#Лабораторная работа 2
#Ряды Фурье
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#Вариант 1

> *#Задание 1.* **Для 2π − периодической кусочно − непрерывной функции $f(x)$ получить разложение в тригонометрический ряд Фурье. Построить графики частичных сумм $S_1(x)$, $S_3(x)$, $S_7(x)$ ряда и его суммы $S(x)$.**

 # Определение кусочной функции
$f := x \rightarrow piecewise(-\mathrm{Pi} \le x < 0, \mathrm{Pi} + 2 \cdot x, 0 \le x < \mathrm{Pi}, -\mathrm{Pi});$

 # Построение графика функции на главном периоде
$plot(f(x), x = -\mathrm{Pi} .. \mathrm{Pi}, discont = true);$

 # Процедура для вычисления коэффициентов Фурье и частичной суммы
$FourierTrigSum := \mathbf{proc}(f, m, a, b)$
$\quad \mathbf{local}\ a0, an, bn, n, l, Sm;$
$\quad l := \frac{(b - a)}{2};$
$\quad assume(n :: posint);$
$\quad a0 := simplify\left(\frac{int(f(x), x = a .. b)}{l}\right);$
$\quad an := simplify\left(\frac{int(f(x)\cos(\mathrm{Pi} \cdot n \cdot x / l), x = a .. b)}{l}\right);$
$\quad bn := simplify\left(\frac{int(f(x)\sin(\mathrm{Pi} \cdot n \cdot x / l), x = a .. b)}{l}\right);$
$\quad Sm := m \rightarrow \frac{1}{2} \cdot a0 + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 .. m\right);$
$\quad \mathbf{return}\ a0, an, bn, Sm, \frac{1}{2} \cdot a0 + Sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 .. m\right);$
$\mathbf{end\ proc}:$

 # Вычисление коэффициентов и частичной суммы
$coeff\_and\_Sm := FourierTrigSum(f, \mathrm{infinity}, -\mathrm{Pi}, \mathrm{Pi}):$
$a0 := coeff\_and\_Sm[1];$
$an := coeff\_and\_Sm[2];$
$bn := coeff\_and\_Sm[3];$
$S := coeff\_and\_Sm[4]:$
$Sx := coeff\_and\_Sm[5];$

 # Определение частичных сумм
$S\_1 := S(1);$
$S\_3 := S(3);$
$S\_7 := S(7);$
$S\_20000 := S(20000):$

```
# Определение диапазона для графиков
 x_range := -3· Pi .. 3· Pi :

# Построение графиков частичных сумм и оригинальной функции
plot( [S_1, S_3, S_7, S_20000], x = x_range,
            legend = ["S_1", "S_3", "S_7", "S_20000"],
            color = ["CadetBlue", "DarkCyan", "DimGray", "DarkGray"]) :
plot(f(x), x = x_range,
            legend = "f(x)", discont = true,
            color = black, thickness = 3 ) :
plots[display](%, %%);

# Нахождение точек разрыва
plots[display](
  plot(S_20000, x = -3·Pi .. 3·Pi),
  plots[pointplot]([(-3·Pi,-Pi), (-2·Pi, 0), (-Pi,-Pi), (0, 0), (Pi,-Pi), (2·Pi, 0), (3·Pi,
     -Pi)], color = "Blue")
 );

# Анимация частичных сумм ряда Фурье
plots[animatecurve]({S(1), S(3), S(7), f(x)}, x = x_range, frames = 50);
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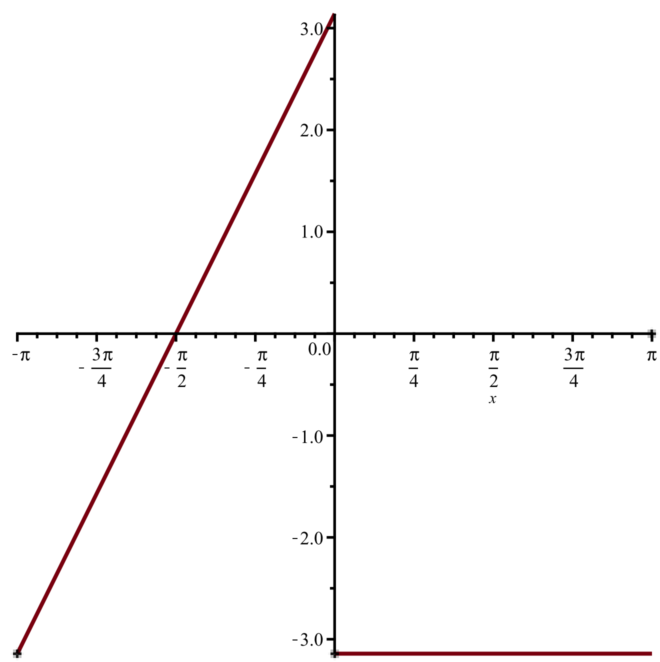
$$f := x \to piecewise(-\pi \le x \textbf{ and } x < 0, \pi + 2\,x, 0 \le x \textbf{ and } x < \pi, -\pi)$$



$$a0 := -\pi$$

$$an := \frac{2\,(-1)^{1+n\sim} + 2}{n\sim^2\,\pi}$$

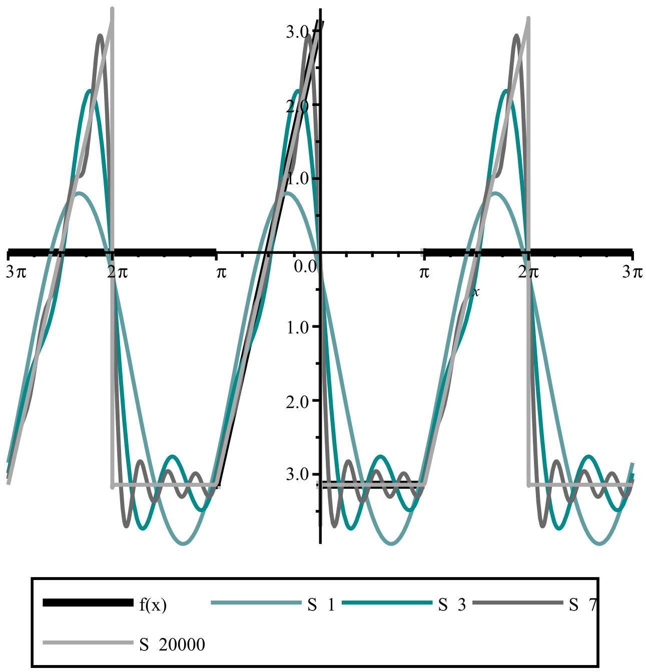
$$bn := -\frac{2}{n\sim}$$

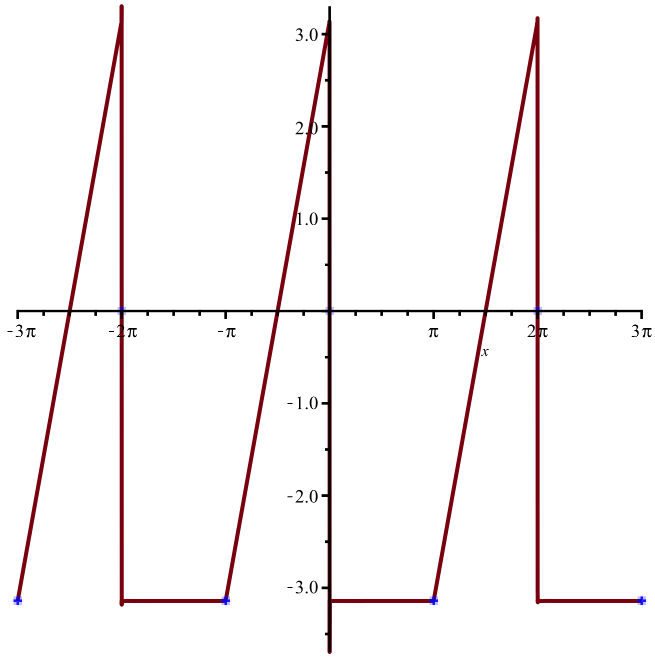
$$Sx := -\frac{1}{2}\pi + \sum_{n\sim=1}^{\infty}\left(\frac{\left(2\left(-1\right)^{1+n\sim}+2\right)\cos(n\sim x)}{n\sim^{2}\pi} - \frac{2\sin(n\sim x)}{n\sim}\right)$$

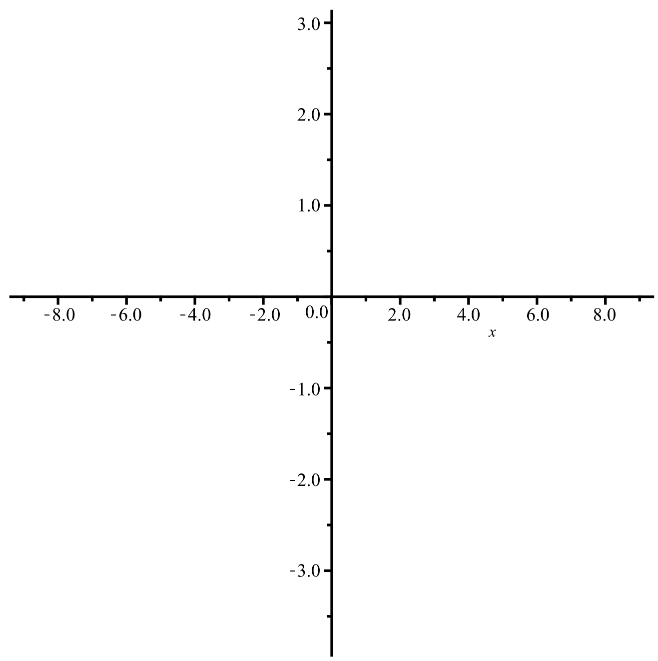
$$S\_1 := -\frac{1}{2}\pi + \frac{4\cos(x)}{\pi} - 2\sin(x)$$

$$S\_3 := -\frac{1}{2}\pi + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2x) + \frac{4}{9}\frac{\cos(3x)}{\pi} - \frac{2}{3}\sin(3x)$$

$$S\_7 := -\frac{1}{2}\pi + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2x) + \frac{4}{9}\frac{\cos(3x)}{\pi} - \frac{2}{3}\sin(3x) - \frac{1}{2}\sin(4x) + \frac{4}{25}\frac{\cos(5x)}{\pi} - \frac{2}{5}\sin(5x) - \frac{1}{3}\sin(6x) + \frac{4}{49}\frac{\cos(7x)}{\pi} - \frac{2}{7}\sin(7x)$$







> *#Задание 2.*

Разложить в ряд Фурье x_2-периодическую функцию y=f(x). Построить графики частичных сумм $S_1(x)$, $S_3(x)$,

$S_7(x)$ ряда и его суммы $S(x)$ на промежутке $[-2\,x_2, 2\,x_2]$

$a := 1 :$

$b := 2 :$

$c := -1 :$

$x\_1 := 2 :$

$x\_2 := 5 :$

$f := x \to piecewise(0 < x < x\_1, a \cdot x + b, x\_1 \le x \le x\_2, c) :$

$f(x);$

