of products they can sell profitably.

Pushing this even further are pure digital services, such as iTunes, which offer the additional savings of delivering their digital goods online at virtually no marginal cost. Since an extra database entry and a few megabytes of storage on a server cost effectively nothing, these retailers have no economic reason not to carry *everything* available.



Read <http://www.wired.com/wired/archive/12.10/tail.html> to learn more!

Types of Recommendations

- **Editorial and hand curated**
 - List of favorites
 - Lists of “essential” items
- **Simple aggregates**
 - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
 - Amazon, Netflix, ...

Formal Model

- X = set of **Customers**
- S = set of **Items**
- **Utility function** $u: X \times S \rightarrow R$
 - R = set of ratings
 - R is a totally ordered set
 - e.g., **0-5** stars, real number in **[0,1]**

Utility Matrix

| | Avatar | LOTR | Matrix | Pirates |
|-------|--------|------|--------|---------|
| Alice | 1 | | 0.2 | |
| Bob | | 0.5 | | 0.3 |
| Carol | 0.2 | | 1 | |
| David | | | | 0.4 |

Key Problems

- **(1) Gathering “known” ratings for matrix**
 - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
 - Mainly interested in high unknown ratings
 - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
 - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**

- Ask people to rate items
- Doesn't work well in practice – people can't be bothered

- **Implicit**

- Learn ratings from user actions
 - E.g., purchase implies high rating
- What about low ratings?

(2) Extrapolating Utilities

- **Key problem:** Utility matrix U is **sparse**
 - Most people have not rated most items
 - **Cold start:**
 - New items have no ratings
 - New users have no history
- **Three approaches to recommender systems:**
 - 1) Content-based
 - 2) Collaborative
 - 3) Latent factor based

