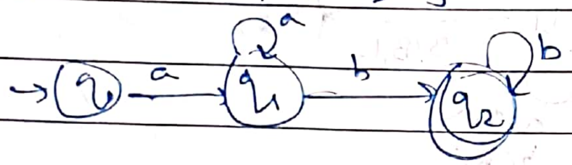
if not happening then lang is RCFL

Pumping lemma of Regular sets

Pumping lemma of states that for a sufficiently long string accepted by FSM, we can find a substring near the beginning of that string, that may be repeated or pumped as many times as you like & still the resulting string is accepted by FSM.

Q. $L = \{a^n b^m \mid n, m \geq 1\}$



① Assume that given lang is RL.

② Given lang will follow pumping lemma.

③ FA, $M = \{Q, \Sigma, \delta, q_0, F\}$

n min no. of states needed to start $q_0 - q_f$

m symbols string accepted by m/c

$\boxed{m \leq n}$

④ $\delta(q_0, q_1) = q_1$ $\delta(q_0, q_1, q_2) = q_2$

$\delta(q_0, q_1 \ldots q_5) = q_5$

⑤ ∃ j & k such that $j < k$ & $q_j = q_k$

Context ~~ary~~ free language & context free grammar

CF language is for PD automata

Grammar means the ~~state~~ set of formal rules generating syntactically correct sentences.

If $CFG = (V, T, P, S)$

V = set of variables

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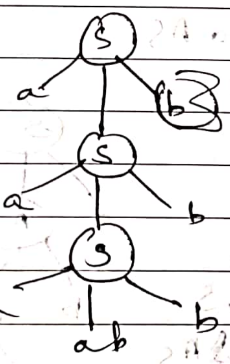
Terminal
T = set of transition symbols
P = set of prod rules
S = starting symbol

Rules are of type $A \rightarrow$ ~~Alpha~~ α
where $\alpha = (V \cup T)^*$

CFG is type 2 grammar

$G = L\{a^n b^n \mid n \geq 1\}$



$V = \{S\}$
$T = \{a, b\}$
$P = \{S \rightarrow aSb,$
$S \rightarrow ab\}$
$S = S$

~~RG~~ RL can be CFL
but CFL cannot be RL

Q. Design the ~~gr.~~ CFG for language $L = \{a^m b^n \mid m, n \geq 0\}$

→

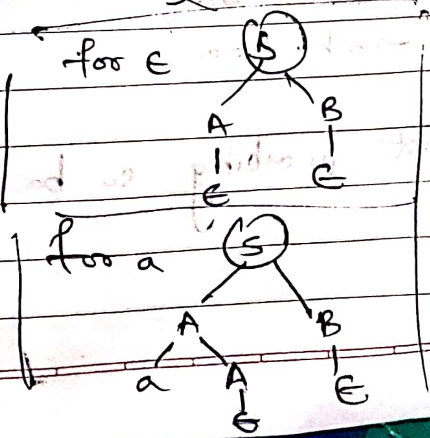
$G = L\{a^m b^n \mid m, n \geq 0\}$

$T = \{$~~a b~~ $a, b\}$ $\quad \epsilon, a, b, ab, aab, \underset{A\ B}{abb}$

$V = \{S, A, B\}$

$P = \{S \rightarrow AB$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid \epsilon\}$
$S = S$

