

TUTORIAL 9

- i) Design PDM for context free language
 $L = \{a^n b^n \mid n \geq 1\}$.

Def: A pushdown automata M is defined

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

Q = set of states

Σ = input alphabets

Γ = stack symbols

$q_0 = q_0 \in Q$ is initial state

δ = transition function is a transition form

$F = F \subseteq Q$ is a set of final states

z_0 = initial state symbol

Features:

i) Has readable pointer

ii) Has stack memory

2.1) Push operation

2.2) Pop operation

2.3) Check empty condition of stack through an initial stack symbol.

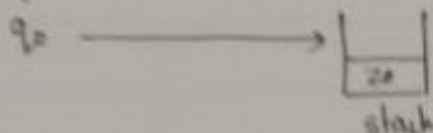
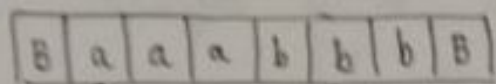
2.4) No operation reads top symbol of stack

iii) Makes state changes

For language = $\{a^n b^n\}$

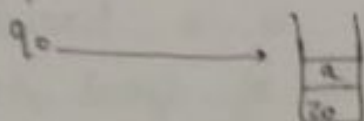
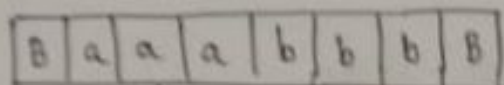
Words = $\{aabb, aaabbb, \dots\}$

Sample string: aaabbb.

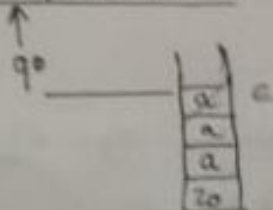
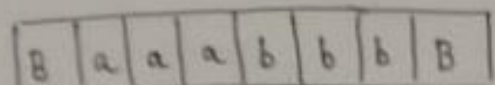
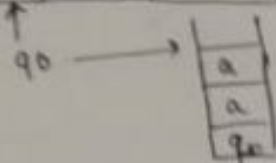
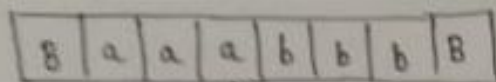


Transition:

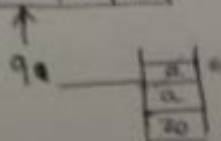
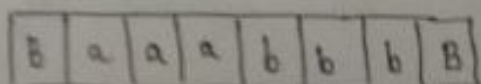
$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$



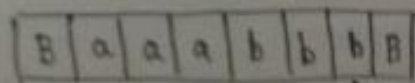
$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$



