① Tm

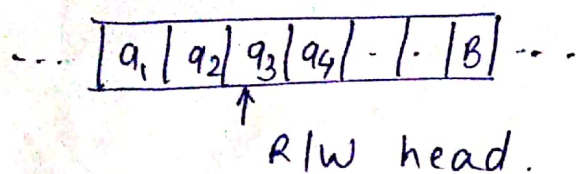| a_1 | a_2 | a_3 | a_4 | · | · | B |

R/W head.

① unlimited capacity
   infinite input tape.

- tape is divided into cells of ∞ length.
- R/W read/write head can move in L/R/N direction
- cell can contain a single symbol.
- Tm is said to be in halt state if it is not able to transit further.

- 7 tuple. $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$

  Q = finite set of states.
  Σ = input alphabet (set of finite input symbols)
  Γ = finite non empty set of tape symbols. that Tm can write on input tape.
  δ = transition function
  $$Q \times \Sigma \to \Gamma \times Q \times \{L, R, N\}$$
  q_0 = Initial state $\delta(q_0, a) \to (q_1, b, R)$

  b = blank symbol.

  F = set of final states.

- same powerful like computer. any problem that can have an algorithm/solvable is solvable by turing machine.
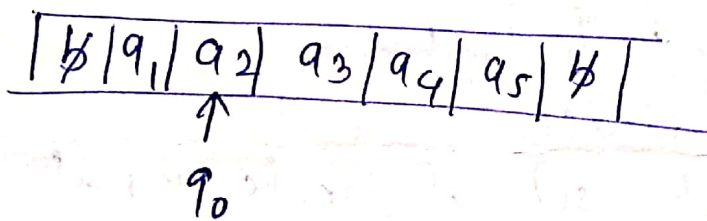
- FSM < PDA < TM [as per the power]
  T^c there is n_c ...

when it reaches to final state it does job of

- acceptor.
- calculator
- recognizer
- computer

Representation Tm is done with
1. ID - instantaneous Description
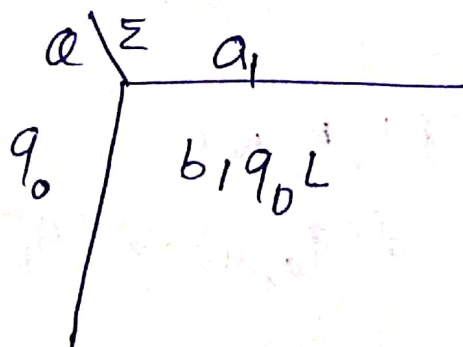2. TD - Transition diagram
3. TT - Transition table

| ¢ | q_1 | q_2 | q_3 | q_4 | q_5 | ¢ |

$\uparrow$
q_0

$ID =$

$$¢\, q_1\, q_0\, a_2\, a_3\, a_4\, a_5\, ¢$$

$$= \overset{\leftrightarrow}{\alpha} \quad \beta \quad \vec{r}$$

$$= ¢q_1 \quad \beta = q_0 \quad r = a_2\, a_3\, a_4\, q$$

TD

$$\begin{array}{c|cc} Q \backslash \Sigma & a_1 \\ \hline q_0 & b_1 q_0 L \end{array}$$

TD

$q_0 \xrightarrow{} (q_0)$ with loop $a_1 / b_1, L$
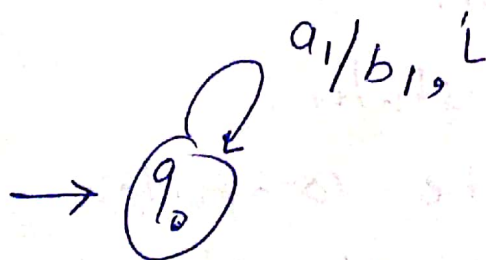
Design a Tm to check whether a string over {a,b} contains equal number of a's and b's.

