#Лабораторная работа 2
#Ряды Фурье
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#Вариант 1

> #Задание 1. **Для 2π – периодической кусочно – непрерывной функции $f(x)$ получить разложение в тригонометрический ряд Фурье. Построить графики частичных сумм $S_1(x)$, $S_3(x)$, $S_7(x)$ ряда и его суммы $S(x)$.**

 # Определение кусочной функции

$f := x \rightarrow piecewise(-\mathrm{Pi} \le x < 0, \mathrm{Pi} + 2 \cdot x, 0 \le x < \mathrm{Pi}, -\mathrm{Pi});$

 # Построение графика функции на главном периоде

$plot(f(x), x = -\mathrm{Pi} .. \mathrm{Pi}, discont = true);$

 # Процедура для вычисления коэффициентов Фурье и частичной суммы

$FourierTrigSum := \mathbf{proc}(f, m, a, b)$

$\quad \mathbf{local}\ a0, an, bn, n, l, Sm;$

$\quad l := \frac{(b - a)}{2};$

$\quad assume(n :: posint);$

$\quad a0 := simplify\left(\frac{int(f(x), x = a .. b)}{l}\right);$

$\quad an := simplify\left(\frac{int(f(x)\cos(\mathrm{Pi} \cdot n \cdot x / l), x = a .. b)}{l}\right);$

$\quad bn := simplify\left(\frac{int(f(x)\sin(\mathrm{Pi} \cdot n \cdot x / l), x = a .. b)}{l}\right);$

$\quad Sm := m \rightarrow \frac{1}{2} \cdot a0 + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 .. m\right);$

$\quad \mathbf{return}\ a0, an, bn, Sm, \frac{1}{2} \cdot a0 + Sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1 .. m\right);$

$\mathbf{end\ proc}:$

 # Вычисление коэффициентов и частичной суммы

$coeff\_and\_Sm := FourierTrigSum(f, \mathrm{infinity}, -\mathrm{Pi}, \mathrm{Pi}):$

$a0 := coeff\_and\_Sm[1];$

$an := coeff\_and\_Sm[2];$

$bn := coeff\_and\_Sm[3];$

$S := coeff\_and\_Sm[4]:$

$Sx := coeff\_and\_Sm[5];$

 # Определение частичных сумм

$S\_1 := S(1);$

$S\_3 := S(3);$

$S\_7 := S(7);$

$S\_20000 := S(20000):$

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# Определение диапазона для графиков
x_range := -3·Pi .. 3·Pi :

# Построение графиков частичных сумм и оригинальной функции
plot([S_1, S_3, S_7, S_20000], x = x_range,
          legend = ["S_1", "S_3", "S_7", "S_20000"],
          color = ["CadetBlue", "DarkCyan", "DimGray", "DarkGray"]) :
plot(f(x), x = x_range,
          legend = "f(x)", discont = true,
          color = black, thickness = 3) :
plots[display](%, %%);

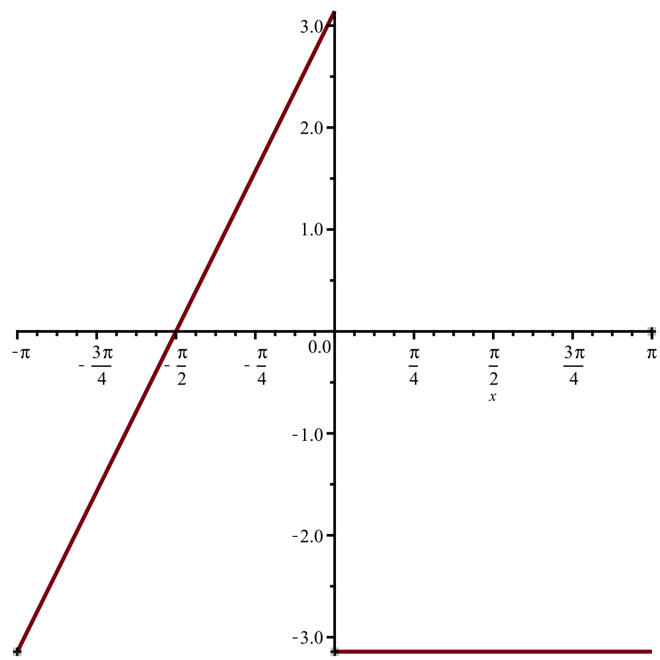
# Нахождение точек разрыва
list_of_disconts := Statistics[Sort](convert(discont(f(x), x), list)) :
discont_points := [ ] :

# Добавление точек разрыва и их средних значений
for n from -1 to 1 do
   for p in list_of_disconts do
      limit_left := limit(f(x), x = p, left);
      limit_right := limit(f(x), x = p, right);
      average := (limit_left + limit_right)/2;
      discont_points := [op(discont_points), [2·Pi·n + p, average]];
   end do;
end do;

# Построение графиков
plots[display](
   plot(S_20000, x = x_range, legend = "S", color = "Blue"),
   plots[pointplot](discont_points, color = red, legend = "Discontinuities")
 );

# Анимация частичных сумм ряда Фурье
plots[animatecurve]({S(1), S(3), S(7), f(x)}, x = x_range, frames = 50);
```

$$f := x \mapsto \begin{cases} \pi + 2\cdot x & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases}$$



$$a0 := -\pi$$

$$an := \frac{-2\,(-1)^{n\sim}+2}{n\sim^2\,\pi}$$

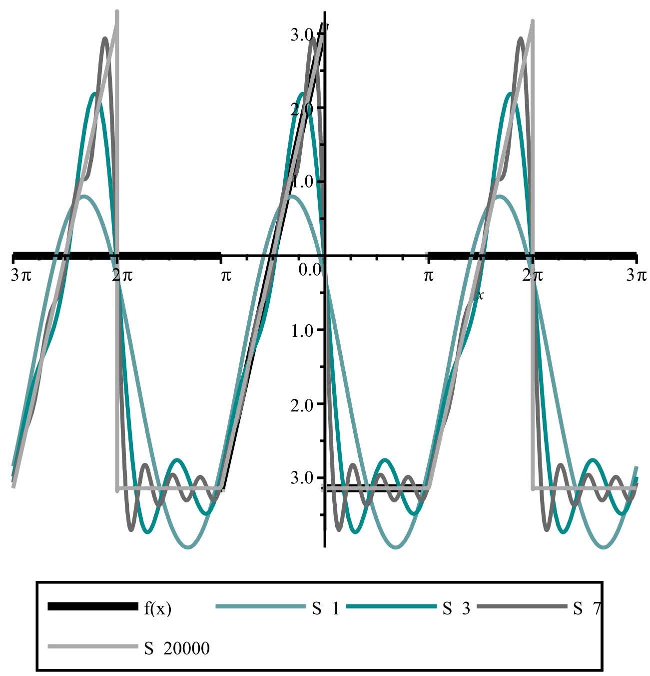
$$bn := -\frac{2}{n\sim}$$

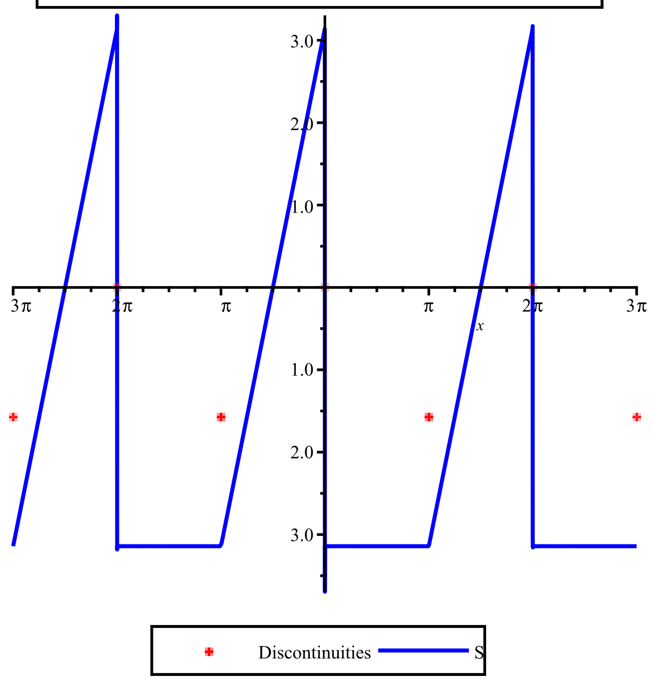
$$Sx := -\frac{\pi}{2} + \sum_{n\sim=1}^{\infty}\left(\frac{\left(-2\,(-1)^{n\sim}+2\right)\cos(n\sim x)}{n\sim^2\,\pi} - \frac{2\sin(n\sim x)}{n\sim}\right)$$

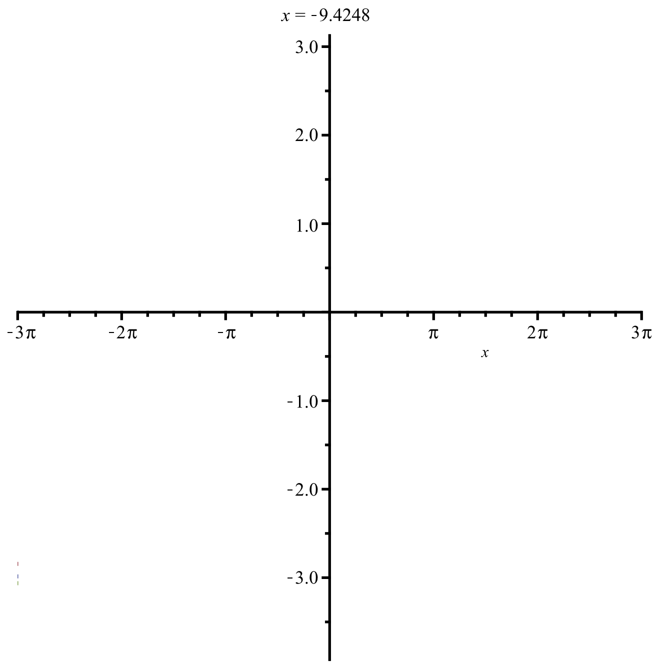
$$S\_1 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x)$$

$$S\_3 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2x) + \frac{4\cos(3x)}{9\pi} - \frac{2\sin(3x)}{3}$$

$$S\_7 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2x) + \frac{4\cos(3x)}{9\pi} - \frac{2\sin(3x)}{3} - \frac{\sin(4x)}{2} + \frac{4\cos(5x)}{25\pi} - \frac{2\sin(5x)}{5} - \frac{\sin(6x)}{3} + \frac{4\cos(7x)}{49\pi} - \frac{2\sin(7x)}{7}$$


3.0
2.0
1.0
0.0
3π
2π
π
π
2π
3π
x
1.0
2.0
3.0
f(x)
S 1
S 3
S 7
S 20000


3.0
2.0
1.0
3π
2π
π
π
2π
3π
x
1.0
2.0
3.0
Discontinuities
S



> #Задание 2.

Разложить в ряд Фурье x_2-периодическую функцию y=f(x). Построить графики частичных сумм $S_1(x)$, $S_3(x)$, $S_7(x)$ ряда и его суммы $S(x)$ на промежутке $[-2x_2, 2x_2]$