~~S → AB~~



Q. ~~$a^n 2^n$~~ $L = \{a^n b^{2n+1} \mid n \geq 1\}$

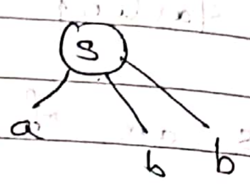
$= \underbrace{a^n (b b)^n}_{X} . b$

$\{abbb, aabbbbb \ldots$

$S \rightarrow Xb$

$X \rightarrow aXbb$

$X \rightarrow abb$



~~CG~~ CFG

derivation ~~abaabaa~~

LMD RMD

$S \rightarrow a \mid aAS$

$A \rightarrow bS$

LMD

$S \rightarrow aAS$

$S \rightarrow abSS$

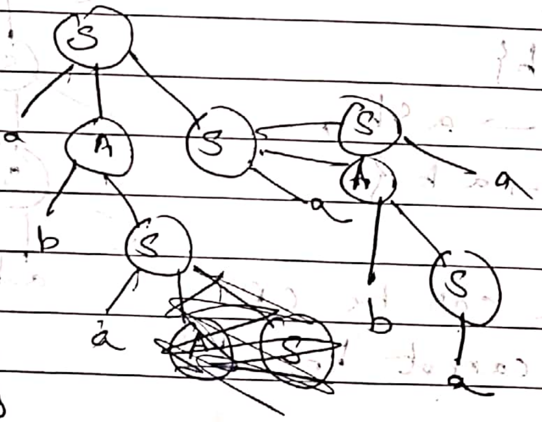
$S \rightarrow abaS$

$S \rightarrow abaaAS$

$S \rightarrow abaabSS$

$S \rightarrow abaabaS$

$S \rightarrow abaabaa$



~~abaabaa~~

RMD

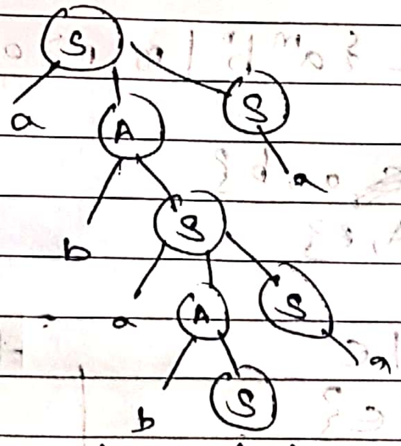
$S \rightarrow aAS$

$S \rightarrow aAa$

$S \rightarrow abSAa$

$S \rightarrow abaASa$

$S \rightarrow abaAaa$



Not working so backtracked

RMD

$S \rightarrow aAS$

$S \rightarrow aAaAS$

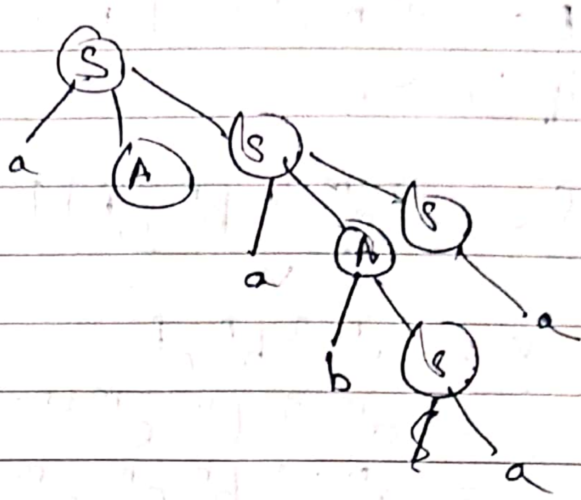
$S \rightarrow aAaAa$

$S \rightarrow aAabSa$

$S \rightarrow aAabaa$

$S \rightarrow abSabaa$

$S \rightarrow abaabaa$



If ambiguous, can generate 2 trees
" unambiguous cannot " " "

→ Type 2 language
PDM
CFL

Q. RE = $(ab + ba)^*$

$S \rightarrow \epsilon | abS | baS$

$G = (V, T, P, S)$

$S \rightarrow \epsilon$

$\rightarrow XS | YS$

$X \rightarrow abX | ab$

$Y \rightarrow baY | ba$

$\{\underbrace{a^n b^n}_{A} \underbrace{c^i}_{C} \mid i \geq 1, n \geq 1\}$

$S \rightarrow AC$

$A \rightarrow aAb | ab$

$C \rightarrow cC | c$

Q. $\{a^n b a^m \mid n, m \geq 1\}$

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$L = \{a^m b^n c^p d^q \mid m+n = p+q\}$

