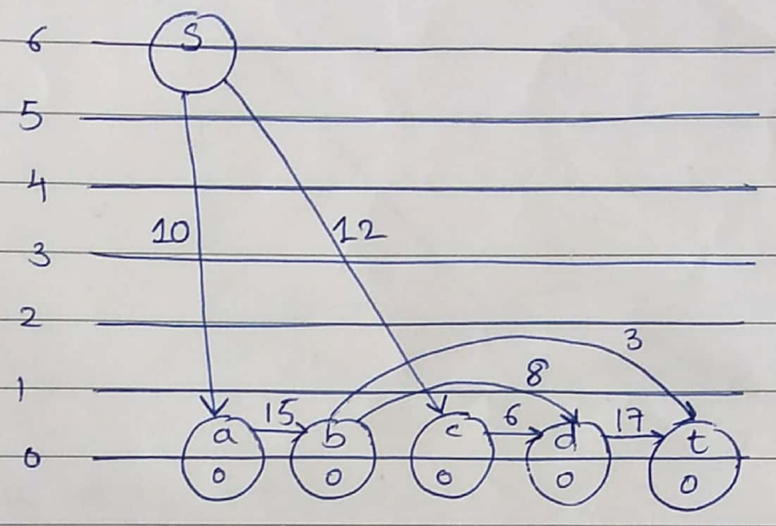
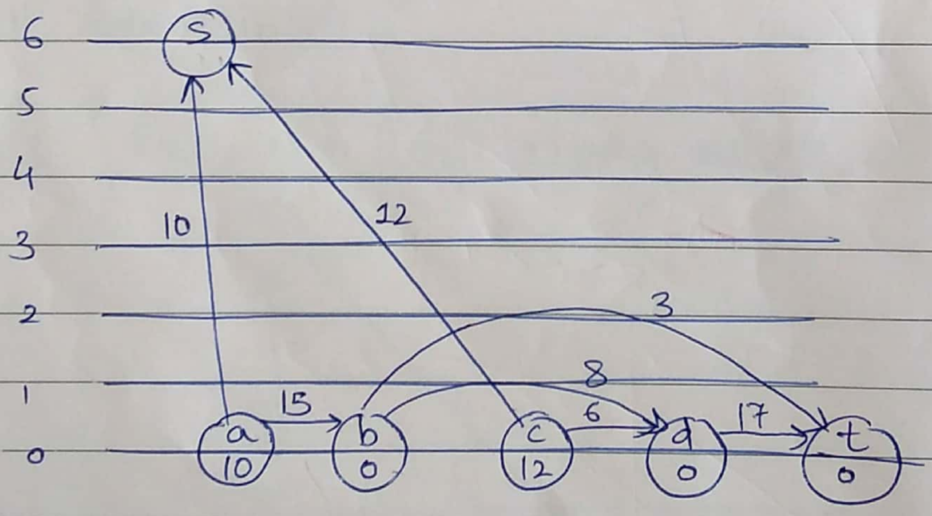
Push Relable

Step 1 :



Step 2 :



Here, $e(a) = 10$, $e(c) = 12$. a, c are overflowing vertices

Now, select 'a'

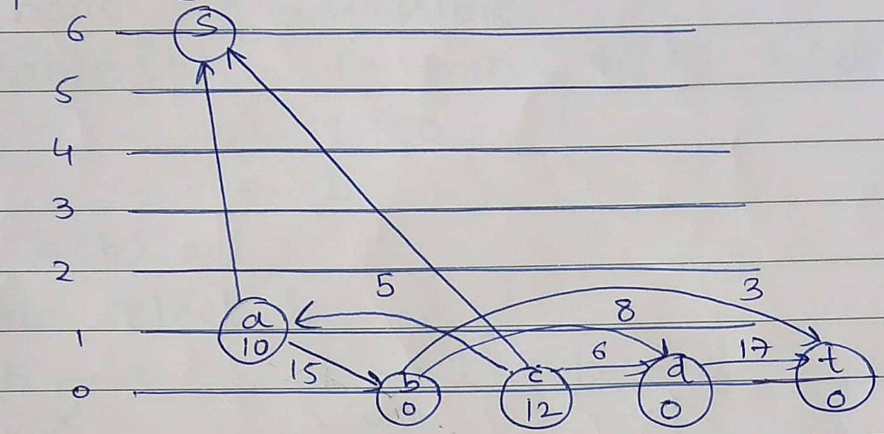
Edges from a :

$\underset{\times}{a \to s}$, $\underset{\times}{a \to b}$ (Both edges not admissible)

Relable (a) : $1 + \min\{h(s), h(b)\}$

$= 1 + 0 = \boxed{1}$

push (a,b) = 10

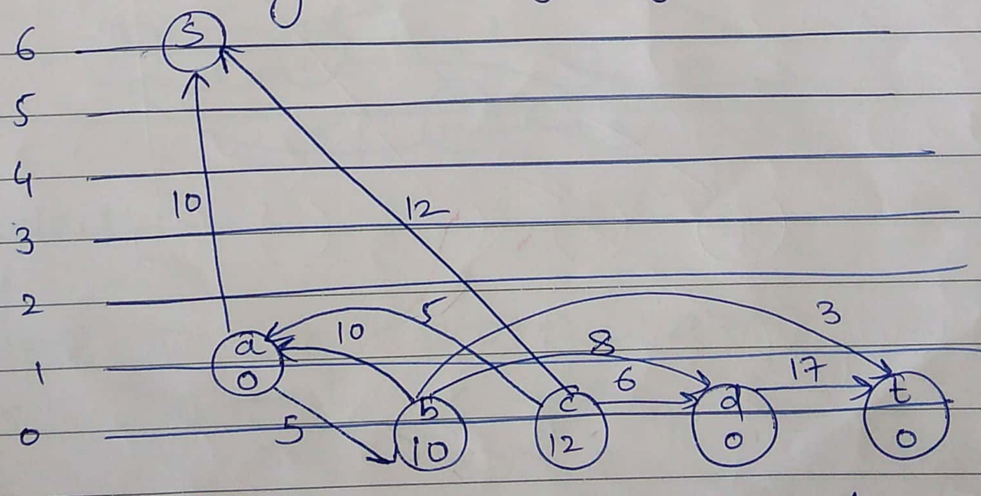


Now, a is relabled, h(a) = 1.

Again, select 'a'.

$a \rightarrow s$ (not admissible) , $a \rightarrow b$ (admissible)

Step 3 : Push the flow along edge $a \rightarrow b$



Again two overflowing vertices — b, c.
Select 'b'.

