TCS - Tutorial III

(1) Write regular expressions.

(a) $L = \{uvu \mid u, v = \{a, b\}^* \text{ and } |u| = 2\}$

ie $u = \{aa, ab, ba, bb\}$

Ans:- $aa(a+b)^*aa + ab(a+b)^*ab + ba(a+b)^*ba + bb(a+b)^*bb$

(b) even no. of zero's. $\Sigma = \{0,1\}$

$\rightarrow 1^*.(1^*01^*01^*)^*$

(c) Not containing two consecutive b's.

$\rightarrow (b+\epsilon)\cdot(a+ab)^*\cdot + (a+ba)^*(b+\epsilon)$

(d) Not containing two consecutive 0's and string ends with 1.

$\rightarrow (1+01)^*$

(e) do not contain two consecutive a's nor b's.

$\rightarrow (b+\epsilon)(ab)^*(a+\epsilon) + (a+\epsilon)(ba)^*(b+\epsilon)$

(f) string starts with abb and ends with bbb.

$\rightarrow abbb + abbbb + abb(a+b)^*bbb$

(g) Not containing three consecutive b's.

$\rightarrow (\epsilon+b+bb)(a+ab+abb)^*(\epsilon+a+aa)$

(h) Not containing three consecutive a's or b's.

$\rightarrow (\epsilon+b+bb)(ab+aab+abb+aabb)^*(\epsilon+a+aa)$

(i) strings of length $\neq 2$ $\Sigma = \{0,1\}$

$\rightarrow \epsilon+0+1+(0+1)(0+1)((0+1))^+$

(i) that have atmost three a's.

$\rightarrow (b+c)^* + (b+c)^* a (b+c)^* + (b+c)^* a (b+c)^* a (b+c)^*$
$+ (b+c)^* a (b+c)^* a (b+c)^* a (b+c)^*$

(K) each symbol at least once $\{a, b, c\}$

$\rightarrow (a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^* +$
$(a+b+c)^* a (a+b+c)^* c (a+b+c)^* b (a+b+c)^* +$
$(a+b+c)^* b (a+b+c)^* a (a+b+c)^* c (a+b+c)^* +$
$(a+b+c)^* b (a+b+c)^* c (a+b+c)^* a (a+b+c)^* +$
$(a+b+c)^* c (a+b+c)^* a (a+b+c)^* b (a+b+c)^* +$
$(a+b+c)^* c (a+b+c)^* b (a+b+c)^* a (a+b+c)^*$

(l) at most one occurance of two consecutive zeros.

$\rightarrow (1+01)^* (\epsilon+0) + (1+01)^* 0 (1+01)^*$

(m) Intersection of $(a+b)^* a$ and $b.(a+b)^*$

$\rightarrow b.(a+b)^* a$

(n) $L = \{a^n \mid n \text{ is divisible by } 2 \text{ or } 3 \text{ or } 5\}$

$(aa)^* + (aaa)^* + (aaaaa)^*$

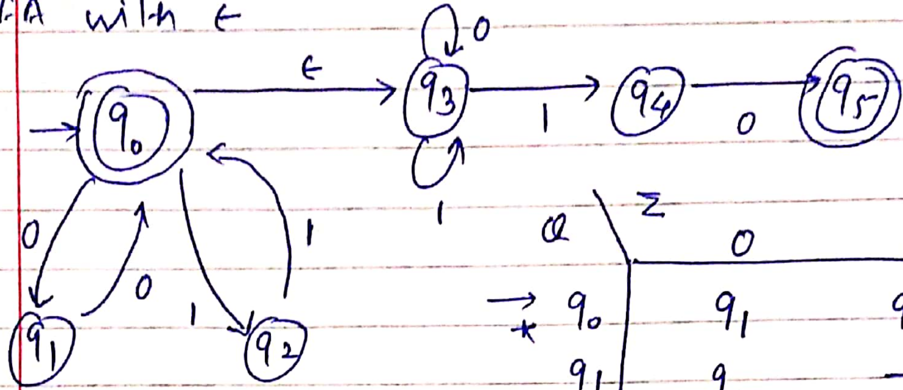
(o) $L = \{a^n b^m \mid m \geq 3,\ n \geq 4\}$