if $m > q$

$S = a^q \underbrace{a^{m-q} b^n c^p}_{X} d^q$ $\qquad S \rightarrow aSd \mid X$

$X = a^{m-q} b^n c^p$

$= a^{m-q} b^n c^{m+n-q}$

$= a^{m-q} \underbrace{b^n c^n}_{Y} c^{m-q}$ $\qquad X \rightarrow aXc \mid Y$

$Y = \cancel{a} b^n c^n$

$Y \rightarrow bYc \mid \epsilon$

if $m < q$

$S = a^m b^n c^p d^q$

$= a^m \underbrace{b^n c^p d^{q-m}}_{M} d^{m}$ $\qquad S \rightarrow aSd \mid M$

$M = b^n c^p d^{q-m}$

$= b^{p+q-m} c^p d^{q-m}$

$= b^{q-m} \underbrace{b^p c^p}_{Y} d^{q-m}$ $\qquad M \rightarrow bMd \mid Y$

if $m = q$

$S = a^m b^n c^p d^q$

$= a^q \underbrace{b^n c^n}_{Y} d^q$ $\qquad S \rightarrow aSD \mid Y$

$Y \rightarrow bYc \mid \epsilon$

Q. $L = \{a^n b^m c^k \mid n = m \text{ or } m \leq k\}$

if $n = m$ $\qquad S \rightarrow XY$

$S = \underbrace{a^n b^m}_{X} \underbrace{c^k}_{Y}$ $\qquad X \rightarrow aXb \mid ab$

$Y \rightarrow cY \mid c$

$= \underbrace{a^n b^n}_{X} \underbrace{c^k}_{Y}$

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every word (string) will have its own parse tree.

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or $\boxed{m \leq k}$

$S = a^n b^m c^k$

$= \underbrace{a^n}_{A} \underbrace{b^m c^m}_{B} \underbrace{c^{k-m}}_{C}$ $S \to ABC$

$A \to aA \mid a$

$B \to bBc \mid bc$

$C \to cC \mid c$

~~$M \to aM \mid$~~

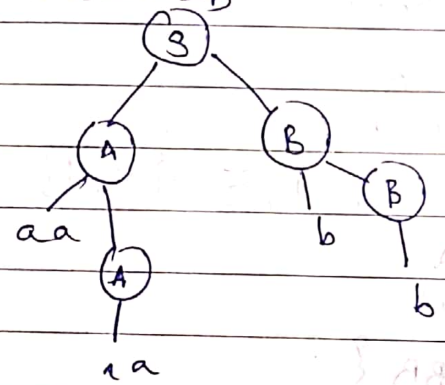
or $m = k$ $S = a^n \underbrace{b^m c^m}_{B}$
A B

$\therefore$ $S \to XY \mid ABC \mid AB$

Q $L = \{a^{2n} b^m \mid n \geq 1, m \geq \_\}$

→ $\{aab, aaaab, aaaabb, \ldots\}$

aaaabb



$S \to AB$

$A \to aaA$

$A \to aa$

$B \to bB$

$B \to b$ (ϵ if $n \geq 0$)

Q. $G = (VTPS)$

P: $S \to a \mid aAb \mid abSb$

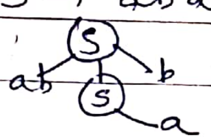
$A \to aAAb \mid bS$

To identify whether ambi or unambi.

→ LMD (Take a random pattern) → abab

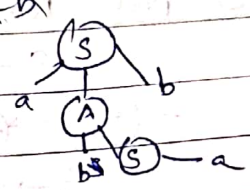
$S \to abSb$

$S \to abab$ $S \to aAb$



$S \to abSb$

$S \to abab$



Pattern generated using 2 diff. parse trees

$\therefore$ ambiguous

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Q. $S \to 0S1S \mid 1S0S \mid \epsilon$

~~0101~~

LMD

~~$S \to 0S$~~

~~$\to 01S0S$~~

~~$\to 01\epsilon$~~

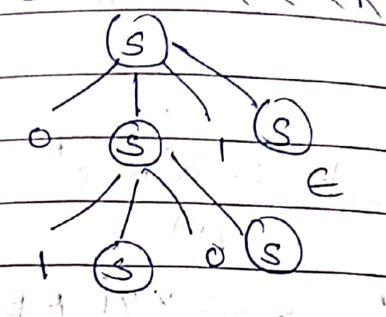
$S \to 0S1S$

$\to 0\epsilon 10S1S$

$\quad\quad \epsilon \; \epsilon$

RMD

$S \Rightarrow$ ~~0101~~



∴ Ambiguous

Simplify G

Step 3 Remove useless symbols

Q for G (V,T,P,S) remove

where $V = \{S, A, B\}$, $T, \Sigma = \{0, 1\}$

$P : \{S \to A \mid 11B \mid 11A$

$S \to B$, $S \to 11$

$A \to 0$, $B \to BB\}$

① Non generating symbols

$\{0, 1\}$

$A \to 0$

$S \to 11$

$\{0, 1, A, S\}$

No symbols generating $\{0, 1, A, S\}$

So remove the rest i.e: B.