```
a := 1 :
b := 2 :
c := -1 :
x_1 := 2 :
x_2 := 5 :
f := x→piecewise(0 < x < x_1, a·x + b, x_1 ≤ x ≤ x_2, c) :
f(x);

plot(f(x), x = 0 ..x_2, discont = true) ;

coeff_and_Sm := FourierTrigSum(f, ∞, 0, x_2) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];

S_1 := S(1);
S_3 := S(3);
S_7 := S(7);
S_20000 := S(20000) :

x_range := -2· x_2 .. 2· x_2 :

plot([S_1, S_3, S_7, S_20000], x = x_range,
        legend = ["S_1", "S_3", "S_7", "S_20000"],
```

```
        color = ["CadetBlue", "DarkCyan", "DimGray", "DarkGray"]) :
plot(f(x), x = x_range,
        legend = "f(x)", discont = true,
        color = black, thickness = 3) :
plots[display](%, %%);

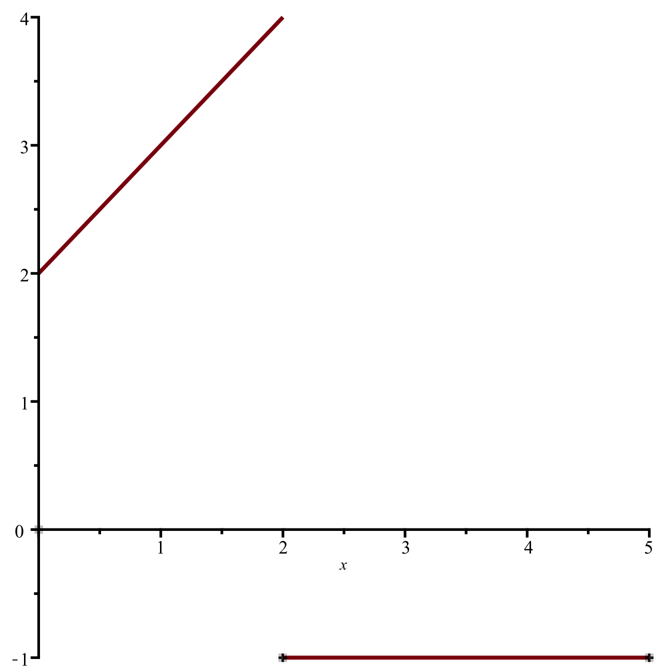
list_of_disconts := Statistics[Sort](convert(discont(f(x), x), list)) :
discont_points := [ ] :

for n from -1 to 1 do
  for p in list_of_disconts do
    limit_left := limit(f(x), x = p, left);
    limit_right := limit(f(x), x = p, right);
    if limit_left ≠ limit_right then
      average := (limit_left + limit_right) / 2;
      discont_points := [op(discont_points), [x_2·n + p, average]];
    end if;
  end do;
end do;

plots[display](
  plot(S_20000, x = x_range, legend = "S", color = "Blue"),
  plots[pointplot](discont_points,  color = red, legend = "Discontinuities")
 );

