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Module 4.0
Angle Modulation (FM & PM)

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Electronic Circuits & Communication Fundamentals
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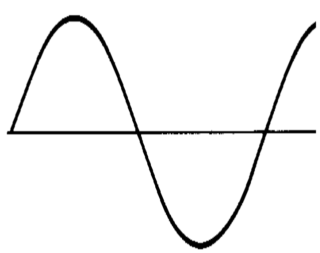
A Brief Review of Modulation

- **Low frequency baseband signal represented by $v_m(t) = V_m \sin\omega_m t$ OR $v_m(t) = V_m \cos\omega_m t$**
- **High frequency carrier signal is represented by $v_c(t) = V_c \sin\omega_c t$ OR $v_c(t) = V_c \cos\omega_c t$**

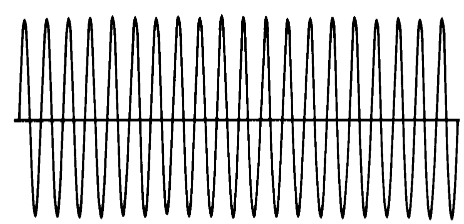
low frequency baseband signal
$v_m(t) = V_m \sin\omega_m t$

→ **Analog Modulation** →

output modulated signal



high frequency carrier signal
$v_c(t) = V_c \sin\omega_c t$



Definition of Modulation

Analog Modulation is defined as a process in which **one** of the **parameters** (characteristics) of a high frequency **carrier** signal **(amplitude, frequency or phase)** is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping other parameters constant.

- Carrier Amplitude – $V_c \propto v_m(t)$
- Carrier Frequency – $f_c \propto v_m(t)$
- Carrier Phase – $\theta_c \propto v_m(t)$

one of them varies while two others remain constant

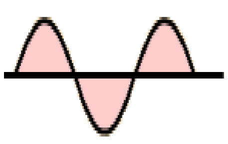
Types of Analog Modulation

- Amplitude Modulation (AM) where the carrier amplitude (V_c) varies with $v_m(t)$
- Frequency Modulation (FM) where the carrier frequency (f_c) varies with $v_m(t)$
- Phase Modulation (PM) where phase of the carrier (θ_c) varies with $v_m(t)$
- Since phase & frequency are directly related, they are angular modulation

Amplitude Modulation (AM)

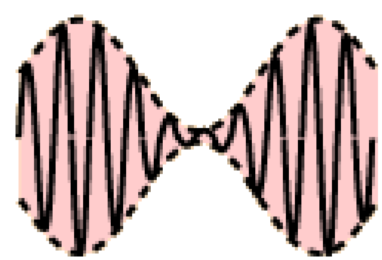
Amplitude Modulation is defined as process in which the **amplitude** of high frequency **carrier** signal is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping **phase** & **frequency** of the **carrier** signal **constant.**

low frequency baseband signal
$v_m(t) = V_m \sin\omega_m t$

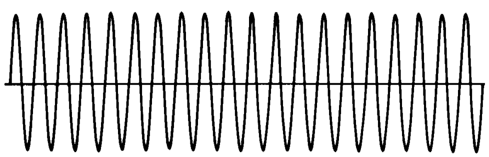


Amplitude Modulation

output amplitude modulated signal
$v_{AM}(t)$

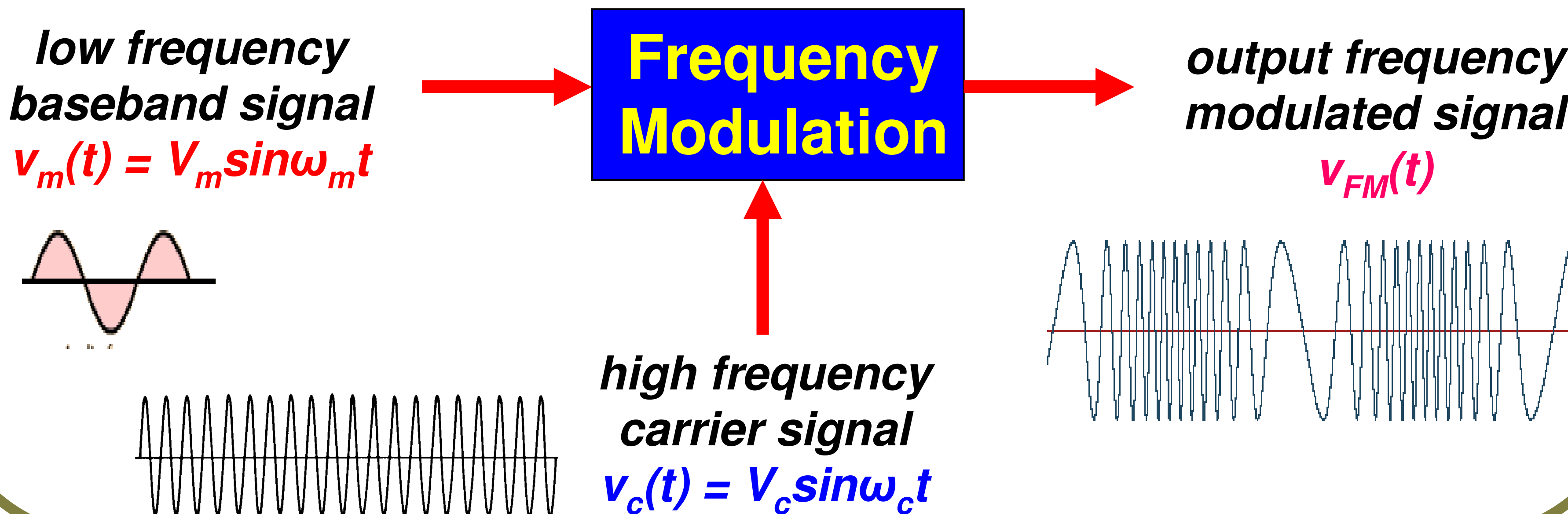


high frequency carrier signal
$v_c(t) = V_c \sin\omega_c t$



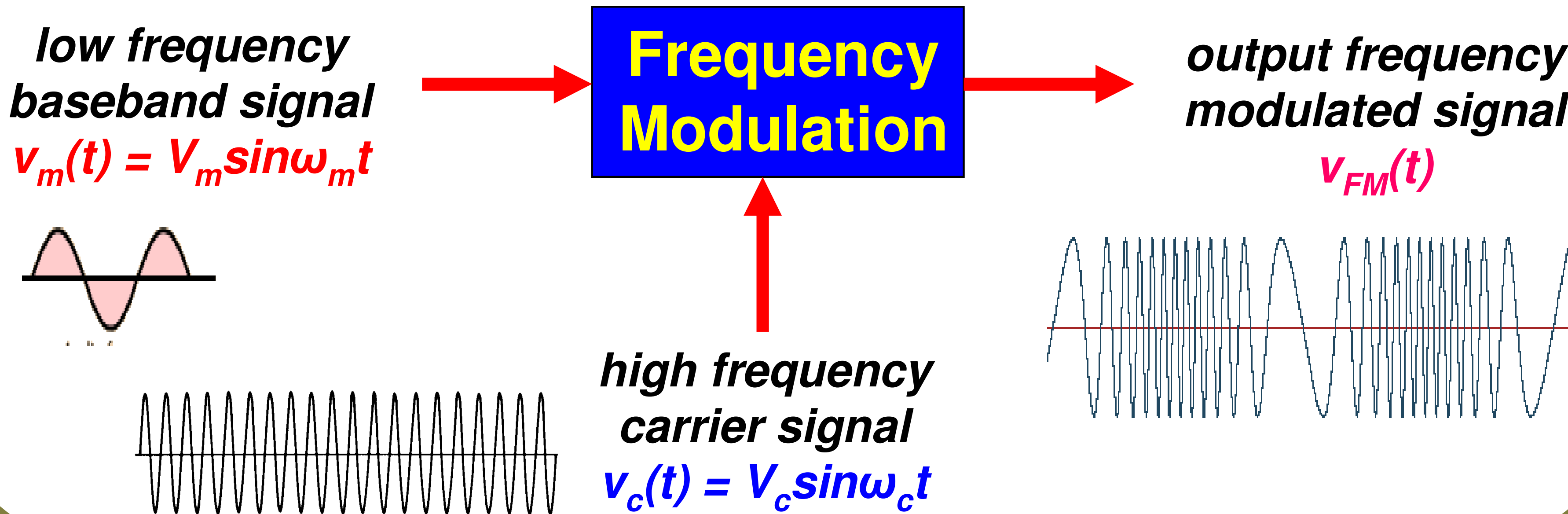
Frequency Modulation (FM)

Frequency Modulation is defined as process in which the **frequency** of high frequency **carrier** signal is **varied** proportionally to **instantaneous** amplitude of **modulating** signal, keeping **phase** & **amplitude** of the **carrier** signal **constant.**

