Completed : Check sum 3 .

TUTORIAL · 6 .

1). Explain Pumping Lemma with help of diagram & prov that given language is not a regular language.

→

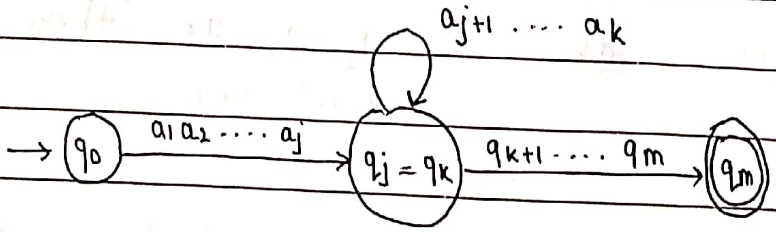
Pumping lemma states that for a sufficiently long string accepted by FSM, there exist a substring at t beginning of the string that can be repeated 'n' no times & is still accepted by FSM.

i). If a language is regular it is accepted by DFA.
$n = (Q, \Sigma, \delta, q_0, F)$ where n is a set of finite stat

ii). Consider an input str with n or more no of symb ie $a_1 a_2 \dots a_m$, $m \geq n$ for $i = 1, 2, 3 \dots m$

let $\delta(q_0, a_1 a_2 \dots a_i) =$

∴ It is not possible for each n+1 states to be distinct ie at least 2 states coincide as there are only n stat



iii) Thus there are 2 integers j & k such that $0 \leq j < k \leq n$ & $q_j = q_k$.

We observe that,

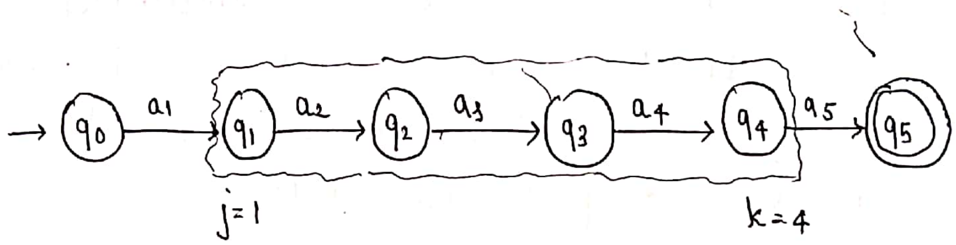
(a) Since $j < k$ the string $a_{j+1} \dots a_k$ is of the length of at least 1.

(b) $k \leq n$, the length is not more than n.

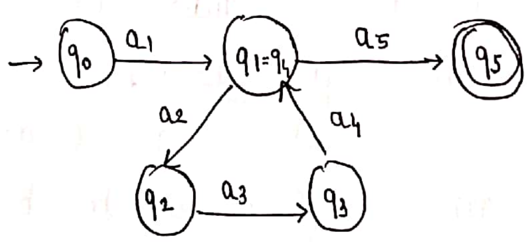
For eg:

n : No of states = 3

m : No of symbols = 5 ie $a_1\ a_2\ a_3\ a_4\ a_5$



If j & k coincide:



So $j < k$ & $a_{j+1} \ldots a_k$. ie a_2, a_3, a_4 (there exist string at least one)

& $k \leq n$

& the string can be repeated any no of times and is still accepted by FSM.

∴ We conclude,

(i) if q_m is in final state F & $a_1 \ldots a_m$ is $L(m)$ then $a_1 \ldots a_j\ a_{k+1} \ldots a_m$ is also in $L(m)$

(ii). We can go around the loop i no of times

ie $\boxed{a_1 \ldots a_j\ (a_{j+1} \ldots a_k)^i\ a_k \ldots a_m \text{ for } i \geq 0}$

Q2)

~~a) $L = \{0^i 1^j \mid i \geq j\}$~~

★★ ③ So if L = regular set

n = pumping lemma constant

z = any word in lang

then $\boxed{|z| \geq n}$ length of z $\geq$ pumping lemma c

We can view $z = uvw$

$\therefore$ for all $i \geq 0$ uv^iw is in L

where $uv \leq n$

$|v| \geq 1$

Q2).

a). $L = \{0^i 1^j \mid i > j\}$

→ Assume $L = \{0^i 1^j \mid i > j\}$ is a regular language.