plots[animatecurve]({S(1), S(3), S(7), f(x)}, x = x_range, frames = 50);
```

$$\begin{cases} x+2 & 0 < x < 2 \\ -1 & 2 \le x \le 5 \end{cases}$$



$$a0 := \frac{6}{5}$$

$$an := \frac{5\left(2\,\pi\,n\sim\sin\left(\frac{4\,\pi\,n\sim}{5}\right)+\cos\left(\frac{4\,\pi\,n\sim}{5}\right)-1\right)}{2\,\pi^2\,n\sim^2}$$

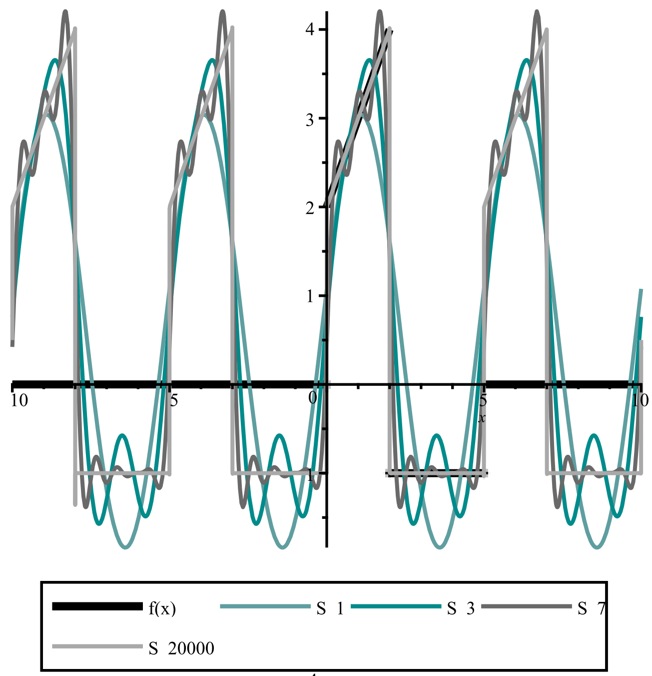
$$bn := \frac{-10\,\pi\,n\sim\cos\left(\frac{4\,\pi\,n\sim}{5}\right)+6\,\pi\,n\sim+5\sin\left(\frac{4\,\pi\,n\sim}{5}\right)}{2\,\pi^2\,n\sim^2}$$

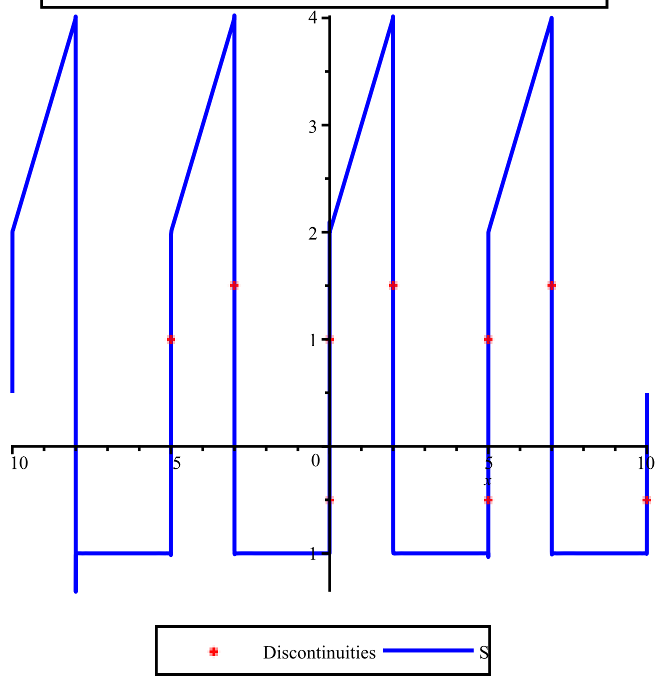
$$Sx := \frac{3}{5}+\sum_{n\sim=1}^{\infty}\left(\frac{5\left(2\,\pi\,n\sim\sin\left(\frac{4\,\pi\,n\sim}{5}\right)+\cos\left(\frac{4\,\pi\,n\sim}{5}\right)-1\right)\cos\left(\frac{2\,\pi\,n\sim\,x}{5}\right)}{2\,\pi^2\,n\sim^2}\right.$$
$$\left.+\frac{\left(-10\,\pi\,n\sim\cos\left(\frac{4\,\pi\,n\sim}{5}\right)+6\,\pi\,n\sim+5\sin\left(\frac{4\,\pi\,n\sim}{5}\right)\right)\sin\left(\frac{2\,\pi\,n\sim\,x}{5}\right)}{2\,\pi^2\,n\sim^2}\right)$$

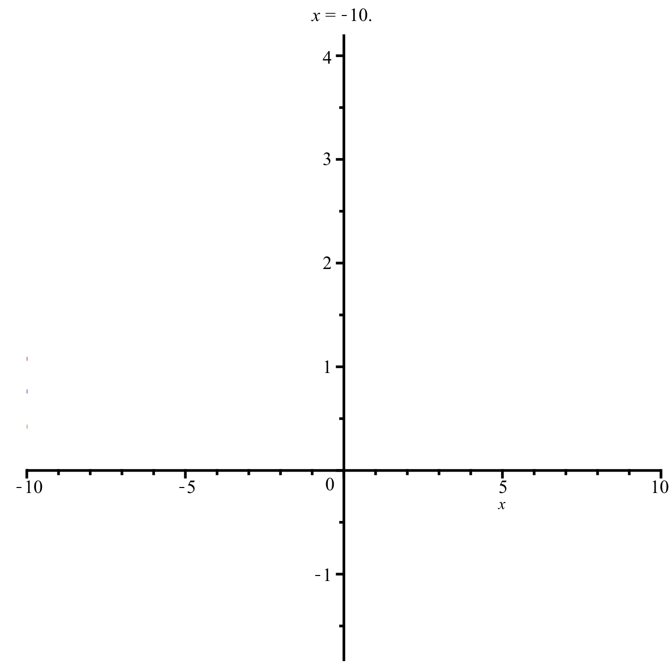
$$S\_1 := \frac{3}{5}+\frac{5\left(2\,\pi\sin\left(\frac{\pi}{5}\right)-\cos\left(\frac{\pi}{5}\right)-1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$
$$+\frac{\left(10\,\pi\cos\left(\frac{\pi}{5}\right)+6\,\pi+5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$

$$S\_3 := \frac{3}{5}+\frac{5\left(2\,\pi\sin\left(\frac{\pi}{5}\right)-\cos\left(\frac{\pi}{5}\right)-1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$
$$+\frac{\left(10\,\pi\cos\left(\frac{\pi}{5}\right)+6\,\pi+5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$
$$+\frac{5\left(-4\,\pi\sin\left(\frac{2\,\pi}{5}\right)+\cos\left(\frac{2\,\pi}{5}\right)-1\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2}$$
$$+\frac{\left(-20\,\pi\cos\left(\frac{2\,\pi}{5}\right)+12\,\pi-5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2}$$
$$+\frac{5\left(6\,\pi\sin\left(\frac{2\,\pi}{5}\right)+\cos\left(\frac{2\,\pi}{5}\right)-1\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$
$$+\frac{\left(-30\,\pi\cos\left(\frac{2\,\pi}{5}\right)+18\,\pi+5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$

$$S\_7 := \frac{3}{5} + \frac{5\left(2\,\pi \sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$
$$+ \frac{\left(10\,\pi\cos\left(\frac{\pi}{5}\right) + 6\,\pi + 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2}$$
$$+ \frac{5\left(-4\,\pi\sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2}$$
$$+ \frac{\left(-20\,\pi\cos\left(\frac{2\,\pi}{5}\right) + 12\,\pi - 5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2}$$
$$+ \frac{5\left(6\,\pi\sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$
$$+ \frac{\left(-30\,\pi\cos\left(\frac{2\,\pi}{5}\right) + 18\,\pi + 5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$
$$+ \frac{5\left(-8\,\pi\sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{8\,\pi\,x}{5}\right)}{32\,\pi^2}$$
$$+ \frac{\left(40\,\pi\cos\left(\frac{\pi}{5}\right) + 24\,\pi - 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{8\,\pi\,x}{5}\right)}{32\,\pi^2} - \frac{2\sin(2\,\pi\,x)}{5\,\pi}$$
$$+ \frac{5\left(12\,\pi\sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{12\,\pi\,x}{5}\right)}{72\,\pi^2}$$
$$+ \frac{\left(60\,\pi\cos\left(\frac{\pi}{5}\right) + 36\,\pi + 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{12\,\pi\,x}{5}\right)}{72\,\pi^2}$$
$$+ \frac{5\left(-14\,\pi\sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{14\,\pi\,x}{5}\right)}{98\,\pi^2}$$
$$+ \frac{\left(-70\,\pi\cos\left(\frac{2\,\pi}{5}\right) + 42\,\pi - 5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{14\,\pi\,x}{5}\right)}{98\,\pi^2}$$


f(x)
S 1
S 3
S 7
S 20000
x


Discontinuities
S
x

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> # Задание 3. Построить три разложения в тригонометрический ряд Фурье функции,
      считая что она определена : на полном периоде, на полупериоде (четная),
      на полупериоде (нечетная). Сравнить результат с порождающей функцией.
  f := x→piecewise(0 < x ≤ 2, x^2 − 2·x + 1, 2 < x < 3, 3 − x);
  plot(f(x), x = 0 ..3, discont = true);

  #На полном периоде
  coeff_and_Sm := FourierTrigSum(f, ∞, 0, 3) :
  a0 := coeff_and_Sm[1];
  an := coeff_and_Sm[2];
  bn := coeff_and_Sm[3];
  S := coeff_and_Sm[4] :
  Sx := coeff_and_Sm[5];

   x_range := -9 ..9 :
  plot(S(20000), x = x_range,
              legend = "S_20000",
              color = "DarkGray") :
  plot(f(x), x = x_range,
             legend = "f(x)", discont = true,
             color = black, thickness = 3) :
  plots[display](%, %%);

  list_of_disconts := Statistics[Sort](convert(discont(f(x), x), list)) :
  discont_points := [ ] :

  for n from -1 to 1 do
     for p in list_of_disconts do
        limit_left := limit(f(x), x = p, left);
        limit_right := limit(f(x), x = p, right);
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      average := (limit_left + limit_right)/2;
      discont_points := [op(discont_points), [3·n + p, average]];
   end do;
end do;

plots[display](
   plot(S(20000), x=x_range, legend="S", color="Blue"),
   plots[pointplot](discont_points,  color=red, legend="Discontinuities")
 );

#На полупериоде (четный способ)
f_even := x→piecewise(0 ≤ x ≤ 3, f(x), -3 ≤ x < 0, f(-x));
plot(f_even(x), x=-3..3);
coeff_and_Sm := FourierTrigSum(f_even, ∞, -3, 3) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];

x_range := -9..9 :
plot(S(20000), x=x_range,
          legend="S_20000",
          color="DarkGray") :
plot(f(x), x=x_range,
         legend="f(x)", discont=true,
         color=black, thickness=3) :
plots[display](%, %%);

list_of_disconts := Statistics[Sort](convert(discont(f(x), x), list)) :
discont_points := [ ] :

for n from -1 to 1 do
   for p in list_of_disconts do
      limit_left := limit(f(x), x=p, left);
      limit_right := limit(f(x), x=p, right);
      average := (limit_left + limit_right)/2;
      discont_points := [op(discont_points), [2·3·n + p, average]];
   end do;
end do;

plots[display](
   plot(S(20000), x=x_range, legend="S", color="Blue"),
   plots[pointplot](discont_points,  color=red, legend="Discontinuities")
 );
```
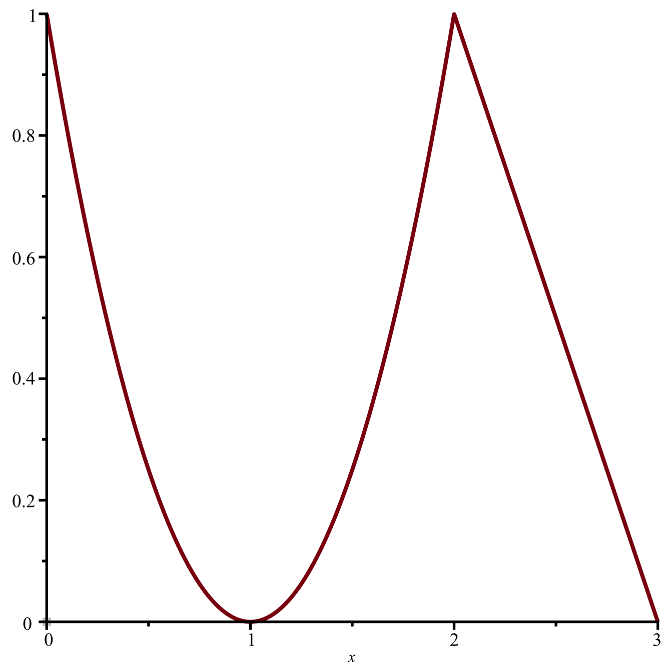

```
#На полупериоде (нечетный способ)
f_odd := x→piecewise(0 ≤ x ≤ 3, f(x), -3 ≤ x < 0, -f(-x));
plot(f_odd(x), x=-3..3);
coeff_and_Sm := FourierTrigSum(f_odd, ∞, -3, 3) :
a0 := coeff_and_Sm[1];
an := coeff_and_Sm[2];
bn := coeff_and_Sm[3];
S := coeff_and_Sm[4] :
Sx := coeff_and_Sm[5];

x_range := -9..9 :
plot(S(20000), x=x_range,
        legend="S_20000",
        color="DarkGray") :
plot(f(x), x=x_range,
        legend="f(x)", discont=true,
        color=black, thickness=3) :
plots[display](%, %%);

list_of_disconts := Statistics[Sort](convert(discont(f(x), x), list)) :
discont_points := [ ] :

for n from -2 to 2 do
  for p in list_of_disconts do
    limit_left := limit(f(x), x=p, left);
    limit_right := limit(f(x), x=p, right);
    average := (limit_left + limit_right)/2;
    discont_points := [op(discont_points), [2·3·n + p, average]];
  end do;
end do;

plots[display](
  plot(S(20000), x=x_range, legend="S", color="Blue"),
  plots[pointplot](discont_points, color=red, legend="Discontinuities")
 );
```

$$f := x \mapsto \begin{cases} x^2 - 2\cdot x + 1 & 0 < x \le 2 \\ 3 - x & 2 < x < 3 \end{cases}$$

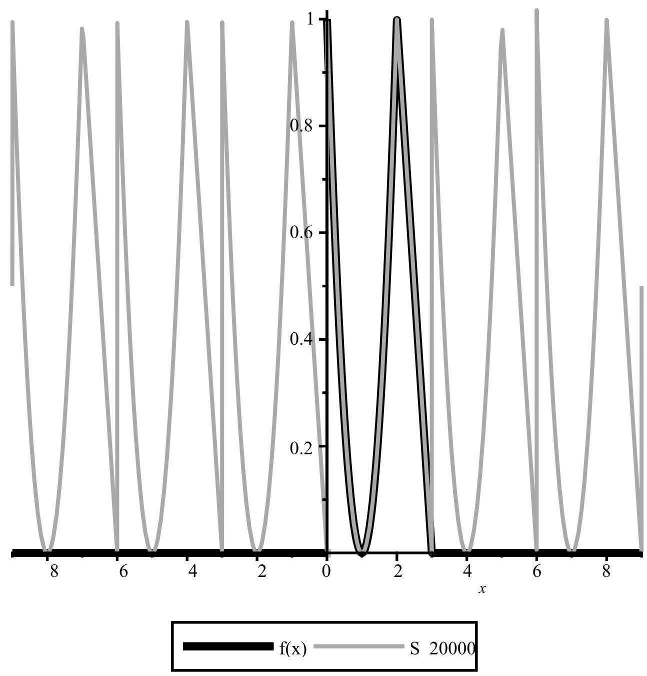


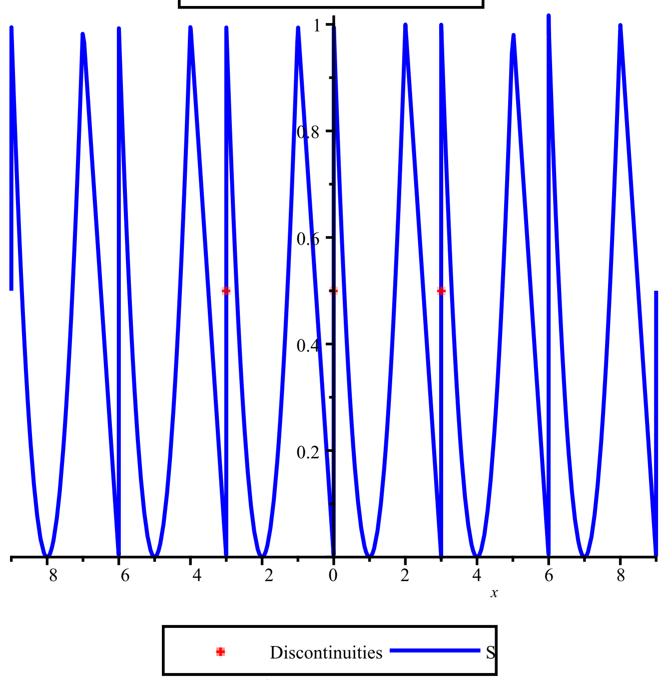
$$a0 := \frac{7}{9}$$

$$an := \frac{9\,\pi\,n{\sim}\cos\left(\frac{4\,\pi\,n{\sim}}{3}\right) + 3\,\pi\,n{\sim} - 9\sin\left(\frac{4\,\pi\,n{\sim}}{3}\right)}{2\,\pi^3\,n{\sim}^3}$$