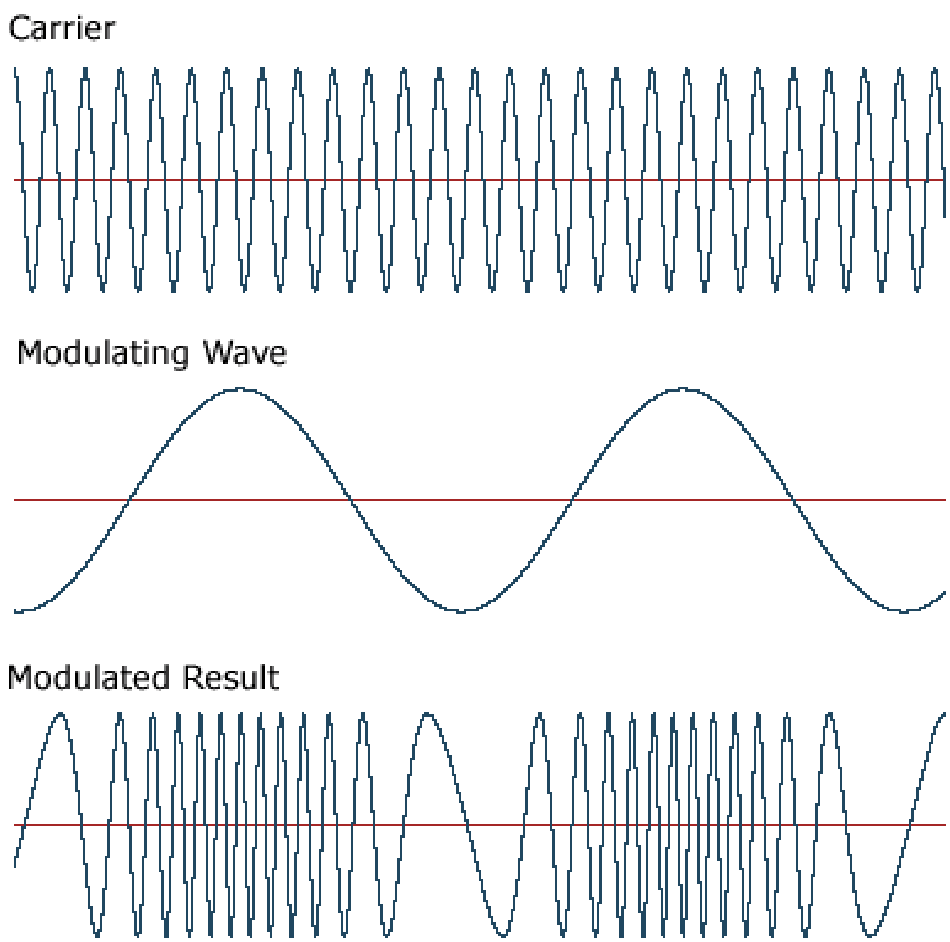


Frequency Modulation (FM)

- **AF modulating (baseband) signal given by** $v_m(t) = V_m \sin\omega_m t$ **OR** $v_m(t) = V_m \cos\omega_m t$
- **HF carrier signal is given by the equation of** $v_c(t) = V_c \sin\omega_c t$ **OR** $v_c(t) = V_c \cos\omega_c t$



Frequency Modulation (FM)



HF carrier signal

$$v_c(t) = V_c \cos\omega_c t$$

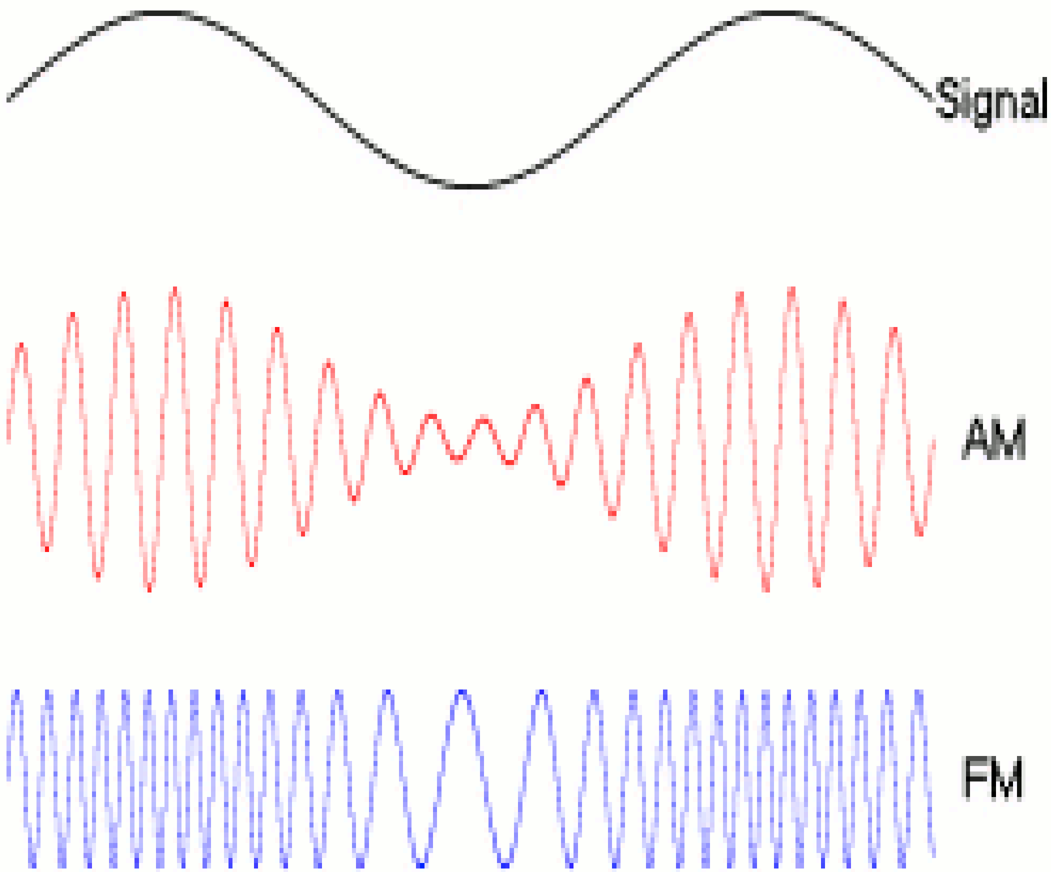
modulating signal

$$v_m(t) = V_m \cos\omega_m t$$

output signal

$$FM = v_{FM}(t)$$

AM & FM Waveforms



modulating or baseband signal

amplitude variations in V_c due to $v_m(t)$

frequency variations in ω_c due to $v_m(t)$

Concept of Frequency Deviation

- **Frequency Deviation** is change in carrier frequency (f_c) with time due to the input modulating baseband signal & expressed mathematically as :-

$$\boxed{\delta(t) = k_F v_m(t)}$$

k_F is constant (Hz/V) frequency sensitivity

- **Maximum Frequency Deviation** refers to the highest change in the carrier signal frequency (fc) due to the input modulating baseband signal given by :-

$$\boxed{\delta_{max} = k_F V_m}$$

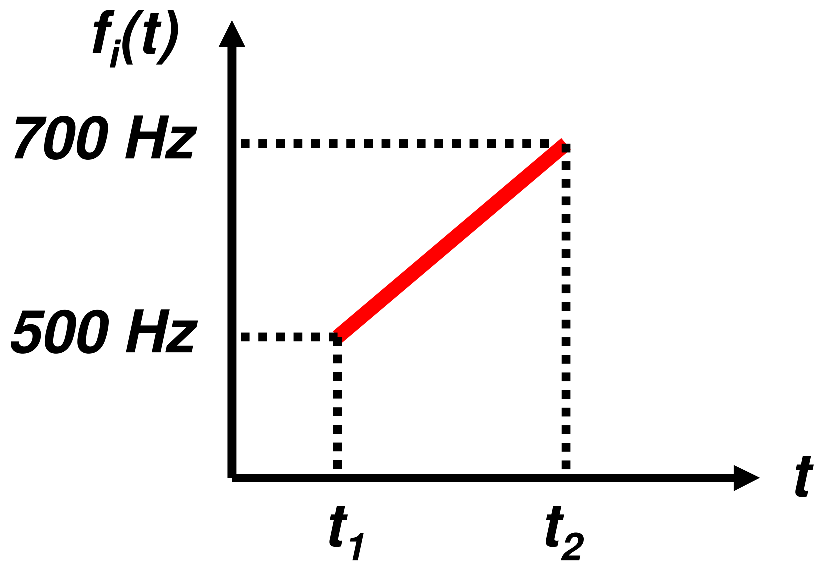
V_m is maximum OR peak signal amplitude

maximum frequency deviation is proportional to the maximum or peak amplitude of input modulating (baseband) signal

Concept of Instantaneous Frequency

Instantaneous Frequency refers to variation in output frequency of FM wave, defined at all the points of time (t) expressed as follows :-

$$\boxed{f_i(t) = f_c + \delta(t)}$$



based on frequency deviation, instantaneous frequency $f_i(t)$ is either above (more) or below (less) the carrier frequency f_c

Modulation Index (m_f)

- **Modulation Index (m_f)** **of FM wave is defined as ratio of the maximum frequency deviation (δ_{max}) to modulating signal frequency (f_m) :-**

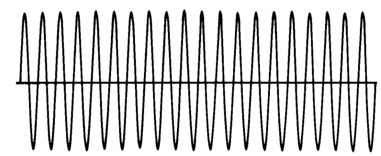
$$\boxed{m_f = \frac{\delta_{max}}{f_m}}$$

since frequency deviation & modulating signal frequency both carry the unit in Hz, it is a dimensionless quantity very much like the modulation index (m_a) of an AM wave

Concept of Angle Modulation

Carrier signal is mathematically represented by the following equation :-

$$v_c(t) = V_c \sin\omega_c t \text{ OR } v_c(t) = V_c \cos\omega_c t$$



In terms of vector carrier signal mathematically is also represented by :-

$$v_c(t) = V_c \sin[\theta(t)]$$

where θ(t) is the angular component incorporating frequency & phase shift

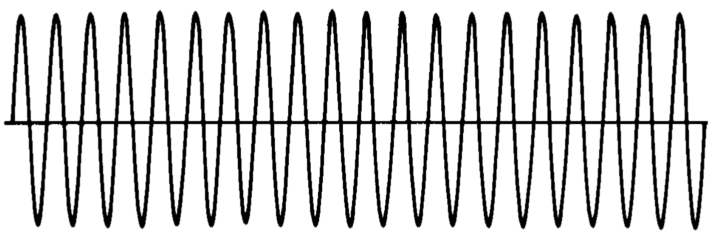
since $\theta(t) = \omega_c t + \Phi(t)$