Edges leaving from b.

$b \rightarrow a$, $b \rightarrow t$, $b \rightarrow d$ (None are admissible)

As none are admissible,

relable(b) : $1 + \min\{h(a), h(d), h(t)\}$

$: 1 + 0$

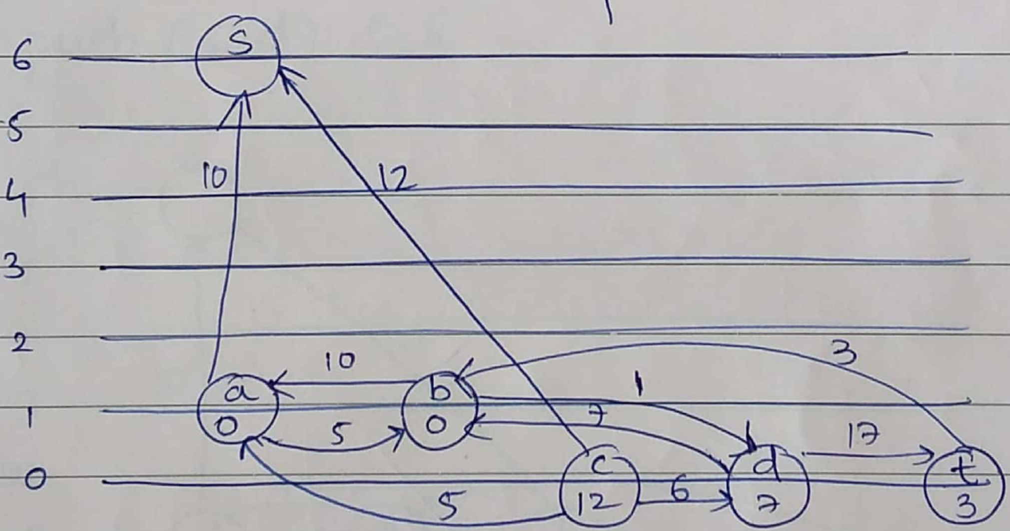
$= 1$

$\therefore h(b) = 1$

Again select b

$b \to a$ ✗ , $b \to t$ ✓ , $b \to d$ ✓

push (b, t) = 3 , push(b, d) = 7



~~Now, select c~~

Step 4:

Select c (as it is overflowing)

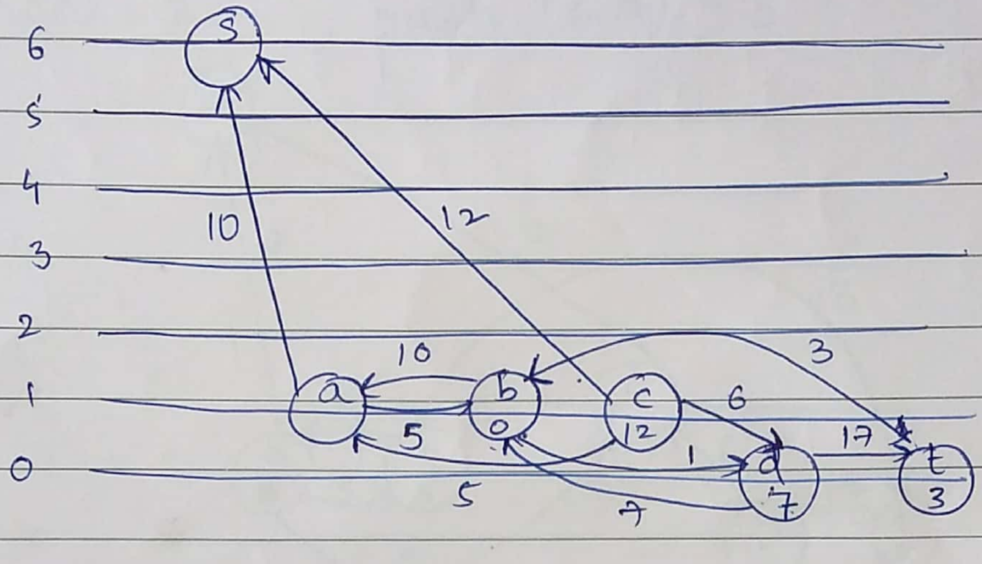
$c \to s$ ✗ , $c \to d$ ✗ , $c \to a$ ✗

Relable (c) = $1 + \min\{h(s), h(d), h(a)\}$

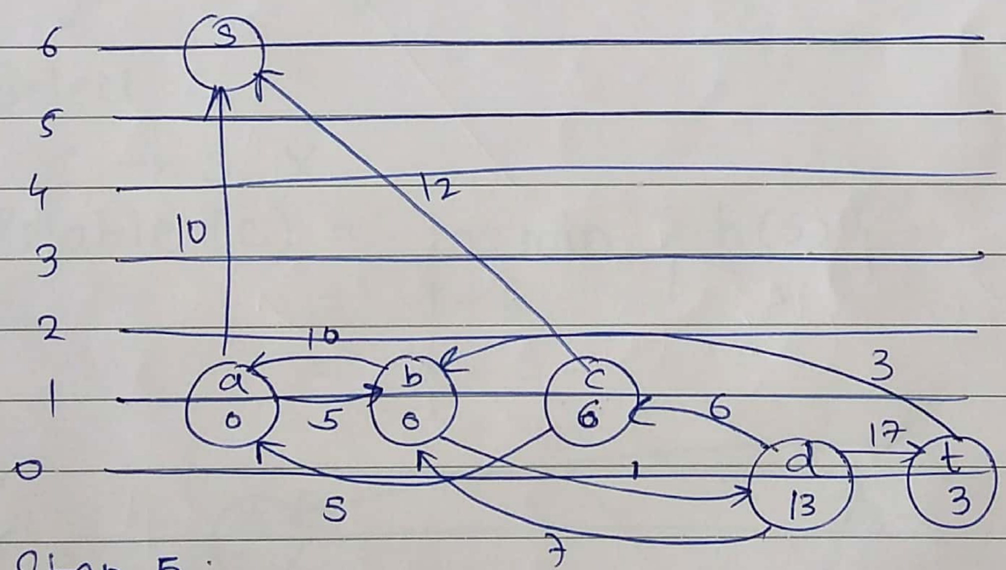
$= 1 + 0$

$= 1$

$h(c) = 1$



push (c,d) = 6



Step 5:

Select c.