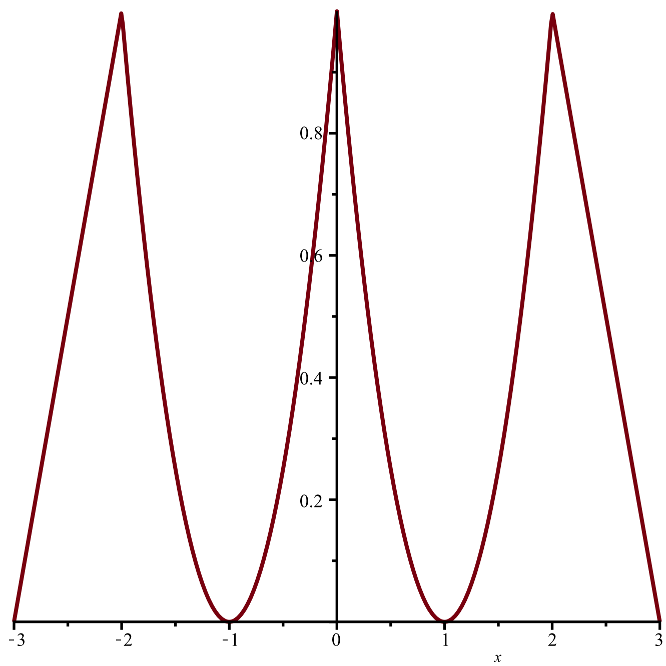
$$bn := \frac{2\,\pi^2\,n{\sim}^2 + 9\,\pi\,n{\sim}\sin\left(\frac{4\,\pi\,n{\sim}}{3}\right) + 9\cos\left(\frac{4\,\pi\,n{\sim}}{3}\right) - 9}{2\,\pi^3\,n{\sim}^3}$$

$$Sx := \frac{7}{18} + \sum_{n{\sim}=1}^{\infty}\left(\frac{\left(9\,\pi\,n{\sim}\cos\left(\frac{4\,\pi\,n{\sim}}{3}\right) + 3\,\pi\,n{\sim} - 9\sin\left(\frac{4\,\pi\,n{\sim}}{3}\right)\right)\cos\left(\frac{2\,\pi\,n{\sim}\,x}{3}\right)}{2\,\pi^3\,n{\sim}^3}\right.$$
$$\left. + \frac{\left(2\,\pi^2\,n{\sim}^2 + 9\,\pi\,n{\sim}\sin\left(\frac{4\,\pi\,n{\sim}}{3}\right) + 9\cos\left(\frac{4\,\pi\,n{\sim}}{3}\right) - 9\right)\sin\left(\frac{2\,\pi\,n{\sim}\,x}{3}\right)}{2\,\pi^3\,n{\sim}^3}\right)$$





$$f\_even := x \mapsto \begin{cases} f(x) & 0 \le x \le 3 \\ f(\;\;x) & -3 \le x < 0 \end{cases}$$

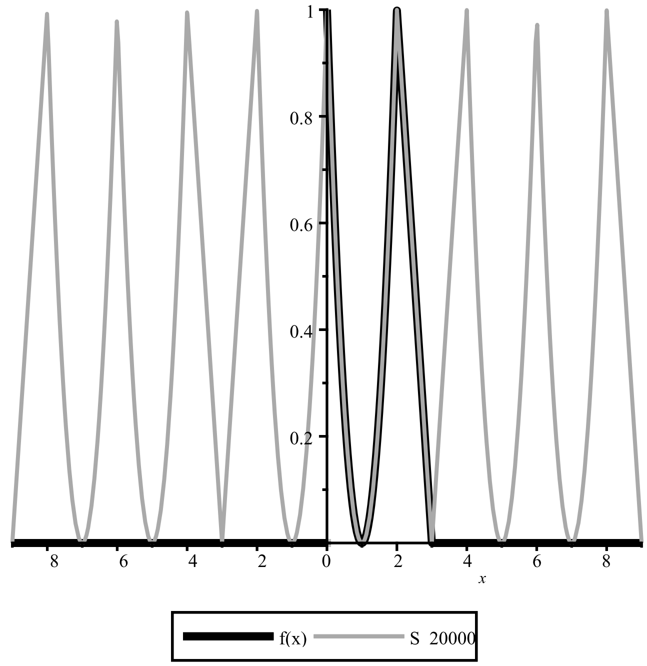


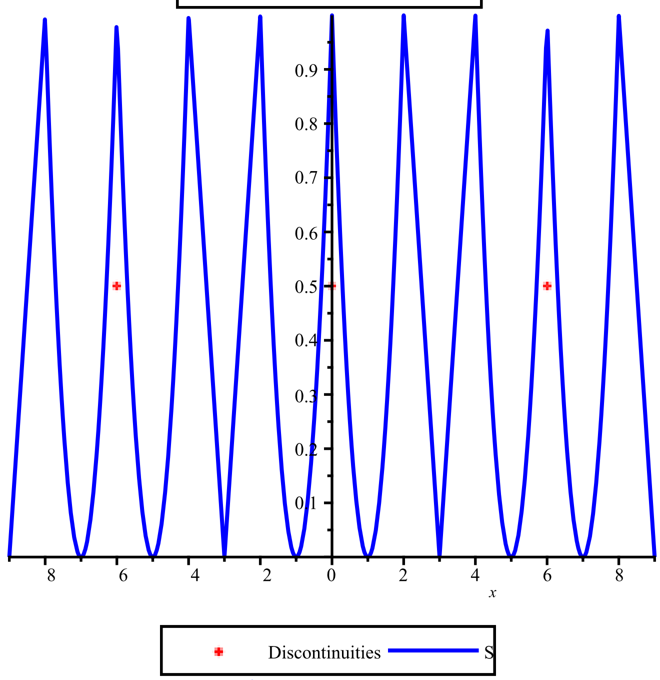
$$a0 := \frac{7}{9}$$

$$an := \frac{18\,\pi\, n\sim \cos\left(\frac{2\,\pi\, n\sim}{3}\right) - 6\,\pi\,(-1)^{n\sim}\, n\sim + 12\,\pi\, n\sim - 36\sin\left(\frac{2\,\pi\, n\sim}{3}\right)}{\pi^3\, n\sim^3}$$

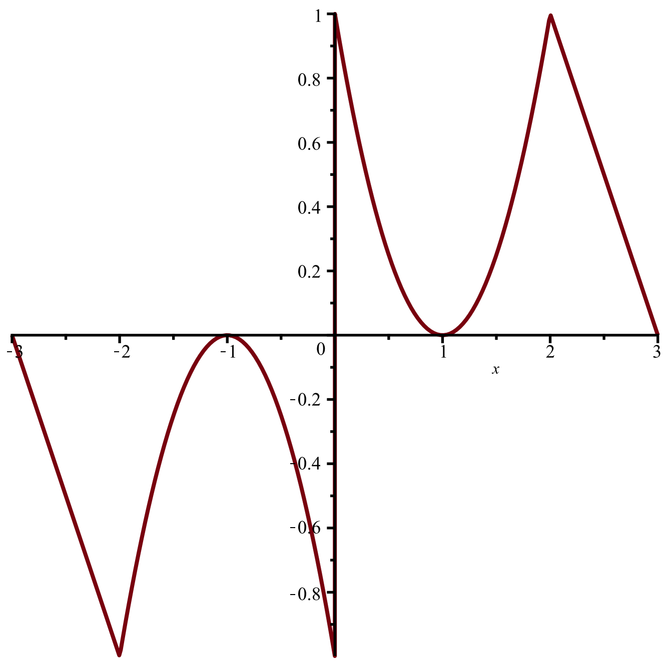
$$bn := 0$$

$$Sx := \frac{7}{18} + \left(\sum_{n\sim=1}^{\infty} \frac{1}{\pi^3\, n\sim^3}\left(\left(18\,\pi\, n\sim \cos\left(\frac{2\,\pi\, n\sim}{3}\right) - 6\,\pi\,(-1)^{n\sim}\, n\sim + 12\,\pi\, n\sim - 36\sin\left(\frac{2\,\pi\, n\sim}{3}\right)\right)\cos\left(\frac{\pi\, n\sim\, x}{3}\right)\right)\right)$$





$$f\_odd := x \mapsto \begin{cases} f(x) & 0 \le x \le 3 \\ f(\ \ x) & -3 \le x < 0 \end{cases}$$