Concept of Angle Modulation

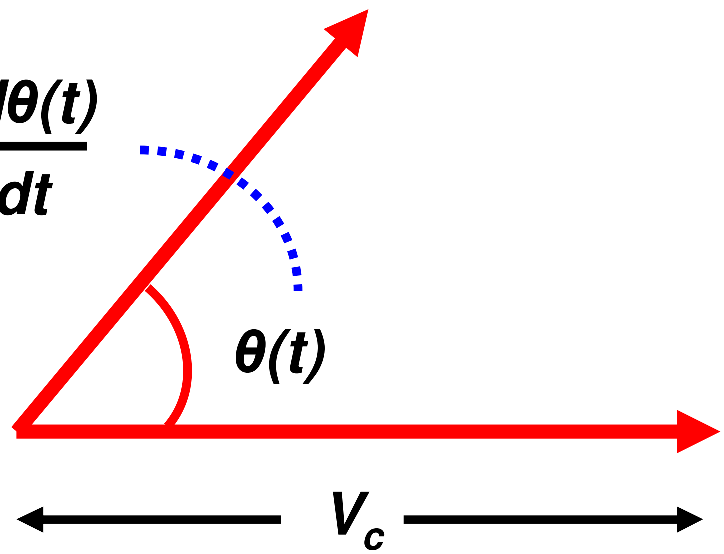
In terms of vector carrier signal mathematically is also represented by :-

$$\boxed{v_c(t) = V_c \sin[\theta(t)]}$$

where $\theta(t) = \omega_c t + \Phi(t)$



$$\omega(t) = \frac{d\theta(t)}{dt}$$



vector representation of carrier waveform

Concept of Angle Modulation

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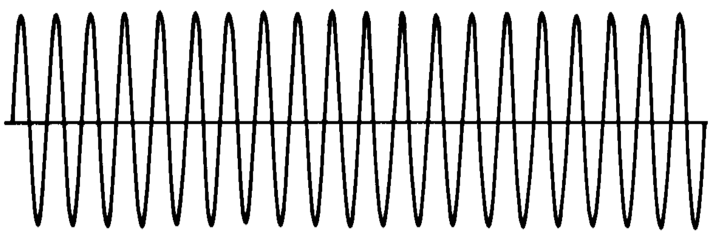
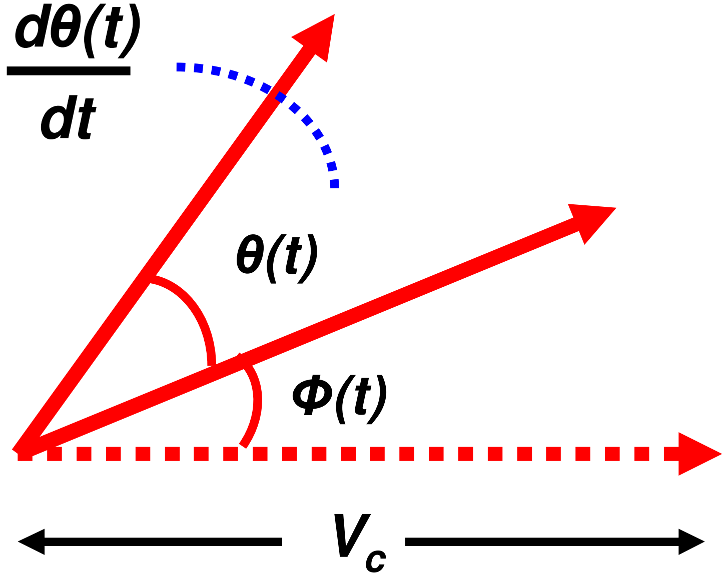


$$\omega(t) = \frac{d\theta(t)}{dt}$$



vector representation of carrier waveform

Concept of Angle Modulation

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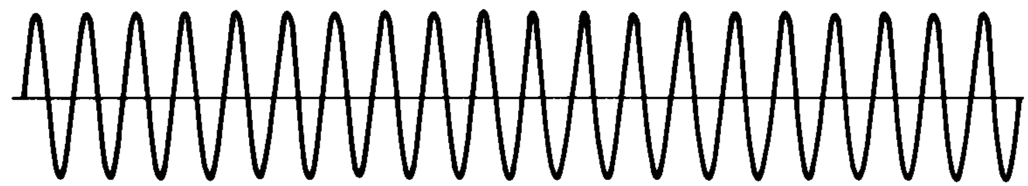


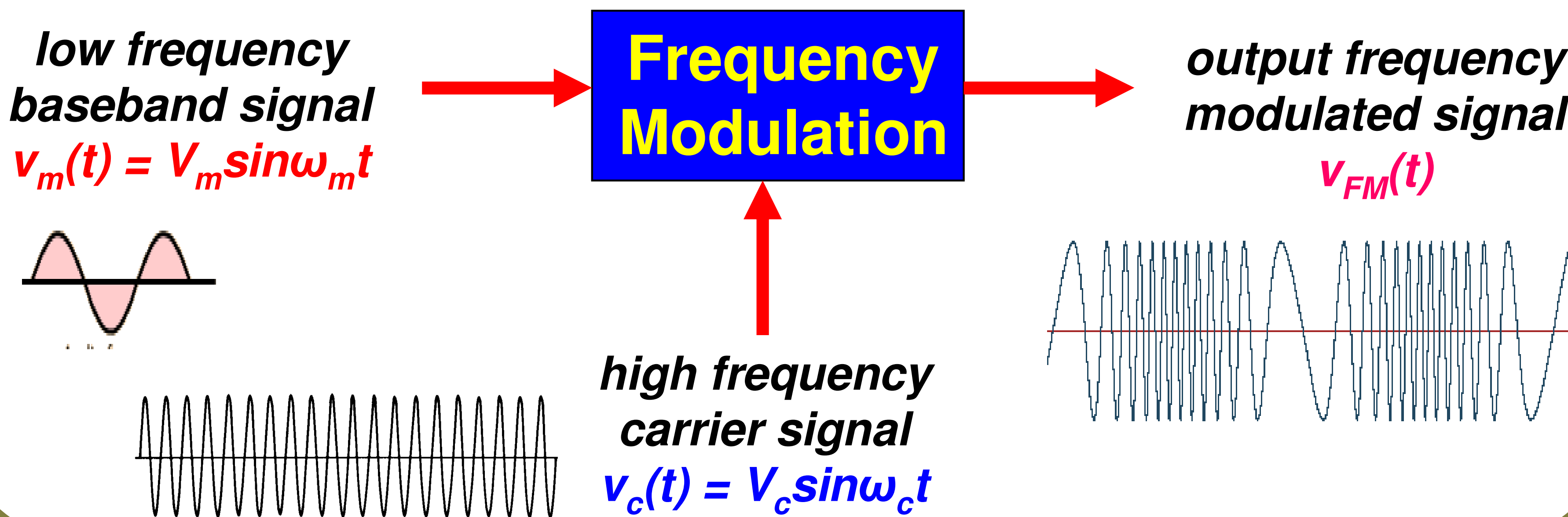
where $\theta(t) = \omega_c t + \Phi(t)$

frequency modulation (FM) make $\omega_c \; \alpha \; v_m(t)$

phase modulation (PM) make $\Phi(t) \; \alpha \; v_m(t)$

Mathematical Analysis of FM

- AF modulating (baseband) signal given by $v_m(t) = V_m \sin\omega_m t$ OR $v_m(t) = V_m \cos\omega_m t$
- HF carrier signal is given by the equation of $v_c(t) = V_c \sin\omega_c t$ OR $v_c(t) = V_c \cos\omega_c t$



Mathematical Analysis of FM

- **The equation of FM wave thus obtained :-**

$$\boxed{v_{FM}(t) = V_c cos(\omega_c t + m_f sin\omega_m t)}$$

- **This equation cannot be analyzed by using simple trigonometric functions but instead can be solved by using Bessel Functions**

thus only by using Bessel Functions the complete equation of FM wave is obtained in terms of carrier frequency & sidebands

Frequency Spectrum of FM

Expanding above equation by using Bessel Functions :-

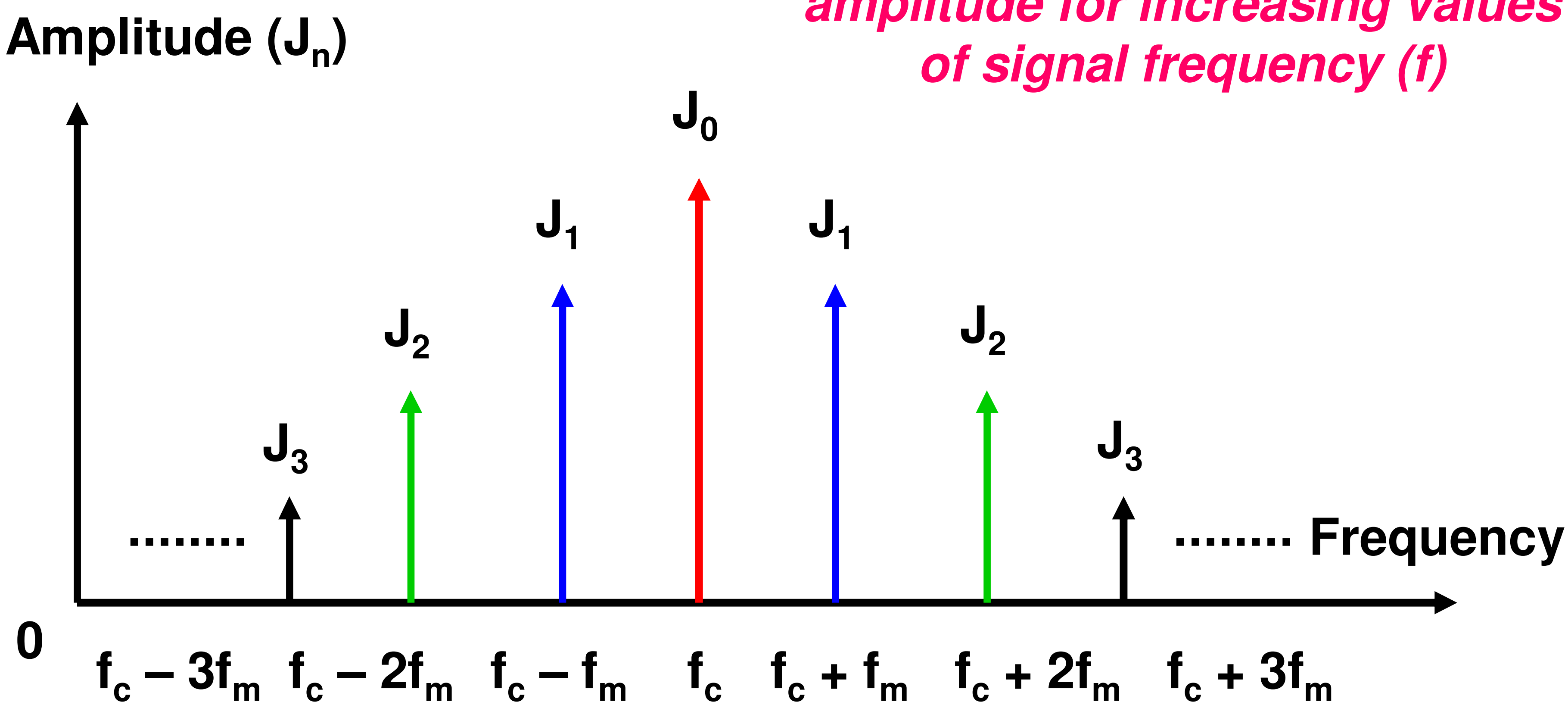
$$v_{FM}(t) = V_c\{\cos\omega_c t[J_0 + J_2\cos 2\omega_m t + J_4\cos 4\omega_m t.......]$$
$$-\sin\omega_c t[J_1\sin\omega_m t + J_3\sin 3\omega_m t.....]\}$$