$\rightarrow (a)^* aaaa\ bbb\ (b)^*$

construct minimized DFA for

$R = (0+1)^* 10 + ((00)^* (11)^*)^*$

NFA with ϵ

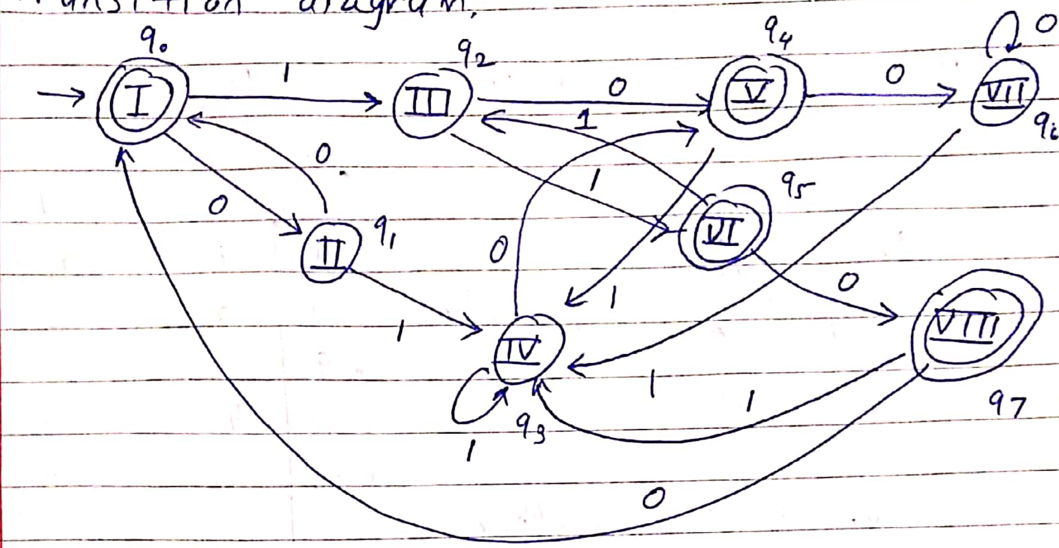


Q \ Σ	0	1
→ * q_0	q_1	q_2
q_1	q_0	—
q_2	—	q_0
q_3	q_3	$\{q_3 q_4\}$
q_4	q_5	—
* q_5	—	—

ϵ closure $(q_0) = \{q_0, q_3\}$
ϵ closure $(q_1) = \{q_1\}$
ϵ closure $(q_2) = \{q_2\}$
ϵ closure $(q_3) = \{q_3\}$
ϵ closure $(q_4) = \{q_4\}$
ϵ closure $(q_5) = \{q_5\}$

Q \ Σ	0	ϵ closure (0)	1	ϵ closure(1)
→ * $[q_0 q_3]$ I	$\{q_1 q_3\}$	$[q_0 q_3]$ II	$\{q_2 q_3 q_4\}$	$[q_2 q_3 q_4]$ III
$[q_1 q_3]$ II	$\{q_0 q_3\}$	$[q_0 q_3]$ I	$\{q_3 q_4\}$	$[q_3 q_4]$ IV
$[q_2 q_3 q_4]$ III	$\{q_3 q_5\}$	$[q_3 q_5]$ V	$\{q_0 q_3 q_4\}$	$[q_0 q_3 q_4]$ VI
$[q_3 q_4]$ IV	$\{q_3 q_5\}$	$[q_3 q_5]$ V	$\{q_3 q_4\}$	$[q_3 q_4]$ IV
* $[q_3 q_5]$ V	$\{q_3\}$	$[q_3]$ VII	$\{q_3 q_4\}$	$[q_3 q_4]$ IV
* $[q_0 q_3 q_4]$ VI	$\{q_1 q_3 q_5\}$	$[q_1 q_3 q_5]$ VIII	$\{q_2 q_3 q_4\}$	$[q_2 q_3 q_4]$ III
$[q_3]$ VII	$\{q_3\}$	$[q_3]$ VII	$\{q_3 q_4\}$	$[q_3 q_4]$ IV
* $[q_1 q_3 q_5]$ VIII	$\{q_0 q_3\}$	$[q_0 q_3]$ I	$\{q_3 q_4\}$	$[q_3 q_4]$ IV

Transition diagram.



q_1	X1						
q_2	X1	X1					
q_3	X1	X1	X1				
q_4	X1	X1	X1	X1			
q_5	X1	?X1	X1	X1	X1		
q_6	X1	X1	X1	X1	X1	X1	
q_7	?XII	X1	X1	X1	X1	X1	X1
	q_0	q_1	q_2	q_3	q_4	q_5	q_6

DFA is already minimized.