① a bb a b a   locate first a or first b.
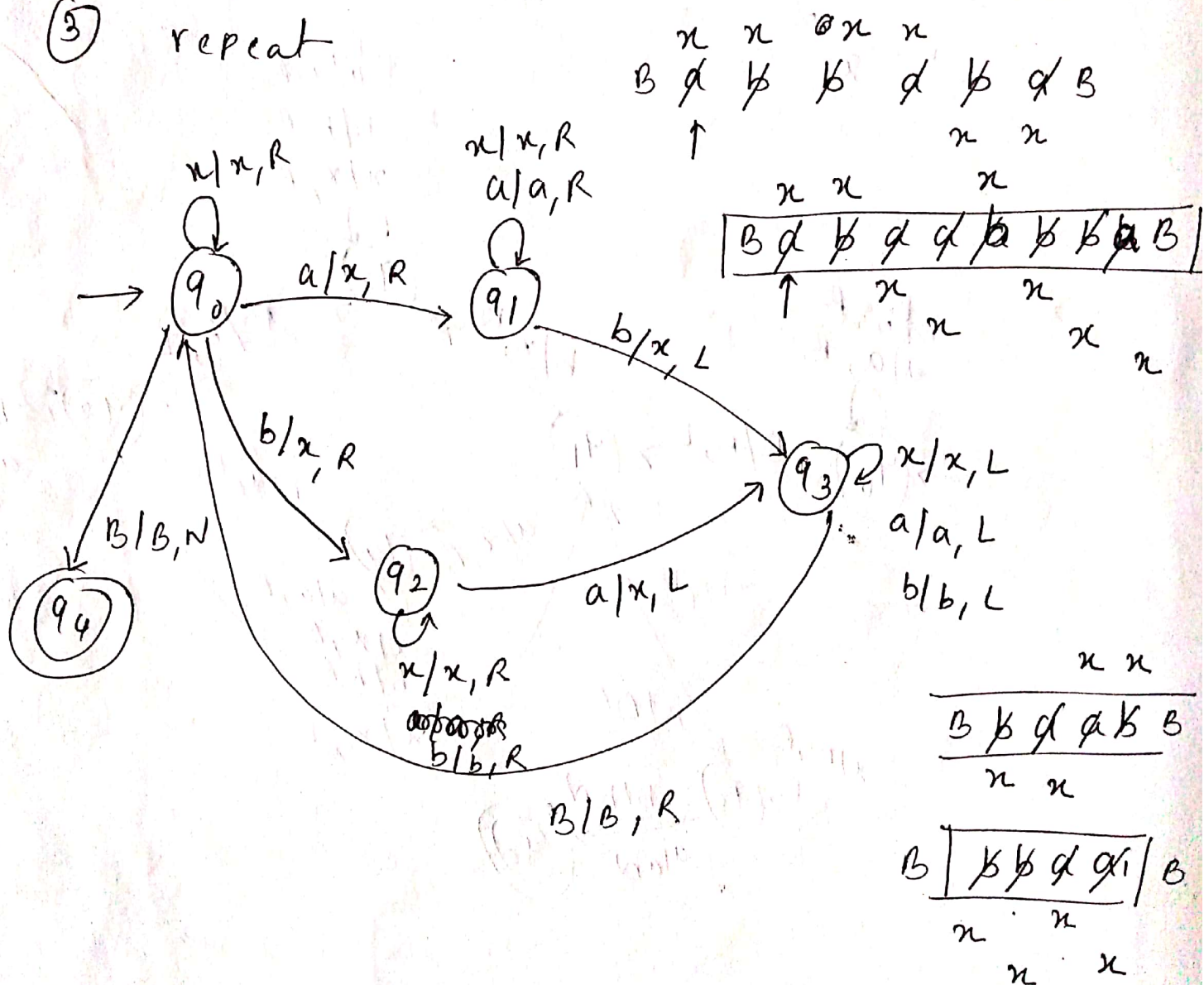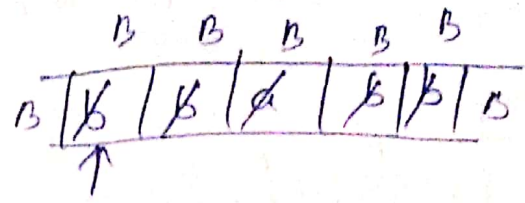
② if it is a then locate first b
   or if it is b then locate first a
   and make them x.