Content-based Recommender Systems

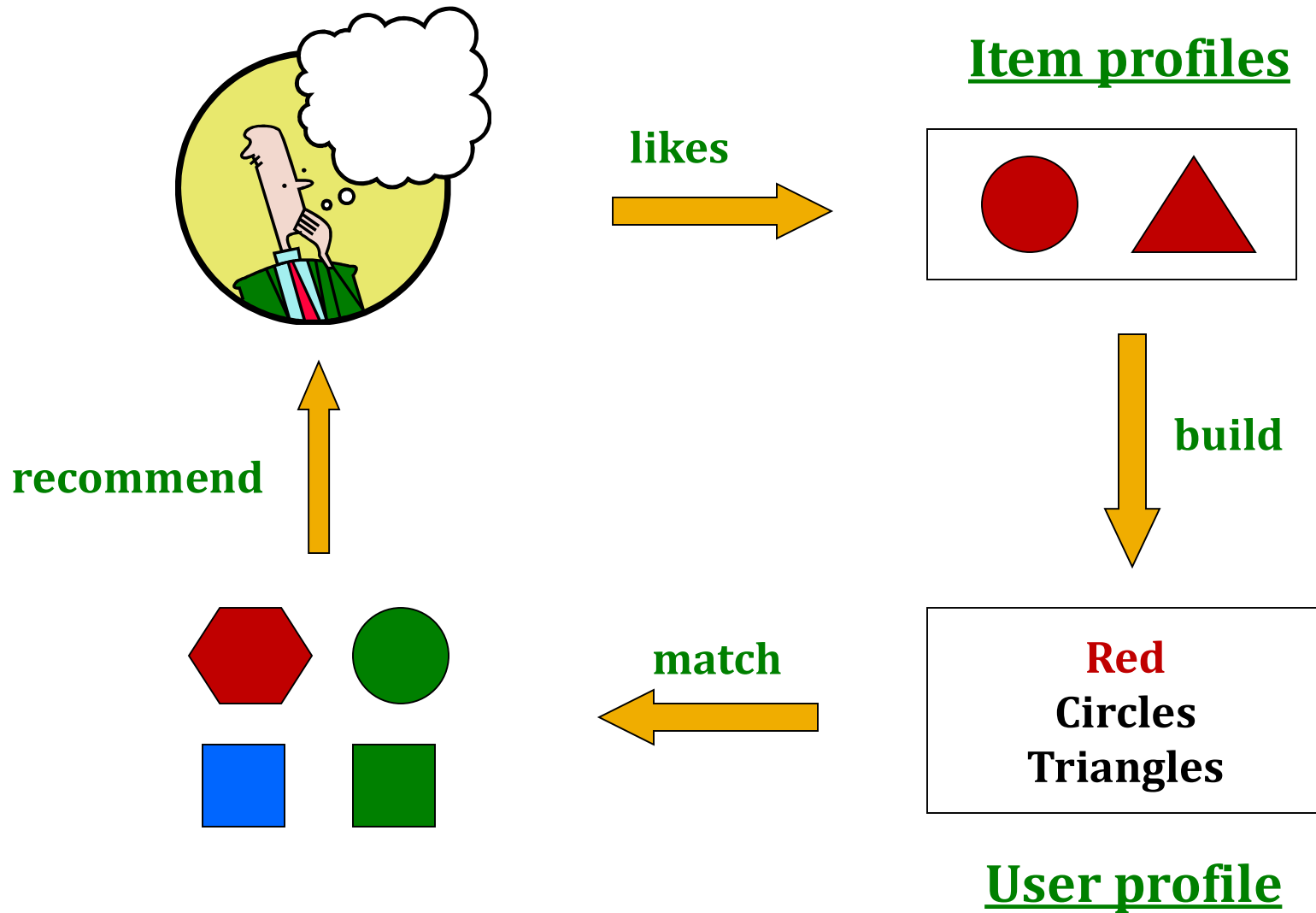
Content-based Recommendations

- **Main idea:** Recommend items to customer x similar to previous items rated highly by x

Example:

- **Movie recommendations**
 - Recommend movies with same actor(s), director, genre, ...
- **Websites, blogs, news**
 - Recommend other sites with “similar” content

Plan of Action



Item Profiles

- For each item, create an **item profile**
- **Profile is a set (vector) of features**
 - **Movies:** author, title, actor, director,...
 - **Text:** Set of “important” words in document
- **How to pick important features?**
 - Usual heuristic from text mining is **TF-IDF**
(Term frequency * Inverse Doc Frequency)
 - **Term ... Feature**
 - **Document ... Item**

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) i in doc (item) j

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

n_i = number of docs that mention term i

N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

Note: we normalize TF to discount for “longer” documents

$\max_k f_{kj}$ = Max number of times Word i appearing in any of document

IDF – high – more weight to less appearing word
- less – less weight to freq. appearing word

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest **TF-IDF** scores, together with their scores

User Profiles and Prediction

- **User profile possibilities:**

- Weighted average of rated item profiles
- **Variation:** weight by difference from average rating for item
- ...

- **Prediction heuristic:**

- Given user profile \mathbf{x} and item profile \mathbf{i} , estimate
- $u(\mathbf{x}, \mathbf{i}) = \cos(\mathbf{x}, \mathbf{i}) = \frac{\mathbf{x} \cdot \mathbf{i}}{||\mathbf{x}|| \cdot ||\mathbf{i}||}$

Pros: Content-based Approach

- **+: No need for data on other users**
 - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
 - No first-rater problem
- **+: Able to provide explanations**
 - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

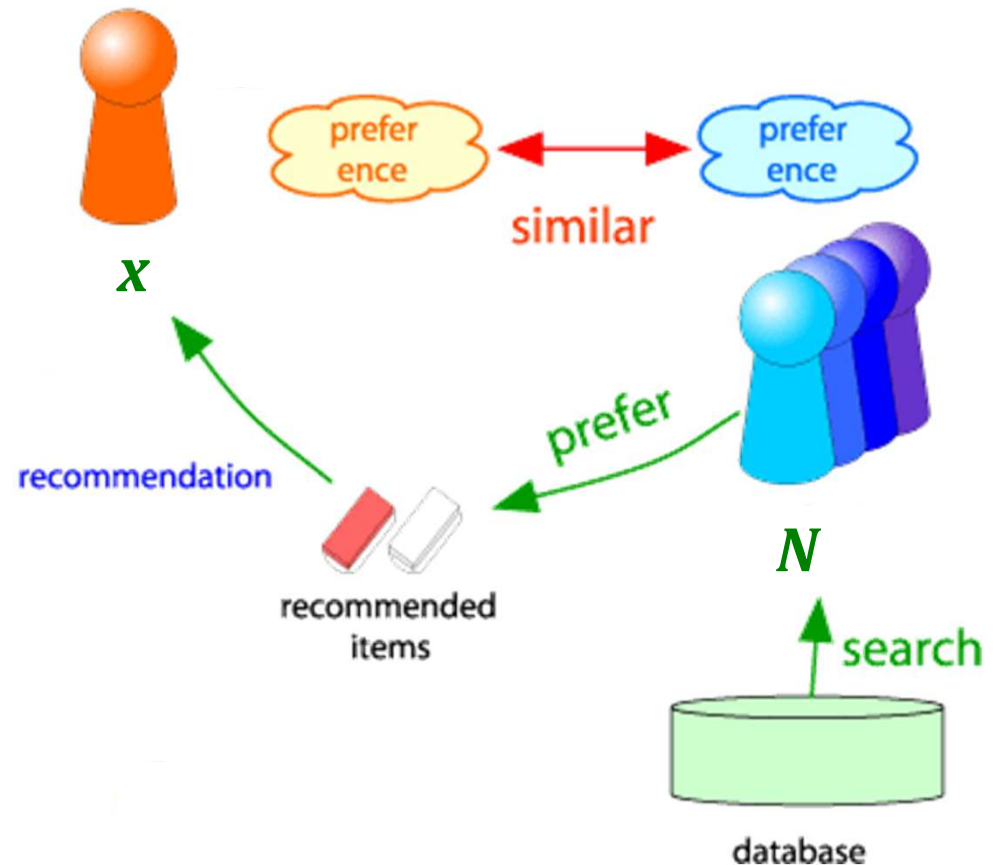
- **–: Finding the appropriate features is hard**
 - E.g., images, movies, music
- **–: Recommendations for new users**
 - **How to build a user profile?**
- **–: Overspecialization**
 - Never recommends items outside user's content profile
 - People might have multiple interests
 - **Unable to exploit quality judgments of other users**