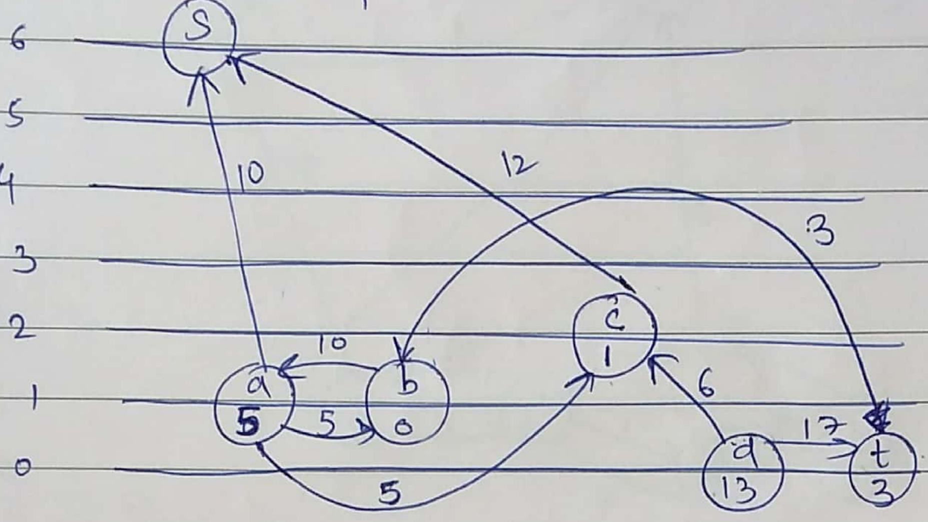
$c \to a$ ✗ $\quad c \to s$ ✗

relable(c) $= 1 + \min\{h(a), h(s)\}$

$= 1 + 1$

$= 2$

$h(c) = 2$, push $(c, a) = 5$



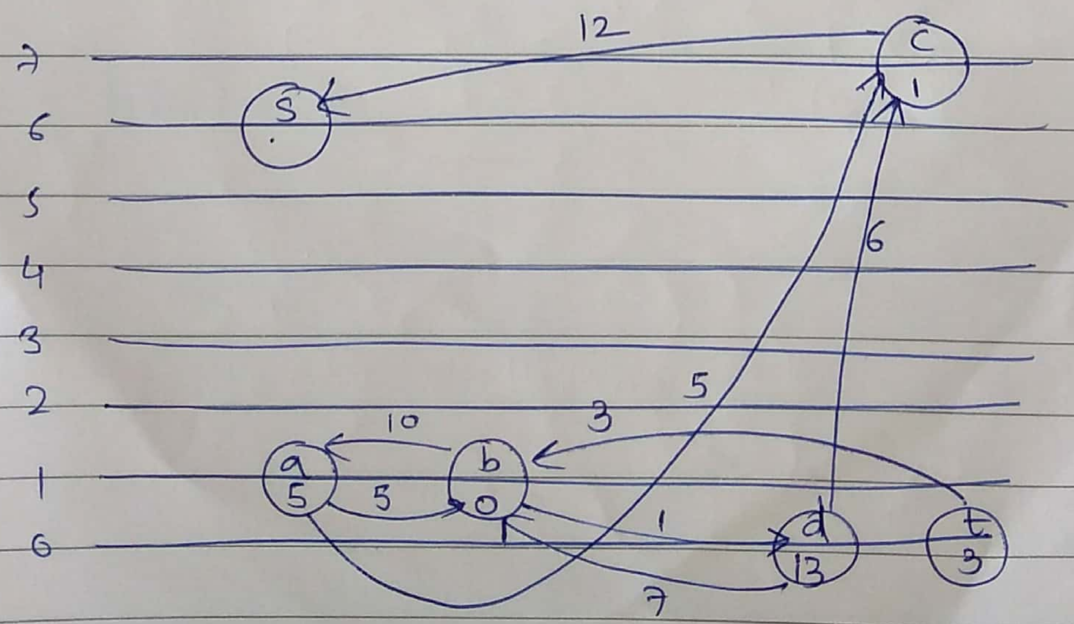
Step 6 :

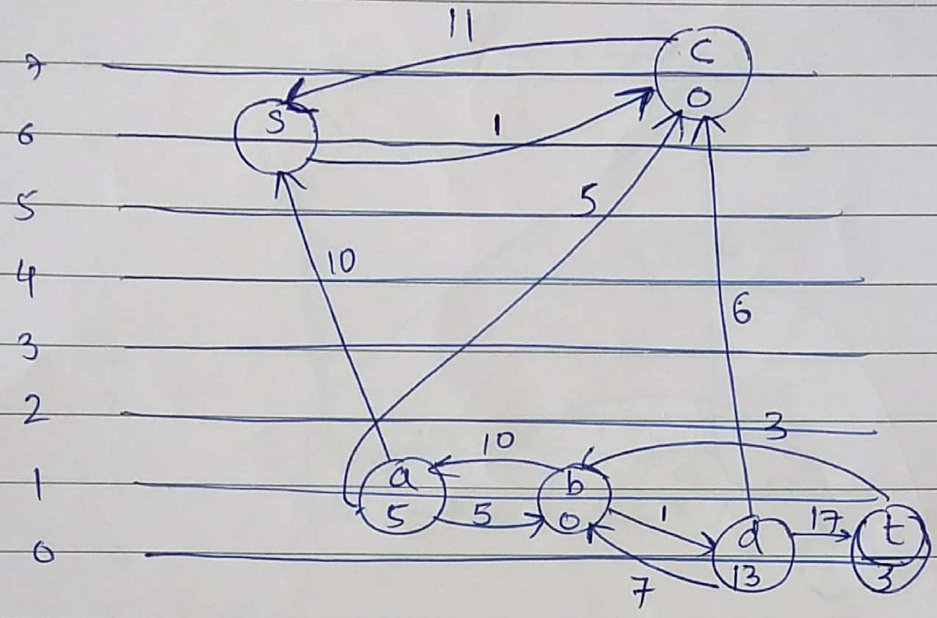
Overflowing ~~edges~~ vertices : a, c, d.

Select c.

$c \rightarrow s$ X

$$\text{Relable}(c) = 1 + \min\{h(s)\} = 1 + 6 = 7$$





push (c, s) = 1

Step 7:

Overflowing vertices : a, d.

Select d.