$(q_0\, q_7) = (q_1\, q_0)\,(q_2\, q_3)$ — 1

$(q_1\, q_2) = (q_0\, q_4)\,(q_3\, q_5)$

$(q_1\, q_3) = (q_0\, q_4)\quad(q_3\, q_3)$

$(q_1\, q_4) = (q_0\, q_6)\quad(q_3\, q_3)$

$(q_1\, q_5) = (q_0\, q_7)\quad(q_3\, q_2)$ — 2

$(q_1\, q_6) = (q_0\, q_6)\quad(q_3\, q_3)$

$(q_2\, q_3) = (q_4\, q_4)\quad(q_5\, q_3)$ — 3

$(q_2\, q_4) = (q_4\, q_6)\quad(q_5\, q_3)$

$(q_2\, q_6) = (q_4\, q_6)\quad(q_5\, q_3)$

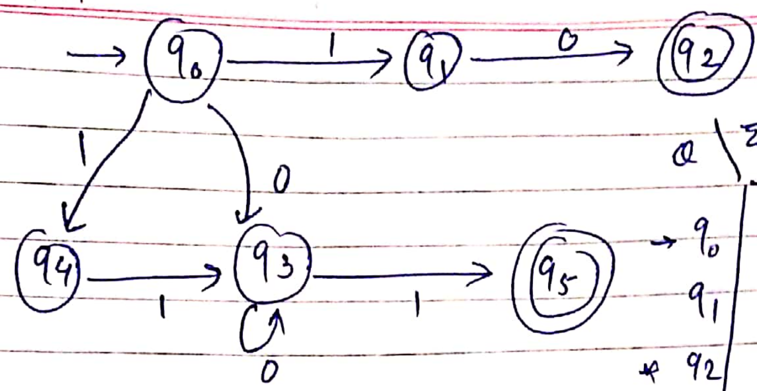
$(q_3\, q_6) = (q_4\, q_6)\quad(q_3\, q_3)$

$(q_4\, q_7) = (q_6\, q_0)\quad(q_3\, q_3)$

$(q_5\, q_7) = (q_7\, q_0)\quad(q_2\, q_3)$

③ $R = 10 + (0+11)\,0^*\,1$

NFA



Q \ Σ	0	1
→ q_0	q_3	$\{q_1, q_4\}$
q_1	q_2	—
* q_2	—	—
q_3	q_3	q_5
q_4	—	q_3
* q_5	—	—

subset conv. method.

DFA $M' = (Q', \Sigma', \delta', q_0', F')$

$q_0' = [q_0]$ I

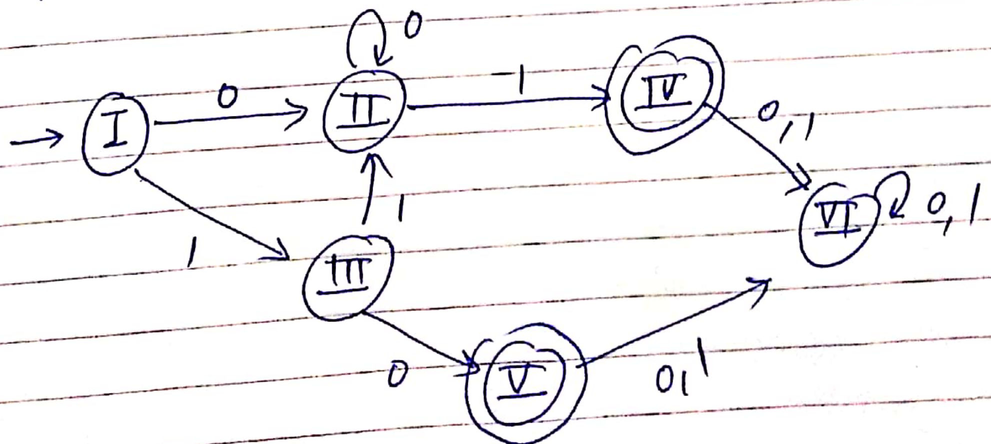
Q \ Σ	0	1
→ $[q_0]$ I	$[q_3]$ II	$[q_1, q_4]$ III
$[q_3]$ II	$[q_3]$ II	$[q_5]$ IV
$[q_1, q_4]$ III	$[q_2]$ V	$[q_3]$ II
* $[q_5]$ IV	[ ] VI	[ ] VI
* $[q_2]$ V	[ ] VI	[ ] VI
[ ] VI	[ ] VI	[ ] VI