② It would follow pumping lemma.

③. Word of language $= 0^i 1^j$

$= uv^iw$ where $uv \leq n$

$|v| \geq 1$.

Eg: 00011 is a string accepted by the string

Case i: $\underbrace{00}_{u}\,\underbrace{(01)^i}_{v}\,\underbrace{1}_{w}$

$\therefore$ if $i = 2$ $\therefore$ $\overline{00\ 0101\ 1}$ = Resulting str.

$\therefore$ Resulting string is not in language.

Case 2 : word : $\underset{u}{\underline{0}}\ \underset{v}{\underline{(0011)}}^{i}\ \underset{w}{\underline{1}}$ where $|uv| \le n$, $|v| \ge 1$

If $i = 2$

Resulting str = 0 0011 0011 1 is not in L

If $i = 3$

Resulting str = 0 0011 0011 00111 is not in L

So L does not follow pumping lemma.

$\therefore$ L is not regular language.

b). $L = \{a^m b^n \mid \gcd(m,n) = 1\}$

→ Assume $L = \{a^m b^n \mid \gcd(m,n) = 1\}$ is a regular language.

② It would follow pumping lemma.

③ Word of the language = $\underset{u}{\underline{aa}}\ \underset{v}{\underline{(a\ b)}}^{i}\ \underset{w}{\underline{b\,b\,b\,b}}$ $m = 3$ $n = 5$

$uv \le n$

$|v| \ge 1$

Case 1 : word : $\underset{u}{\underline{aa}}\ \underset{v}{\underline{(ab)}}^{i}\ \underset{w}{\underline{bbbb}}$

If $i = 2$

Resulting str = aa ab ab bbbb is not in language

If $i = 4$

Resulting str = aa ab ab ab ab bbbb is not in language

Case 2 : $\underset{u}{\underline{a}}\ \underset{v}{\underline{(aab\,bb)}}^{i}\ \underset{w}{\underline{bb}}$.

If $i = 2$ a aabbb aabbb bb is not in language

So L does not follow pumping lemma

ie L is not a regular language

Check : saying it is regular

c). $L = \{ 0^{2n} \mid n \geq 1 \}$

→ Assume $L = \{0^{2n} \mid n \geq 1\}$ is a regular language.

② It would follow pumping lemma.

③. Word of language = $\underset{u}{0}(\underset{v}{00})^i\underset{w}{0}$ $n = 2$ $|uv| \leq n$, $|v| \geq 1$

Case I : If $i = 2$

Resulting str = 0 000 000 is not in language → ①

If $i = 3$

Resulting str = 0 000 000 000 is in language.

Word of language = $\underset{u}{00}(\underset{v}{0})^i\underset{w}{0}$

Case I: If $i = 2$

Resulting str = 00 000 is not in language → ②

If $i = 6$.

Resulting str = 00 0000000 is not in language → ③

From ①, ②, ③ we can conclude L does not follow pumping lemma

∴ L is not a regular language.

$a^{n-1}.a$

$2n \geq r$

$|xy| \leq n$ a^s $|y| > 0$ $a^1 \mid x > 0$

$xyz =$ $a^s . a^x . a^{n-s-x-1} . a$

$xy^2z =$ $a^s a^{2x} a^{n-s-x-1} . a = a^{n+x-1} a$

a^{n+1}

d). $L = \{0^{2n+1} \mid n \geq 0\}$.

→ Assume $L = \{0^{2n+1} \mid n \geq 0\}$ is a regular language, It follows pumping lemma.