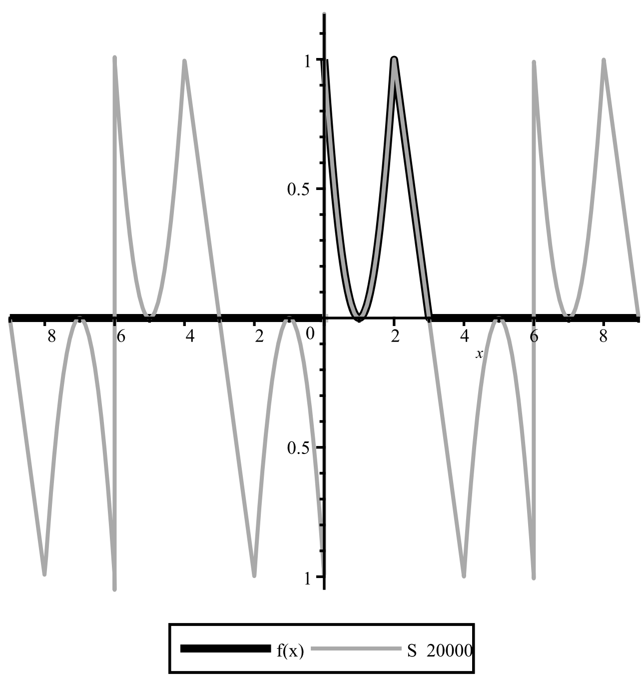


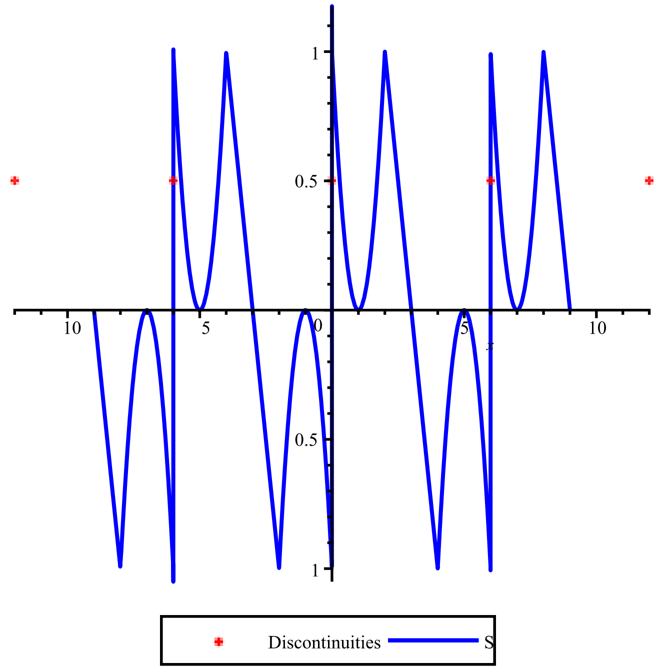
$$a0 := 0$$

$$an := 0$$

$$bn := \frac{2\,\pi^2\,n\sim^2 + 18\,\pi\,n\sim \sin\left(\frac{2\,\pi\,n\sim}{3}\right) + 36\cos\left(\frac{2\,\pi\,n\sim}{3}\right) - 36}{\pi^3\,n\sim^3}$$

$$Sx := \sum_{n\sim=1}^{\infty} \frac{\left(2\,\pi^2\,n\sim^2 + 18\,\pi\,n\sim \sin\left(\frac{2\,\pi\,n\sim}{3}\right) + 36\cos\left(\frac{2\,\pi\,n\sim}{3}\right) - 36\right)\sin\left(\frac{\pi\,n\sim\,x}{3}\right)}{\pi^3\,n\sim^3}$$





> #Задание 4.
#***Разложить функцию* в *ряд Фурье по многочленам Лежандра* и *Чебышёва на промежутке* [1, 1],**
***экспериментально найти наименьший порядок частичных сумм равномерно аппроксимирующей на всем промежутке заданную функцию* с *точностью* 0, 1.**

$f := x \rightarrow (\sin(2\,x))^3;$
$plot(f(x), x = \ \ 1..1);$

$aproxis := plot([f(x) + 0.1, f(x) \quad 0.1], x = \ \ 1..1, linestyle = dash, color = blue) :$

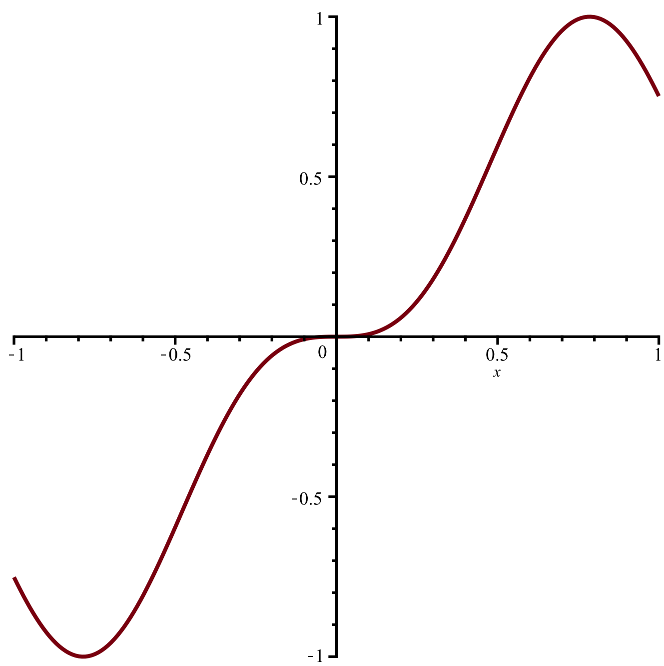
$with(orthopoly) :$
#**Полином** *Лежандра*
$LegendrePolynom :=$ **proc**(f, k, a, b)
local $c\_n, n;$
$$c\_n := \frac{int(f(x) \cdot P(n, x), x = a..b)}{int\left(P(n, x)^2, x = a..b\right)};$$
return $sum(c\_n \cdot P(n, x), n = 0..k);$
end proc:

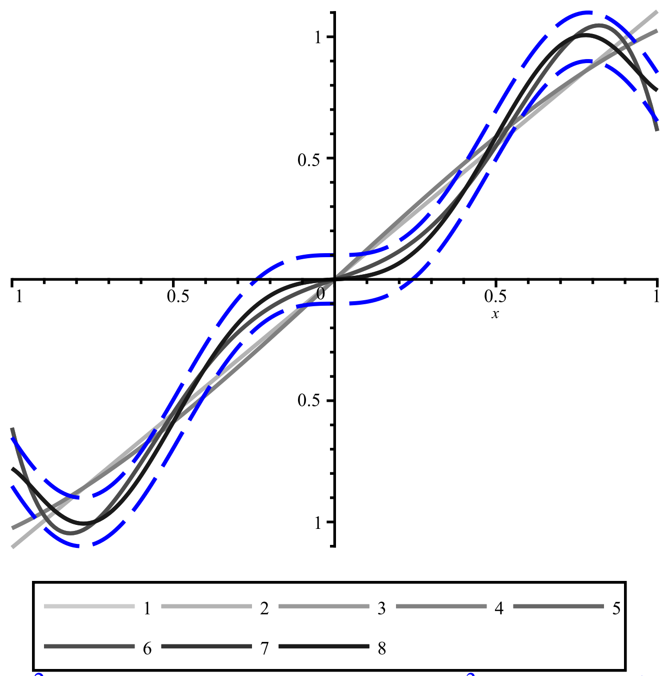
$seq(plot(LegendrePolynom(f, k, \ \ 1, 1), x = \ \ 1..1, color = COLOUR(RGB, 0.9 \quad 0.1 \cdot k, 0.9 \quad 0.1 \cdot k, 0.9 \quad 0.1 \cdot k), legend = k), k = 1..8) :$
$plots[display](\%, aproxis);$
$S\_Suff1 := LegendrePolynom(f, 7, \ \ 1, 1);$ **#*n* = 7,**
это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке **[1, 1]** ***функцию* с *точностью* 0, 1 *для полинома Лежандра***
plot(f(x), x = 1 ..1, color = black, legend = "f(x)") :
plot(S_Suff1, x = 1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);

#Полином Чебышева

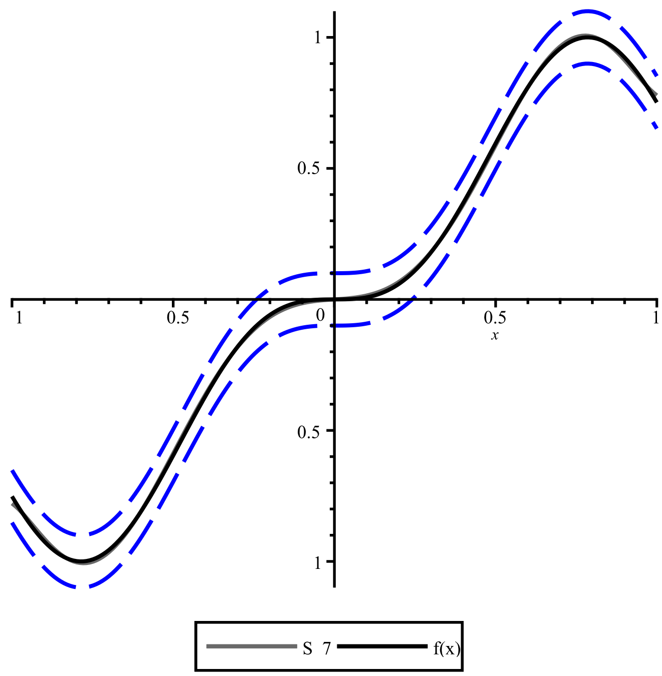
```
ChebyshevPolynom := proc(f, k, a, b)
  local c_n, c_0, n;
  c_n := 2/Pi·int(f(x)·T(n, x)/sqrt(1 − x^2), x = a..b);
  c_0 := 2/Pi·int(f(x)·T(0, x)/sqrt(1 − x^2), x = a..b);
  return c_0/2 + sum(c_n·T(n, x), n = 1..k);
end proc:
seq(plot(ChebyshevPolynom(f, k, -1, 1), x = -1..1, color = COLOUR(RGB, 0.9 − 0.1·k, 0.9
    − 0.1·k, 0.9 - 0.1·k), legend = k), k = 1..8) :
plots[display](%, aproxis);
S_Suff2 := ChebyshevPolynom(f, 7, -1, 1); #n = 7,
    это наименьший порядок частичной суммы равномерно аппроксимирующей на
    промежутке [ -1, 1] функцию с точностью 0, 1 для полинома Чебышева
plot(f(x), x = -1..1, color = black, legend = "f(x)") :
plot(S_Suff2, x = -1..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);
```