```
plot(f(x), x = 0 ..x_2, discont = true) ;

coeff_and_Sm := FourierTrigSum(f, ∞, 0, x_2) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];

S_1 := S(1);
S_3 := S(3);
S_7 := S(7);
S_20000 := S(20000) :

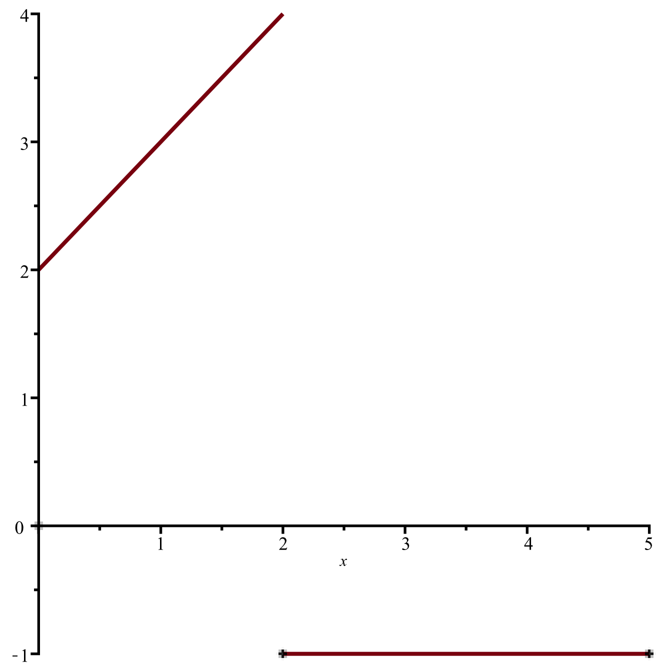
x_range := -2·x_2 .. 2·x_2 :

plot([S_1, S_3, S_7, S_20000], x = x_range,
        legend = ["S_1", "S_3", "S_7", "S_20000"],
        color = ["CadetBlue", "DarkCyan", "DimGray", "DarkGray"]) :
plot(f(x), x = x_range,
        legend = "f(x)", discont = true,
        color = black, thickness = 3) :
plots[display](%, %%);

# Нахождение точек разрыва
plots[display](
  plot(S_20000, x = -3·Pi .. 3·Pi),
  plots[pointplot]([(-8, 1.5), (-5, 0.5), (-3, 1.5), (0, 0.5), (2, 1.5), (5, 0.5), (7, 1.5)], color
     = "Blue")
 );