Edges from d:

d → b , d → c , d → t

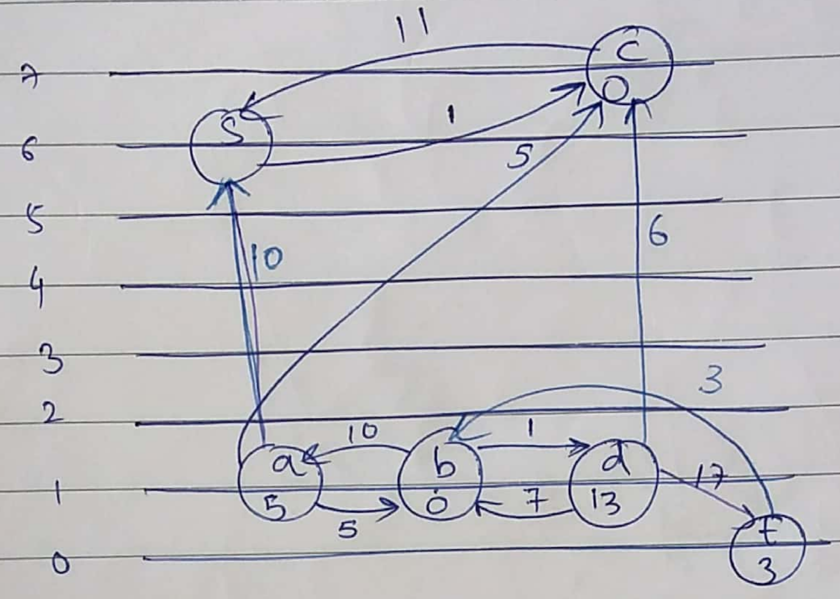
X X X

Relable (d) = 1 + min { h(b), h(c), h(t) }

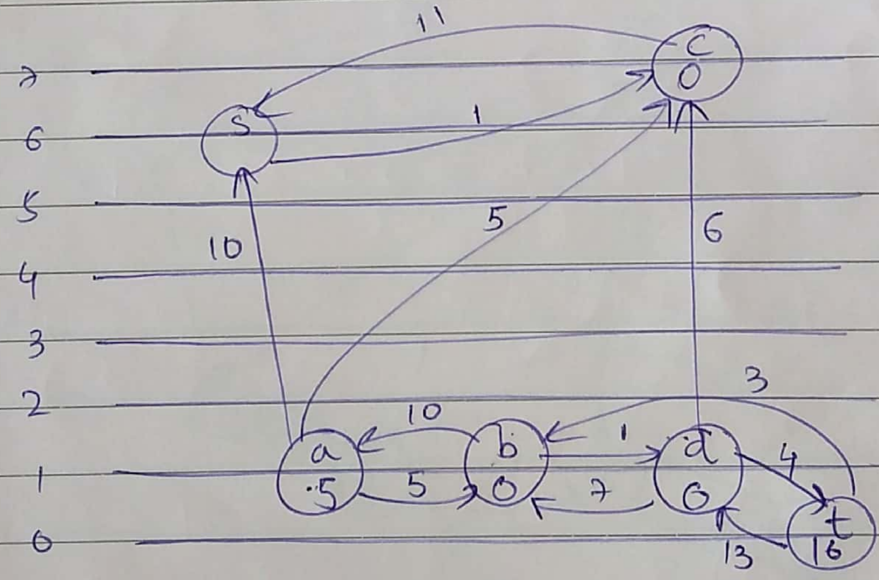
= 1 + 0

= 1

h(d) = 1



Push (d, t) = 13

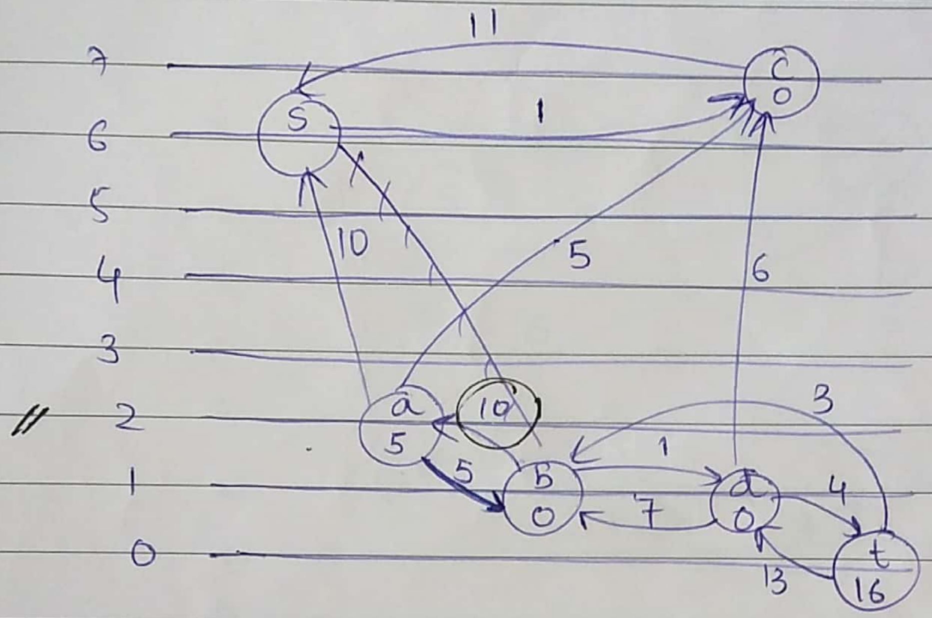


~~Se~~ Step 8 : First select a, a → b, a → c, a → s

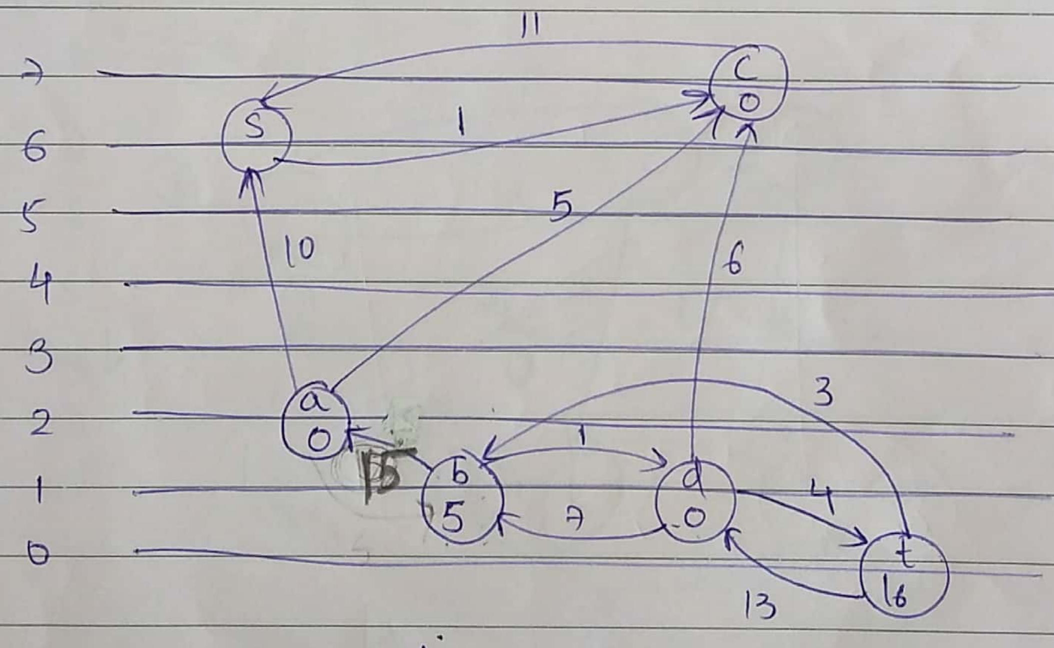