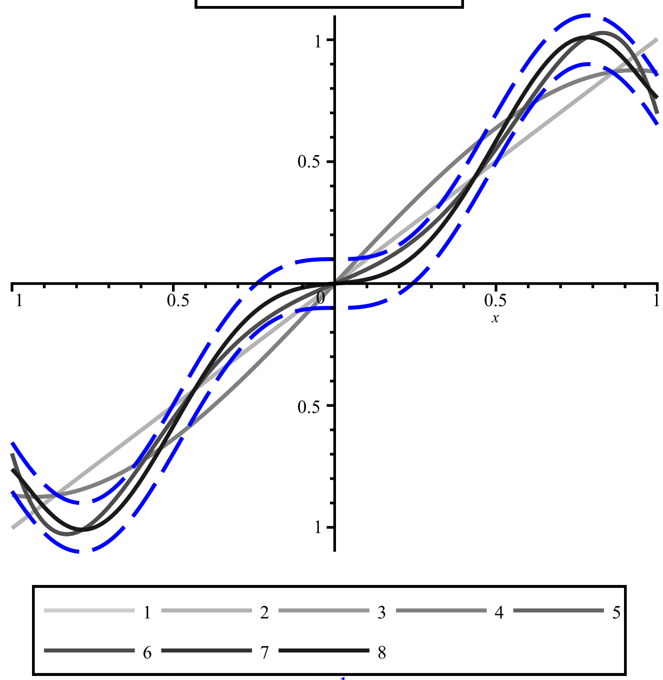
$$f := x \mapsto \sin(2 \cdot x)^3$$





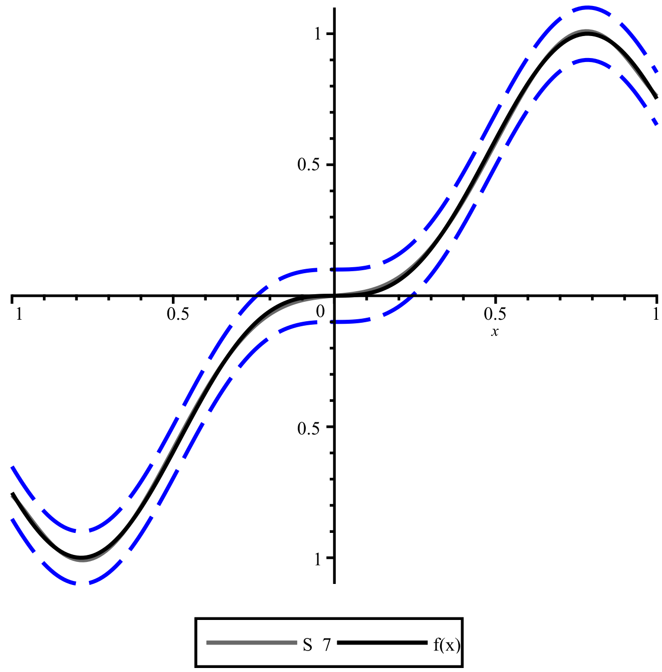
$$
\begin{aligned}
S\_Suff1 := {} & \frac{3\left(\;\frac{\sin(2)^2\cos(2)}{3}\quad\frac{2\cos(2)}{3}+\frac{\sin(2)^3}{18}+\frac{\sin(2)}{3}\right)x}{2} \\
& + \frac{1}{2}\left(7\left(\;\frac{7\sin(2)^2\cos(2)}{36}+\frac{19\cos(2)}{9}+\frac{11\sin(2)}{18}+\frac{67\sin(2)^3}{216}\right)\left(\frac{5}{2}x^3\quad\frac{3}{2}x\right)\right) + \frac{1}{2}\left(11\left(\frac{19\sin(2)^2\cos(2)}{48}+\frac{65\sin(2)^3}{288}\quad\frac{565\sin(2)}{48}\quad\frac{611\cos(2)}{24}\right)\left(\frac{63}{8}x^5\quad\frac{35}{4}x^3+\frac{15}{8}x\right)\right) \\
& + \frac{1}{2}\left(15\left(\;\frac{1679\sin(2)^2\cos(2)}{5184}+\frac{499961\sin(2)}{1296}+\frac{136357\cos(2)}{162}\quad\frac{24661\sin(2)^3}{31104}\right)\left(\frac{429}{16}x^7\quad\frac{693}{16}x^5+\frac{315}{16}x^3\quad\frac{35}{16}x\right)\right)
\end{aligned}
$$





$$S\_Suff2 := \frac{2 \left(\int_{-1}^{1} \frac{\sin(2\,x)^3\, x}{\sqrt{x^2+1}} \, \mathrm{d}x \right) x}{\pi} + \frac{2 \left(\int_{-1}^{1} \frac{\sin(2\,x)^3 \left(4\,x^3 \quad 3\,x\right)}{\sqrt{x^2+1}} \, \mathrm{d}x \right) \left(4\,x^3 \quad 3\,x\right)}{\pi}$$
$$+ \frac{2 \left(\int_{-1}^{1} \frac{\sin(2\,x)^3 \left(16\,x^5 \quad 20\,x^3 + 5\,x\right)}{\sqrt{x^2+1}} \, \mathrm{d}x \right) \left(16\,x^5 \quad 20\,x^3 + 5\,x\right)}{\pi}$$

$$+\frac{1}{\pi}\left(2\left(\int_{-1}^{1}\frac{\sin(2x)^3\,(64x^7-112x^5+56x^3-7x)}{\sqrt{-x^2+1}}\,dx\right)(64x^7-112x^5+56x^3-7x)\right)$$



> *#Разложить функцию в тригонометрический и степенной ряды на промежутке* [**1, 1**] *, Найти наименьший порядок частичных суммы, равномерно аппроксимирующих на всем промежутке заданную функцию с точностью 0, 1.*

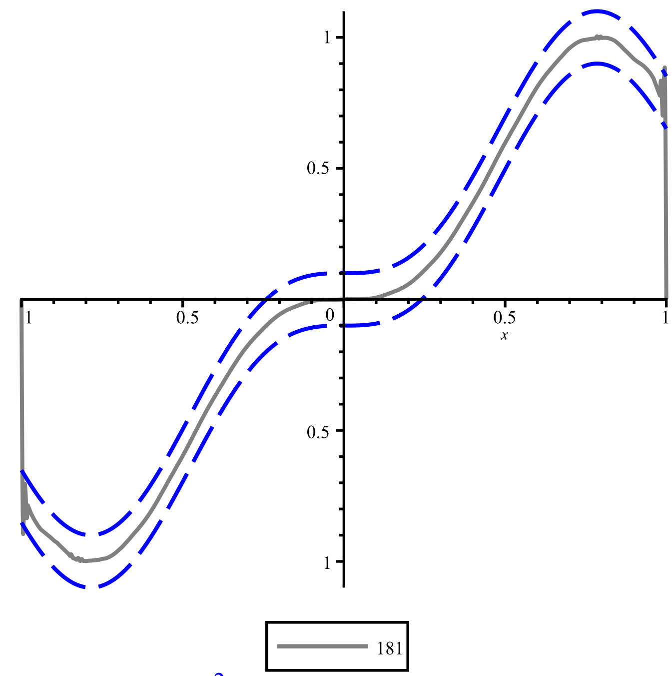
$seq(plot(FourierTrigSum(f, k+180,\ \ 1, 1)[5], x=\ \ 1..1, color=COLOUR(RGB, 0.9\ \ \ 0.4\cdot k, 0.9\ \ \ 0.4\cdot k, 0.9\ \ \ 0.4\cdot k), legend=k+180), k=1..1):$

$plots[display](\%, aproxis);$

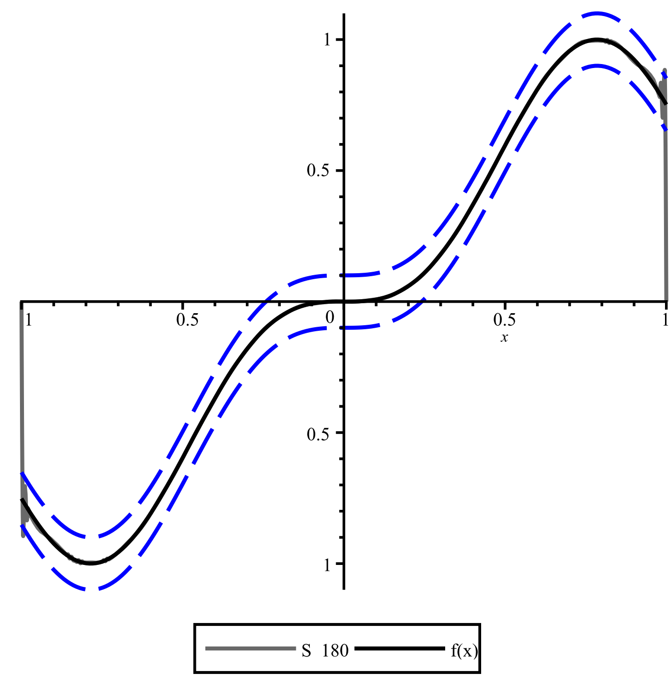
S_Suff3 := FourierTrigSum(f, 180,   1, 1) [5] :**#*n* = 180, *это порядок частичной суммы почти равномерно аппроксимирующей на промежутке* [1, 1] *функцию* с *точностью* 0, 1 *для Тригонометрического ряда Фурье***

FourierTrigSum(f, 180,   1, 1) [1 ..3];

plot(f(x), x =   1 ..1, color = black, legend = "f(x)") :
plot(S_Suff3, x =   1 ..1, color = "DimGray", legend = "S_180") :
plots[display](%, %%, aproxis);



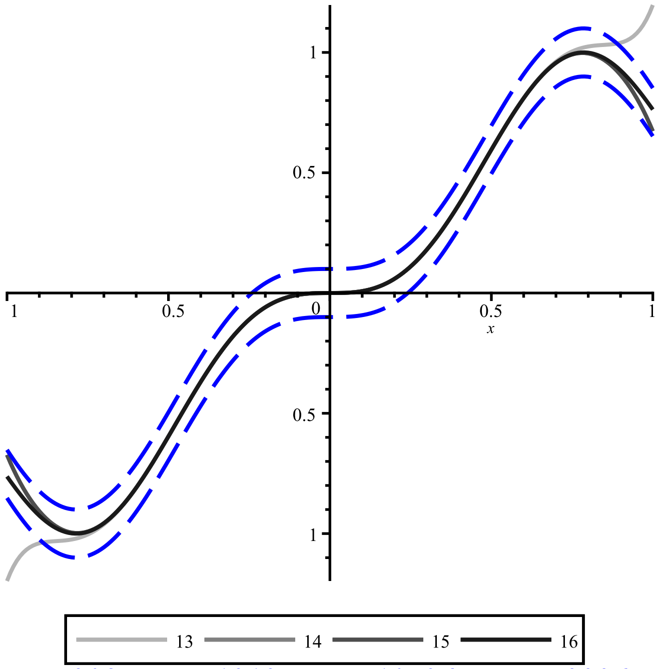
$$0,\ 0,\quad \frac{3\,n\sim\left(\pi^{2}\sin(2)\,n\sim^{2}\quad \dfrac{\pi^{2}\sin(6)\,n\sim^{2}}{3}\quad 36\sin(2)+\dfrac{4\sin(6)}{3}\right)\pi\,(-1)^{n\sim}}{2\,\pi^{4}\,n\sim^{4}\quad 80\,\pi^{2}\,n\sim^{2}+288}$$