plots[animatecurve]({S(1), S(3), S(7), f(x)}, x = x_range, frames = 50);
```

$$\begin{cases} x+2 & 0 < x \textbf{ and } x < 2 \\ -1 & 2 \le x \textbf{ and } x \le 5 \end{cases}$$



$$a0 := \frac{6}{5}$$

$$an := \frac{5}{2}\,\frac{2\sin\left(\frac{4}{5}\,\pi\,n\sim\right)\pi\,n\sim+\cos\left(\frac{4}{5}\,\pi\,n\sim\right)-1}{\pi^2\,n\sim^2}$$

$$bn := -\frac{1}{2}\,\frac{10\cos\left(\frac{4}{5}\,\pi\,n\sim\right)\pi\,n\sim-6\,\pi\,n\sim-5\sin\left(\frac{4}{5}\,\pi\,n\sim\right)}{\pi^2\,n\sim^2}$$

$$Sx := \frac{3}{5}+\sum_{n\sim=1}^{\infty}\left(\frac{5}{2}\,\frac{\left(2\sin\left(\frac{4}{5}\,\pi\,n\sim\right)\pi\,n\sim+\cos\left(\frac{4}{5}\,\pi\,n\sim\right)-1\right)\cos\left(\frac{2}{5}\,\pi\,n\sim\,x\right)}{\pi^2\,n\sim^2}\right.$$
$$\left.-\frac{1}{2}\,\frac{\left(10\cos\left(\frac{4}{5}\,\pi\,n\sim\right)\pi\,n\sim-6\,\pi\,n\sim-5\sin\left(\frac{4}{5}\,\pi\,n\sim\right)\right)\sin\left(\frac{2}{5}\,\pi\,n\sim\,x\right)}{\pi^2\,n\sim^2}\right)$$

$$S\_1 := \frac{3}{5}+\frac{5}{2}\,\frac{\left(2\sin\left(\frac{1}{5}\,\pi\right)\pi-\cos\left(\frac{1}{5}\,\pi\right)-1\right)\cos\left(\frac{2}{5}\,\pi\,x\right)}{\pi^2}$$
$$-\frac{1}{2}\,\frac{\left(-10\cos\left(\frac{1}{5}\,\pi\right)\pi-6\,\pi-5\sin\left(\frac{1}{5}\,\pi\right)\right)\sin\left(\frac{2}{5}\,\pi\,x\right)}{\pi^2}$$

$$S\_3 := \frac{3}{5}+\frac{5}{2}\,\frac{\left(2\sin\left(\frac{1}{5}\,\pi\right)\pi-\cos\left(\frac{1}{5}\,\pi\right)-1\right)\cos\left(\frac{2}{5}\,\pi\,x\right)}{\pi^2}$$

$$- \frac{1}{2} \frac{\left(-10 \cos\left(\frac{1}{5}\pi\right)\pi - 6\pi - 5\sin\left(\frac{1}{5}\pi\right)\right)\sin\left(\frac{2}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{8} \frac{\left(-4 \sin\left(\frac{2}{5}\pi\right)\pi + \cos\left(\frac{2}{5}\pi\right) - 1\right)\cos\left(\frac{4}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{8} \frac{\left(20 \cos\left(\frac{2}{5}\pi\right)\pi - 12\pi + 5\sin\left(\frac{2}{5}\pi\right)\right)\sin\left(\frac{4}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{18} \frac{\left(6 \sin\left(\frac{2}{5}\pi\right)\pi + \cos\left(\frac{2}{5}\pi\right) - 1\right)\cos\left(\frac{6}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{18} \frac{\left(30 \cos\left(\frac{2}{5}\pi\right)\pi - 18\pi - 5\sin\left(\frac{2}{5}\pi\right)\right)\sin\left(\frac{6}{5}\pi x\right)}{\pi^2}$$

$$S\_7 := \frac{3}{5} + \frac{5}{2} \frac{\left(2 \sin\left(\frac{1}{5}\pi\right)\pi - \cos\left(\frac{1}{5}\pi\right) - 1\right)\cos\left(\frac{2}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{2} \frac{\left(-10 \cos\left(\frac{1}{5}\pi\right)\pi - 6\pi - 5\sin\left(\frac{1}{5}\pi\right)\right)\sin\left(\frac{2}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{8} \frac{\left(-4 \sin\left(\frac{2}{5}\pi\right)\pi + \cos\left(\frac{2}{5}\pi\right) - 1\right)\cos\left(\frac{4}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{8} \frac{\left(20 \cos\left(\frac{2}{5}\pi\right)\pi - 12\pi + 5\sin\left(\frac{2}{5}\pi\right)\right)\sin\left(\frac{4}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{18} \frac{\left(6 \sin\left(\frac{2}{5}\pi\right)\pi + \cos\left(\frac{2}{5}\pi\right) - 1\right)\cos\left(\frac{6}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{18} \frac{\left(30 \cos\left(\frac{2}{5}\pi\right)\pi - 18\pi - 5\sin\left(\frac{2}{5}\pi\right)\right)\sin\left(\frac{6}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{32} \frac{\left(-8 \sin\left(\frac{1}{5}\pi\right)\pi - \cos\left(\frac{1}{5}\pi\right) - 1\right)\cos\left(\frac{8}{5}\pi x\right)}{\pi^2}$$

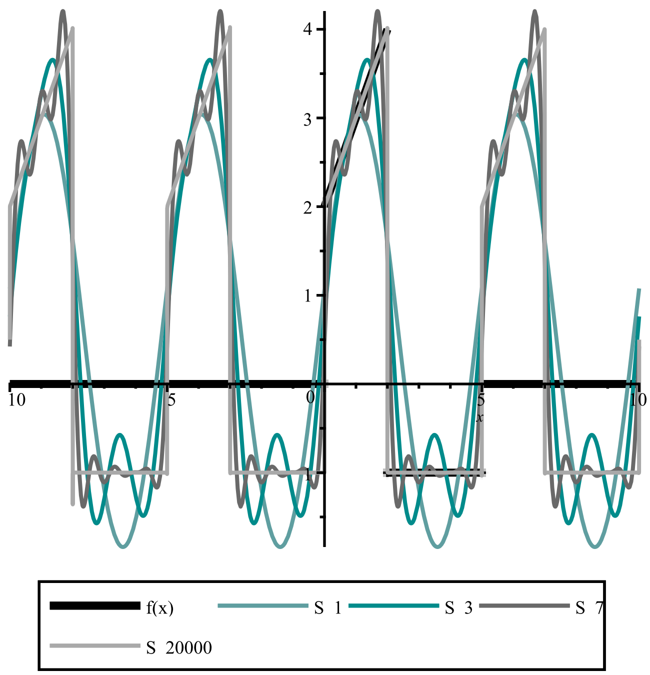
$$- \frac{1}{32} \frac{\left(-40 \cos\left(\frac{1}{5}\pi\right)\pi - 24\pi + 5\sin\left(\frac{1}{5}\pi\right)\right)\sin\left(\frac{8}{5}\pi x\right)}{\pi^2} - \frac{2}{5}\frac{\sin(2\pi x)}{\pi}$$

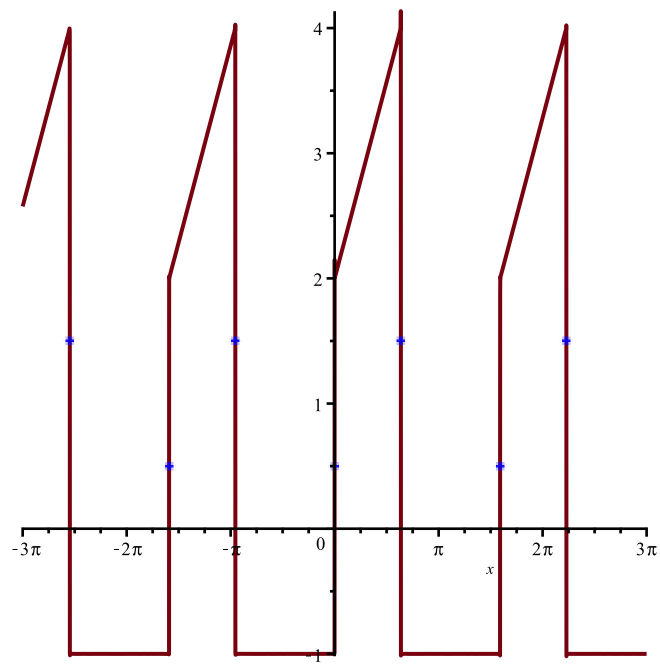
$$+ \frac{5}{72} \frac{\left(12 \sin\left(\frac{1}{5}\pi\right)\pi - \cos\left(\frac{1}{5}\pi\right) - 1\right)\cos\left(\frac{12}{5}\pi x\right)}{\pi^2}$$

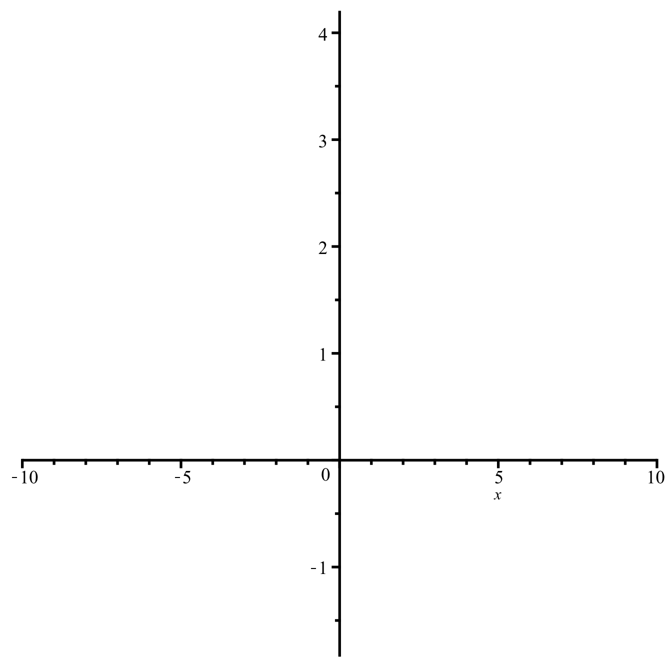
$$- \frac{1}{72} \frac{\left(-60 \cos\left(\frac{1}{5}\pi\right)\pi - 36\pi - 5\sin\left(\frac{1}{5}\pi\right)\right)\sin\left(\frac{12}{5}\pi x\right)}{\pi^2}$$

$$+ \frac{5}{98} \frac{\left(-14 \sin\left(\frac{2}{5}\pi\right)\pi + \cos\left(\frac{2}{5}\pi\right) - 1\right)\cos\left(\frac{14}{5}\pi x\right)}{\pi^2}$$

$$- \frac{1}{98} \frac{\left(70 \cos\left(\frac{2}{5}\pi\right)\pi - 42\pi + 5\sin\left(\frac{2}{5}\pi\right)\right)\sin\left(\frac{14}{5}\pi x\right)}{\pi^2}$$







> \# *Задание 3.* ***Построить три разложения*** **в** ***тригонометрический ряд Фурье функции, считая что она определена*** **: *на полном периоде, на полупериоде*** (***четная***), ***на полупериоде*** (***нечетная***)**. *Сравнить результат*** **с** ***порождающей функцией*.**

$f := x \rightarrow piecewise\left(0 < x \leq 2, x^2 - 2 \cdot x + 1, 2 < x < 3, 3 - x\right);$
$plot(f(x), x = 0 ..3, discont = true);$

\#***На полном периоде***
$coeff\_and\_Sm := FourierTrigSum(f, \infty, 0, 3):$
$a0 := coeff\_and\_Sm[1];$
$an := coeff\_and\_Sm[2];$
$bn := coeff\_and\_Sm[3];$
$S := coeff\_and\_Sm[4]:$

```
Sx := coeff_and_Sm[5];
S_20000 := S(20000) :

 x_range := -9 ..9 :
plot(S_20000, x = x_range,
             legend = "S_20000",
             color = "DarkGray") :
plot(f(x), x = x_range,
            legend = "f(x)", discont = true,
            color = black, thickness = 3) :
plots[display](%, %%);

