

CFG

Grammar - set of formal rules generating syntactically correct sentence.

$$G = (V, T, P, S)$$

V = set of variables
 $A \dots Z$

CFG = TYPE 2

T = set of terminals

\downarrow
lang \rightarrow CFL
 \downarrow of

$a, b \dots$ or $1, 2 \dots$

PDA

P = set of production rules

S = start symbol.

rules are of type

$$A \rightarrow \alpha$$

\downarrow

(left side

= single var)

$$\rightarrow (VUT)^*$$

*

$L = \{0^n 1^n \mid n \geq 0\}$ is not a regular lang.

Assume $0^n 1^n$ is a regular lang.

Then it should follow pumping lemma.

$$\rightarrow (9_0) \sim 0^3 1^3 \sim (9_f)$$

$u \quad (v)^i \quad w$

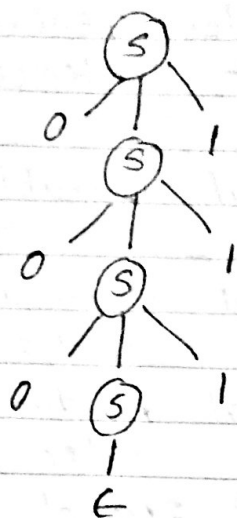
$$0^2 (0)^1 1^3$$

can be repeated zero/1/more than 1 times.
if zero

resulting string = $0^2 1^3$ which is not a part of L . so contradiction.

so $L = \{0^n 1^n\}$ is not a regular lang. $\{$

$$L = \underline{0^3 1^3} \quad \xrightarrow[\text{G}]{\text{find}} \quad G = (V, T, P, S)$$



$$V = \{S\} \quad S = S$$

$$T = \{0, 1\}$$

$$P: \begin{cases} S \rightarrow 0S1 \\ S \rightarrow \epsilon \end{cases}$$

① Identify language

$$S \rightarrow aB / bA$$

$$A \rightarrow a / aS / bAA$$

$$B \rightarrow b / bS / aBB$$

$$G = (\{S, A, B\}, \{a, b\}, P, S)$$

Identify words.

$$\begin{array}{lll} \textcircled{1} & S \rightarrow aB & S \rightarrow aB & S \rightarrow aB \\ & \rightarrow ab & \rightarrow abS & \rightarrow aBB \\ & & \rightarrow abab & \rightarrow aabB \\ & & \rightarrow abab & \rightarrow aabb \end{array}$$

$$\begin{array}{lll} \textcircled{2} & S \rightarrow bA & S \rightarrow bA & S \rightarrow bA \\ & \rightarrow ba & \rightarrow baS & \rightarrow bAA \\ & & \rightarrow baab & \rightarrow bbaA \\ & & \rightarrow baab & \rightarrow bbaa \end{array}$$

$$\begin{array}{l} \textcircled{3} \\ S \rightarrow aB \\ \rightarrow abS \\ \rightarrow abbaA \\ \rightarrow abbaS \\ \rightarrow abbaaB \\ \rightarrow abbaaBB \\ \rightarrow abbaaabb \end{array}$$

$L =$ set of words where number of a's and number of b's are same.

2) Q2 design a context free grammar for a string

$$L = \{a^{2n}b^m; n \geq 0, m \geq 0\}$$

$$G = (V, T, P, S)$$

$$P: \rightarrow S \rightarrow \epsilon \mid AB$$

$$A \rightarrow aaA \mid \epsilon$$

$$B \rightarrow bB \mid \epsilon$$

Grammar $G = (V, T, P, S)$

$$V = \{S, A, B\} \quad T = \{a, b\} \quad S = S.$$

words
 $\epsilon, aab,$
 $aaaaabbb \dots$
 $\underbrace{(aa)^n}_A \underbrace{b^m}_B$

Q3 Give CFG for $(baa + abb)^+$

$$S \rightarrow AS \mid BS \mid \epsilon$$

$$A \rightarrow baa$$

$$B \rightarrow abb.$$

$$V = \{S, A, B\}$$

$$T = \{a, b\}$$

$\underbrace{baa}_A \quad \underbrace{abb}_B$

Q4 Give CFG for equal number of a's & b's.

$$S \rightarrow aSbS$$

$$S \rightarrow bSaS$$

$$S \rightarrow \Lambda.$$

Q
③

for a given CFG find out leftmost derivation and rightmost derivation, and parse tree. (00110101)