Collaborative Filtering

Harnessing quality judgments of other users

Collaborative Filtering

- Consider user x
- Find set N of other users whose ratings are “**similar**” to x ’s ratings
- Estimate x ’s ratings based on ratings of users in N



Finding “Similar” Users

$$\begin{aligned} r_x &= [*, _, _, *, ***] \\ r_y &= [*, _, **, **, _] \end{aligned}$$

- Let r_x be the vector of user x 's ratings
- Jaccard similarity measure**
 - Problem:** Ignores the value of the rating

r_x, r_y as sets:

$$r_x = \{1, 4, 5\}$$

$$r_y = \{1, 3, 4\}$$

- Cosine similarity measure**
 - $\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{\|r_x\| \cdot \|r_y\|}$
 - Problem:** Treats missing ratings as “negative”

r_x, r_y as points:

$$r_x = \{1, 0, 0, 1, 3\}$$

$$r_y = \{1, 0, 2, 2, 0\}$$

- Pearson correlation coefficient**
 - S_{xy} = items rated by both users x and y

$$\text{sim}(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)(r_{ys} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \bar{r}_y)^2}}$$

$\bar{r}_x, \bar{r}_y \dots$ avg.
rating of x, y

Similarity Metric

$$\text{Cosine sim: } \text{sim}(x, y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

| | HP1 | HP2 | HP3 | TW | SW1 | SW2 | SW3 |
|---|-----|-----|-----|----|-----|-----|-----|
| A | 4 | | | 5 | 1 | | |
| B | 5 | 5 | 4 | | | | |
| C | | | | 2 | 4 | 5 | |
| D | | 3 | | | | | 3 |

- **Intuitively we want:** $\text{sim}(A, B) > \text{sim}(A, C)$
- **Jaccard similarity:** $1/5 < 2/4$
- **Cosine similarity:** $0.386 > 0.322$
 - Considers missing ratings as “negative”
 - **Solution: subtract the (row) mean**

$$4+5+1=10 \quad \text{Avg}=10/3 \quad 4-10/3=2/3$$

| | HP1 | HP2 | HP3 | TW | SW1 | SW2 | SW3 |
|---|-----|-----|------|------|------|-----|-----|
| A | 2/3 | | | 5/3 | -7/3 | | |
| B | 1/3 | 1/3 | -2/3 | | | | |
| C | | | | -5/3 | 1/3 | 4/3 | |
| D | | 0 | | | | | 0 |

sim A,B vs. A,C:
 $0.092 > -0.559$

Notice cosine sim. is
 correlation when
 data is centered at 0

Rating Predictions

From similarity metric to recommendations:

- Let \mathbf{r}_x be the vector of user x 's ratings
- Let N be the set of k users most similar to x who have rated item I
- **Prediction for item s of user x :**
 - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
 - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$
 - Other options?
- **Many other tricks possible...**

Shorthand:

$$s_{xy} = \text{sim}(x, y)$$

Item-Item Collaborative Filtering

- So far: **User-user collaborative filtering**
- **Another view: Item-item**
 - For item i , find other similar items
 - Estimate rating for item i based on ratings for similar items
 - Can use same similarity metrics and prediction functions as in user-user model