 # Нахождение точек разрыва
plots[display](
  plot(S_20000, x = -3·Pi .. 3·Pi),
  plots[pointplot]([(-9, 0.5), (-7, 1), (-6, 0.5), (-4, 1), (-3, 0.5), (-1, 1), (0, 0.5), (2,
     1), (3, 0.5), (5, 1), (6, 0.5), (8, 1), (9, 0.5)], color = "Blue")
 );
```

#На полупериоде (четный способ)

```
f_even := x→piecewise(0 ≤ x ≤ 3, f(x), -3 ≤ x < 0, f(-x));
plot(f_even(x), x =-3 ..3);
coeff_and_Sm := FourierTrigSum(f_even, ∞, -3, 3) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];
S_20000 := S(20000) :

x_range := -9 ..9 :
plot(S_20000, x = x_range,
             legend = "S_20000",
             color = "DarkGray") :
plot(f(x), x = x_range,
            legend = "f(x)", discont = true,
            color = black, thickness = 3) :
plots[display](%, %%);

# Нахождение точек разрыва
plots[display](
  plot(S_20000, x = -3·Pi .. 3·Pi),
  plots[pointplot]([(-9, 0), (-8, 1), (-6, 1), (-4, 1), (-3, 0), (-2, 1), (0, 1), (2, 1), (3,
     0), (4, 1), (6, 1), (8, 1), (9, 0)], color = "Blue")
 );
```

#На полупериоде (нечетный способ)

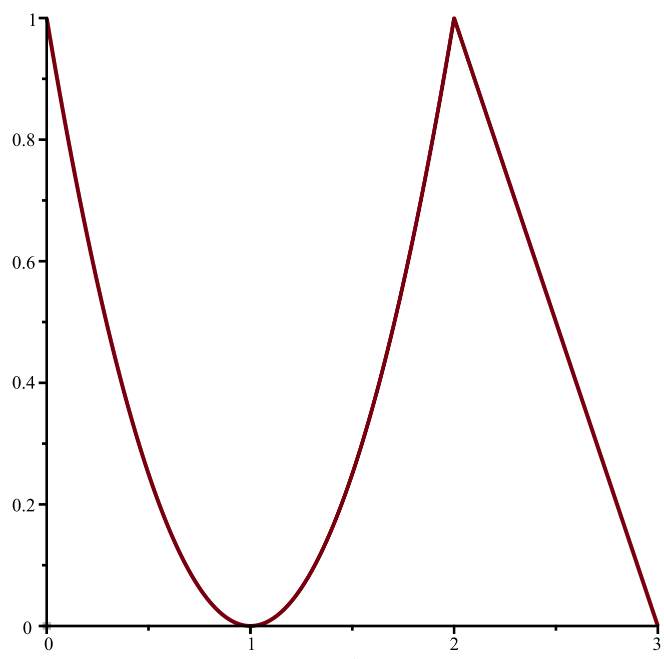
```
f_odd := x→piecewise(0 ≤ x ≤ 3, f(x), -3 ≤ x < 0, -f(-x));
```

```
plot( f_odd(x), x=-3 ..3);
coeff_and_Sm := FourierTrigSum( f_odd, ∞, -3, 3) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];
S_20000 := S(20000) :

x_range := -9 ..9 :
plot(S_20000, x = x_range,
          legend = "S_20000",
          color = "DarkGray") :
plot(f(x), x = x_range,
         legend = "f(x)", discont = true,
         color = black, thickness = 3) :
plots[display](%, %%);