$G = (V, T, P, S)$ $V = \{S, A, B\}$ $T = \{0, 1\}$

$S \rightarrow 0B \mid 1A$

$A \rightarrow 0 \mid 0S \mid 1AA$

$B \rightarrow 1 \mid 1S \mid 0BB$

Leftmost derivation
(LMD) 00110101

$S \rightarrow 0B$

$S \rightarrow 00BB$

$S \rightarrow 001B$

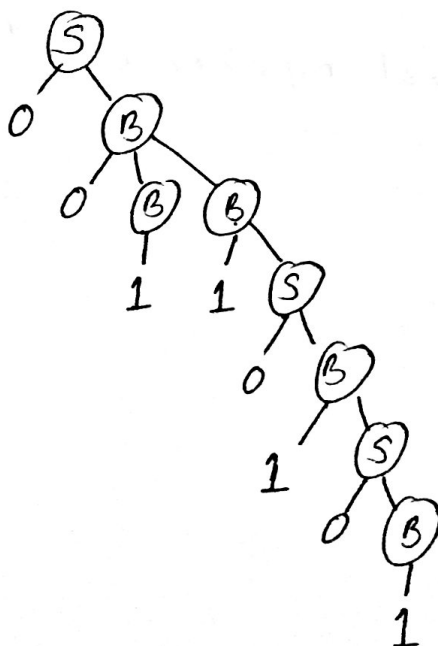
$S \rightarrow 0011S$

$S \rightarrow 00110B$

$S \rightarrow 001101S$

$S \rightarrow 0011010B$

$S \rightarrow 00110101$



Rightmost derivation
(RMD) 00110101

$S \rightarrow 0B$

$S \rightarrow 00BB$

$S \rightarrow 00B1$

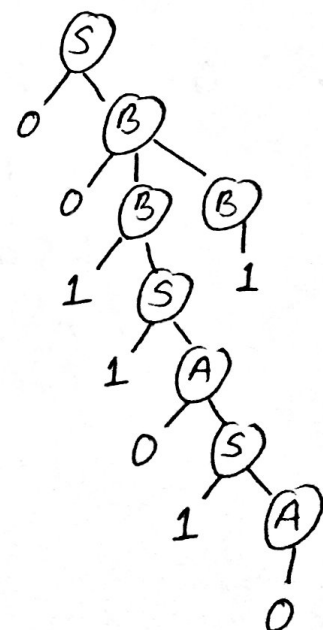
$S \rightarrow 001S1$

$S \rightarrow 0011AS$

$S \rightarrow 00110B1$

$S \rightarrow 001101A1$

$S \rightarrow 00110101$



Removal of Ambiguity

G: $E \rightarrow E + E \mid E * E \mid (E) \mid I$
 $I \rightarrow a \mid b$

① show that G is ambiguous.

② Remove ambiguity.

Solution -

Step 1 - Identify a string belongs to lang. of this grammar [\neq Identify language]

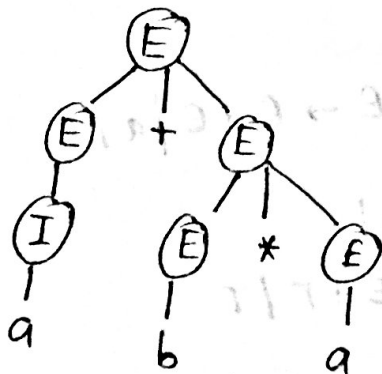
$a + b * a$ is a string belongs to this G. [\neq not in this Problem]

Step 2 - Try to find 2 left-most derivations i.e. different parse trees for the string.

~~$a * b * a$~~

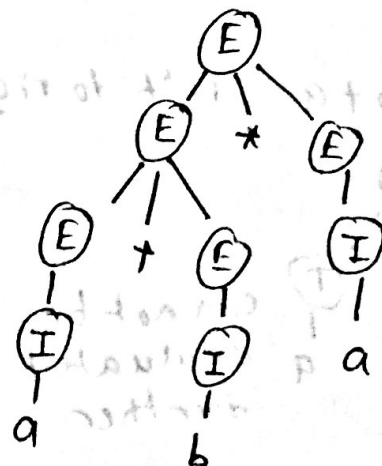
LMD1 \rightarrow

$E \rightarrow E + E$
 $E \rightarrow I + E$
 $E \rightarrow a * E$
 $\rightarrow a + E * E$
 $\rightarrow a + I * E$
 $\rightarrow a + b * E$
 $\rightarrow a + b * I$
 $\rightarrow a + b * a$



LMD2 \rightarrow

$E \rightarrow E * E$
 $E \rightarrow E + E * E$
 $\rightarrow I + E * E$
 $\rightarrow a + E * E$
 $\rightarrow a + I * E$
 $\rightarrow a + b * E$
 $\rightarrow a + b * I$
 $\rightarrow a + b * a$



There are 2 different parse trees for a sentence $a + b * a$ for left most derivation.

So the grammar is ambiguous.
 For removing ambiguity for the lang.
 that has precedence of $*$ operator higher
 than that of $+$ operator, we introduce
 new variable T (term) that should be
 evaluated first. F (factor) for $()$

$$E \rightarrow E + T \mid T$$

$$I \rightarrow a \mid b \mid c$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid a \mid b \mid I$$

LMD $a + b * a$

$$E \rightarrow E + T$$

$$E \rightarrow T + T$$

$$\rightarrow F + T$$

$$\rightarrow I + T$$

$$\rightarrow a + T$$

$$\rightarrow a + T * F$$

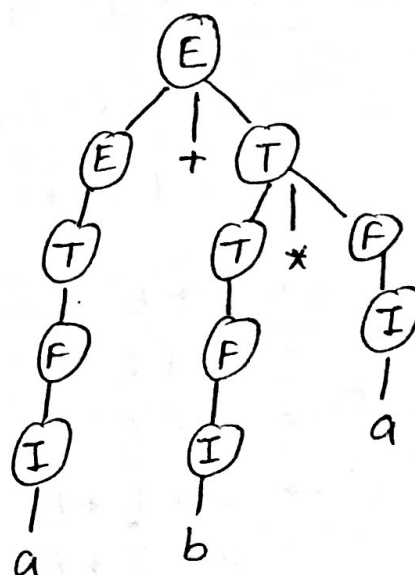
$$\rightarrow a + F * F$$

$$\rightarrow a + I * F$$

$$\rightarrow a + b * F$$

$$\rightarrow a + b * I$$

$$\boxed{E \rightarrow a + b * a}$$



$a + b + a$ left to right

$$E \rightarrow E + E \mid a \mid b$$

$$T \rightarrow a \mid b$$

$$E \rightarrow E + T \mid T$$

