Solution to 1.5.10

Question: Verify that:

$$AE_3 = AF_3 = m, BD_3 = BF_3 = n, CD_3 = CE_3 = p. \tag{1}$$

Solution:

The coordiantes of the points of contact of the circle and the triangle are:

$$\mathbf{D}_3 = \begin{pmatrix} \frac{-366\sqrt{74}-406\sqrt{122}-488\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \\ \frac{-610\sqrt{74}-170\sqrt{122}+732\sqrt{32}}{122(\sqrt{74}+\sqrt{32}+\sqrt{122})} \end{pmatrix} from\ 1.5.8 \tag{2}$$

$$\mathbf{E}_3 = \begin{pmatrix} \frac{-111-20\sqrt{37}+5\sqrt{2257}}{74} \\ \frac{185+28\sqrt{37}-7\sqrt{2257}}{74} \end{pmatrix} from\ 1.5.9 \tag{3}$$

$$\mathbf{F}_3 = \begin{pmatrix} \frac{-2-\sqrt{37}+\sqrt{61}}{2} \\ \frac{-6-\sqrt{37}+\sqrt{61}}{2} \end{pmatrix} from\ 1.5.9 \tag{4}$$

Now we have to find m,n and p.
We can find that by using the formula for magnitude of a vector:

Maginutde of Vector

$$\|\mathbf{AE_3}\| = \sqrt{\mathbf{AE_3}^\top.\mathbf{AE_3}}$$

$$\mathbf{AE_3} = \begin{pmatrix} -0.136256 - 1 \\ -2.136256 + 1 \end{pmatrix} \tag{5}$$

$$\|\mathbf{AE_3}\| = \sqrt{\begin{pmatrix} -1.136256 & -1.136256 \end{pmatrix} \begin{pmatrix} -1.136256 \\ -1.136256 \end{pmatrix}} \tag{6}$$

$$\|\mathbf{AE_3}\| = 1.6069092 \tag{7}$$

$$\mathbf{AF_3} = \begin{pmatrix} 0.066003 - 1 \\ 0.307596 + 1 \end{pmatrix} \tag{8}$$

$$\|\mathbf{AF_3}\| = \sqrt{\begin{pmatrix} -0.933997 & 1.307596 \end{pmatrix} \begin{pmatrix} -0.933997 \\ 1.307596 \end{pmatrix}} \tag{9}$$

$$\|\mathbf{AF_3}\| = 1.6069092 \tag{10}$$

Therefore $\|\mathbf{AE_3}\| = \|\mathbf{AF_3}\| = m$ verified.

$$\mathbf{BD_3} = \begin{pmatrix} -3.36666 + 4 \\ -0.96669 - 6 \end{pmatrix} \tag{11}$$

$$\|\mathbf{BD_3}\| = \sqrt{\begin{pmatrix} 0.63334 & 6.96669 \end{pmatrix} \begin{pmatrix} 0.63334 \\ 6.96669 \end{pmatrix}} \tag{12}$$

$$\|\mathbf{BD_3}\| = 6.9954191 \tag{13}$$

$$\mathbf{BF_3} = \begin{pmatrix} 0.066003 + 4 \\ 0.307596 - 6 \end{pmatrix} \tag{14}$$

$$\|\mathbf{BF_3}\| = \sqrt{\begin{pmatrix} 4.066003 & -5.692404 \end{pmatrix} \begin{pmatrix} 4.066003 \\ -5.692404 \end{pmatrix}} \tag{15}$$

$$\|\mathbf{BF_3}\| = 6.9954191 \tag{16}$$

Therefore $\|\mathbf{BD_3}\| = \|\mathbf{BF_3}\| = n$ verified.

$$\mathbf{CD_3} = \begin{pmatrix} -3.36666 + 3 \\ -0.96669 + 5 \end{pmatrix} \tag{17}$$

$$\|\mathbf{CD_3}\| = \sqrt{\begin{pmatrix}-0.36666 & 4.03331\end{pmatrix}\begin{pmatrix}-0.36666 \\ 4.03331\end{pmatrix}} \tag{18}$$

$$\|\mathbf{CD_3}\| = 4.049942 \tag{19}$$

$$\mathbf{CE_3} = \begin{pmatrix}-0.136256 + 3 \\ -2.136256 + 5\end{pmatrix} \tag{20}$$

$$\|\mathbf{CE_3}\| = \sqrt{\begin{pmatrix}2.863744 & 2.863744\end{pmatrix}\begin{pmatrix}2.863744 \\ 2.863744\end{pmatrix}} \tag{21}$$

$$\|\mathbf{CE_3}\| = 4.049942 \tag{22}$$

Therefore $\|\mathbf{CD_3}\| = \|\mathbf{CE_3}\| = p$ verified.