~~Select b~~ Relable (a) = 1 + 1 = 2

~~b → a~~

X



Push (a, b) = 5



Step 9 : Select b

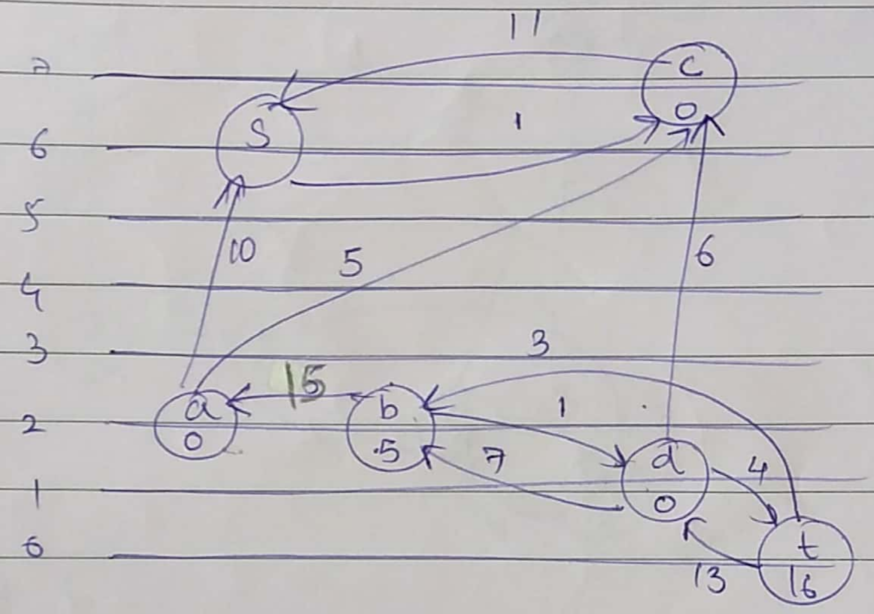
b → d , b → a

X X

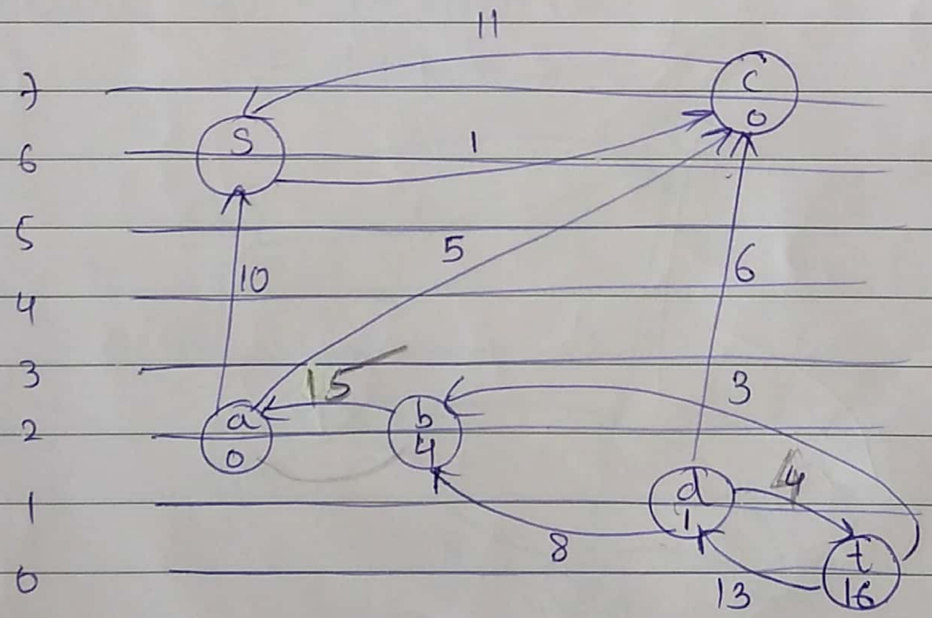
Relable (b) = 1+min {h(d), h(a)}

= 2

$h(b) = 2$



Push $(b, d) = 1$

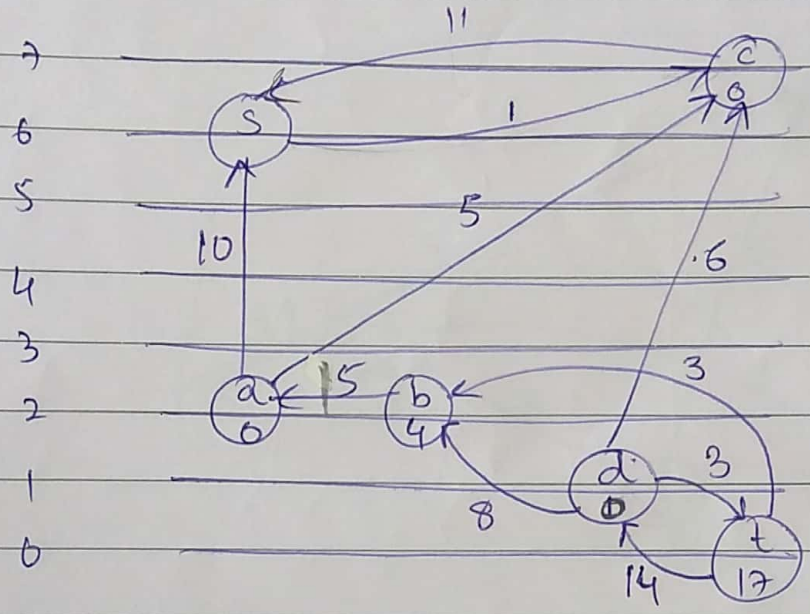


Select d.