③ repeat

x x $⊗x$ x
B $\not{a}$ $\not{b}$ $\not{b}$ $\not{a}$ $\not{b}$ $\not{a}$ B
↑
x x

x x x
B $\not{a}$ $\not{b}$ $\not{a}$ $\not{a}$ $\not{a}$ $\not{b}$ $\not{b}$ $\not{a}$ B
↑ x x x x x

$x/x,R$

$x/x,R$
$a/a,R$

→ (q₀) $a/x,R$ → (q₁) $b/x,L$ → (q₃) $x/x,L$
                                      $a/a,L$
$b/x,R$ $b/b,L$

$B/B,N$

(q₄)

(q₂) $a/x,L$

$x/x,R$
$b/b,R$

$B/B,R$

x x
B $\not{b}$ $\not{a}$ $\not{a}$ $\not{b}$ B
x x

B | $\not{b}$ $\not{b}$ $\not{a}$ $\not{a}$ | B
x x
x x

a a b c a b b b a

(6) Palindromes
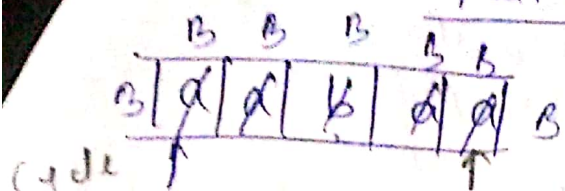
Cycle ①

first char ——— matched with last
& both erased.   char

when all blanks — end

State transition diagram:

- $q_0 \xrightarrow{b/B, R} q_1$
- $q_0 \xrightarrow{B/B, N} q_8$
- $q_0 \xrightarrow{b/B, R} q_2$
- q_1: $b/b, R$, $a/a, R$ self-loop; $\xrightarrow{b/b,R \; a/a,R} q_3$
- $q_1 \xrightarrow{B/B, N} q_8$
- q_3: $a/a, R$, $b/b, R$ self-loop; $q_3 \xrightarrow{B/B, L} q_4$
- $q_4 \xrightarrow{a/B, L} q_5$
- q_5: $a/a, L$, $b/b, L$ self-loop
- $q_2 \xrightarrow{B/B, N} q_8$
- $q_2 \xrightarrow{b/b, R \;\; a/a, R} q_6$
- q_6: $a/a, R$, $b/b, R$ self-loop; $q_6 \xrightarrow{B/B, L} q_7$
- $q_7 \xrightarrow{b/B, L} q_5$
- $q_5 \xrightarrow{B/B, R} q_0$ (via bottom curve)

Tm for well formed ness

$$\mathcal{I} = \{(, [,])\}$$

$$\overset{n}{\{} [\overset{n}{[} [\overset{n}{[} E \overset{n}{]} \overset{n}{E} \overset{n}{)}]]-(\overset{n}{\{})\overset{n}{E} \overset{n}{]})^{n} B$$