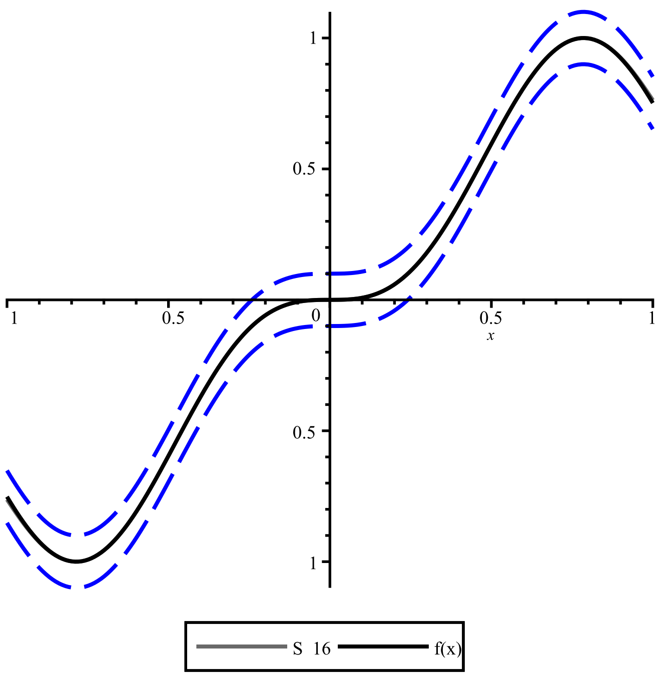
> *seq*(*plot*(*convert*(*taylor*($f(x)$, $x=0$, $k+12$), *polynom*), $x=\ \ 1..1$, *color* = *COLOUR*(*RGB*, 0.9 $\ \ 0.2\cdot k$, 0.9 $\ \ 0.2\cdot k$, 0.9 $\ \ 0.2\cdot k$), *legend* = $k+12$), $k=1..4$) :
plots[*display*](%, *aproxis*);

S_Suff4 := *convert*(*taylor*($f(x)$, $x=0$, 16), *polynom*) ;
#n = 16, это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке [1, 1] функцию с точностью 0, 1 для степенного ряда (Маклорена)

plot(f(x), x =- 1 ..1, color = black, legend = "f(x)") :
plot(S_Suff4, x =- 1 ..1, color = "DimGray", legend = "S_16") :
plots[display](%, %%, aproxis);



$S\_Suff4 := 8\,x^3 \quad 16\,x^5 + \frac{208}{15}\,x^7 \quad \frac{1312}{189}\,x^9 + \frac{10736}{4725}\,x^{11} \quad \frac{2336}{4455}\,x^{13} + \frac{19131872}{212837625}\,x^{15}$



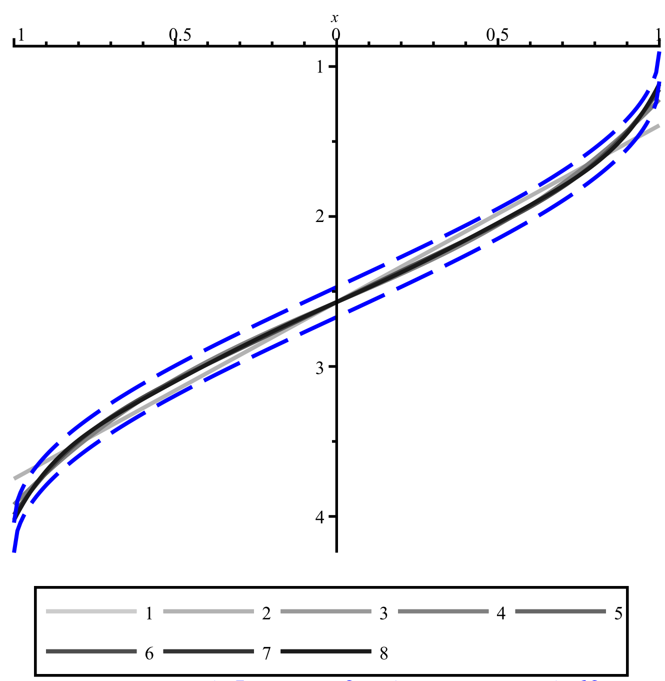
> $f := x \rightarrow \ \arccos(x) \quad 1;$
$plot(f(x), x = \ 1 ..1);$

$aproxis := plot([f(x) + 0.1, f(x) \quad 0.1], x = \ 1 ..1, linestyle = dash, color = blue) :$

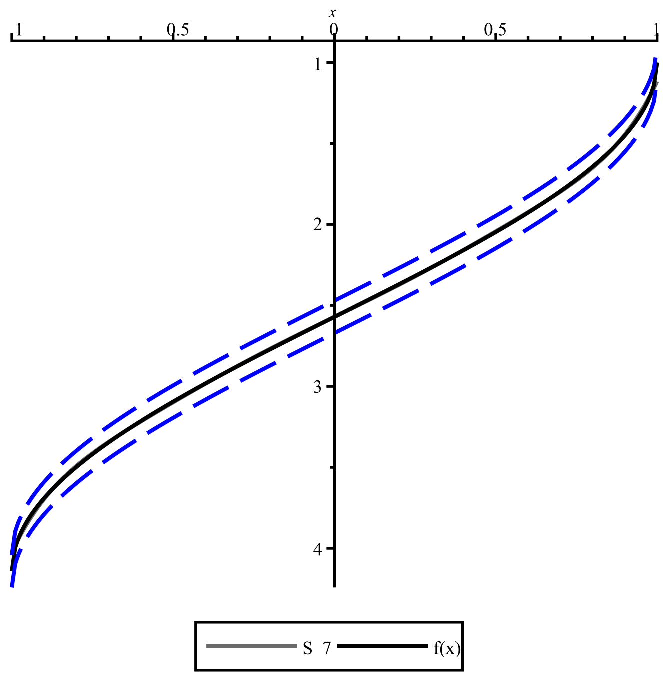
$seq(plot(LegendrePolynom(f, k, \ 1, 1), x = \ 1 ..1, color = COLOUR(RGB, 0.9 \quad 0.1 \cdot k, 0.9$

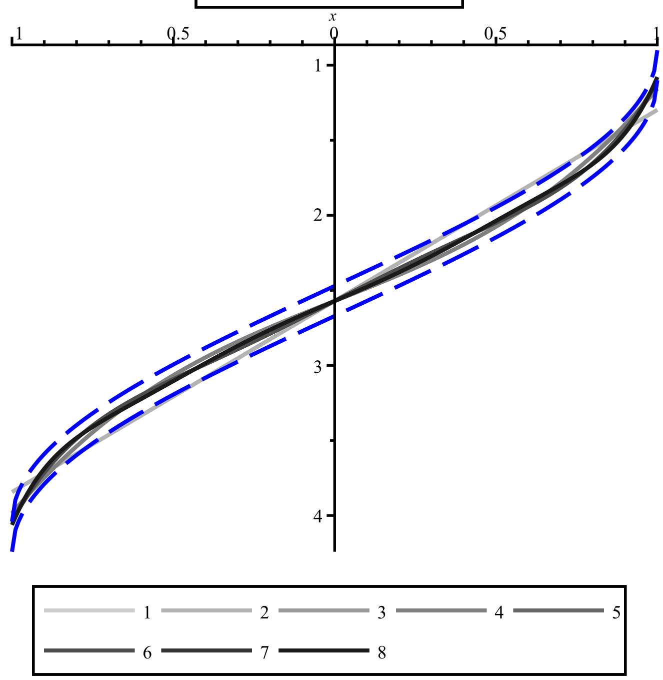
− 0.1·*k*, 0.9 − 0.1·*k*), *legend* = *k*), *k* = 1 ..8) :
plots[*display*](%, *aproxis*);
S_Suff1 ≔ *LegendrePolynom*(f, 7, −1, 1); **#*n* = 7,**
это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке* [−1, 1] *функцию* с *точностью* 0, 1 *для полинома Лежандра
plot(f(x), x = −1 ..1, color = black, legend = "f(x)") :
plot(S_Suff1, x = −1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);
seq(*plot*(*ChebyshevPolynom*(*f*, *k*, −1, 1), *x* = −1 ..1, *color* = *COLOUR*(*RGB*, 0.9 − 0.1·*k*, 0.9 − 0.1·*k*, 0.9 − 0.1·*k*), *legend* = *k*), *k* = 1 ..8) :
plots[*display*](%, *aproxis*);
S_Suff2 ≔ ChebyshevPolynom(f, 7, −1, 1); **#*n* = *7*,**
это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке* [−*1*, *1*] *функцию с точностью 0*, *1 для полинома Чебышева
plot(f(x), x = −1 ..1, color = black, legend = "f(x)") :
plot(S_Suff2, x = −1 ..1, color = "DimGray", legend = "S_7") :
plots[display](%, %%, aproxis);