② Non reachable symbol

$S \to 11A \quad S \to 11 \quad A \to 0$

→ S → A both are reachable

$S \to 11A$

∴ Ans: $\boxed{\begin{array}{l} S \to 11A \\ S \to 11 \\ \cancel{A \to 0} \end{array}}$

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Q. Remove useless. $S \to aC \mid \cancel{S}B$
$A \to aSCa$
$B \to aSB \mid bBC$
$C \to a\cancel{B}C \mid ad$

→ Non gen. symbols:
Assume G symbols all $\{a, b, d\}$
$\downarrow C \to ad$
$\{a, b, d, C\}$
$\downarrow S \to aC$
$\{a, b, d, C, S\}$
$\downarrow A \to aSCa$

Set of G symbols $= \{a, b, d, C, S, A\}$

∴ B is removed.

→ ② Non reachable

$S \xrightarrow{S \to aC} C$

A is not reachable, so removed.

∴ Answer $S \to aC$
$C \to ad$

Q. Remove ~~useless~~ symbols ~~from CFG~~: $S \to aC \mid SB$
$A \to aSCa$
$B \to aSB \mid bBC$
$C \to$

Q. ~~$S \to aC \mid SP$~~ $S \to aAa$
$A \to Sb \mid bCC \mid D\cancel{a}A$
$C \to abb \mid \cancel{D}D$
$\cancel{E} \to aC$
$\cancel{D \to} aDa$

① Non gen:
gen symbols, $\{a, b\}$
Remove D
$\downarrow C \to abb$
$\{a, b, C\}$
$\downarrow E \to aC, A \to bCC$
$\{a, b, C, E, A\}$
$\downarrow S \to aAa$
$\{a, b, C, E, A, S\}$

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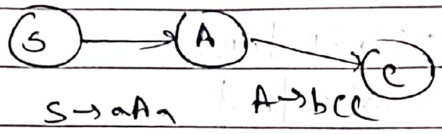
(c) Non reachable.



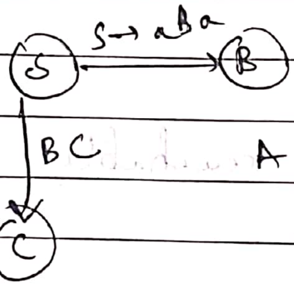
Final : S → aAa
A → Sb | bCC
C → abb

~~Remove E~~

Q. S → aBa | BC
~~A~~ → aC | BCC
C → a
B → bCC
~~D~~ → E
~~E~~ → d

① Non gen.
generating sym. – {a, b, d}
↓ E → d, C → a
{a, b, d, E, C}
↓ B → bCC, D → E
A → aC
{a, b, d, A, B, C, D, E}
↓ S → aBa | BC
{a, b, d, A, B, C, D, E, S}

② Non reachable



A, D, E are removed

Step 1 Remove ϵ productions.
S → ABA
A → aA | ϵ
B → bB | ϵ

A → ϵ | B → ϵ

(1) B → bB | b
(2) ~~A →~~ S → ABA | AA
(3) A → aA | a
(4) S → ABA | AA | B | AB | BA | A

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Q. $S \to AB \mid \epsilon$
$A \to aASb \mid a$
$B \to bS$

$S \to AB$ $\quad S \to \epsilon$
$A \to aASb \mid a \mid aAb$
$B \to bS \mid b$

Q. $S \to AB \mid ABC$
$A \to BA \mid BC \mid a \mid \epsilon$
$B \to AC \mid CB \mid b \mid \epsilon$
$C \to A \mid C \mid BC \mid AB$