$[|E,R$
$n/n,R$ $[|(,R$

$]/n,L$ q_1 $n/n,L$

$B|B,L$

$(|n,R$ $[|n,R$

$)/n,L$

q_3

$x|B,L$ q_2

$B|B$ $n/n,L$
  N

q_4

$$\overset{x}{\mathcal{I}} \overset{n}{\mathcal{E}} \overset{x}{\mathcal{E}} \overset{x}{\mathcal{(}} \overset{x}{\mathcal{)}} \overset{n}{\mathcal{]}} B$$
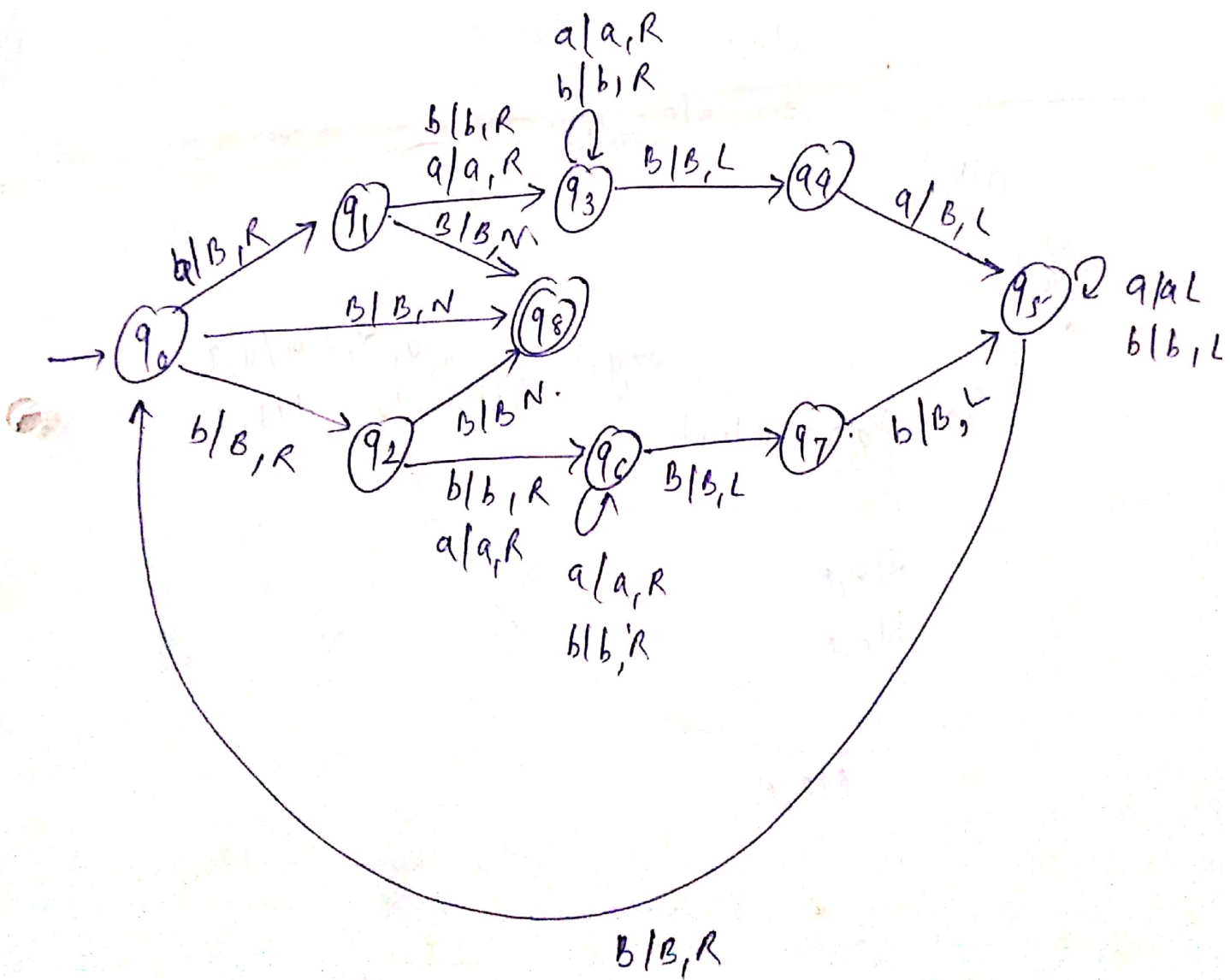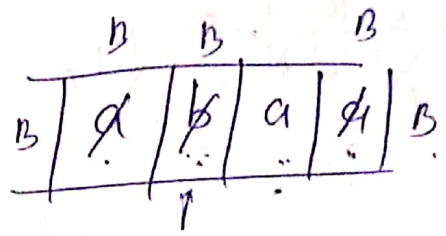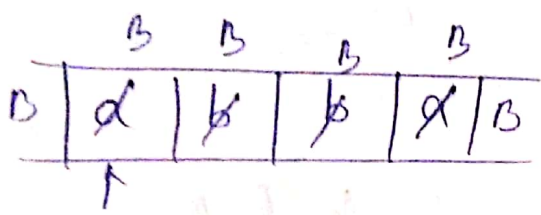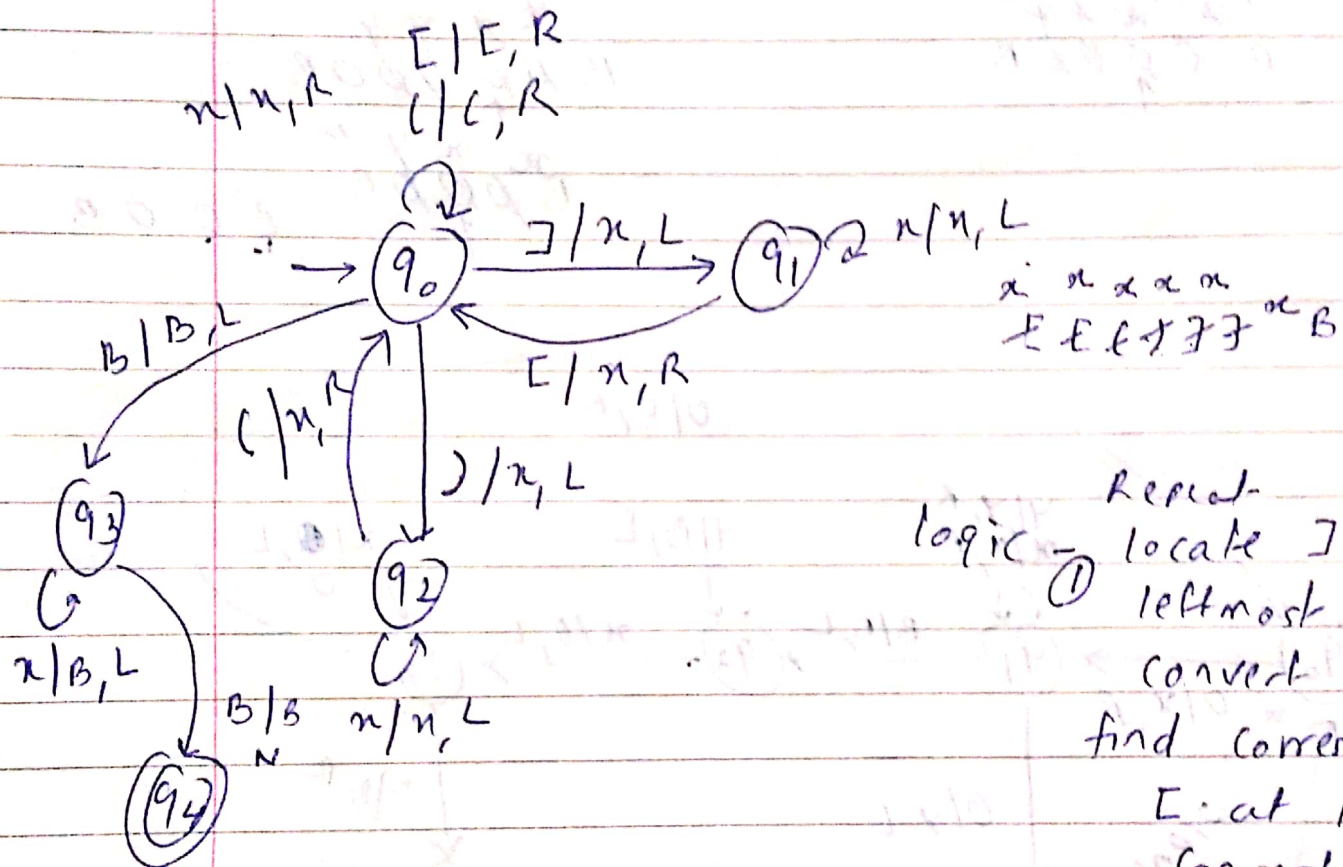
Repeat