verify

10

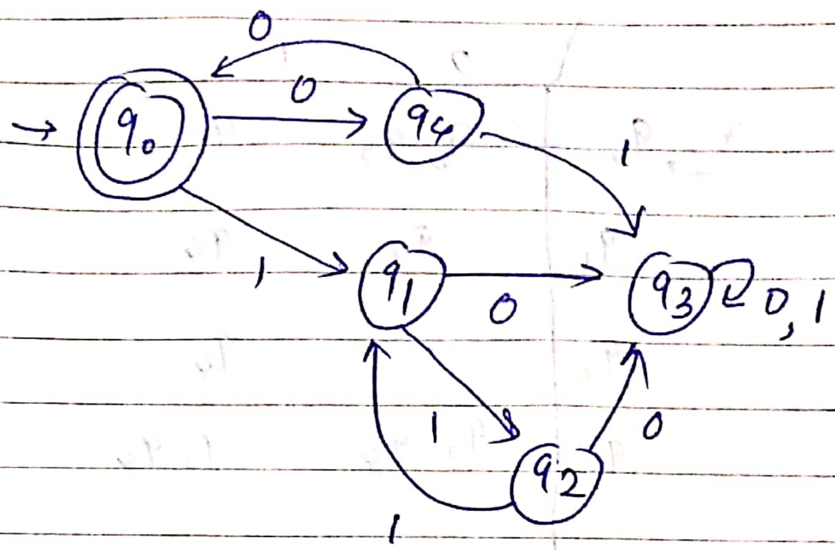
00001

110001



Describe lang. given by DFA

	0	1
$\rightarrow * q_0$	q_4	q_1
q_1	q_3	q_2
q_2	q_3	q_1
q_3	q_3	q_3
q_4	q_0	q_3



$q_0 = \epsilon + q_4 0$ — (1) $\quad q_3 = q_2 0 + q_4 1 + q_3 0 + q_3 1$

$q_1 = q_0 1 + q_2 1$ — (2) $\quad q_4 = q_0 0$ — (4) $\quad + q_1 0$ — (5)

$q_2 = q_1 1$ — (3)

Put eq^n 4 into eq^n 1

$$q_0 = \epsilon + q_0 0 0$$

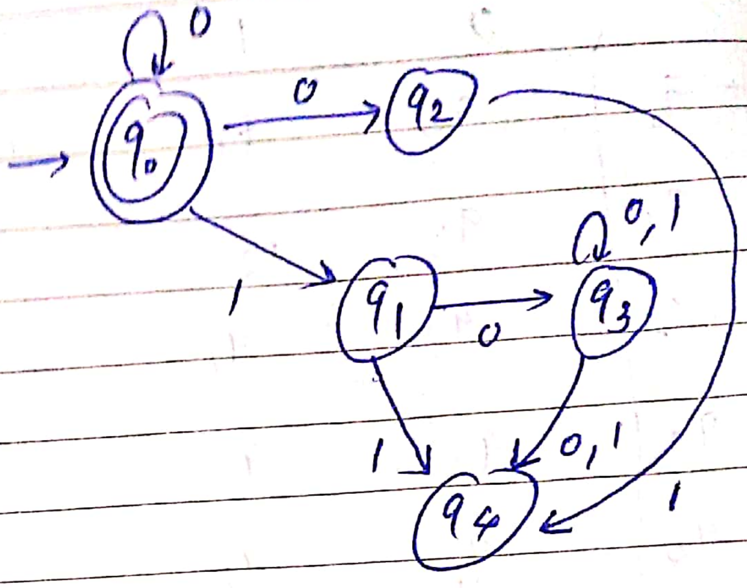
$$R = Q + R P$$

$$q_0 = \epsilon (00)^* = (00)^*$$

Scanned by CamScanner

(5) describe language given by FA

	0	1
$* \to q_0$	$q_0 q_2$	q_1
q_1	q_3	q_4
q_2	—	q_4
q_3	$q_3 q_4$	$q_3 q_4$
q_4	—	—



$$q_0 = \epsilon + q_0 0$$

$$q_1 = q_0 1$$

$$q_2 = q_0 0$$

$$q_3 = q_1 0 + q_3 0 + q_3 1$$

$$q_4 = q_1 1 + q_3 0 + q_3 1 + q_2 1$$

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\,\underbrace{0}_{P}$$

$$q_0 = \epsilon (0)^* = 0^*$$