$A \to \epsilon \qquad B \to \epsilon$

$S \to AB \mid ABC \mid B \mid A \mid C \mid \cancel{AB} \mid BC \mid AC$ $\quad S \to \epsilon \quad C \to \epsilon$
$A \to BA \mid BC \mid a \mid A \mid B \mid C$
$B \to AC \mid CB \mid b \mid A \mid B \mid C$
$C \to A \mid C \mid BC \mid AB \mid B$

Q. $S \to XYX$
$X \to 0X \mid \epsilon$ $\quad X \to \epsilon \quad Y \to \epsilon$
$Y \to 1Y \mid \epsilon$

$Y \to 1Y \mid 1$

$S \to XYX \mid XX \mid \cancel{X}$

$X \to 0X \mid 0$

$S \to \cancel{XYX} \mid XX \mid X \mid Y$

② Remove Unit productions

$S \to AB \mid A$

$A \to ab \mid cd$

then $S \to AB \mid ab \mid cd$

Q. $S \to aS \mid A \mid D$

$A \to a$

$B \to aa$

$D \to aDb$

① remove ϵ product

② remove unit prod

$S \to aS \mid a \mid aDb$ $\quad S \to A \quad S \to D$

$\boxed{A \to a}$

③ Remove useless

$\{a, b\}$ $\quad S \to aS \mid a \mid aDb$

$\downarrow B \to aa, S \to a, A \to a$ $\quad B \to aa \quad A \to a$

$\{a, b, B, A, S\}$ $\quad D \to aDb$

Remove D

$S \to S \to$

Remove A & B

Ans: $\boxed{S \to aS \mid a}$

Q. $S \to AB \mid aB$

$A \to BC \mid B \mid a$

$B \to C$

$C \to b \mid \epsilon$

① remove ϵ prod.

$C \to b$ $\quad C \to \epsilon \quad B \to \epsilon \quad A \to \epsilon$

$B \to \epsilon$ $\quad S \to \epsilon$

$A \to BC \mid B \mid a \mid C$

$S \to AB \mid aB \mid A \mid a \mid B$

(3) Remove unit prod

$S \rightarrow AB \mid aB \mid A \mid a \mid B$

$A \rightarrow BC \mid B \mid a \mid c$

$B \rightarrow C\cancel{b}$

$C \rightarrow b$

$S \rightarrow A \quad S \rightarrow B \quad A \rightarrow B \quad A \rightarrow C \quad B \rightarrow C \quad \cancel{S \rightarrow}$

$B \rightarrow \cancel{Cb} \; \cancel{C} \mid b$

$A \rightarrow \cancel{B}\, BC \mid \cancel{C} \mid b \mid a \mid c \mid \cancel{B}$

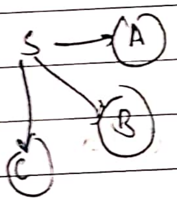
$S \rightarrow AB \mid aB \mid \cancel{A} \mid a \mid \cancel{C} \mid b \mid BC \mid \cancel{B}$

$\cancel{S \rightarrow AB}$

(5) G symbols $\{a, b\}$

↓ $S \rightarrow a,\ A \rightarrow a,\ B \rightarrow b,$
$C \rightarrow b$

$\{a, b, S, A, B, C\}$



$S \rightarrow AB \mid aB \mid a \mid b \mid BC$

$A \rightarrow BC \mid a \mid b$

$B \rightarrow b$

$C \rightarrow b$

unit prod

$S \rightarrow AB \mid aB \mid A \mid a \mid B$

$A \rightarrow BC \mid B \mid a \mid c$

$B \rightarrow C$

$C \rightarrow b$

$\boxed{S \rightarrow A} \; \boxed{S \rightarrow B} \; \boxed{A \rightarrow B} \; \boxed{A \rightarrow C} \; \boxed{B \rightarrow C}$

$B \rightarrow b$

$A \rightarrow BC \mid B \mid a \mid b$

$A \rightarrow BC \mid \cancel{B}b \mid a \mid \cancel{b}$

$S \rightarrow AB \mid aB \mid A \mid a \mid b$

$S \rightarrow AB \mid aB \mid BC \mid B \mid a \mid b \mid$