# Нахождение точек разрыва
plots[display](
  plot(S_20000, x = -3·Pi .. 3·Pi),
  plots[pointplot]([(9, 0), (-8, -1), (-6, 0), (-4, 1), (-3, 0), (-2, -1), (0, 0), (2, 1), (3,
    0), (4, -1), (6, 0), (8, 1), (9, 0)], color = "Blue")
 );
```

$$f := x \to piecewise\left(0 < x \textbf{ and } x \leq 2, x^2 - 2x + 1, 2 < x \textbf{ and } x < 3, 3 - x\right)$$

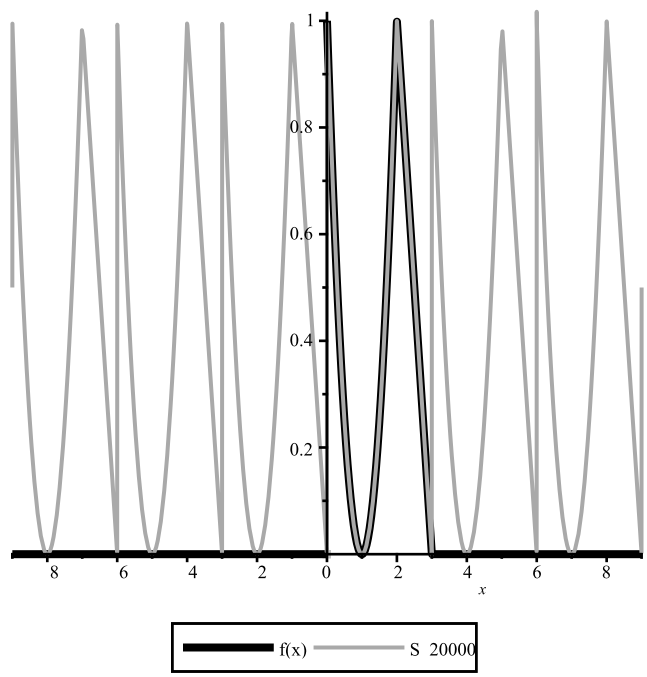


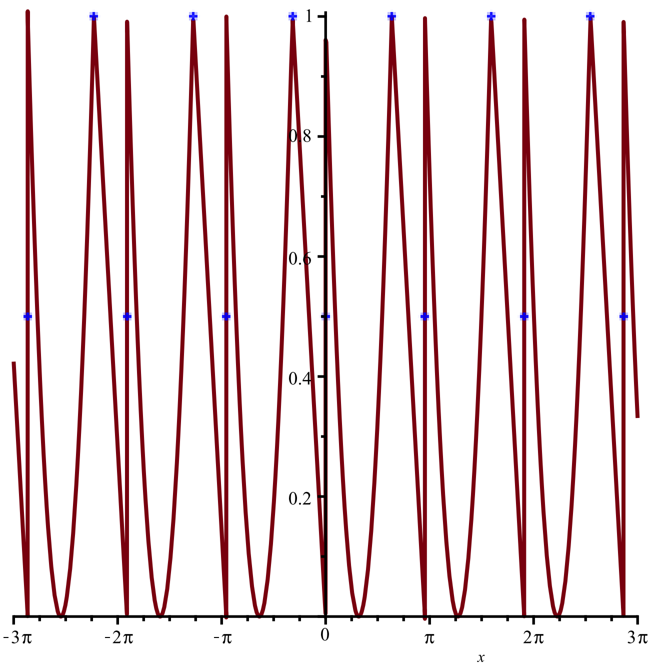
$$a0 := \frac{7}{9}$$

$$an := \frac{3}{2}\,\frac{3\,\pi\,n\sim\cos\left(\frac{4}{3}\,\pi\,n\sim\right)+\pi\,n\sim-3\sin\left(\frac{4}{3}\,\pi\,n\sim\right)}{\pi^3\,n\sim^3}$$

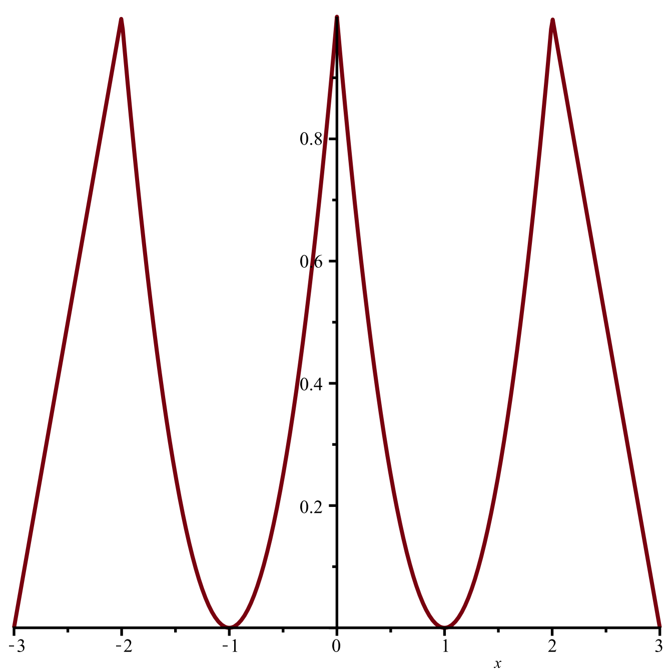
$$bn := \frac{1}{2}\,\frac{2\,\pi^2\,n\sim^2+9\sin\left(\frac{4}{3}\,\pi\,n\sim\right)\pi\,n\sim+9\cos\left(\frac{4}{3}\,\pi\,n\sim\right)-9}{\pi^3\,n\sim^3}$$

$$Sx := \frac{7}{18}+\sum_{n\sim=1}^{\infty}\left(\frac{3}{2}\,\frac{\left(3\,\pi\,n\sim\cos\left(\frac{4}{3}\,\pi\,n\sim\right)+\pi\,n\sim-3\sin\left(\frac{4}{3}\,\pi\,n\sim\right)\right)\cos\left(\frac{2}{3}\,\pi\,n\sim x\right)}{\pi^3\,n\sim^3}\right.$$
$$\left.+\frac{1}{2}\,\frac{\left(2\,\pi^2\,n\sim^2+9\sin\left(\frac{4}{3}\,\pi\,n\sim\right)\pi\,n\sim+9\cos\left(\frac{4}{3}\,\pi\,n\sim\right)-9\right)\sin\left(\frac{2}{3}\,\pi\,n\sim x\right)}{\pi^3\,n\sim^3}\right)$$





$$f\_even := x \to piecewise(0 \le x \textbf{ and } x \le 3, f(x), -3 \le x \textbf{ and } x < 0, f(-x))$$



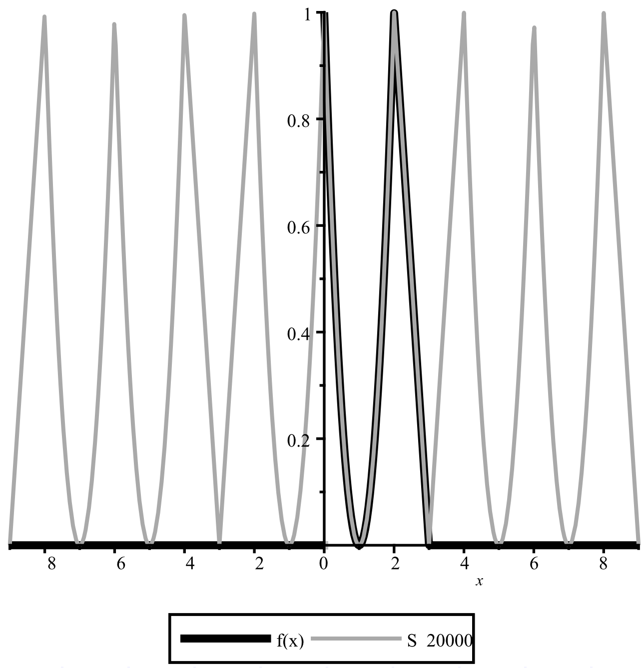
$$a0 := \frac{7}{9}$$

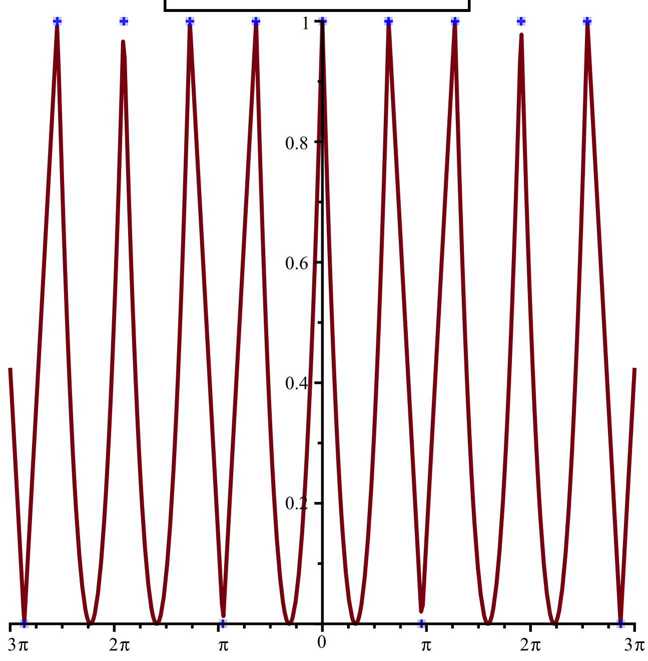
$$an := \frac{6\,(-1)^{1+n\sim}\,\pi\, n\sim + 18\,\pi\, n\sim \cos\left(\frac{2}{3}\,\pi\, n\sim\right) + 12\,\pi\, n\sim - 36\,\sin\left(\frac{2}{3}\,\pi\, n\sim\right)}{\pi^3\, n\sim^3}$$

$$bn := 0$$

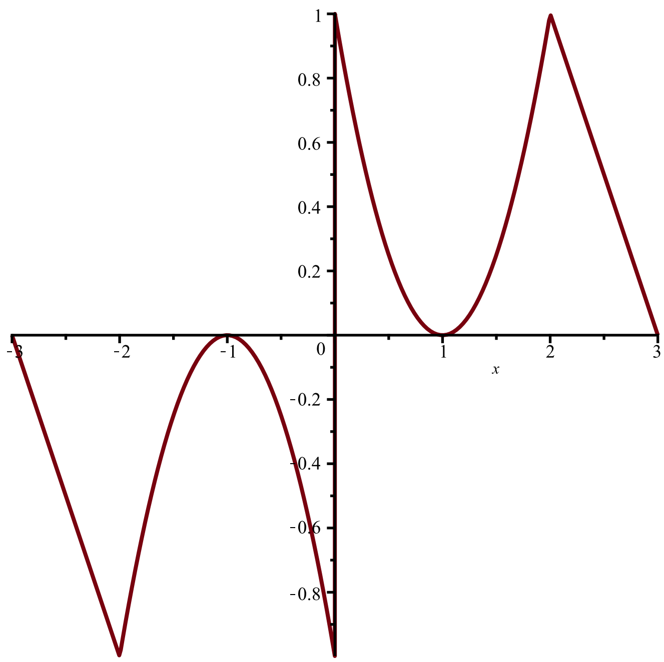
$$Sx := \frac{7}{18} + \sum_{n\sim=1}^{\infty}$$

$$\frac{1}{\pi^3 n\sim^3}\left(\left(6\,(-1)^{1+n\sim}\,\pi\,n\sim + 18\,\pi\,n\sim\cos\left(\frac{2}{3}\,\pi\,n\sim\right) + 12\,\pi\,n\sim - 36\sin\left(\frac{2}{3}\,\pi\,n\sim\right)\right)\cos\left(\frac{1}{3}\,\pi\,n\sim x\right)\right)$$