$$v_{FM}(t) = V_c\{J_0\cos\omega_c t + J_1[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t]$$
$$+ J_2[\cos(\omega_c + 2\omega_m)t - \cos(\omega_c - 2\omega_m)t]..... + J_5....\}$$

- **where J_0 is Bessel Function of first type & n^{th} order**
- **J_0 – amplitude of the carrier signal**
- **J_n – amplitude of the sidebands, with frequency $\omega_c + n\omega_m$**

Frequency Spectrum of FM

FM spectrum contains infinite sidebands with a decreasing amplitude for increasing values of signal frequency (f)



Bessel Functions – Introduction

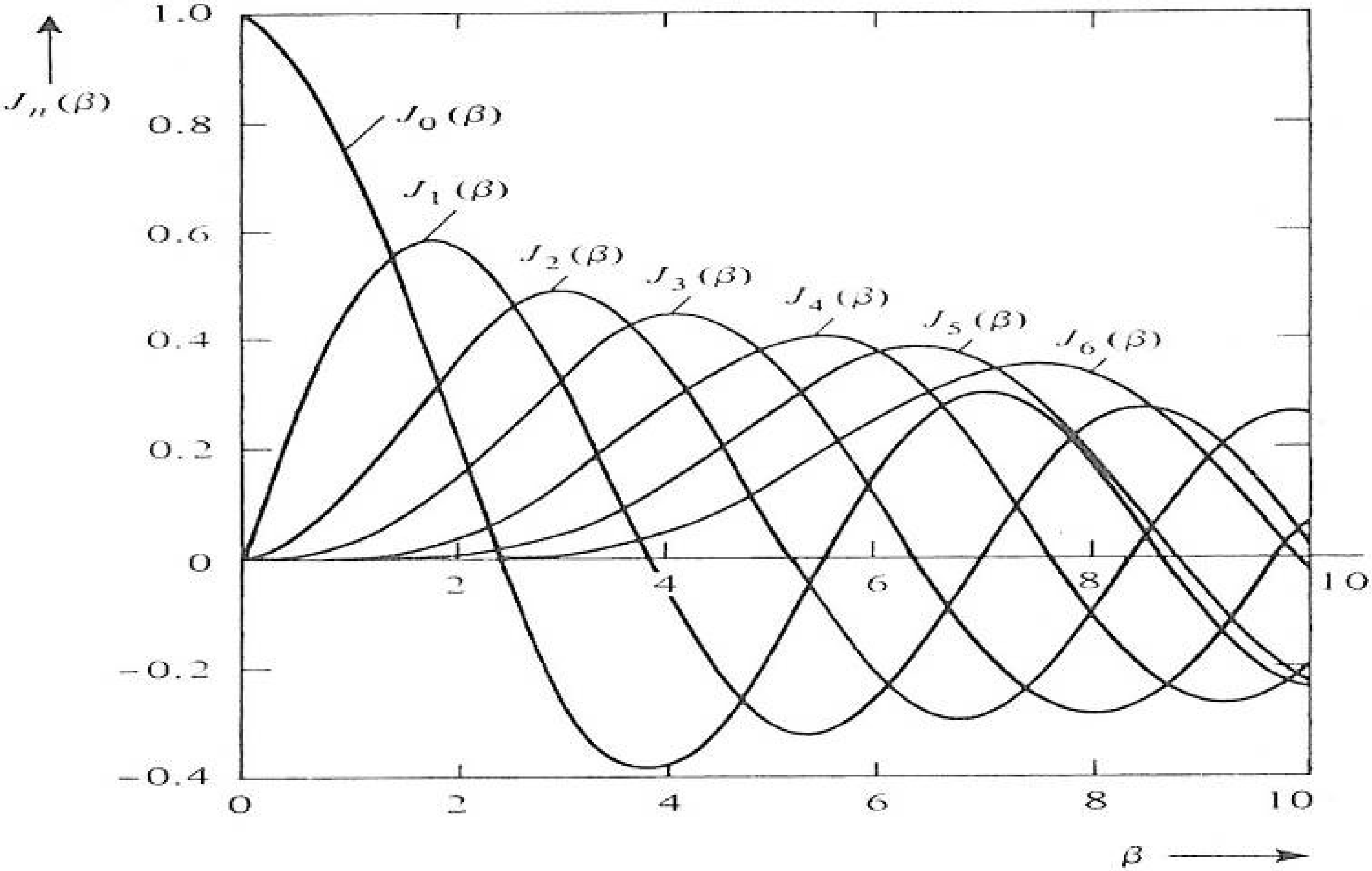
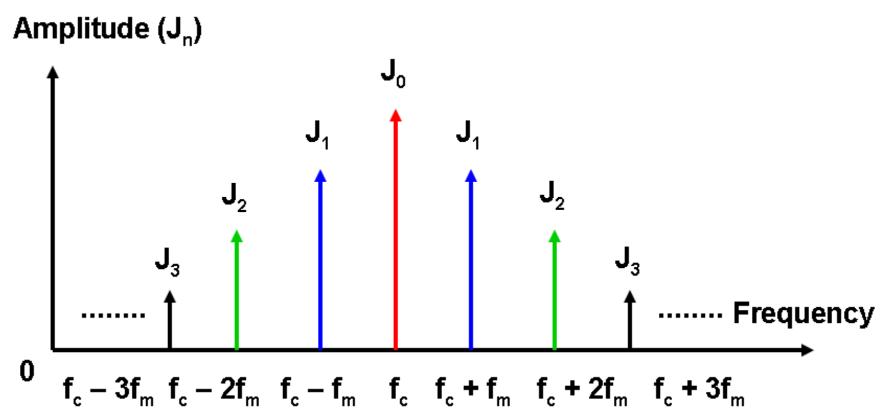


Table of Bessel Functions

n	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.7$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 3.0$	$\beta = 5.0$	$\beta = 7.0$	$\beta = 8.0$	$\beta = 10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001

Bessel Functions – Introduction

- First column gives the sideband number & the first row gives the modulation index (m_f)
- Other columns indicate amplitude of carrier signal & the various pair of sidebands
- Sidebands with a relative magnitude of less than 0.01 can be neglected



Frequency Spectrum of FM

- **Some of the sidebands & carrier signal has negative amplitudes for some values of 'm_f'**
- **This indicates that the signal represented by that amplitude is only 180° out of phase**
- **FM spectrum varies considerably in terms of bandwidth based upon modulation index**
- **The higher the modulation index (m_f) more is the bandwidth of the FM wave**
- **As 'm_f' increases, carrier amplitude decrease & sideband amplitude will increase**

Bandwidth of FM wave

- Theoretically FM waves consists an infinite number of sidebands (both LSB & USB)
- So, ideally the frequency modulated (FM) should have an infinite bandwidth
- However Bessel Functions for some higher 'm_f' values become relatively insignificant
- Hence only those terms should taken into consideration whose amplitudes or Bessel Function coefficients are significant

Bandwidth of FM wave

From Bessel Function analysis, theoretical bandwidth of FM wave taking into account only the significant sidebands is given by :-

$$\boxed{BW = 2nf_m}$$

However a more practical approach uses 'approximation' of the FM bandwidth given by Carson's rule as :-

$$\boxed{BW = 2(\delta_{max} + f_m)}$$

Carson's rule approximates bandwidth reasonably well only for higher modulation index values (m_f)

Power Equations in FM wave

- **In frequency modulation (FM) amplitude of FM wave is same as that of the carrier**
- **Amplitude of sidebands is relatively smaller compared to the carrier amplitude (V_c)**
- **Since they are relatively insignificant, power content of sidebands can be ignored too**

$$\boxed{P_T = P_C = \frac{V_C^2}{2R}}$$

this equation indicates that total power in FM is equal to the carrier power itself where R = resistance of the transmitting (Tx) antenna

Types of FM – NBFM & WBFM

- Practical FM systems used in all commercial applications are of two main types :-
  1. Narrow Band FM (NBFM)
  2. Wide Band FM (WBFM)
- These FM systems are classified at following points based on service operation :-
  1. Modulation Index (m_f)
  2. Frequency Deviation (δ)
  3. Bandwidth (BW)

Types of Frequency Modulation (FM) – 1. Narrow Band FM (NBFM)

- Modulation index of unity (m = 1)
- Frequency Deviation (δ) = 5 kHz – 10 kHz
- Bandwidth between BW = 10 kHz – 30 kHz
- Modulating Frequency f_m = 30 Hz – 3 kHz
- Typical applications include :-

(a) Short range communication
(b) Mobile communication (radio/wireless)
(c) Police, Ambulance, Taxicabs etc.
(d) Coast Guard & Maritime Communication

