

# Solution to 1.5.10

**Question:** Verify that:

$$AE_3 = AF_3 = m, BD_3 = BF_3 = n, CD_3 = CE_3 = p. \quad (1)$$

**Solution:**

The coordinates of the points of contact of the circle and the triangle are:

$$\mathbf{D}_3 = \begin{pmatrix} \frac{-366\sqrt{74}-406\sqrt{122}-488\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \\ \frac{-610\sqrt{74}-170\sqrt{122}+732\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \end{pmatrix} \text{ from 1.5.8} \quad (2)$$

$$\mathbf{E}_3 = \begin{pmatrix} \frac{-111-20\sqrt{37}+5\sqrt{2257}}{74} \\ \frac{185+28\sqrt{37}-7\sqrt{2257}}{74} \end{pmatrix} \text{ from 1.5.9} \quad (3)$$

$$\mathbf{F}_3 = \begin{pmatrix} \frac{-2-\sqrt{37}+\sqrt{61}}{2} \\ \frac{-6-\sqrt{37}+\sqrt{61}}{2} \end{pmatrix} \text{ from 1.5.9} \quad (4)$$

Length of line segment between two points is given by:

$$AE_3 = \sqrt{(\mathbf{E}_3 - \mathbf{A})^\top (\mathbf{E}_3 - \mathbf{A})}$$

1)

$$\mathbf{E}_3 - \mathbf{A} = \begin{pmatrix} -0.136 - 1 \\ -2.136 + 1 \end{pmatrix} \quad (5)$$

$$\Rightarrow AE_3 = \sqrt{\begin{pmatrix} -1.136 & -1.136 \end{pmatrix} \begin{pmatrix} -1.136 \\ -1.136 \end{pmatrix}} \quad (6)$$

$$= 1.607 \quad (7)$$

$$\mathbf{F}_3 - \mathbf{A} = \begin{pmatrix} 0.066 - 1 \\ 0.308 + 1 \end{pmatrix} \quad (8)$$

$$\Rightarrow AF_3 = \sqrt{\begin{pmatrix} -0.934 & 1.308 \end{pmatrix} \begin{pmatrix} -0.934 \\ 1.308 \end{pmatrix}} \quad (9)$$

$$= 1.607 \quad (10)$$

$\therefore AE_3 = AF_3 = m$  verified.

2)

$$\mathbf{D}_3 - \mathbf{B} = \begin{pmatrix} -3.367 + 4 \\ -0.967 - 6 \end{pmatrix} \quad (11)$$

$$\Rightarrow BD_3 = \sqrt{\begin{pmatrix} 0.633 & 6.967 \end{pmatrix} \begin{pmatrix} 0.633 \\ 6.967 \end{pmatrix}} \quad (12)$$

$$= 6.995 \quad (13)$$

$$\mathbf{F}_3 - \mathbf{B} = \begin{pmatrix} 0.066 + 4 \\ 0.308 - 6 \end{pmatrix} \quad (14)$$

$$\Rightarrow BF_3 = \sqrt{\begin{pmatrix} 4.066 & -5.692 \end{pmatrix} \begin{pmatrix} 4.066 \\ -5.692 \end{pmatrix}} \quad (15)$$

$$= 6.995 \quad (16)$$

$\therefore BD_3 = BF_3 = n$  verified.

3)

$$\mathbf{D}_3 - \mathbf{C} = \begin{pmatrix} -3.367 + 3 \\ -0.967 + 5 \end{pmatrix} \quad (17)$$

$$\Rightarrow CD_3 = \sqrt{\begin{pmatrix} -0.367 & 4.033 \end{pmatrix} \begin{pmatrix} -0.367 \\ 4.033 \end{pmatrix}} \quad (18)$$

$$= 4.0499 \quad (19)$$

$$\mathbf{E}_3 - \mathbf{C} = \begin{pmatrix} -0.136 + 3 \\ -2.136 + 5 \end{pmatrix} \quad (20)$$

$$\Rightarrow CE_3 = \sqrt{\begin{pmatrix} 2.864 & 2.864 \end{pmatrix} \begin{pmatrix} 2.864 \\ 2.864 \end{pmatrix}} \quad (21)$$

$$= 4.0499 \quad (22)$$

$\therefore CD_3 = CE_3 = p$  verified.