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Vertex cover Problem.

→ Defⁿ: $G = (V, E)$ → undirected.

vertex cover is a subset $V' \subseteq V$ such that if $(u,v) \in E$ then $u \in V'$ or $v \in V'$ (or both).
∴ All the edges of the graph are covered by the vertices in V'.

Size of vertex cover of a Graph $= |V'|$

Vertex cover problem is to minimize the no. of vertices present in V'.

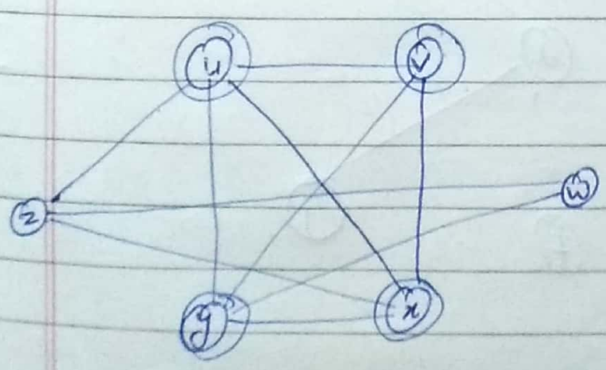
→ Proof that vertex cover is NP complete

Given : $G = (V,E)$ & integer K.

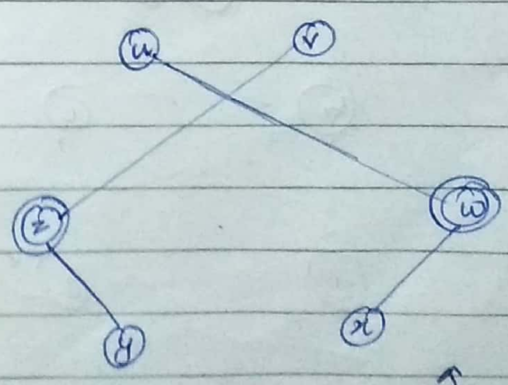
→

$V' \subseteq V$
$|V'| = k$
$(u,v) \in E$, $u \in V'$ or $v \in V'$ (or both)

} verification conditions.
↓
Takes polynomial time.



(a)
clique



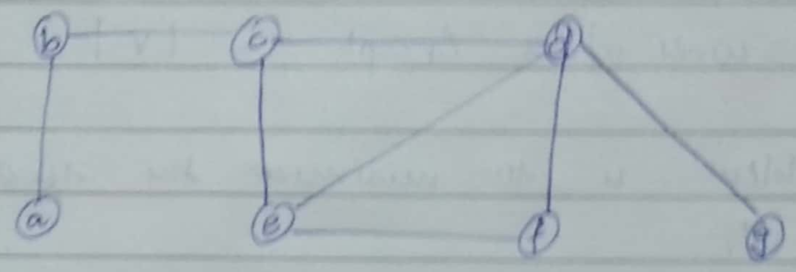
(b)
vertex cover

→ Approximation algorithm for vertex cover.

↓

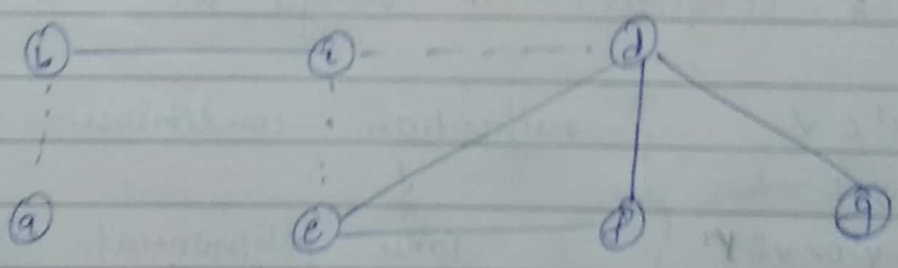
~~Bour~~ ~~Gr~~ Guarantees to be no more than twice the size of ~~vertex~~ the optimal vertex cover.

eg

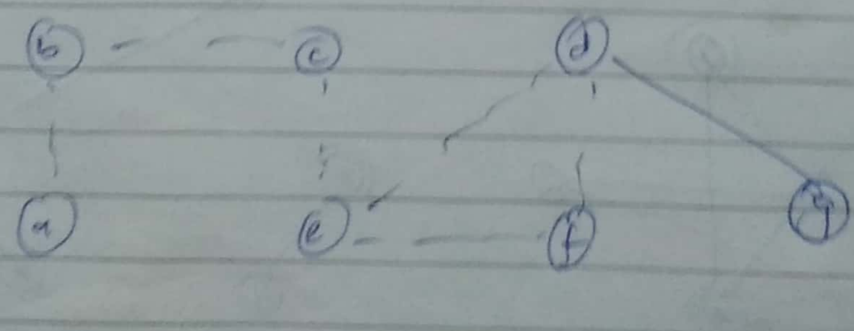


Randomly select edge (b, c)

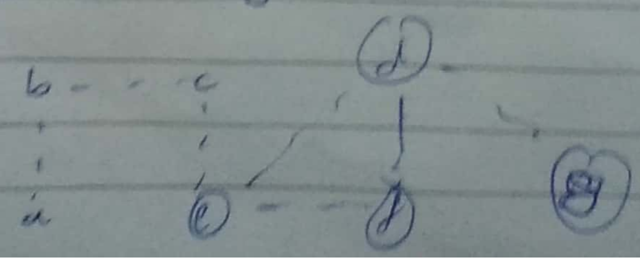
put b & c in ~~c~~ ~~C~~ cover, remove incident edges



select (e, f)



select d, g



$\therefore$ $v' = \{ b, c, d, e, g, f \}$.

$v' \leq 2v''$ ($v'' \rightarrow$ optimal vertex cover).
where here $v'' = 3$ (b, d & e).

Algo

$C = 0$
$E' = G(E)$
while $E' \neq 0$
let (u, v) be an arbitrary edge of E'
$C = C \cup \{u, v\}$
remove from E', every edge incident on either u or v.
return C

→ Proof of not more than 2 x (optimal vertex cover size).

- In order to cover the edges in A, any vertex cover, particularly optimal vertex cover C^* must include atleast one endpoint of edges in A

$\therefore$ $|C^*| \geq |A|$ — ①

- No two edges in A share the same endpoint in C.

$|C| = 2|A|$ — ②

from ① + ②

$|C| \leq 2|C^*|$