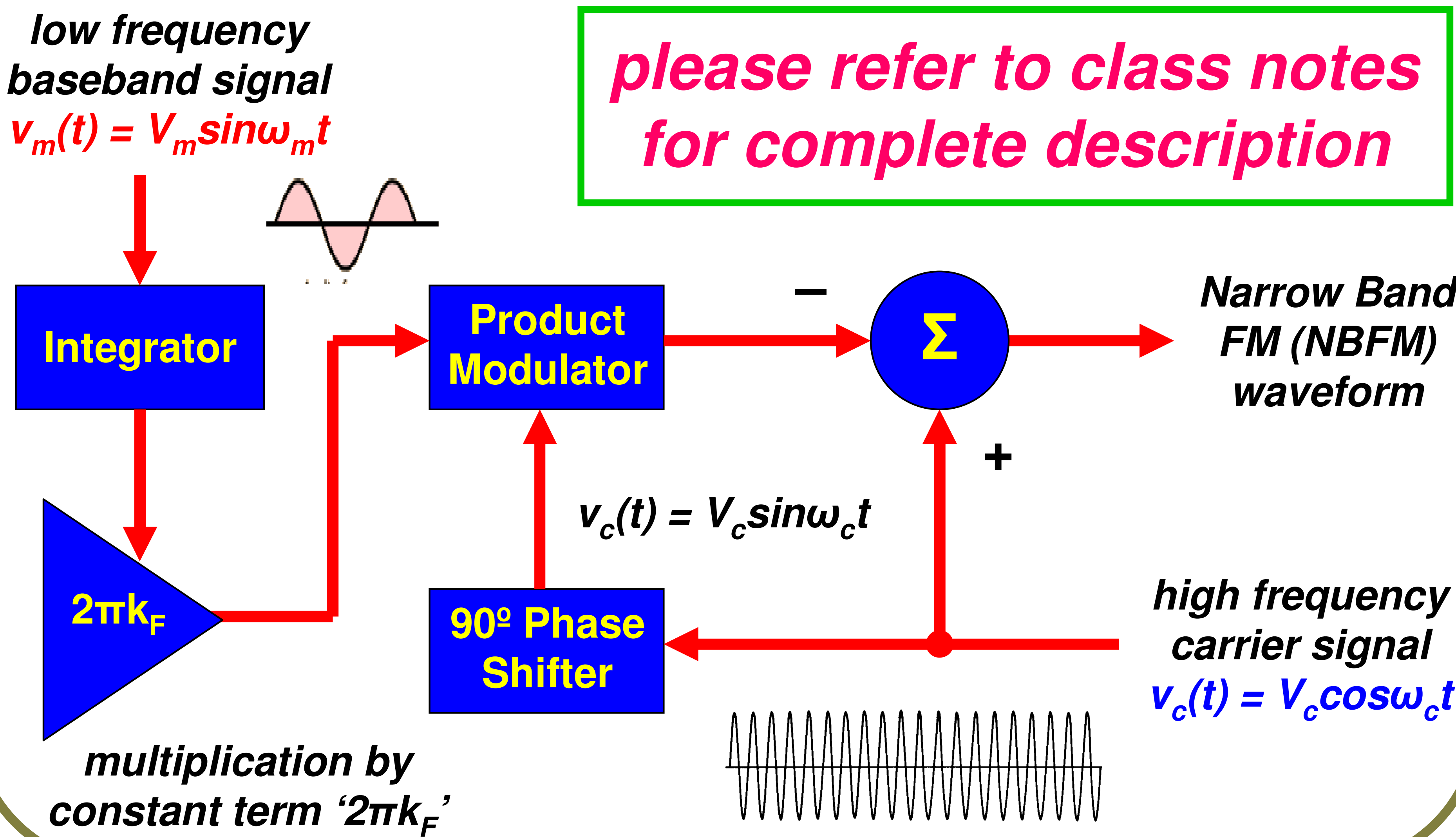
Types of Frequency Modulation (FM) – 2. Wide Band FM (WBFM)

- Modulation index (m) from $1 \le m \le 2500$
- Frequency Deviation (δ) = 75 kHz
- Bandwidth 10 to 15 times higher than NBFM
- Modulating Frequency f_m = 3 kHz – 15 kHz
- Typical applications include :-

  (a) Stereo Multiplexing
  (b) Commercial Broadcasting (FM Radio)
  (c) TV Broadcasting (sound with picture)

Generation of Narrow Band FM (NBFM)

please refer to class notes for complete description

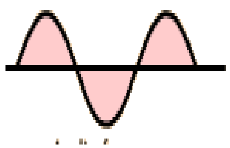


Generation of Wide Band FM (WBFM)

please refer to class notes for complete description

Modulating Signal

$v_m(t) = V_m \sin\omega_m t$



NBFM Generator → **Frequency Multiplier** → **Product Modulator** → **Bandpass Filter** → ***Wide Band FM (WBFM) waveform***

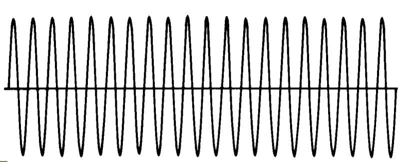
changes both frequency & deviation by factor 'n'

Local Oscillator → **Product Modulator**: $\cos\omega_o t$

generates highly stable & high frequency wave

Carrier Signal

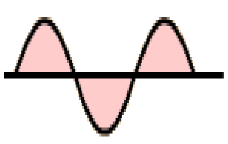
$v_c(t) = V_c \cos\omega_c t$



Phase Modulation (PM)

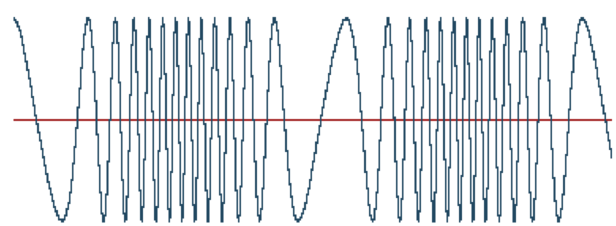
Phase Modulation is defined as the process in which the phase of the high frequency carrier signal is varied proportionally to instantaneous amplitude of modulating signal, with frequency & amplitude of the carrier signal constant.

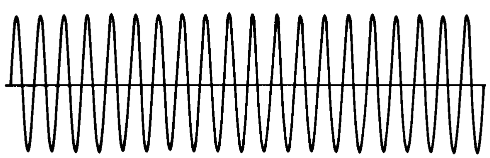
low frequency baseband signal
$v_m(t) = V_m \sin\omega_m t$



Phase Modulation

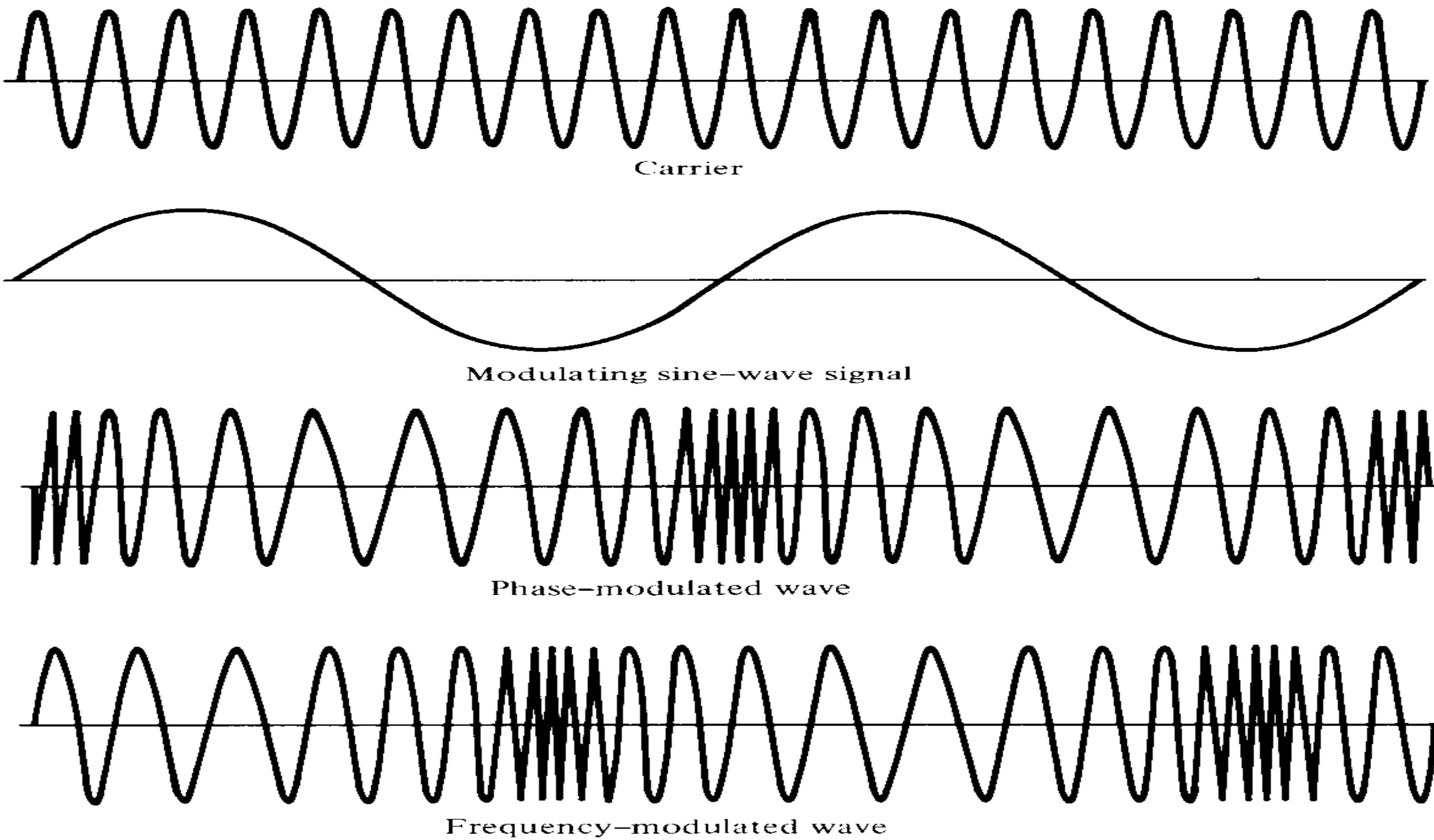
output phase modulated signal
$v_{PM}(t)$





high frequency carrier signal
$v_c(t) = V_c \sin\omega_c t$

Phase Modulation (PM)