logic → ① locate] and
          leftmost
          convert it to x
          find corresponding
          [at left and
          convert to n.

or

① locate more leftmost),
   convert it to x and find
   corresponding (at left
   and convert it to x

Repeat ① when all blanks are
left on tape then accept and
reach to final state.

Design a Tm for n^2 where n is integer $n \geq 0$

→ computation of 3^2 :-
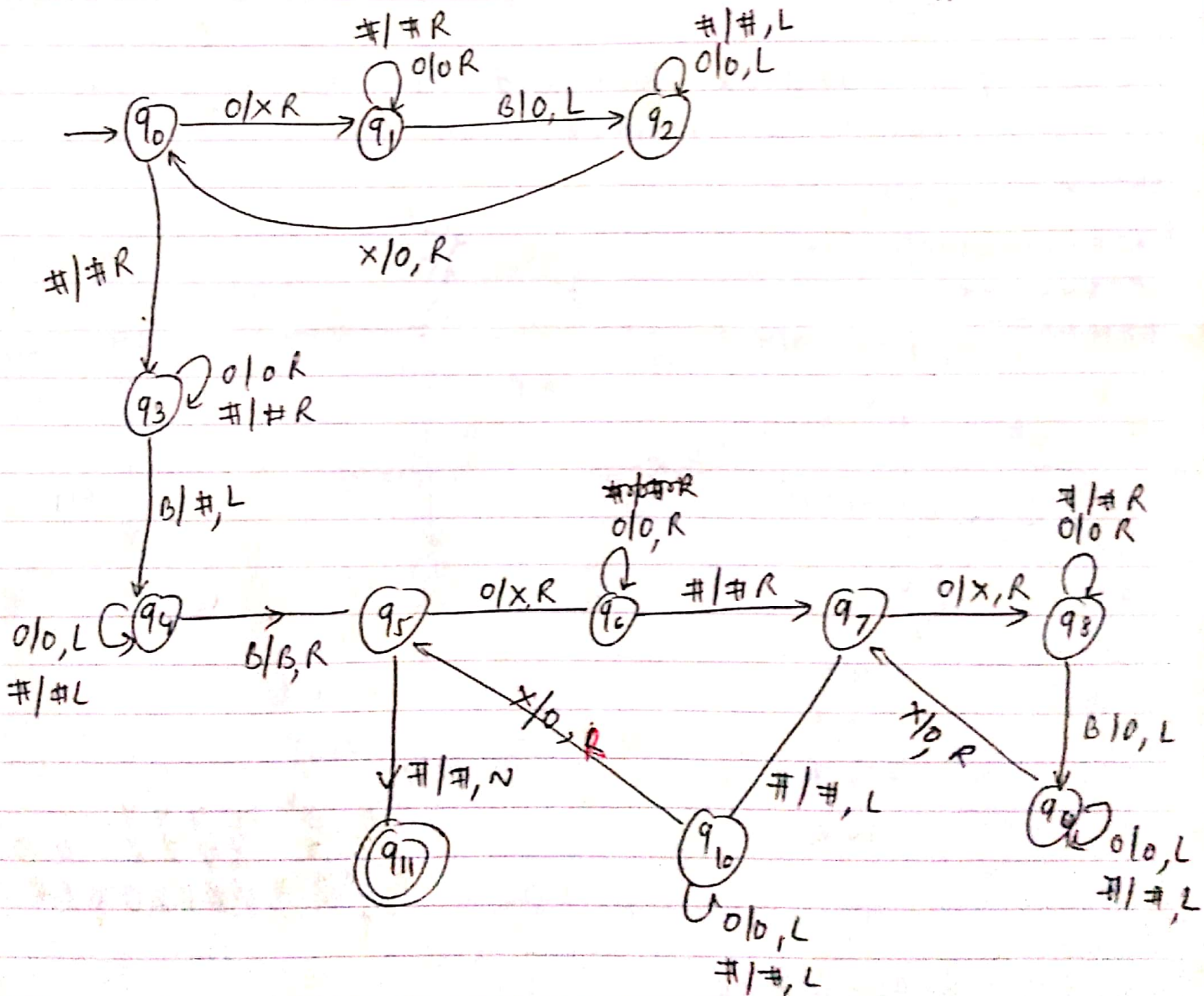   3 : 000
   ① tape   B 000 # B
   ② make copy of it   B 000 # 000 B
   ③ multiplication —

$$B \overset{x}{0}00 \# 000 \# 000 \ B$$
$$B xx0 \# 000 \# 000000 B$$
$$B xxx \# 000 \# \underbrace{000\,0000008}_{n^2}$$

④ Design a Tm to compute for n^2 where n is integer and $n \geq 0$.
remainder and quotient when a unary number is divided by another unary number.