$d \rightarrow b$, $d \rightarrow c$, $d \rightarrow t$

× × ✓

push $(d, t) = 1$

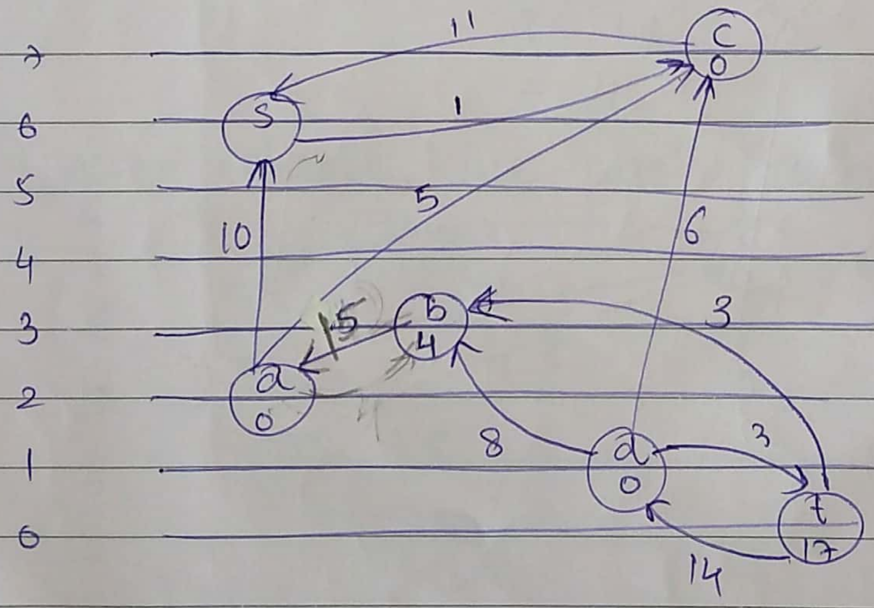


Select b.

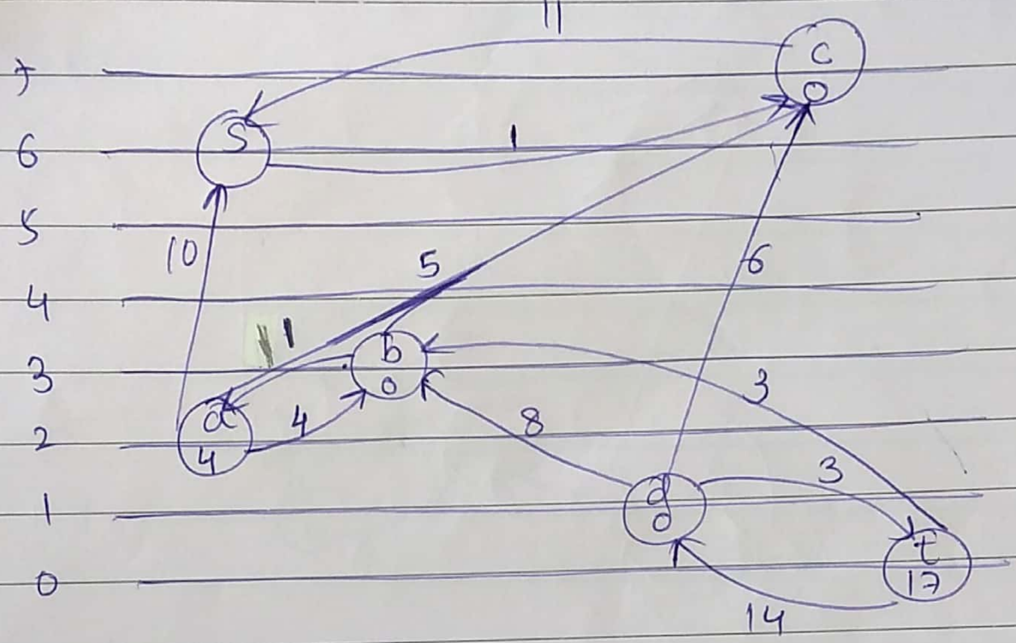
∵ $b \rightarrow a$ ✗

Relable (b) = 1 + min (a)

= 1 + 2 = 3



push (b, a) = 4



Select a

$a \rightarrow b$

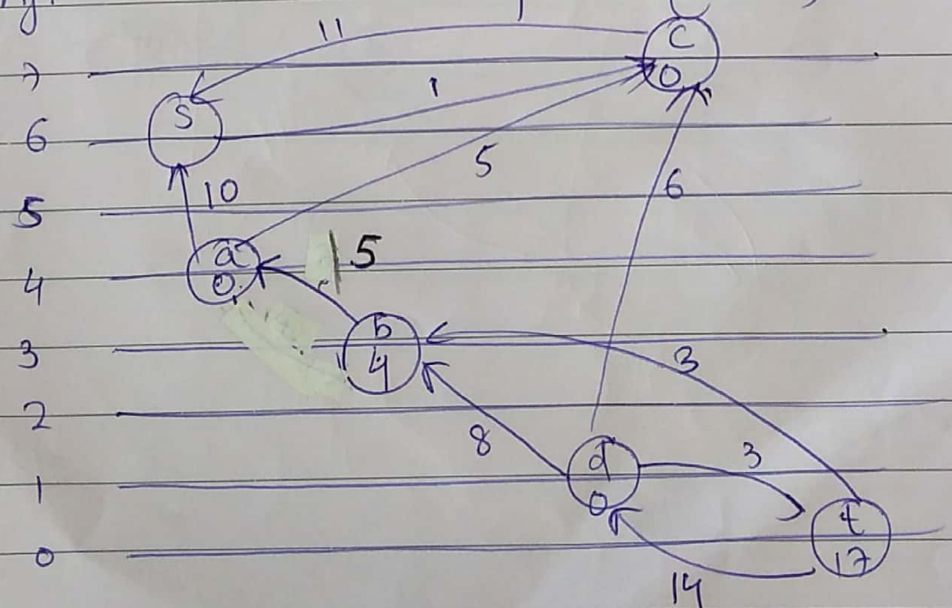
X

~~push~~. Relabel(a) = 1 + min(h(b))