① Word of the language : $\underbrace{0}_{u}\underbrace{(0)^i}_{v}\underbrace{0\,0\,0}_{w}$ where $n = 3$.

Case 1: If $i = 2$ — $uv \leq$ length of str, $|v| \geq 1$

Resulting str = 0 000 000 is not in language L

If $i = 4$

Resulting str = 0 0000 000 is not in language L

Word of language : $(000)^i\,00$ where $n = 3$

$uv \leq 3$

$|v| \geq 1$

Case 1: If $i = 2$

Resulting str = 000 000 00 is not in language L

If $i = 4$

Resulting str = 000 000 000 000 00 is not in language L

ie L does not follow pumping lemma

L is not regular language.

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Q. $L = \{a^n (ab)^m c^n \mid n, m \geq 1\}$

→ ① Assume $L = \{a^n (ab)^m c^n \mid n, m \geq 1\}$ is a regular language.

② It follows pumping lemma

③ Word of this language : $\underbrace{aa}_{u}\,\underbrace{(ab)^i}_{v}\,\underbrace{cccc}_{w}$

$|uv| \leq n$

Case 1 : If $i = 2$ $\qquad |v| \geq 1$

Resulting string = aa ab ab ccccc is not in L

If $i = 3$

Resulting string = aa ab ab ab ccccc is not in L

Word of language = $\underbrace{a}_{u}\,\underbrace{(aabc)^i}_{v}\,\underbrace{cccc}_{w}$ $\qquad uv \leq n$

$|v| \geq 1$

Case 2 : If $i = 2$

Resulting str = a aabc aabc cccc is not in L

If $i = 3$

Resulting str = aaabc aabc cccc is not in L

Thus L does not follow pumping lemma

So it is not regular language.

Q. $L = \{a^n \mid n \geq 0\}$

→ ① Assume $L = \{a^n \mid n \geq 0\}$ is a regular language.

② It follows pumping lemma.

w { Word of language } = a(aaa)a $n = 5$

$\underset{u}{a}\ \underset{v}{(aaa)^i}\ \underset{w}{a}$

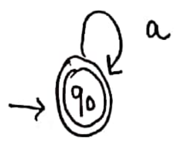
If $i = 2$

Resulting string = aaaaaaa a is in language L.

If $i = 3$

Resulting string = aaaaaaaaa a is in language L

FA is given:



Language is regular.

g). $L = \{ww \mid w \in (a,b)^*\}$

→ ① Assume language $L = \{ww \mid w \in (a,b)^*\}$ is a regular language

② It follows pumping lemma.

Word of language $w = \overset{n}{a}ab\overset{n}{a}ab$.

$\therefore \quad w = \underset{u}{\underline{a^s}} \quad \underset{v}{\underline{a^r}} \quad \underset{w}{\underline{a^{n-s-r} \, b \, a^n \, b}}$. $\qquad |y| > 0$

$|xy| \leq n$

Case 1: If $i = 2$

$w = a^s \; a^{2r} \; a^{n-s-r} \; b \; a^n \; b$

$w = a^{n+r} \; b \; a^n \; b$

Since $r > 0$, $a^{n+r} \, b \, a^n \, b$ is not of the form ww

$\therefore$ L does not following pumping lemma

$\therefore$ L is not a regular language

h) $L = \{x \mid x$ is palindrome, $x \in (0,1)^*\}$

→ ① Assume $L = \{x \mid x$ is palindrome, $x \in (0,1)^*\}$ is a regular language

② It follows pumping lemma

Word of language = $\underset{u}{\underline{0}}\underset{v}{\underline{(111)}}\underset{w}{\underline{\overset{1}{0}\,111\,0}}$

Case i: If $i = 2$

$w = 0111\,111\;0\,111\,0$ is not in language L

If $i = 3$

$w = 0111111111\;0111\,0$ is not in language L

$\therefore$ L does not follow pumping lemma

L is not a regular language

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i). L = equal no of 0's and 1's.