$$\delta(q_0, b, a) \rightarrow (q_1, \epsilon)$$



$$\delta(q_1, b, a) \rightarrow \delta(q_1, \epsilon)$$

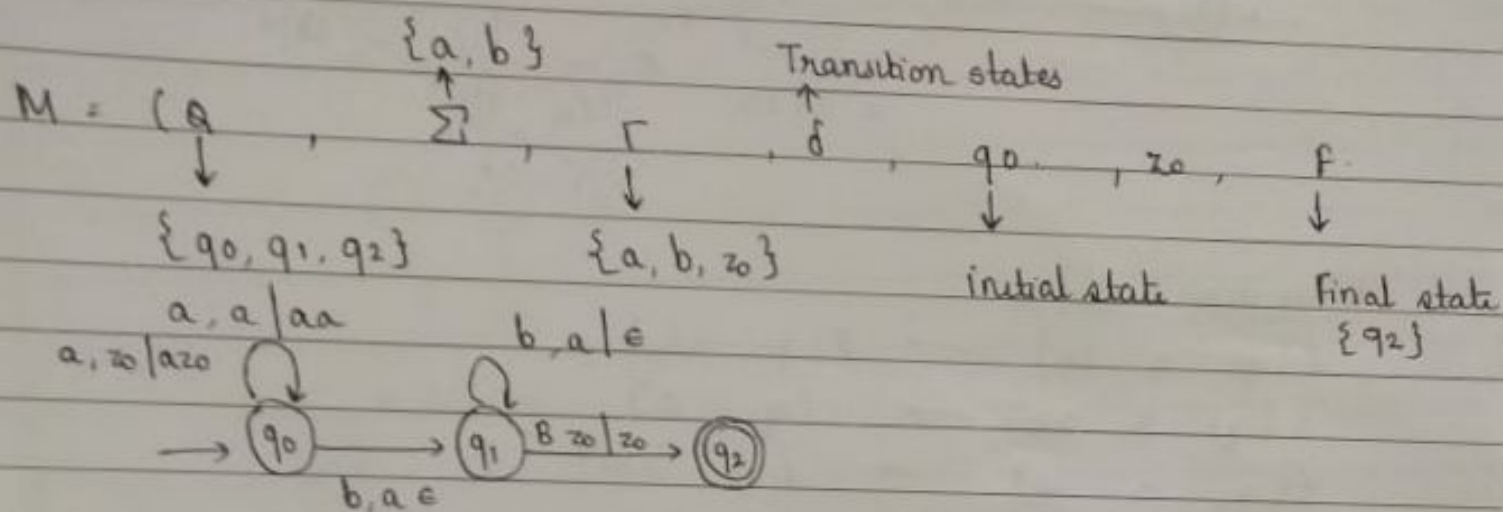


$$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$$

B a a a b b b B

q_0 — $\boxed{z_0}$

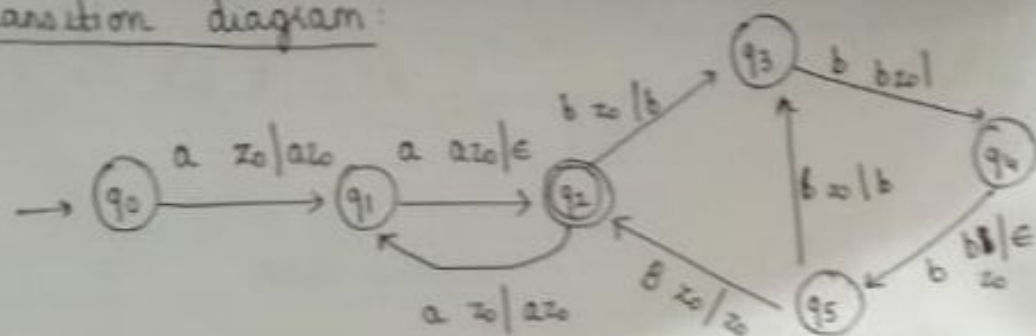
$$\delta(q_1, B, z_0) \rightarrow \delta(q_2, z_0)$$



2) Design PDA for $L = \{a^{2n} b^{3m} \mid n \geq 1\}$
 words = $\{aa, aaaa, aaaaabbb, \dots\}$

- Logic:
- ① For 1st 'a' push to the stack
 - ② For 2nd 'a' pop from stack
 - ③ For 1st 'b' push on to the stack
 - ④ For 2nd 'b' no-operation
 - ⑤ For 3rd 'b' pop from stack

Transition diagram:



Transition states:

$$\delta(q_0, a, z_0) \rightarrow \delta(q_1, az_0)$$

$$\delta(q_1, a, az_0) \rightarrow (q_2, \epsilon)$$

$$\delta(q_2, a, z_0) \rightarrow (q_1, az_0)$$

$$\delta(q_2, b, z_0) \rightarrow (q_3, b)$$

$$\delta(q_3, b, bz_0) \rightarrow (q_3) \text{ NOP}$$

$$\delta(q_4, b, bz_0) \rightarrow (q_5, \epsilon)$$

$$\delta(q_5, b, z_0) \rightarrow (q_3, b)$$

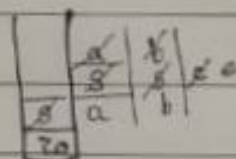
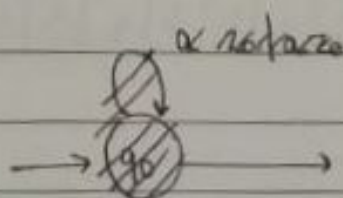
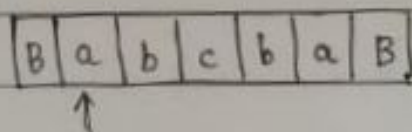
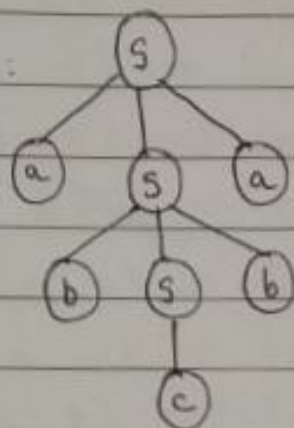
$$\delta(q_5, \epsilon, z_0) \rightarrow (q_2, z_0)$$