Phase Shift Deviation (ΔΦ)

- **Phase shift deviation is defined as resulting change in the carrier signal phase shift (ΔΦ)**

$$\boxed{\Delta\varphi_{max} = k_P V_m}$$

k_P is constant (rad/V) of phase sensitivity

- **Now the instantaneous phase shift of the PM wave is given by the following equation :-**

$$\boxed{\theta(t) = \omega_c t + k_P v_m(t)}$$

based on the phase deviation, instantaneous phase θ(t) is either above (more) or below (less) the carrier phase Φ(t)

Modulation Index (m_p)

- **Modulation Index (m_p) of PM wave is defined as maximum possible phase change due to input modulating signal & is given as :-**

$$\boxed{m_p = k_p V_m = \Delta\varphi_{max}}$$

unlike both amplitude modulation (AM) & frequency modulation (FM), modulation index for phase modulation (PM) carries units of radians (rad) which maximum ranges from – π ≤ ΔΦ ≤ π

Mathematical Analysis of PM

- **The equation of PM wave thus obtained :-**

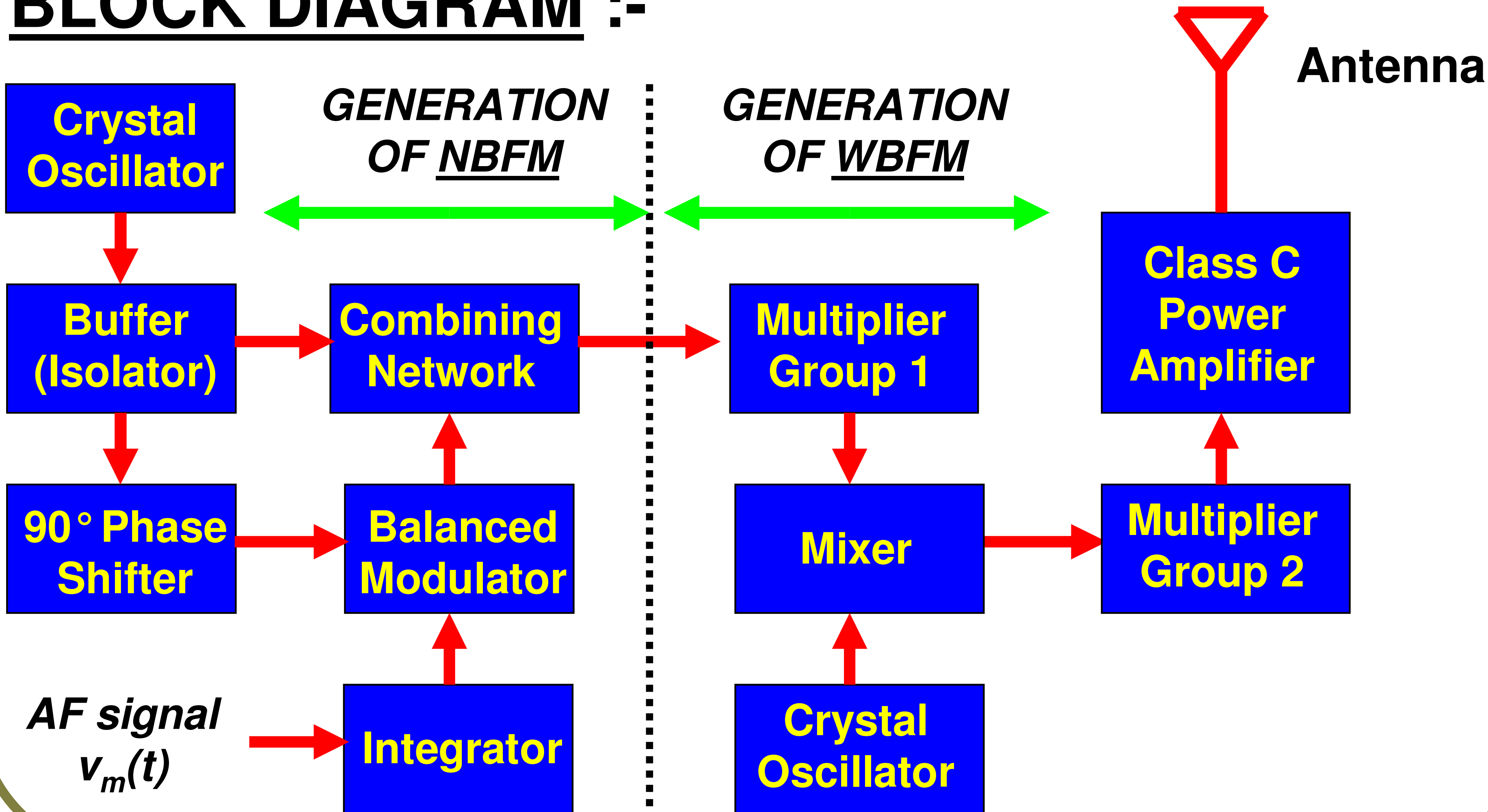
$$\boxed{v_{PM}(t) = V_c cos(\omega_c t + m_p sin\omega_m t)}$$

- **This equation cannot be analyzed by using simple trigonometric functions but instead can be solved by using Bessel Functions**

thus only by using Bessel Functions the complete equation of PM wave is obtained in terms of carrier frequency & sidebands

2. Indirect (Armstrong) Method

BLOCK DIAGRAM :-



2. Indirect (Armstrong) Method

- Here a narrowband FM (NBFM) is generated indirectly by using phase modulation (PM)
- This NBFM signal converted into wideband FM (WBFM) by frequency multiplication
- For NBFM, input modulating signal is first integrated & then phase modulated
- The resulting NBFM is given to a group of multipliers to generate wideband frequency modulated wave (WBFM)

Demodulation of FM

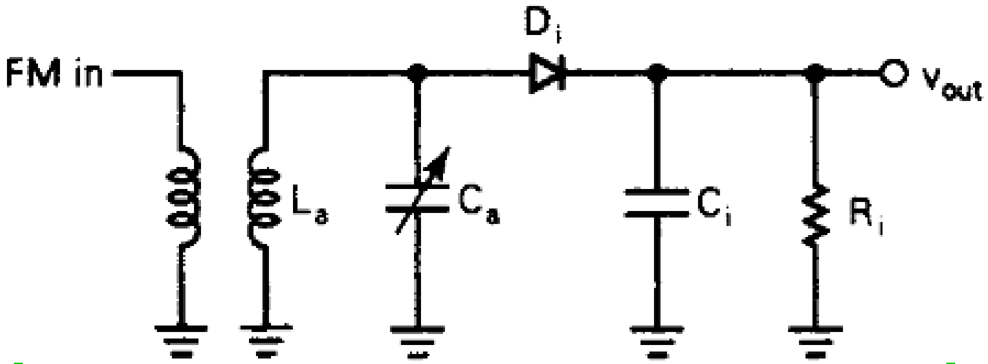
For detection (demodulation) of FM waves the following four methods are common :-

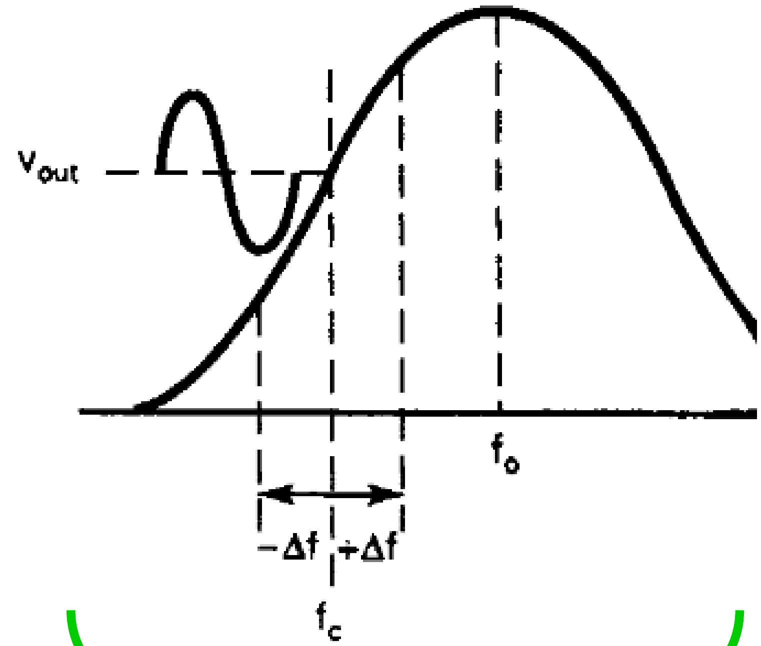
- Single Slope Detector (Demodulator)
- Balanced Slope Detector (Demodulator)
- Foster – Seeley Discriminator (Detector)
- The Ratio Detector (Demodulator)

All of the above follow a common principle :-

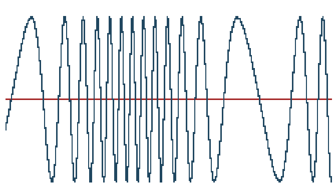
Input FM signal $v_{FM}(t)$ → **FM to AM Conversion** → **AM Diode Detector** → modulating signal $v_m(t)$