→ Assume L = Equal no of 0's & 1's is a regular language

(ii) It follows pumping lemma.

(iii) word of the language = $\underbrace{00}_{u}(\underbrace{11}_{v})^{i}\underbrace{001011}_{w}$ accepted by fsm.

$uv \leq n$

$\& \; v \geq 1$

Case i : If $v = 2$

w = 00111100 1011 is not in L

If $v = 3$

w = 00 1111 11 001011 is not in L

∴ It doesn't follow pumping lemma

∴ It is not a regular language.

j). $L = \{a^p \mid p \text{ is prime}\}$

→ Assume $L = \{a^p \mid p \text{ is prime}\}$ is a regular language

It follows pumping lemma.

$w = \underbrace{a}_{u}(\underbrace{aa}_{v})^{i}\underbrace{aa}_{w}$ $p = 5$ $uv \leq p$

$|v| \geq 1$

Case 1: If $i = 3$

w = aaaaaaaaa is not in language L

$w = \underbrace{a}_{u}(\underbrace{a}_{v})^{i}\underbrace{aaa}_{w}$

Case 1: If $i = 3$ w = aaaa aaaa is not in Language L

It doesnt follow pumping Lemma ∴ Not a regular language

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Completed. Doubt: 0.5

TUTORIAL 7.

1) Show that following grammar is ambiguous

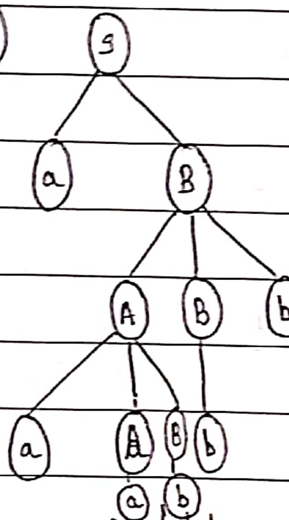
S → aB | ab

A → aAB | a

B → ABb | b

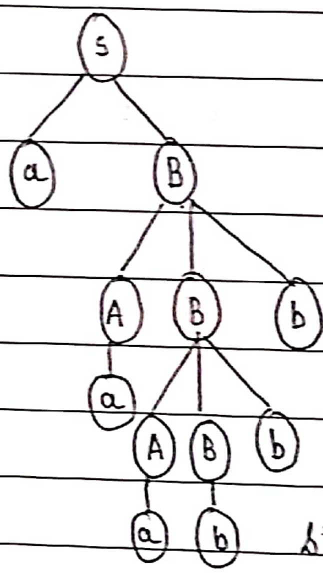
→ A grammar is said to be ambiguous if the language generated by the grammar contains strings that have different parse trees.

For eg. 1)



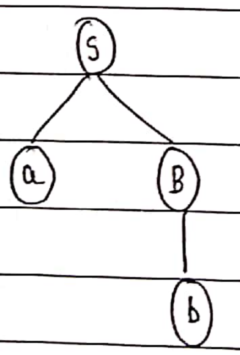
String : aaaabbb

2)



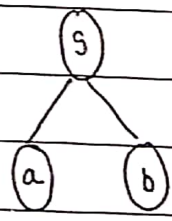
String : aaabbb

eg. 1)



String : ab

2)



String : ab

Two different parse strings generate the same string

∴ Grammar is ambiguous

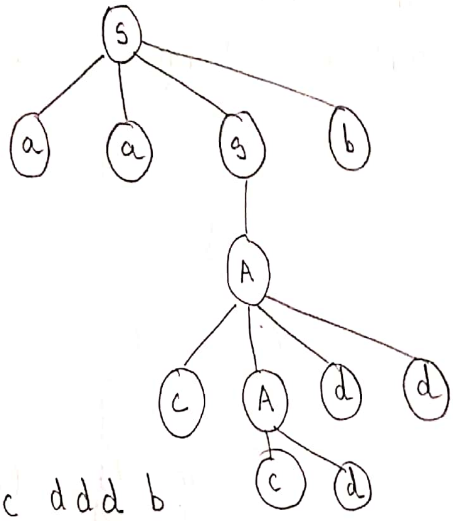
2). Describe the language generated :