B 00 # 00000 # B B
    2   5

B 00 # 00000 # B B → (q_0)
$X0$#Z
$0X$#ZZ
$B00$#$YY000$#$0B$
$X0$#$YY2$
$0X$#$YY22$
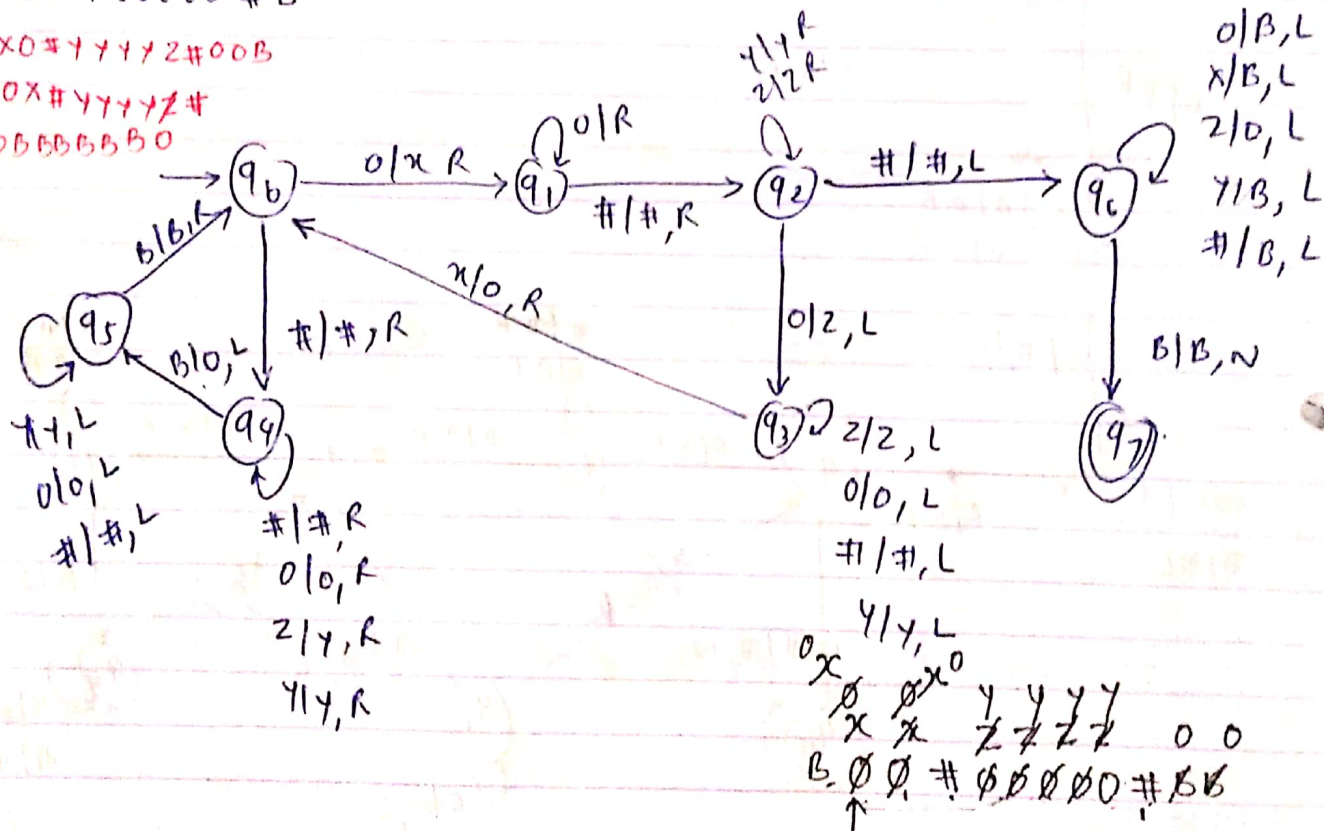00#$YYYY0$#$00B$

① division is performed thru repeated subtractions.

② To perform $x \div y$ both x & y are in unary system. x = dividend
                    y = divisor.

③ y is repeatedly subtracted from x, as long as $x \geq y$ & quotient is incremented by 1.

B#00#00000#B
$BX0$#$YYYY2$#$00B$
$0X$#$YYYYZ$#
$0BBBBBB0$

④/Y R
2/2 R

o/R
(q_b) $\xrightarrow{o/x\ R}$ (q_1) $\xrightarrow{\#/\#,R}$ (q_2) $\xrightarrow{\#/\#,L}$ (q_6)

$o/B,L$
$x/B,L$
$2/0,L$
$Y/B,L$
$\#/B,L$

$B/B,N$

$b/b,L$
(q_5)
$\#$/$\#$,R
$B/0,L$ (q_4) $x/0,R$
x/y,L
$o/0$,L
$\#/\#,L$

$o/2,L$
(q_3) $2/2,L$
$o/0,L$
$\#/\#,L$
$Y/Y,L$

$\#/\#,R$
$o/0,R$
$2/Y,R$
$Y/Y,R$

o o
X X
Y Y Y Y
B 0 0 # 0 0 0 0 0 # B B
    ↑

① $B00$#0000#B
② $B00$#22000#B 2 subtracted from 5.
③ $B00$#$YY000$#$0B$
④ $B00$#$YY220$#$0B$
⑤ $B00$#$YYYY0$#$00B$
⑥ $BX0$#$YYYY2$#$00B$ — $B0$#$00B$
                              R Quotient