Principle of FM Demodulation



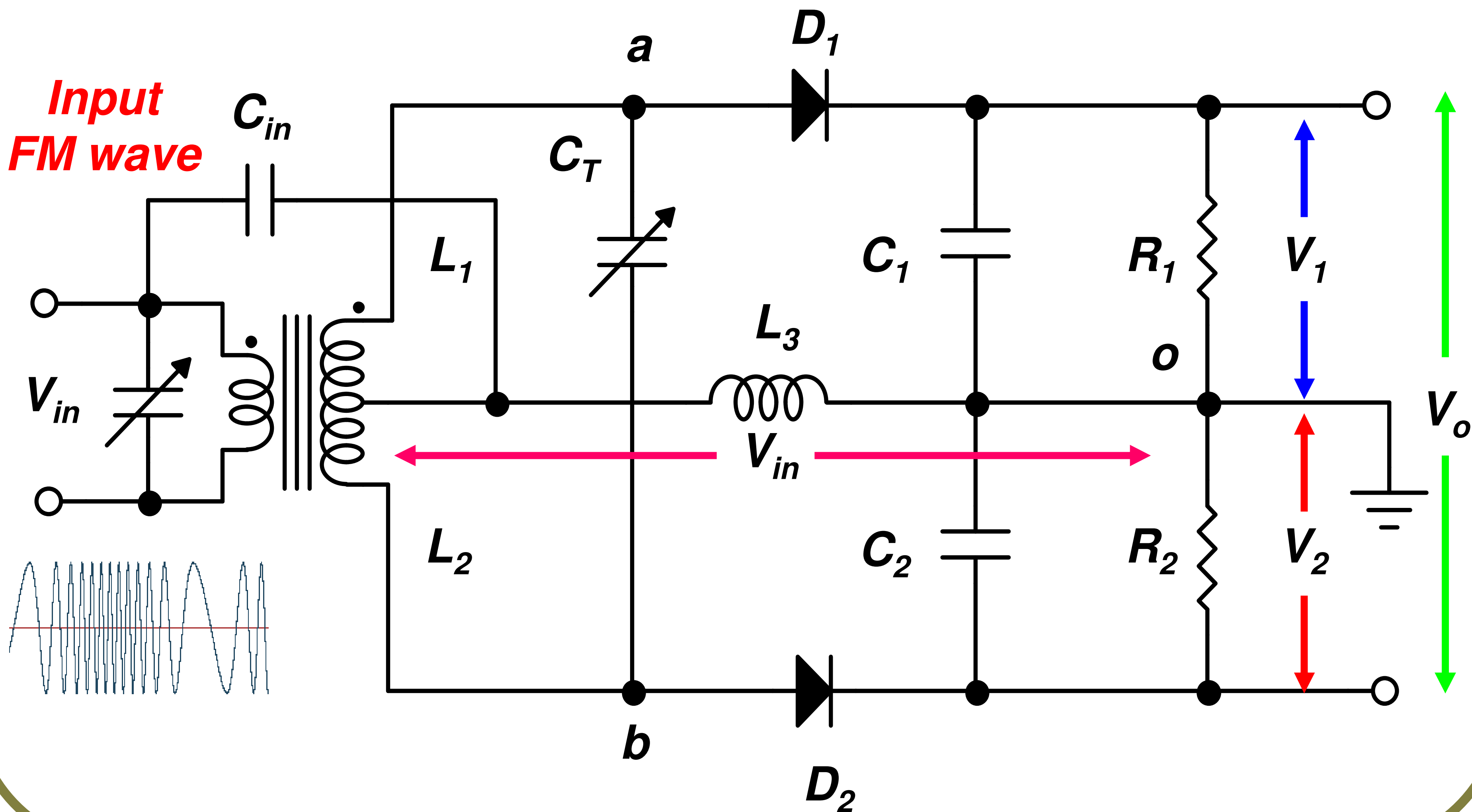


basic principle of operation similar to AM diode detector

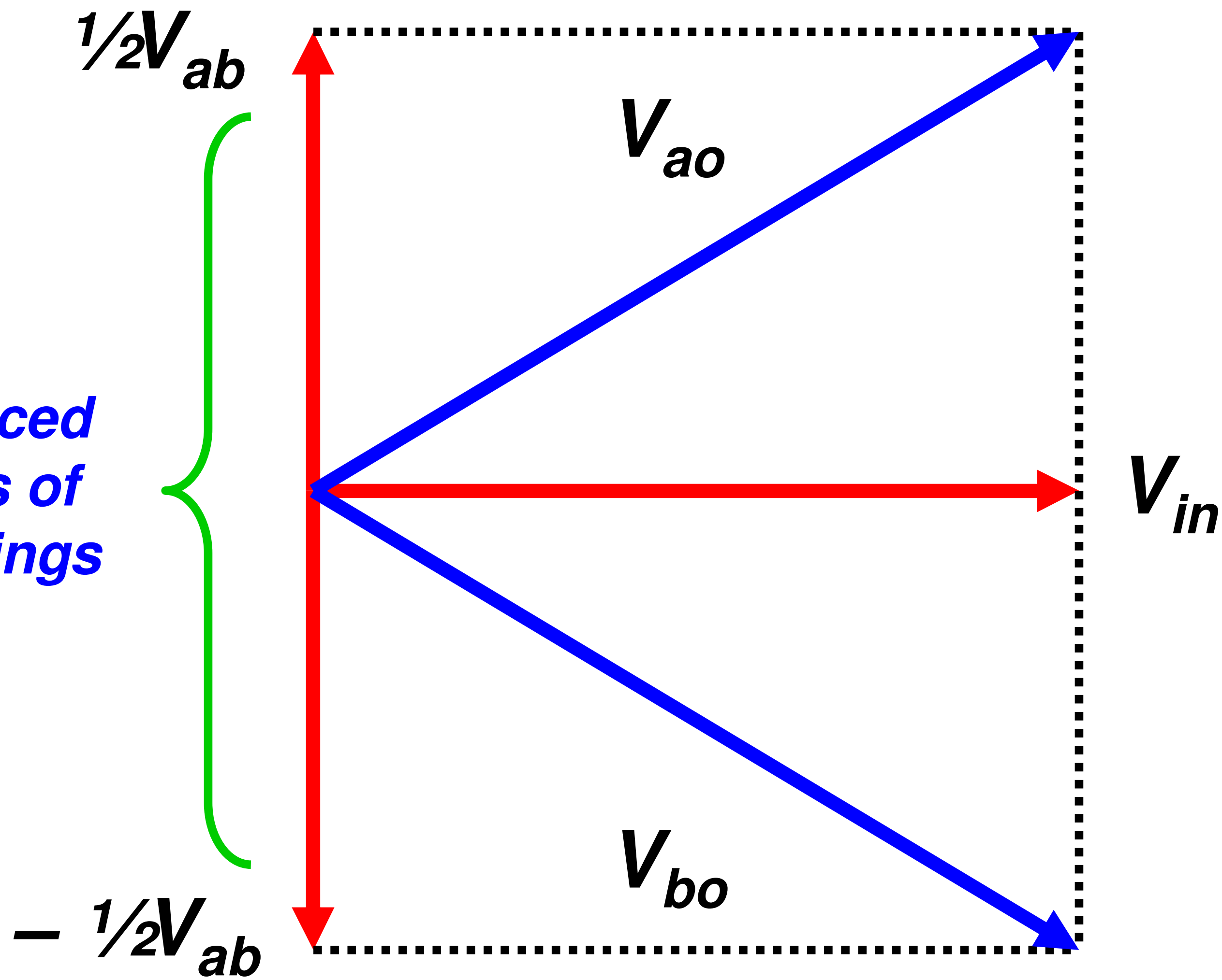


variation in the frequency of FM produces proportional variation in amplitude, applied to detector

