Solution to 1.5.10

Question: Verify that:

$$AE_3 = AF_3 = m, BD_3 = BF_3 = n, CD_3 = CE_3 = p. \tag{1}$$

Solution:

The coordinates of the points of contact of the circle and the triangle are:

$$\mathbf{D}_3 = \begin{pmatrix} \frac{-366\sqrt{74}-406\sqrt{122}-488\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \\ \frac{-610\sqrt{74}-170\sqrt{122}+732\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \end{pmatrix} \text{ from 1.5.8} \tag{2}$$

$$\mathbf{E}_3 = \begin{pmatrix} \frac{-111-20\sqrt{37}+5\sqrt{2257}}{74} \\ \frac{185+28\sqrt{37}-7\sqrt{2257}}{74} \end{pmatrix} \text{ from 1.5.9} \tag{3}$$

$$\mathbf{F}_3 = \begin{pmatrix} \frac{-2-\sqrt{37}+\sqrt{61}}{2} \\ \frac{-6-\sqrt{37}+\sqrt{61}}{2} \end{pmatrix} \text{ from 1.5.9} \tag{4}$$

Now we have to find m,n and p. We can find that by using the formula for magnitude of a vector:

1) $AE_3 = \sqrt{(\mathbf{E_3} - \mathbf{A})^\top (\mathbf{E_3} - \mathbf{A})}$

2)

$$\mathbf{E_3} - \mathbf{A} = \begin{pmatrix} -0.136 - 1 \\ -2.136 + 1 \end{pmatrix} \tag{5}$$

$$\implies AE_3 = \sqrt{\begin{pmatrix} -1.136 & -1.136 \end{pmatrix}\begin{pmatrix} -1.136 \\ -1.136 \end{pmatrix}} \tag{6}$$

$$= 1.607 \tag{7}$$

3)

$$\mathbf{F_3} - \mathbf{A} = \begin{pmatrix} 0.066 - 1 \\ 0.308 + 1 \end{pmatrix} \tag{8}$$

$$\implies AF_3 = \sqrt{\begin{pmatrix} -0.934 & 1.308 \end{pmatrix}\begin{pmatrix} -0.934 \\ 1.308 \end{pmatrix}} \tag{9}$$

$$= 1.607 \tag{10}$$

$\therefore AE_3 = AF_3 = m$ verified.

4)

$$\mathbf{D_3} - \mathbf{B} = \begin{pmatrix} -3.367 + 4 \\ -0.967 - 6 \end{pmatrix} \tag{11}$$

$$\implies BD_3 = \sqrt{\begin{pmatrix} 0.633 & 6.967 \end{pmatrix}\begin{pmatrix} 0.633 \\ 6.967 \end{pmatrix}} \tag{12}$$

$$= 6.995 \tag{13}$$

5)

$$\mathbf{F_3} - \mathbf{B} = \begin{pmatrix} 0.066 + 4 \\ 0.308 - 6 \end{pmatrix} \tag{14}$$

$$\implies BF_3 = \sqrt{\begin{pmatrix} 4.066 & -5.692 \end{pmatrix}\begin{pmatrix} 4.066 \\ -5.692 \end{pmatrix}} \tag{15}$$

$$= 6.995 \tag{16}$$

$\therefore BD_3 = BF_3 = n$ verified.

6)

$$\mathbf{D_3} - \mathbf{C} = \begin{pmatrix} -3.367 + 3 \\ -0.967 + 5 \end{pmatrix} \tag{17}$$

$$\implies CD_3 = \sqrt{\begin{pmatrix} -0.367 & 4.033 \end{pmatrix}\begin{pmatrix} -0.367 \\ 4.033 \end{pmatrix}} \tag{18}$$

$$= 4.0499 \tag{19}$$

7)

$$\mathbf{E_3} - \mathbf{C} = \begin{pmatrix} -0.136 + 3 \\ -2.136 + 5 \end{pmatrix} \tag{20}$$

$$\implies CE_3 = \sqrt{\begin{pmatrix} 2.864 & 2.864 \end{pmatrix}\begin{pmatrix} 2.864 \\ 2.864 \end{pmatrix}} \tag{21}$$

$$= 4.0499 \tag{22}$$

$\therefore CD_3 = CE_3 = p$ verified.