Basic Matrix Operations

1 Consisent and Inconsistent Systems

A system of linear equations is consistent if it has at least one solution. It is inconsistent if it has no solutions. A system is consistent if and only if the rank of the coefficient matrix is equal to the rank of the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right]$$

2 Rank and Nullity

The ranks of a matrix is the number of leading ones in RREF.
For a m x n matrix A:

1. $\text{rank}(A) \leq m$
2. $\text{rank}(A) \leq n$
3. $\text{rank}(A) \leq \min(m, n)$
4. if $[A|b]$ is a system of linear equations and $\text{rank}(A) = n$, the system has unique solutions
5. if $\text{rank}(A) = k < n$, the system has solutions with $n - k$ parameters