

Brain storming problems.

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section 35

① uniform distribution:-

Problem 1:-

Consider a parking lot with 100 spaces. arrive at random times and park in a random available space. model the number of empty spaces in the parking lot as a uniform distribution. Discuss the implications of this model and calculate the probability of having fewer than 20 empty spaces.

Sol:- Given

total no. of spaces in the parking lot = 100

Probability density function for a uniform

distribution is $f(x) = \frac{1}{b-a}$

$$a = 0$$

$$b = 100$$

$$f(x) = \frac{1}{100-0} = \frac{1}{100}$$

$$P(x < 20) = \int_0^{20} \frac{1}{100} dx$$

$$= \frac{1}{100} (20-0)$$

$$= \frac{20}{100} = 0.2$$

Problem 2

Sol:- Given that the ages of respondents between 20 & 50 years are uniformly distributed.

$$\text{mean age} \Rightarrow \frac{\text{min age} + \text{max age}}{2} = \frac{20+50}{2} = 35.$$

$$\text{median} = \frac{20+50}{2} = 35$$

→ 20 years

50 years.

$$\text{Range} = 50 - 20 = 30.$$

Exponential Distribution

Problem-3:

Sol: The probability density function (pdf) of the exponential distribution is

$$f(x) = \beta e^{-\beta x}$$

$$\beta = 1/\mu$$

$$\beta = \frac{1}{100} = 0.01$$

$$P(X < 50) = \int_0^{50} \beta e^{-\beta x} dx$$

$$\Rightarrow -e^{-0.01 \cdot 50} + e^0$$

$$\Rightarrow -e^{-0.5} + 1$$

$$= 1 - 0.6065 \Rightarrow 0.3935$$

Problem-4:

Sol:

Given,

the average completion time (μ) is 15 minutes

$$\beta = 1/\mu$$

$$\beta = \frac{1}{15} = 0.0667$$

Probability that a customer takes more than the domain

$$P(X > 20) = \int_{20}^{\infty} \beta e^{-\beta x} dx$$

$$P(X > 20) = \int_{20}^{\infty} 0.0667 + e^{-0.0667x} dx$$

$$\Rightarrow 1 - P(20)$$

$$P(20) = 1 - e^{-1.334}$$

$$P(20) \Rightarrow 0.736$$

$$P(X > 20) = 1 - 0.736 = 0.2633$$

② Normal Distribution:

Problems:

sol: Given that x is normally distributed

with mean (μ) = 100

standard deviation = 15

$$z = \frac{x - \mu}{\sigma}$$

$$z_{85} = \frac{85 - 100}{15} = -1$$

$$z_{115} = \frac{115 - 100}{15} = 1$$

$$P(x < 85) = P(z < -1)$$

$$P(x < 115) = P(z < 1)$$

by normal distribution

$$P(z < -1) = P(z < -1)$$

$$P(x < 115) = P(z < 1)$$

$$P(z < 1) = 0.8413$$

$$P(85 < x < 115) = P(z < 1) - P(z < -1)$$

$$\rightarrow 0.6826$$

\therefore approximately 68.26% of population has on 12
Score between 85 & 115.

Problem 6:-

Standard deviation (σ) = 20

$$z = \frac{x - \mu}{\sigma}$$

$$z_{170} = \frac{170 - 150}{10} = 2$$

$$P(x > 170) = 1 - P(z < 2)$$

using distribution table

$$P(Z < 1) = 0.8413$$

$$P(X > 140) = 1 - 0.8413$$

$$\Rightarrow 0.1587$$

∴ The probability that a randomly selected bird weight not more than 140 grams is 0.1587

u) Comparative Analysis

Problem 7

* uniform distribution

→ Assume all waiting times with a given range are equally likely.

* exponential distribution

→ Describe waiting time b/w event in a random process.

* scenarios

→ Use uniform distribution when buses arrive predictably, like on a fixed schedule

Problem 8

normal distribution

Advantages - Represents how heights typically vary in a population.

Limitations

- Assume a bell-shaped curve.

Comparison

normal distribution - Better modelling student heights as it reflects real-world variation.

Interactive exploration:-

CLT:- It states that as you take larger and larger samples from any population

Impact of sample size:-

* for small sample sizes the sampling distribution of the mean may not look perfectly normal.

Practical implication:-

→ CLT allows us to use the properties of the normal distribution to make statistical inferences, even when the population distribution is not normal.

Critical thinking scenario:-

Problem 10L

uniform distribution:- unlikely as it assumes all lifespans are equally likely within range.

Normal distribution:- not ideal, as it's not typical used for modelling finite process like light bulb lifespans.

Exponential distribution:- Best choice, It represents the constant and random failure rate of light bulbs overtime.

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