$f\_odd := x \rightarrow piecewise(0 \le x$ **and** $x \le 3, f(x),\ \ 3 \le x$ **and** $x < 0,\ \ f(\ \ x))$

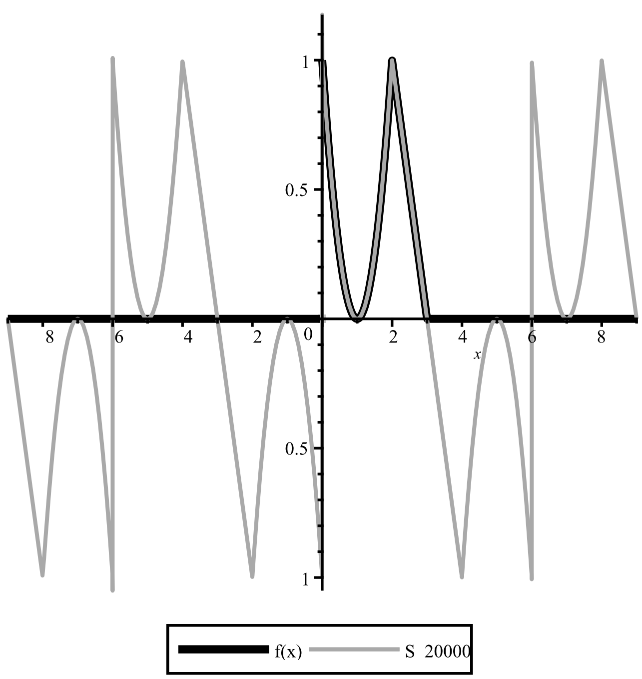


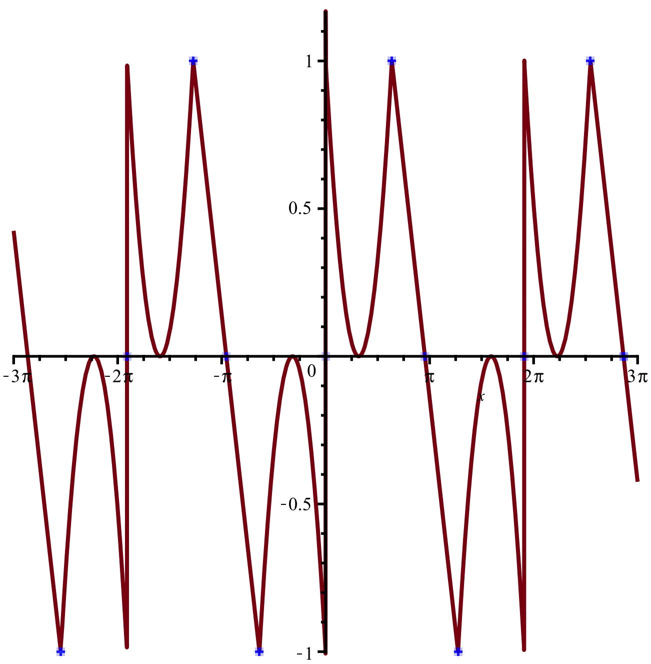
$$a0 := 0$$

$$an := 0$$

$$bn := \frac{2\,\pi^2\,n\sim^2 + 18\,\sin\left(\frac{2}{3}\,\pi\,n\sim\right)\pi\,n\sim + 36\,\cos\left(\frac{2}{3}\,\pi\,n\sim\right) - 36}{\pi^3\,n\sim^3}$$

$$Sx := \sum_{n\sim=1}^{\infty} \frac{\left(2\,\pi^2\,n\sim^2 + 18\,\sin\left(\frac{2}{3}\,\pi\,n\sim\right)\pi\,n\sim + 36\,\cos\left(\frac{2}{3}\,\pi\,n\sim\right) - 36\right)\sin\left(\frac{1}{3}\,\pi\,n\sim\,x\right)}{\pi^3\,n\sim^3}$$





> #*Задание 4.*
> #***Разложить функцию* в *ряд Фурье по многочленам Лежандра* и *Чебышёва на промежутке*** [**-1, 1**]**,**
> ***экспериментально найти наименьший порядок частичных сумм равномерно аппроксимирующей на всем промежутке заданную функцию* с *точностью* 0, 1.**

$f := x \rightarrow (\sin(2\,x))^3;$
$plot(f(x), x = -1 ..1);$

$aproxis := plot([f(x) + 0.1, f(x) - 0.1], x = -1 ..1, linestyle = dash, color = blue) :$

$with(orthopoly) :$
#*Полином* *Лежандра*
$LegendrePolynom :=$ **proc**(f, k, a, b)
local $c\_n, n;$
$$c\_n := \frac{int(f(x) \cdot P(n, x), x = a ..b)}{int\left(P(n, x)^2, x = a ..b\right)};$$
return $sum(c\_n \cdot P(n, x), n = 0 ..k);$
end proc:

$S\_Suff1 := LegendrePolynom(f, 7, -1, 1);$ **#*n* = 7,**
это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке [**-1, 1**] ***функцию* с *точностью* 0, 1 *для полинома Лежандра***
plot(f(x), x = -1 ..1, color = black, legend = "f(x)") :
plot(S_Suff1, x = -1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);

#*Полином Чебышева*
$ChebyshevPolynom :=$ **proc**(f, k, a, b)
local $c\_n, c\_0, n;$

$$c\_n := \frac{2}{\mathrm{Pi}} \cdot int\left(\frac{f(x) \cdot T(n, x)}{\sqrt{1 - x^2}}, x = a..b\right);$$
$$c\_0 := \frac{2}{\mathrm{Pi}} \cdot int\left(\frac{f(x) \cdot T(0, x)}{\sqrt{1 - x^2}}, x = a..b\right);$$
return $\frac{c\_0}{2} + sum(c\_n \cdot T(n, x), n = 1..k);$

end proc:

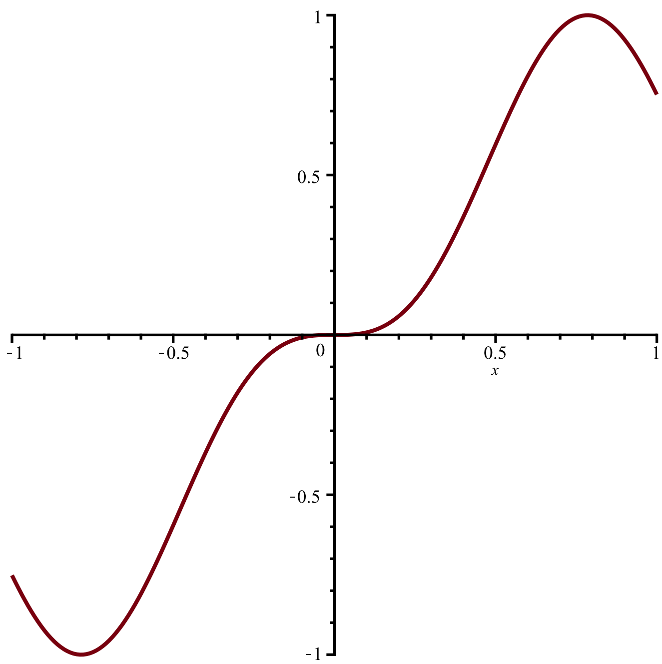
S_Suff2 := ChebyshevPolynom(f, 7, -1, 1); ***#n = 7, это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке*** [***-1, 1***] ***функцию с точностью 0, 1 для полинома Чебышева***