$$M = \{ Q, \overset{\substack{\uparrow \\ \{a, b\}}}{\Sigma}, \overset{\substack{\uparrow \\ \{a, b, z_0\}}}{\delta}, \overset{\substack{\uparrow \text{Transition} \\ \delta}}{\delta}, \overset{\substack{\downarrow \\ q_0}}{q_0}, \overset{\substack{\uparrow \text{initial state} \\ q_0}}{q_0}, \overset{\substack{\uparrow \text{initial stack} \\ z_0}}{z_0}, \overset{\substack{\uparrow \text{symbol} \\ z_0}}{z_0}, \overset{\substack{\downarrow \\ \{q_2\}}}{F} \}$$

$$\{ q_0, q_1, q_2, q_3, q_4, q_5 \}$$

3). Find PDM of CFG
 $S \rightarrow aSa \mid bSb \mid c$

Parse tree:



$$\delta(q_0, B, z_0) \rightarrow (q_0, Sz_0)$$

$$\delta(q_0, a, S) \rightarrow (q_0, aSa)$$

$$\delta(q_0, a, a) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, b, S) \rightarrow (q_0, bSb)$$

$$\delta(q_0, b, b) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, c, S) \rightarrow (q_0, c)$$

$$\delta(q_0, c, c) \rightarrow (q_0, \epsilon)$$

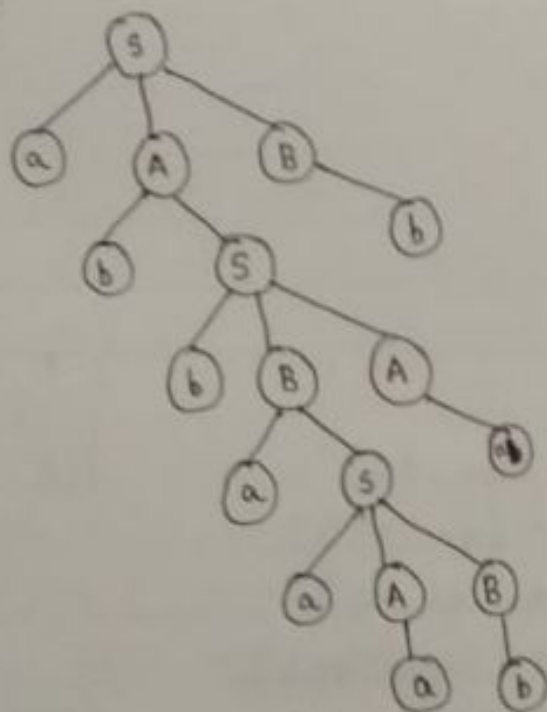
$$\delta(q_0, b, b) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, a, a) \rightarrow (q_0, \epsilon)$$

$$\delta(q_0, B, Sz_0) \rightarrow (q_0, z_0)$$

- 4) PDM for CFG:
 $S \rightarrow aAB \mid bBA$
 $A \rightarrow bS \mid a$
 $B \rightarrow aS \mid b$

Parse string:



B a b b a a a b a b B

Diagram showing the stack and input tape during the parsing process. The stack contains 'B' and the input tape contains 'a b b a a a b a b B'.

- $\delta(q_0, B, z_0) \rightarrow (q_1, S)$
 $\delta(q_1, a, S) \rightarrow (q_1, aAB)$
 $\delta(q_1, a, a) \rightarrow (q_1, \epsilon)$
 $\delta(q_1, b, A) \rightarrow (q_1, bS)$
 $\delta(q_1, b, b) \rightarrow (q_1, \epsilon)$
 $\delta(q_1, b, S) \rightarrow (q_1, bBA)$
 $\delta(q_1, b, b) \rightarrow (q_1, \epsilon)$
 $\delta(q_1, a, B) \rightarrow (q_1, aS)$
 $\delta(q_1, a, a) \rightarrow (q_1, \epsilon)$

$$\delta(q_1, a, s) \rightarrow (q_1, a, aAB)$$

$$\delta(q_1, a, a) \rightarrow (q_1, \epsilon)$$

$$\delta(q_1, a, A) \rightarrow (q_1, a)$$

$$\delta(q_1, a, a) \rightarrow (q_1, \epsilon)$$

$$\delta(q_1, b, B) \rightarrow (q_1, b)$$

$$\delta(q_1, b, b) \rightarrow (q_1, \epsilon)$$

$$\delta(q_1, b, a, A) \rightarrow (q_1, a)$$

$$\delta(q_1, a, a) \rightarrow (q_1, \epsilon)$$

$$\delta(q_1, b, B) \rightarrow (q_1, b)$$

$$\delta(q_1, b, b) \rightarrow (q_1, \epsilon)$$