Basic Matrix Operations

1 Gaussian Elimination and Reduced Row Echelon Form

1. Any two rows can be interchanged.
2. Any row can be multiplied by a nonzero number.
3. Any number of rows can be added and subtracted from any other row.

$$\begin{bmatrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ -2 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. The reduced row echelon form of a matrix is when each row has a leading one.

2 Matrix Multiplication

1. $AB \neq BA$
2. $A(BC) = (AB)C$
3. $A(B + C) = AB + AC$
4. $AI = A$

3 Inverse of a Matrix

To find the inverse of a matrix A solve the augmented matrix $[A|I]$ using Gaussian elimination. If A is invertible, then the reduced row echelon form of $[A|I]$ will be $[I|A^{-1}]$.

1. $A^{-1}A = I$
2. $AA^{-1} = I$
3. $(AB)^{-1} = B^{-1}A^{-1}$