$$f := x \mapsto -\arccos(x) - 1$$

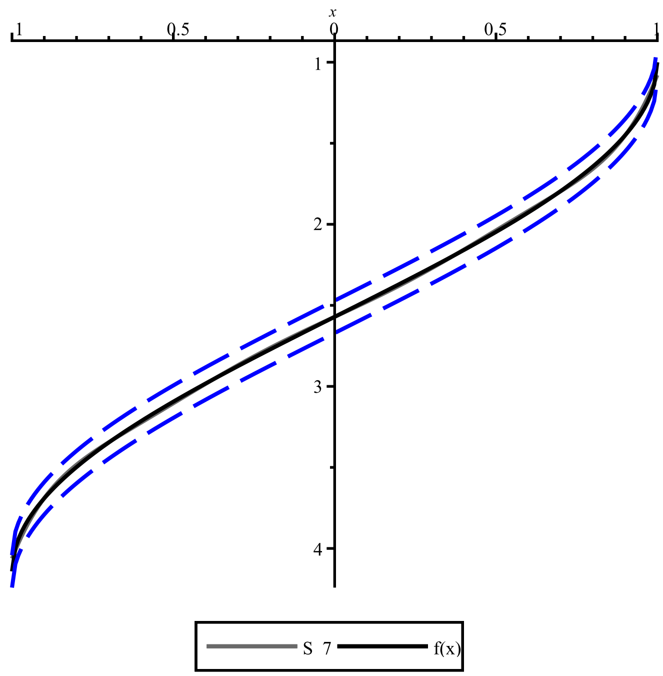


$$S\_Suff1 := 1 \quad \frac{\pi}{2} + \frac{3\,\pi\,x}{8} + \frac{7\,\pi\left(\frac{5}{2}\,x^3 \quad \frac{3}{2}\,x\right)}{128} + \frac{11\,\pi\left(\frac{63}{8}\,x^5 \quad \frac{35}{4}\,x^3 + \frac{15}{8}\,x\right)}{512}$$
$$+ \frac{375\,\pi\left(\frac{429}{16}\,x^7 \quad \frac{693}{16}\,x^5 + \frac{315}{16}\,x^3 \quad \frac{35}{16}\,x\right)}{32768}$$





$$S\_Suff2 := \frac{\frac{1}{2}\,\pi^2 \quad \pi}{\pi} + \frac{4\,x}{\pi} + \frac{4\,\left(4\,x^3 \quad 3\,x\right)}{9\,\pi} + \frac{4\,\left(16\,x^5 \quad 20\,x^3 + 5\,x\right)}{25\,\pi}$$
$$+ \frac{4\,\left(64\,x^7 \quad 112\,x^5 + 56\,x^3 \quad 7\,x\right)}{49\,\pi}$$

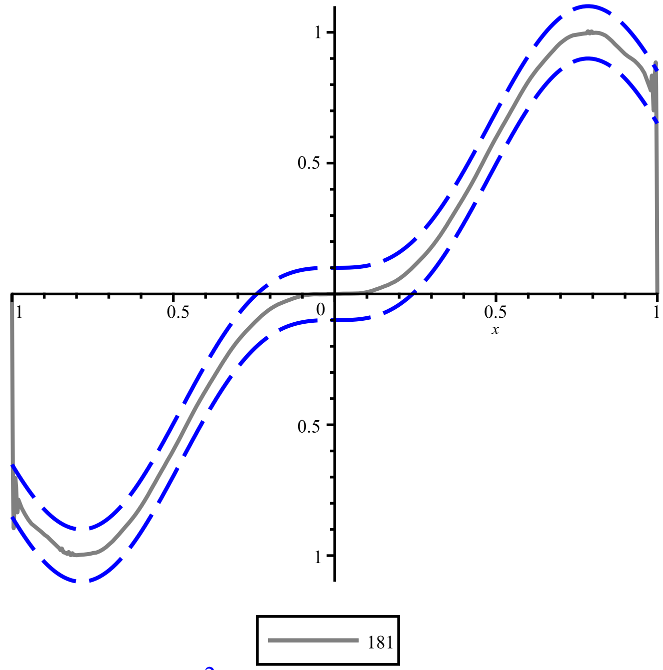


> ***#Разложить функцию в тригонометрический и степенной ряды на промежутке** [**1, 1**] **, Найти наименьший порядок частичных суммы, равномерно аппроксимирующих на всем промежутке заданную функцию с точностью 0, 1.***

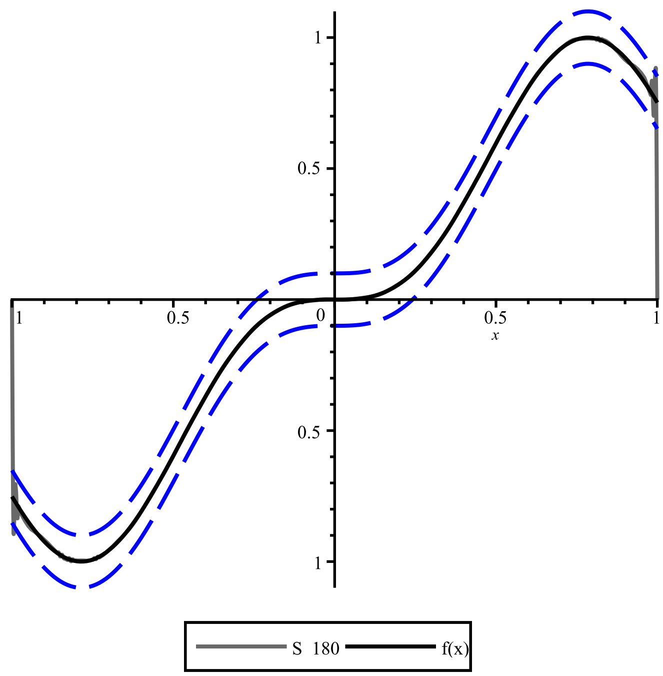
seq(*plot*(*FourierTrigSum*(*f*, *k* + 180, 1, 1) [5], *x* = 1 ..1, *color* = *COLOUR*(*RGB*, 0.9 0.4·*k*, 0.9 0.4·*k*, 0.9 0.4·*k*), *legend* = *k* + 180), *k* = 1 ..1) :
plots[*display*](%, *aproxis*);

S_Suff3 ≔ FourierTrigSum(f, 180, 1, 1) [5] : **#*n* = 180, *это порядок частичной суммы почти равномерно аппроксимирующей на промежутке* [1, 1] *функцию* с *точностью* 0, 1 *для Тригонометрического ряда Фурье***
FourierTrigSum(f, 180, 1, 1) [1 ..3];

plot(f(x), x = 1 ..1, color = black, legend = "f(x)") :
plot(S_Suff3, x = 1 ..1, color = "DimGray", legend = "S_180") :
plots[display](%, %%, aproxis);



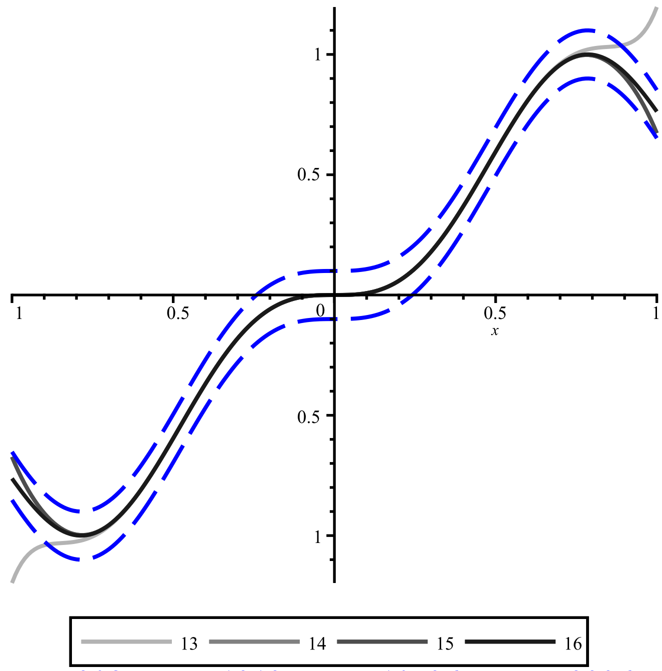
$$0,\ 0,\ \frac{3\, n\sim \left(\pi^{2} \sin(2)\, n\sim^{2} \quad \frac{\pi^{2} \sin(6)\, n\sim^{2}}{3} \quad 36 \sin(2) + \frac{4 \sin(6)}{3} \right) \pi\, (-1)^{n\sim}}{2\, \pi^{4}\, n\sim^{4} \quad 80\, \pi^{2}\, n\sim^{2} + 288}$$



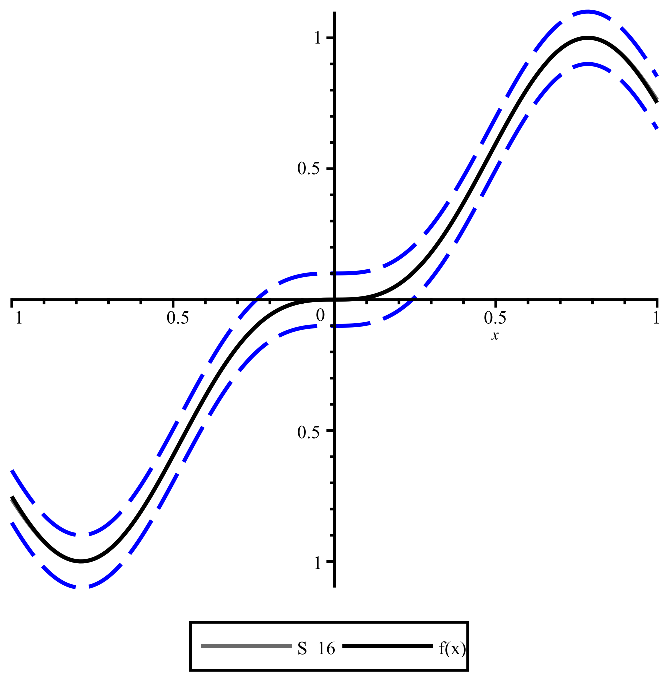
> *seq*(*plot*(*convert*(*taylor*($f(x)$, $x=0$, $k+12$), *polynom*), $x =\ 1\,..1$, *color* = *COLOUR*(*RGB*, 0.9 $\quad$ $0.2 \cdot k$, 0.9 $\quad$ $0.2 \cdot k$, 0.9 $\quad$ $0.2 \cdot k$), *legend* = $k+12$), $k = 1\,..4$) :
plots[*display*](%, *aproxis*);

S_Suff4 := *convert*(*taylor*($f(x)$, $x=0$, 16), *polynom*) ;
#n = 16, это наименьший порядок частичной суммы равномерно аппроксимирующей на промежутке [1, 1] функцию с точностью 0, 1 для степенного ряда (Маклорена)

```
plot(f(x), x =-1 ..1, color = black, legend = "f(x)") :
plot(S_Suff4, x =-1 ..1, color = "DimGray", legend = "S_16") :
plots[display](%, %%, aproxis);
```



$$S\_Suff4 := 8\,x^3 \quad 16\,x^5 + \frac{208}{15}\,x^7 \quad \frac{1312}{189}\,x^9 + \frac{10736}{4725}\,x^{11} \quad \frac{2336}{4455}\,x^{13} + \frac{19131872}{212837625}\,x^{15}$$



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