= 1 + 3

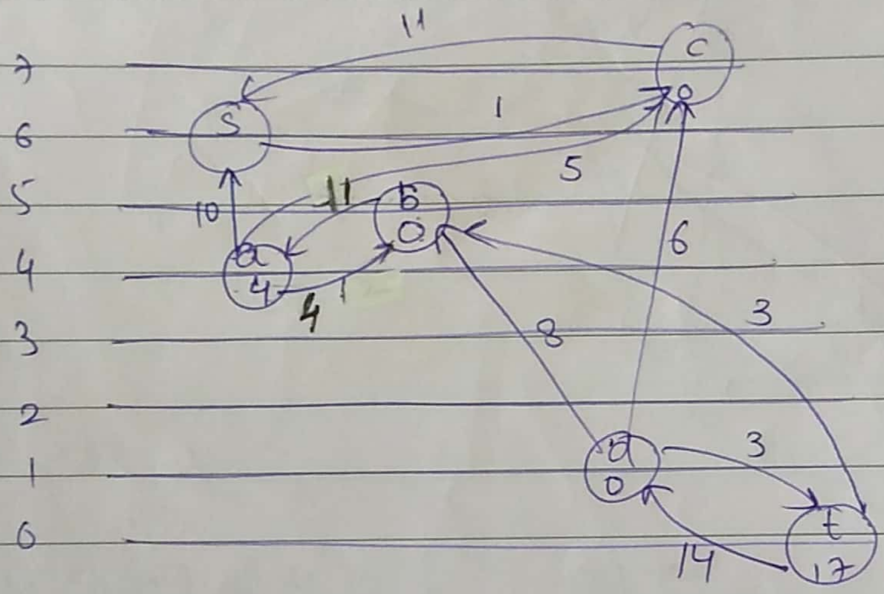
= 4.