$S \rightarrow aaSb$

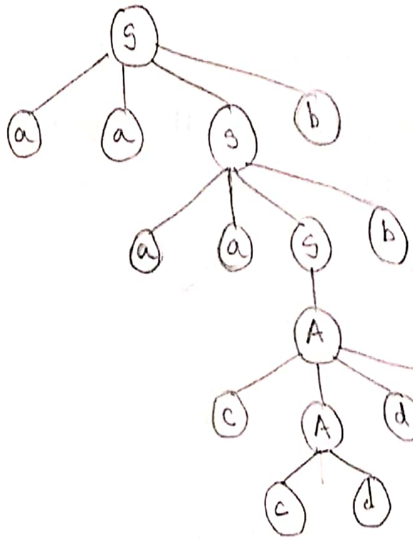
$S \rightarrow A$

$A \rightarrow cAdd$

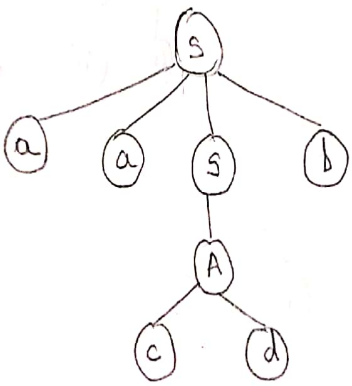
$A \rightarrow cd$.



aaccdddb



aaaaccddddbb



aacdb

$\therefore L = \{a^n c^m d^m b^{2n} \mid m, n \geqslant 1\}$ is the language generated.

3). Eliminate ϵ productions from :

a). $S \to aSa \mid bSb \mid \epsilon$

$\therefore \quad S \to aSa \mid bSb \mid aa \mid bb. \qquad \mid S \to \epsilon$

b). $A \to aBb \mid bBa$

$B \to aB \mid bB \mid \epsilon$

$\therefore \quad A \to aBb \mid bBa$

$B \to aB \mid bB \mid a \mid b. \qquad \mid B \to \epsilon$

4). Eliminate unit productions :

$S \to Aa \mid B$

$A \to a \mid bc \mid B$

$B \to A \mid bb.$

$S \to Aa \mid bb \mid a \mid bc$	$S \to B$
$A \to a \mid bc \mid bb$	$A \to B$
$B \to bb \mid a \mid bc$	$B \to A$

5). Consider CFG, $G = (V, \Sigma, P, S)$ where $V = \{S, A, B\}$

$\Sigma = \{0, 1\}$ and $P = \{ S \to A11B \mid 11A \mid B \mid 11$

$A \to 0$

$B \to BB \}$

Remove useless symbols.

Q) For removing useless symbols do we need to remove ϵ & unit production as well ??

If: Step 1: Remove ϵ productions

- No ϵ productions present

Step 2: Remove unit production

$S \to A11B \mid 11A \mid 11 \mid BB$ $\qquad$ $S \to B$

$A \to 0$

$B \to BB$

Step 3: Remove useless symbols.

G (Generating symbols) = $\{0, 1\}$

$\because A \to 0$

$\therefore G = \{0, A, 1\}$

$\because S \to 11A$

$G = \{0, 1, A, S\}$

$\therefore$ Generating symbols are $\{0, 1, A, S\}$

6). Construct CFG without null production.

$S \to a \mid Ab \mid aBa$

$A \to b \mid \epsilon$

$B \to b \mid A$.

$\therefore S \to a \mid Ab \mid aBa \mid b \mid aa$

$A \to b$

$B \to b \mid A$

$A \to \epsilon$

$\therefore B \to \epsilon$