plot(f(x), x =-1 ..1, color = black, legend = "f(x)") :

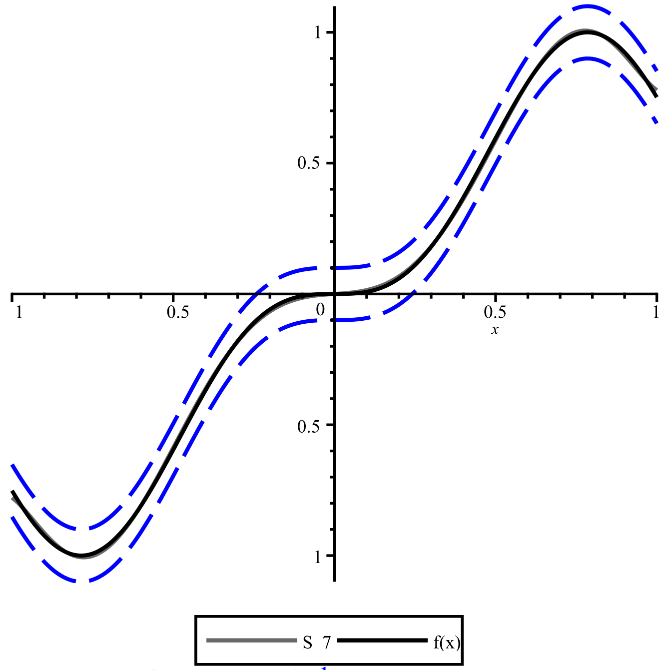
plot(S_Suff2, x =-1 ..1, color = "DimGray", legend = "S_7") :

plots[display](%, %%, aproxis);

$$f := x \to \sin(2\,x)^3$$



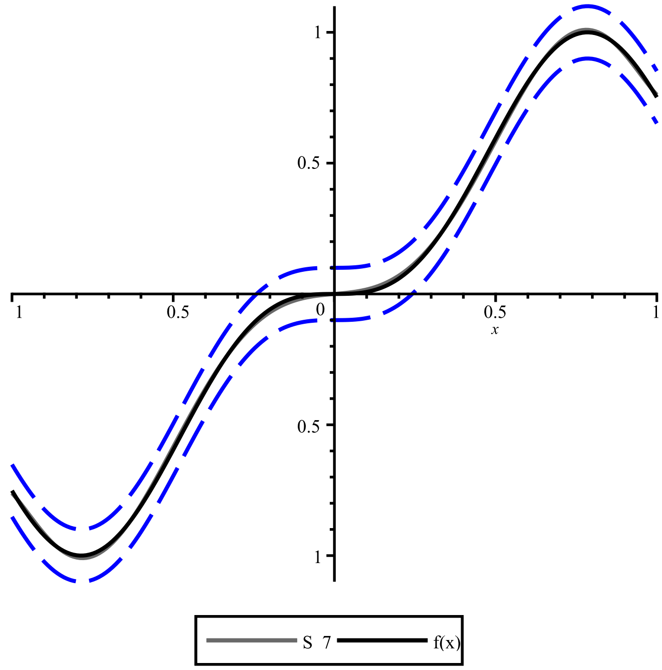
$$S\_Suff1 := \frac{3}{2}\left(-\frac{1}{3}\sin(2)^2\cos(2) - \frac{2}{3}\cos(2) + \frac{1}{18}\sin(2)^3 + \frac{1}{3}\sin(2)\right)x + \frac{7}{2}\left(-\frac{7}{36}\sin(2)^2\cos(2) + \frac{19}{9}\cos(2) + \frac{67}{216}\sin(2)^3 + \frac{11}{18}\sin(2)\right)\left(\frac{5}{2}x^3 - \frac{3}{2}x\right) + \frac{11}{2}\left(-\frac{565}{48}\sin(2) - \frac{611}{24}\cos(2) + \frac{65}{288}\sin(2)^3 + \frac{19}{48}\sin(2)^2\cos(2)\right)\left(\frac{63}{8}x^5 - \frac{35}{4}x^3 + \frac{15}{8}x\right) + \frac{15}{2}\left(\frac{499961}{1296}\sin(2) + \frac{136357}{162}\cos(2) - \frac{1679}{5184}\sin(2)^2\cos(2) - \frac{24661}{31104}\sin(2)^3\right)\left(\frac{429}{16}x^7 - \frac{693}{16}x^5 + \frac{315}{16}x^3 - \frac{35}{16}x\right)$$



$$S\_Suff2 := \frac{2\left(\displaystyle\int_{1}^{1} \frac{\sin(2\,x)^3\,x}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)x}{\pi} + \frac{2\left(\displaystyle\int_{1}^{1} \frac{\sin(2\,x)^3\,\left(4\,x^3\quad 3\,x\right)}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)\left(4\,x^3\quad 3\,x\right)}{\pi}$$

$$+\frac{2\left(\displaystyle\int_{1}^{1} \frac{\sin(2\,x)^3\,\left(16\,x^5\quad 20\,x^3+5\,x\right)}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)\left(16\,x^5\quad 20\,x^3+5\,x\right)}{\pi}$$

$$+\frac{2\left(\displaystyle\int_{1}^{1} \frac{\sin(2\,x)^3\,\left(64\,x^7\quad 112\,x^5+56\,x^3\quad 7\,x\right)}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)\left(64\,x^7\quad 112\,x^5+56\,x^3\quad 7\,x\right)}{\pi}$$



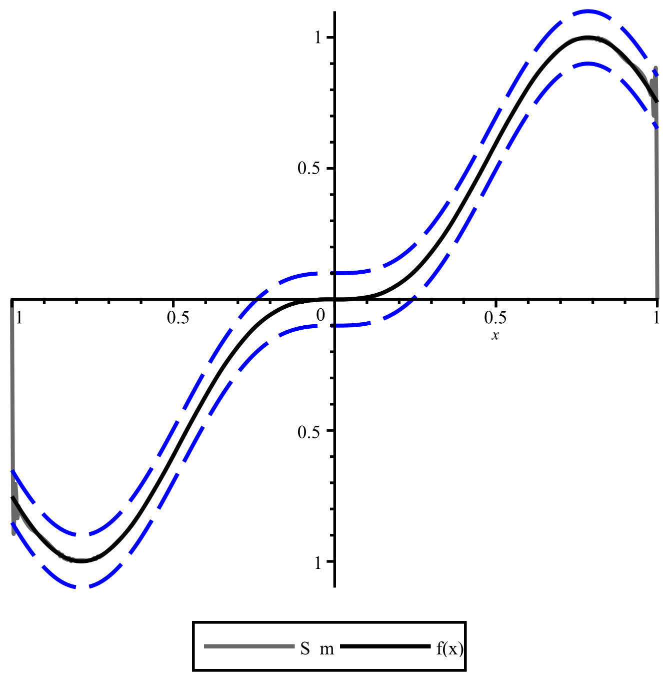
> ***#Разложить функцию в тригонометрический и степенной ряды на промежутке** [**1, 1**] **, Найти наименьший порядок частичных суммы, равномерно аппроксимирующих на всем промежутке заданную функцию с точностью 0, 1.***