After relabel, we push (a, b) = 4



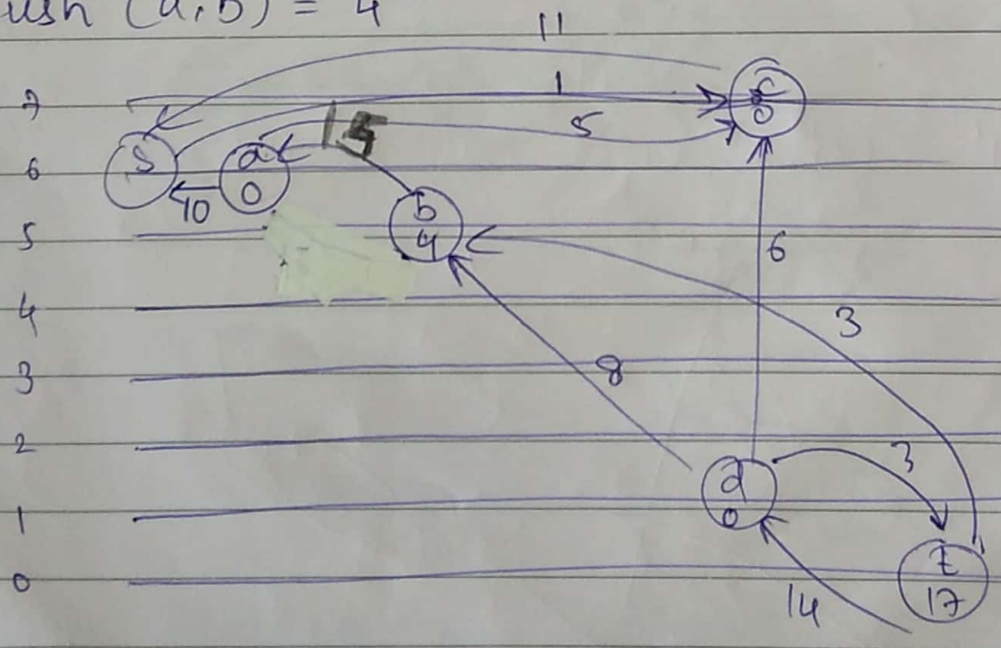
· Relable b → 5

push (b, a) = 4



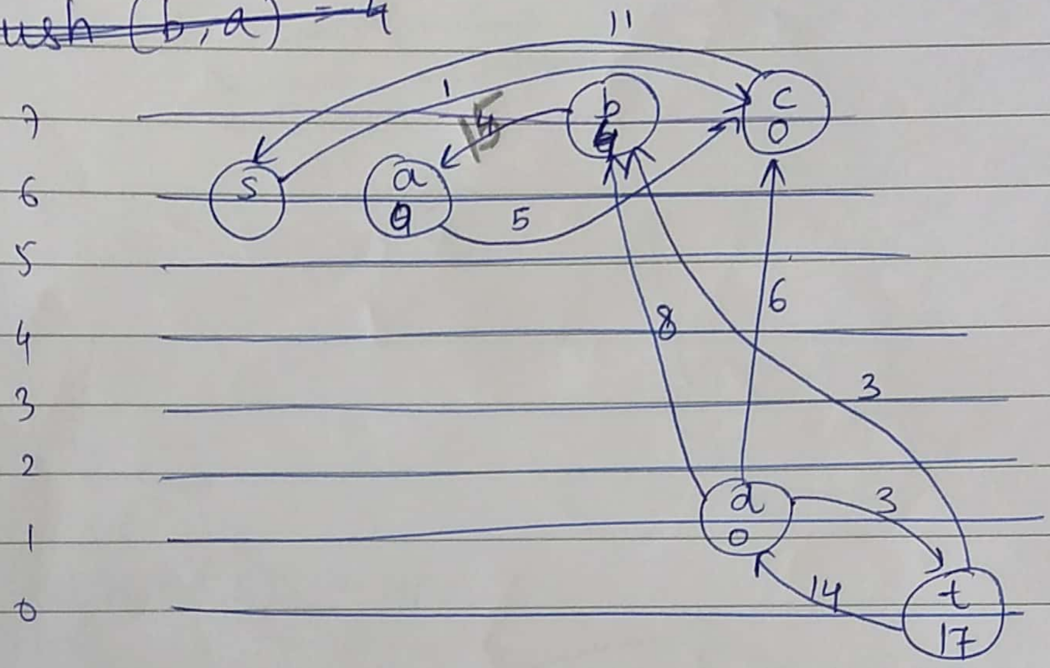
Relable a → 6

push (a, b) = 4



Relable b → 7

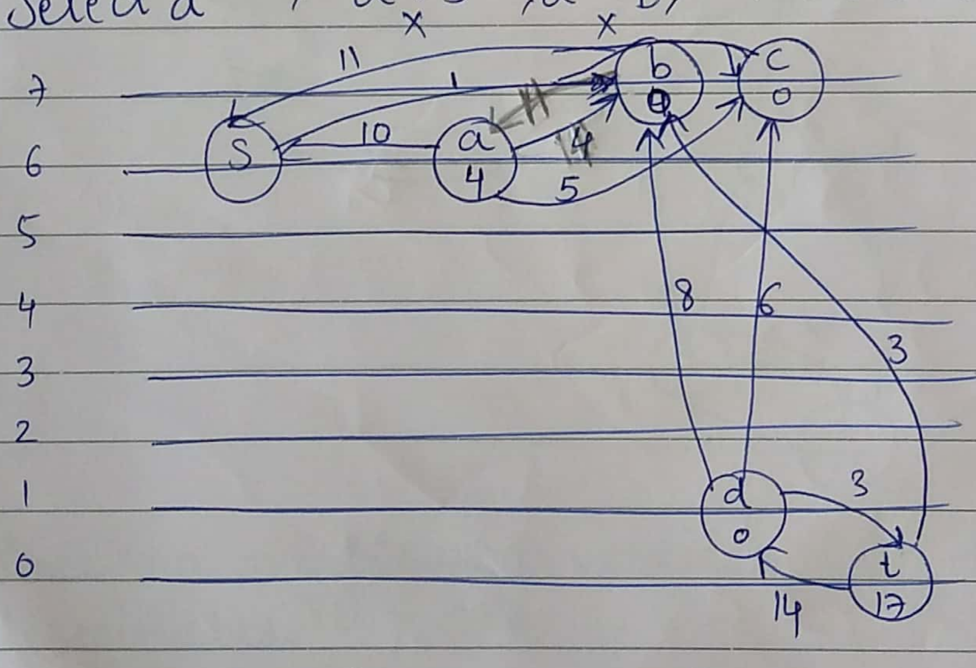
~~push (b, a) = 4~~



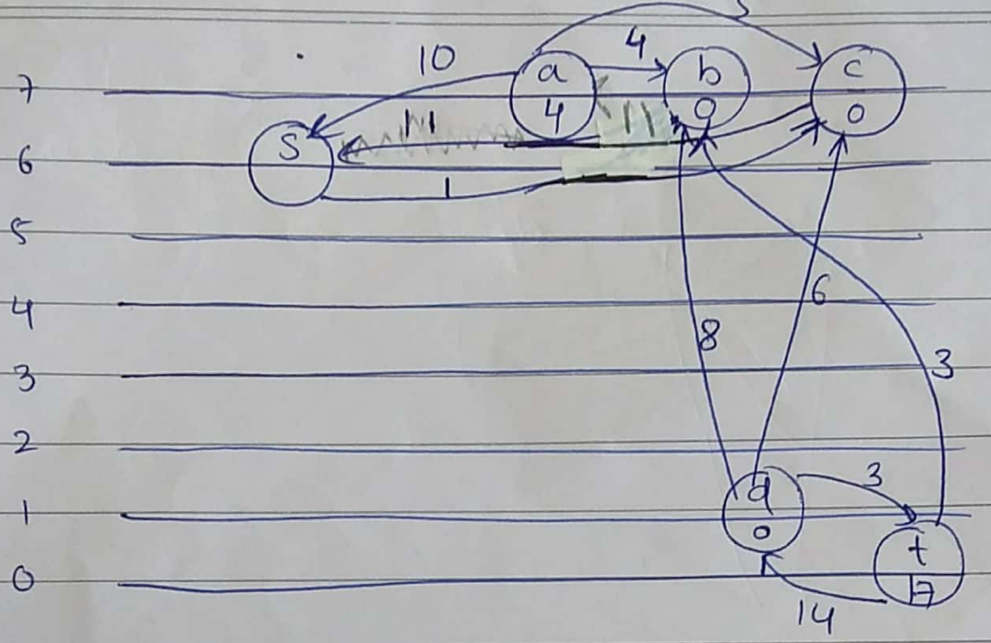
push (b, a) = 4   a → c

Select a , a → s , a → b, × Relable (a) = 1 + min((s), (b), (c))

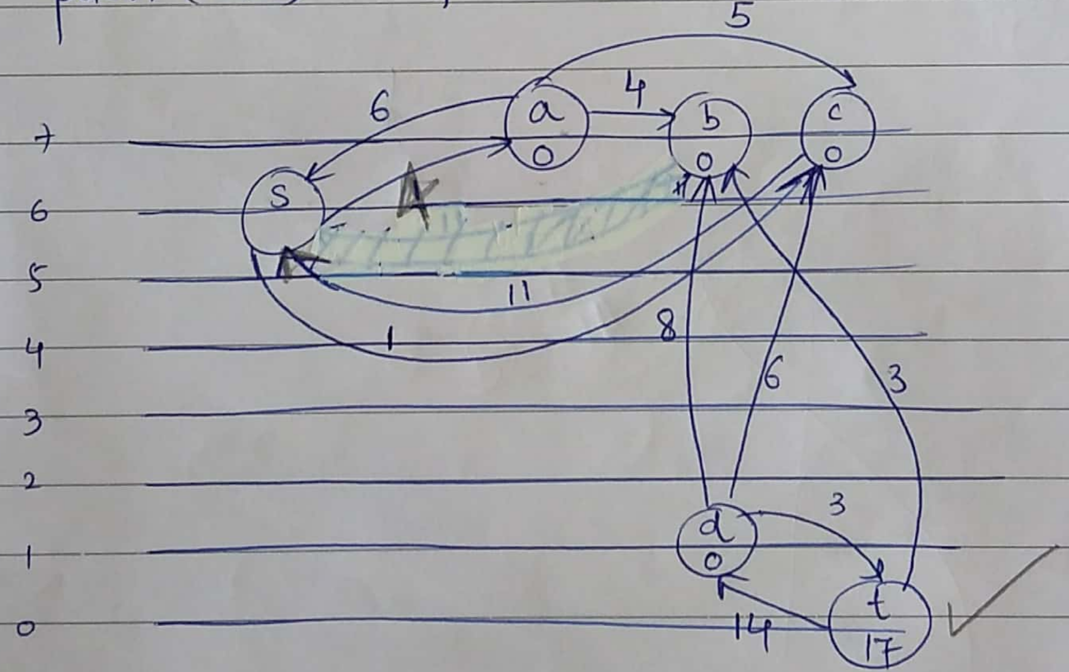
= 1 + 6 = 7



After relabeling a we have :



push (a,s) = 4



Since no overflowing vertices left, the algorithm terminates.