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij} ... similarity of items i and j

r_{xj} ... rating of user u on item j

$N(i;x)$... set items rated by x similar to i

Item-Item CF ($|N|=2$)

users

movies

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 1 | | 3 | | | 5 | | | 5 | | 4 | |
| 2 | | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| 3 | 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| 4 | | 2 | 4 | | 5 | | | 4 | | | 2 | |
| 5 | | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| 6 | 1 | | 3 | | 3 | | | 2 | | | 4 | |

- unknown rating

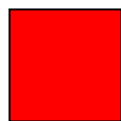
- rating between 1 to 5

Item-Item CF ($|N|=2$)

users

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|
| 1 | 1 | | 3 | | ? | 5 | | | 5 | | 4 | |
| 2 | | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| 3 | 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| 4 | | 2 | 4 | | 5 | | | 4 | | | 2 | |
| 5 | | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| 6 | 1 | | 3 | | 3 | | | 2 | | | 4 | |

movies



- estimate rating of movie **1** by user **5**

Item-Item CF ($|N|=2$)

| | | users | | | | | | | | | | | | |
|--------|----------|-------|---|---|---|---|---|---|---|---|----|----|----|-------------------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $\text{sim}(1,m)$ |
| movies | 1 | 1 | | 3 | | ? | 5 | | | 5 | | 4 | | 1.00 |
| | 2 | | | 5 | 4 | | | 4 | | | 2 | 1 | 3 | -0.18 |
| | <u>3</u> | 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | | <u>0.41</u> |
| | 4 | | 2 | 4 | | 5 | | | 4 | | | 2 | | -0.10 |
| | 5 | | | 4 | 3 | 4 | 2 | | | | | 2 | 5 | -0.31 |
| | <u>6</u> | 1 | | 3 | | 3 | | | 2 | | | 4 | | <u>0.59</u> |

Neighbor selection:

Identify movies similar to movie **1**, rated by user **5**

Here we use Pearson correlation as similarity:

1) Subtract mean rating m_i from each movie i

$$m_1 = (1+3+5+5+4)/5 = 3.6$$

row 1: $[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]$

2) Compute cosine similarities between rows

Item-Item CF ($|N|=2$)

| | | users | | | | | | | | | | | | |
|--------|----------|-------|---|---|---|---|---|---|---|---|----|----|----|-------------------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $\text{sim}(1,m)$ |
| movies | 1 | 1 | | 3 | | ? | 5 | | | 5 | | 4 | | 1.00 |
| | 2 | | | 5 | 4 | | | 4 | | | 2 | 1 | 3 | -0.18 |
| | <u>3</u> | 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | | <u>0.41</u> |
| | 4 | | 2 | 4 | | 5 | | | 4 | | | 2 | | -0.10 |
| | 5 | | | 4 | 3 | 4 | 2 | | | | | 2 | 5 | -0.31 |
| | <u>6</u> | 1 | | 3 | | 3 | | | 2 | | | 4 | | <u>0.59</u> |

Compute similarity weights:

$$s_{1,3}=0.41, s_{1,6}=0.59$$

Item-Item CF ($|N|=2$)

| | | users | | | | | | | | | | | |
|--------|----------|-------|---|---|---|-----|---|---|---|---|----|----|----|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| movies | 1 | 1 | | 3 | | 2.6 | 5 | | | 5 | | 4 | |
| | 2 | | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| | <u>3</u> | 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| | 4 | | 2 | 4 | | 5 | | | 4 | | | 2 | |
| | 5 | | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| | <u>6</u> | 1 | | 3 | | 3 | | | 2 | | | 4 | |

Predict by taking weighted average:

$$r_{1.5} = (0.41 \cdot 2 + 0.59 \cdot 3) / (0.41 + 0.59) = 2.6$$

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

Item-Item vs. User-User

| | Avatar | LOTR | Matrix | Pirates |
|-------|--------|------|--------|---------|
| Alice | 1 | | 0.8 | |
| Bob | | 0.5 | | 0.3 |
| Carol | 0.9 | | 1 | 0.8 |
| David | | | 1 | 0.4 |

- In practice, it has been observed that item-item often works better than user-user
- **Why?** Items are simpler, users have multiple tastes

Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
 - No feature selection needed
- **- Cold Start:**
 - Need enough users in the system to find a match
- **- Sparsity:**
 - The user/ratings matrix is sparse
 - Hard to find users that have rated the same items
- **- First rater:**
 - Cannot recommend an item that has not been previously rated
 - New items, Esoteric items
- **- Popularity bias:**
 - Cannot recommend items to someone with unique taste
 - Tends to recommend popular items