```
m := 180 :

S_Suff3 := FourierTrigSum(f, m,  1, 1) [5] :
FourierTrigSum(f, m,  1, 1) [1 ..3];

plot(f(x), x =  1 ..1, color = black, legend = "f(x)") :
plot(S_Suff3, x =  1 ..1, color = "DimGray", legend = "S_m") :
plots[display](%, %%, aprox is);
```

$$0, 0, \frac{1}{2} \quad \frac{(\quad 1)^{1 + n\sim} \pi\, n\sim \left(3\, \pi^2\, n\sim^2 \sin(2) \quad \pi^2\, n\sim^2 \sin(6) \quad 108 \sin(2) + 4 \sin(6)\right)}{\pi^4\, n\sim^4 \quad 40\, \pi^2\, n\sim^2 + 144}$$



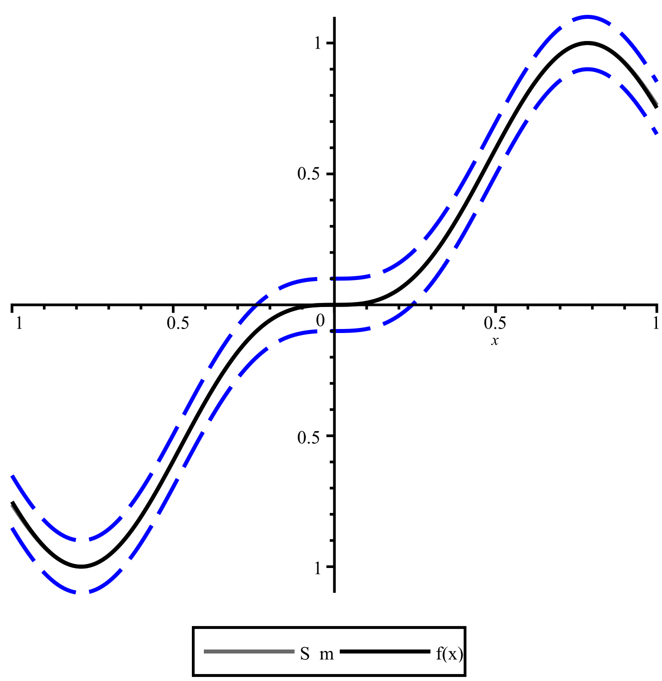
> $m := 16 :$

$S\_Suff4 := convert(taylor(f(x), x = 0, m), polynom)\ ;$

#n = 16, это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке [1, 1] функцию с точностью 0, 1 для степенного ряда (Маклорена)

plot(f(x), x =   1 ..1, color = black, legend = "f(x)") :
plot(S_Suff4, x =   1 ..1, color = "DimGray", legend = "S_m") :
plots[display](%, %%, aproxis);

$$S\_Suff4 := 8\,x^3 \quad 16\,x^5 + \frac{208}{15}\,x^7 \quad \frac{1312}{189}\,x^9 + \frac{10736}{4725}\,x^{11} \quad \frac{2336}{4455}\,x^{13} + \frac{19131872}{212837625}\,x^{15}$$



> $g := x \rightarrow \ \arccos(x) \quad 1;$

$plot(g(x), x = -1 ..1);$

$aproxis := plot([g(x) + 0.1, g(x) - 0.1], x = -1 ..1, linestyle = dash, color = blue) :$

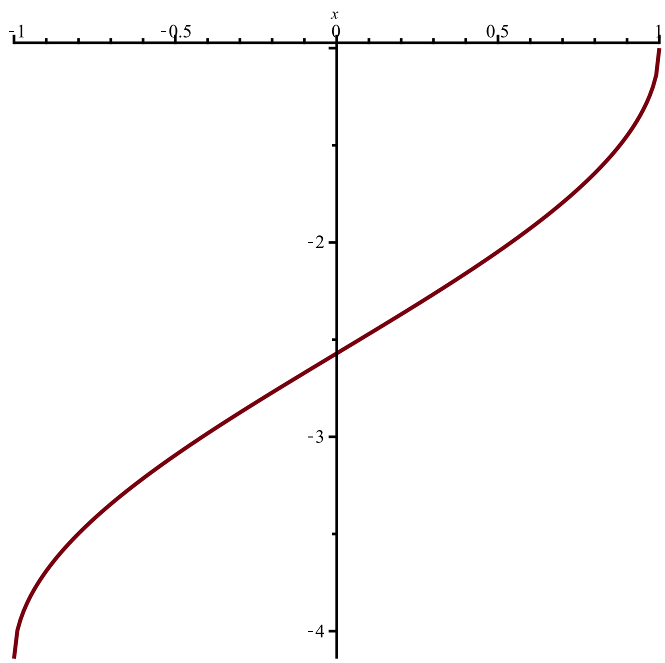
$S\_Suff1 := LegendrePolynom(g, 7, -1, 1);$ **#n = 7, *это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке* [-1, 1] *функцию* с *точностью* 0, 1 *для полинома Лежандра***

plot(g(x), x = -1 ..1, color = black, legend = "g(x)") :
plot(S_Suff1, x = -1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);

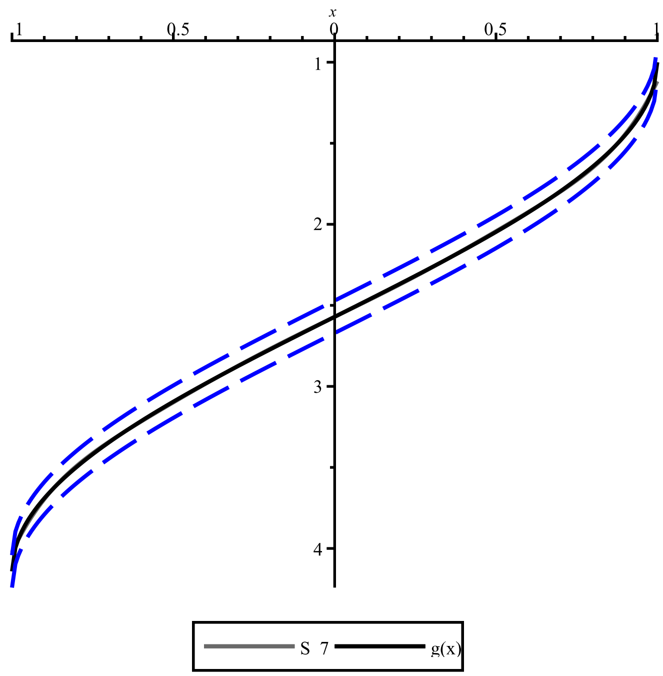
S_Suff2 := ChebyshevPolynom(g, 7, -1, 1); ***#n = 7, это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке [-1, 1] функцию с точностью 0, 1 для полинома Чебышева***

plot(g(x), x = -1 ..1, color = black, legend = "g(x)") :
plot(S_Suff2, x = -1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);

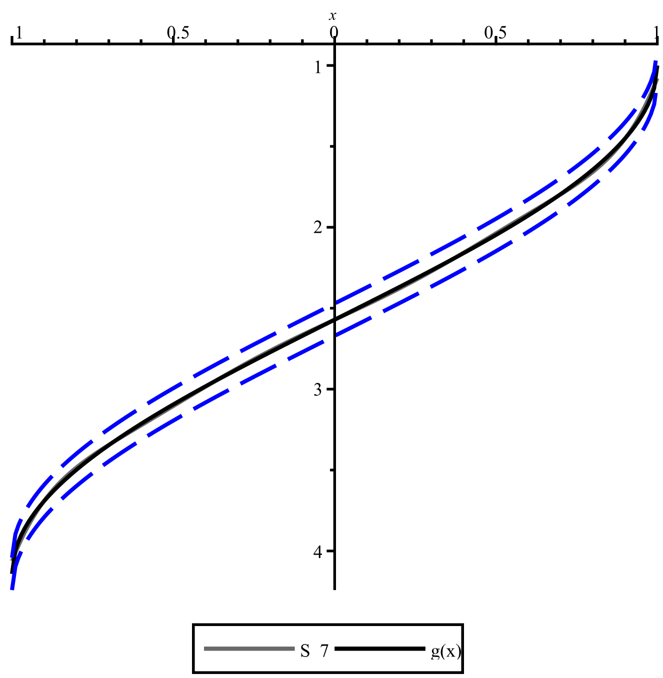
$$g := x \to -\arccos(x) - 1$$



$$S\_Suff1 := -\frac{1}{2}\pi - 1 + \frac{3}{8}\pi x + \frac{7}{128}\pi\left(\frac{5}{2}x^3 - \frac{3}{2}x\right) + \frac{11}{512}\pi\left(\frac{63}{8}x^5 - \frac{35}{4}x^3 + \frac{15}{8}x\right) + \frac{375}{32768}\pi\left(\frac{429}{16}x^7 - \frac{693}{16}x^5 + \frac{315}{16}x^3 - \frac{35}{16}x\right)$$



$$S\_Suff2 := \frac{\frac{1}{2}\pi^2 \quad \pi}{\pi} + \frac{4\,x}{\pi} + \frac{4}{9}\,\frac{4\,x^3 \quad 3\,x}{\pi} + \frac{4}{25}\,\frac{16\,x^5 \quad 20\,x^3 + 5\,x}{\pi} + \frac{4}{49}\,\frac{64\,x^7 \quad 112\,x^5 + 56\,x^3 \quad 7\,x}{\pi}$$



> ***#Разложить функцию в тригонометрический и степенной ряды на промежутке [1, 1], Найти наименьший порядок частичных суммы, равномерно аппроксимирующих на всем промежутке заданную функцию с точностью 0, 1.***

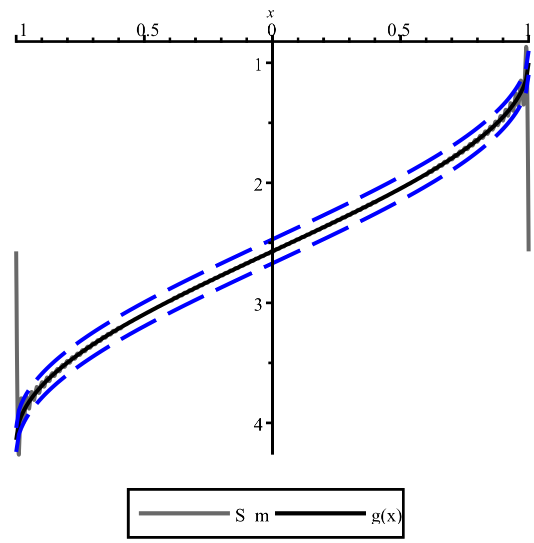
$m := 100:$

S_Suff3 := FourierTrigSum(g, m, 1, 1)[5]:

```
FourierTrigSum(g, m, -1, 1)[1..3];

plot(g(x), x = -1..1, color = black, legend = "g(x)") :
plot(S_Suff3, x = -1..1, color = "DimGray", legend = "S_m") :
plots[display](%, %%, aproxis);
```

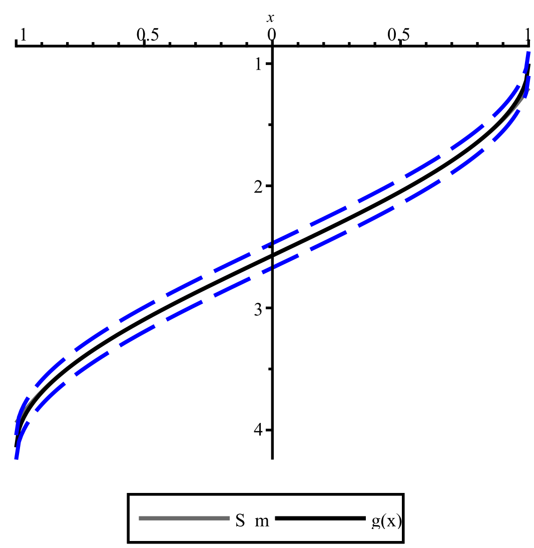
$$-\pi - 2, 0, -\left(\int_{-1}^{1} (\arccos(x) + 1) \sin(\pi\, n{\sim}\, x)\, dx\right)$$




```
> m := 16 :
  S_Suff4 := convert(taylor(g(x), x = 0, m), polynom) ;

  plot(g(x), x =  1..1, color = black, legend = "g(x)") :
  plot(S_Suff4, x =  1..1, color = "DimGray", legend = "S_m") :
  plots[display](%, %%, aproxis);
```

$$S\_Suff4 := \frac{1}{2}\pi \quad 1 + x + \frac{1}{6}x^{3} + \frac{3}{40}x^{5} + \frac{5}{112}x^{7} + \frac{35}{1152}x^{9} + \frac{63}{2816}x^{11} + \frac{231}{13312}x^{13} + \frac{143}{10240}x^{15}$$



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