3. Foster – Seeley Detector

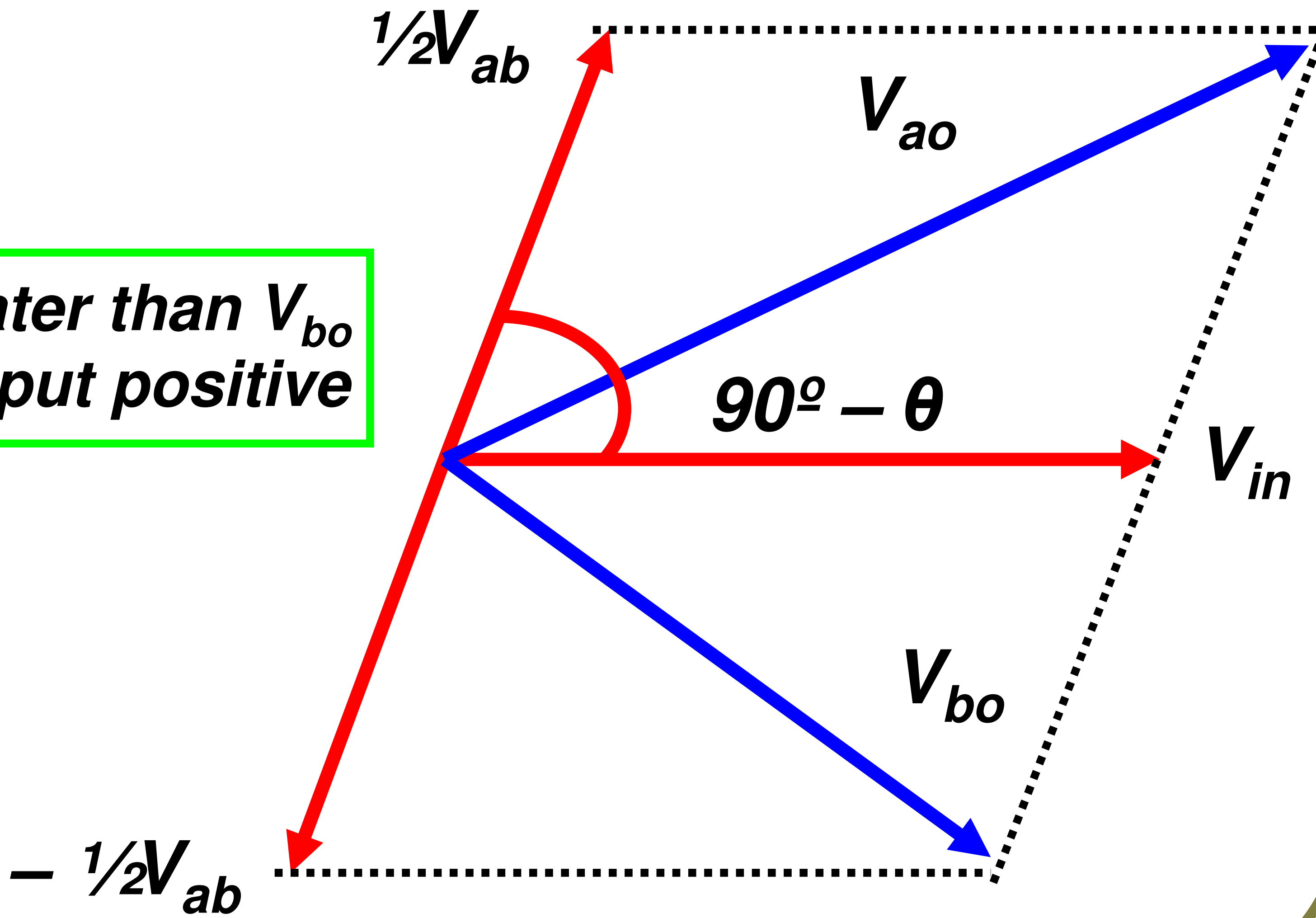


(a) When $f_{in} = f_c$



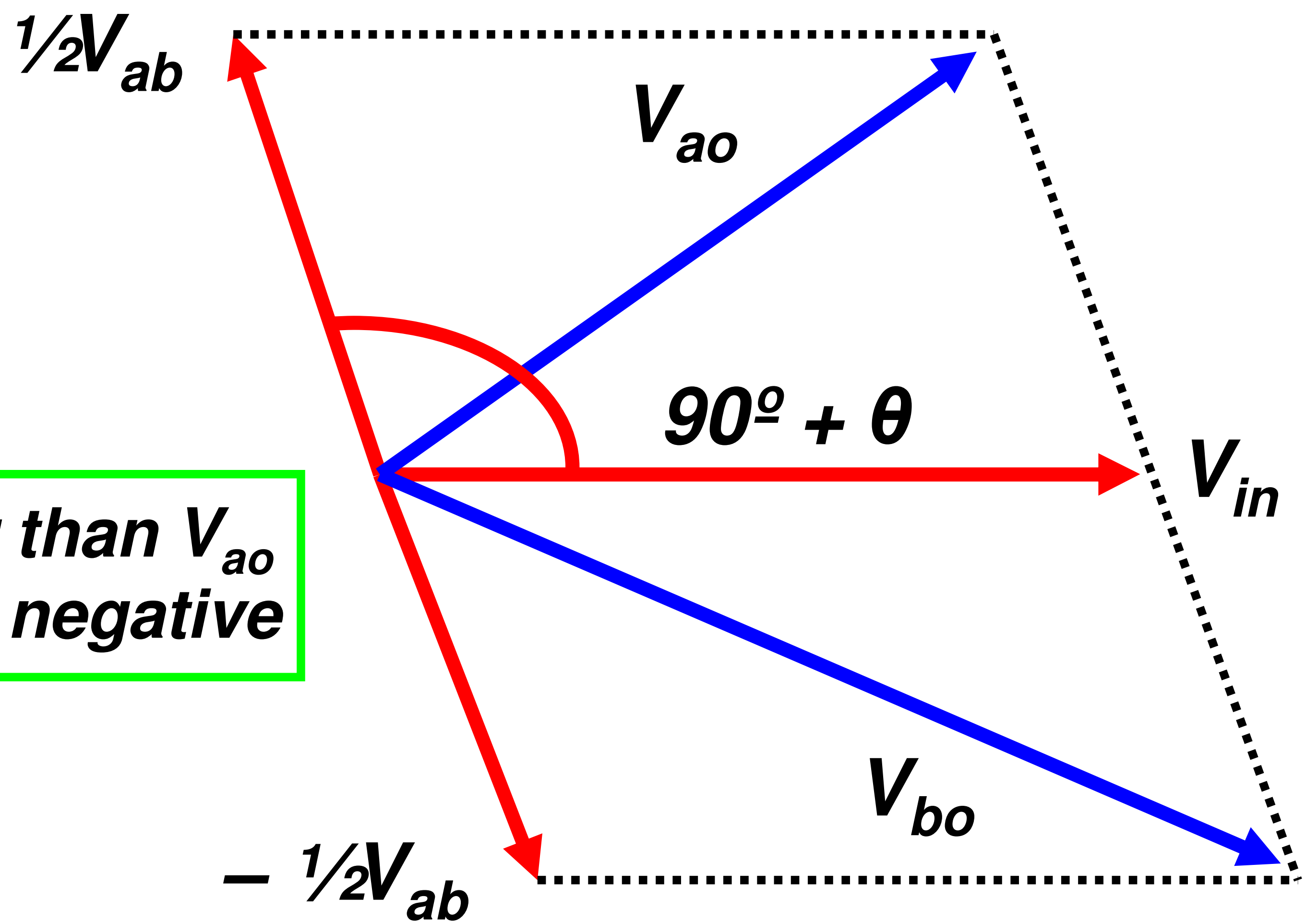
equal voltages induced in two (both) halves of the secondary windings hence $V_{ao} = V_{bo}$

(b) When $f_{in} > f_c$



V_{ao} is greater than V_{bo} hence output positive

(c) When $f_{in} < f_c$



V_{bo} is greater than V_{ao} hence output negative

3. Foster – Seeley Detector

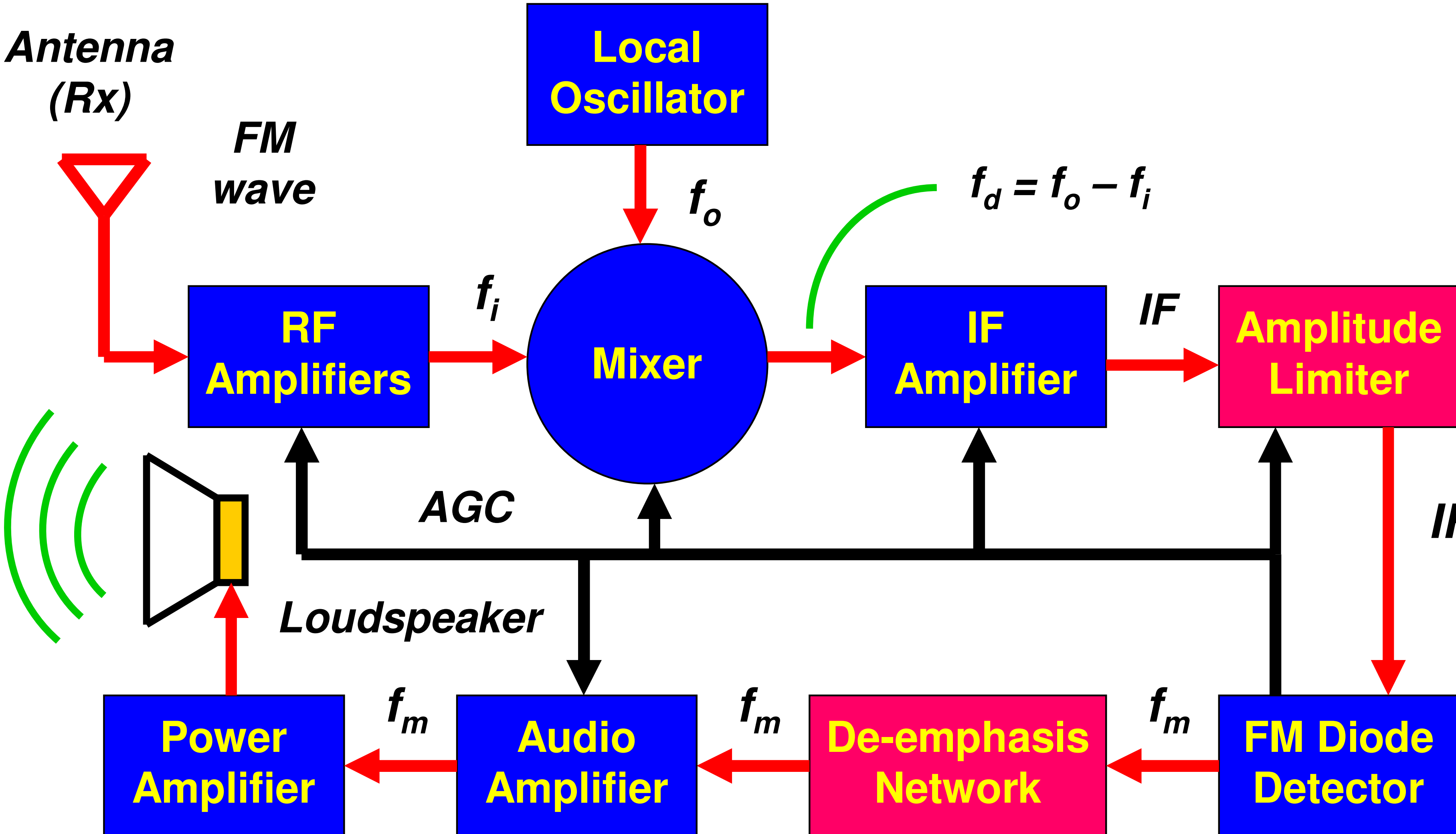
Advantages :-

1. Better linearity than balanced slope detector as circuit depends on a primary – secondary phase relationship, which is almost linear
2. More easy to align (tune) than the balanced slope detector, single frequency for tuning

Disadvantages :-

No amplitude limiting provided hence presence of noise causes errors in the output voltage

The FM Superhetrodyne Receiver



The FM Superhetrodyne Receiver

Block Diagram Description :-

- **Input RF amplifier stages, all tuned together used to select & amplify the input frequency**
- **FM diode detector used to demodulate FM wave to recover modulating signal $v_m(t)$**
- **Audio amplifier amplifies the modulating received signal (increases the amplitude)**
- **Power amplifier raises the power level to a sufficient stage to drive the loudspeakers**

The FM Superhetrodyne Receiver

Block Diagram Description :-

- RF amplifier stages designed for frequency selection between 88 MHz to 108 MHz
- Local oscillator tuning mechanically linked with RF amplifier from 98.7 MHz – 118.7 MHz
- Mixer produces a single constant frequency (IF) of 10.7 MHz over entire FM tuning range
- IF amplifier is narrow-band amplifier having high selectivity to select only IF frequency