MATLAB Transient Circuit Simulator

Veena Bellamkonda
Electrical and Computer Engineering
University of California
Santa Barbara, California

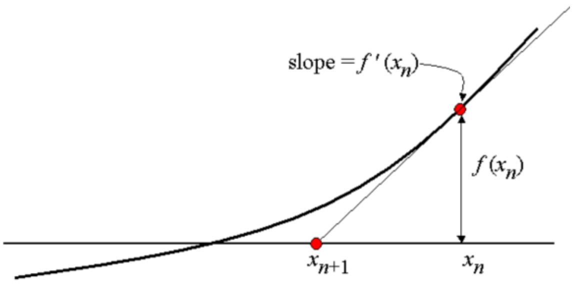
***Abstract*—This report discusses the methods employed to develop a nonlinear transient circuit simulator in MATLAB. The concepts of modified nodal analysis, trapezoidal approximation and multidimensional Newton Raphson algorithm are used to perform nonlinear transient analysis of various circuits. The simulator successfully performs linear transient analysis and nonlinear DC analysis and displays the nodal voltages of the circuit. The speed and efficiency of the simulator is dependent on the convergence rate of the Newton Raphson algorithm and the time step used. The key challenges to accurately simulate the circuit are good estimation of initial values in Newton Raphson iterations and round-off errors due to sparsity of the admittance matrix.**

Keywords—transient circuit analysis, DC analysis, Newton Raphson

I. Introduction

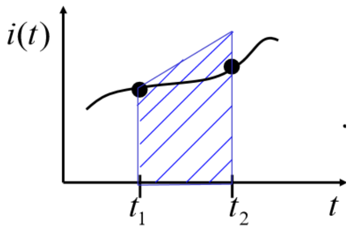
Though the node voltage method and loop current method are the most widely used in literature for circuit analysis, another powerful method is modified nodal analysis (MNA). Regular nodal analysis works by applying Kirchoff's Current Laws (KCL) in each and every node of the circuit. MNA is an extension of nodal analysis determines both the node voltages and branch currents. To automate circuit analysis, converting the data (the topology of the circuit, the nature and the numerical values of the components, etc.) to a machine-amenable form is crucial. The core of MNA is element-by-element stamping concept, which is based on the incomplete Kirchhoff's circuit law. For each node, we stamp a circuit element contribution into the matrix. MNA results in larger systems of equations compared to other methods, but is easier to implement algorithmically which is a substantial advantage when an automated solution is required.

The Newton-Raphson method (also known as Newton's method) is used in nonlinear DC and transient analysis. This method allows to find a good approximation for a nonlinear function. It uses the idea that a continuous and differentiable function can be approximated by a straight-line tangent to it as shown below.



Newton Raphson model may not always converge, techniques like damping have to be applied in order to assist the convergence. In damping, the change in x is limited to a certain tolerance limit, which means x is not allowed to changed more than a certain value between successive iterations.

In numerical analysis, the trapezoidal approximation is a technique for approximating the definite integral. The trapezoidal rule works by approximating the region under the graph of the function *f(x)* as a trapezoid and computing its area as shown below.



The definite integral of the function *i(t)* shown above is approximated to the area of the shaded area between discrete time steps.

$$\int_{t_1}^{t_2} i(t)dt \cong \frac{\Delta t}{2}\cdot\left(i(t_1)+i(t_2)\right)$$

The combination of multidimensional Newton Raphson algorithm, Trapezoidal approximation and Modified nodal analysis is the foundation of this simulator.

II. Simulator Fucntions

A. Linear DC Analysis

A simple linear time-invariant circuit comprises of the basic two terminal elements: resistors, ideal independent current and voltage sources. To ensure all the branches in a circuit are evaluated in a consistent and systematic manner, associated reference directions are used. The positive (+) reference for node voltage at the tail of the branch current reference arrow and negative (-) reference for node voltage at the head of the branch current refence arrow. The circuit being analysed is described in terms of a netlist file. To the nodal admittance equations, each branch of the circuit contributes different terms in a procedural manner. The branches containing resistor element contribute to the Y matrix, while the branches with independent current source elements contribute to the J vector. From the input circuit netlist file, each branch is characterized in terms of matrix stamps. The initial size of the Y matrix is *n x n*, where *n* is the number of linear components in the circuit. The size of the Y matrix increases with the increase in independent voltage sources and controlled sources (current controlled-current source (CCCS), voltage-controlled voltage source (VCVS) and current-controlled voltage source (CCVS)). For a resistor component between node *p* and node *q*, positive conductance value is added to *(i, i)* and *(j, j)* locations of the Y matrix and negative conductance value is added to *(i, j)* and *(j, i)* locations. Similarly, other linear components are added to the Y matrix depending on their contribution to the nodal voltage equations. After incorporating all the circuit components in the Y matrix and J vector, the node voltages are computed. In order to compute the node voltages, one way is to compute the inverse of Y matrix and multiply it with the J vector. Since the Y matrix is a sparse matrix for most of the electronic circuits, an efficient way to compute the node voltages is by using matrix manipulations such as LU decomposition followed by forward and backward substitution.

B. *Linear transient analysis*

In transient analysis, numerical integration is performed to determine the response of RLC circuits. Numerical integration allows for one to solve for discrete moments of time and in effect integrate their response. Linear transient analysis is used to evaluate the large signal behaviour of the circuit as a function of time. When the circuit contains lumped, linear, time-invariant capacitors and inductors, the nodal equations form a set of first order differential equations. The voltage at time $(t+\Delta t)$ depends on the voltage at time t and the current through the component at times t and $(t+\Delta t)$ as shown in the equation below.

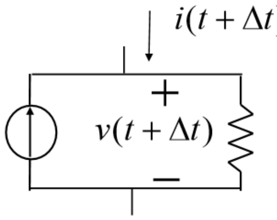
$$v(t+\Delta t) = v(t) + \frac{1}{C}\int_{t}^{t+\Delta t} i(\tau)d\tau$$

The value of current for the entire time interval is unknown, therefore the integral of the current $i(t)$ is estimated by means of trapezoidal approximation.

$$v(t+\Delta t) \approx v(t) + \frac{\Delta t}{2C}[i(t) + i(t+\Delta t)]$$

Both the voltage and current are unknown at the future time $(t+\Delta t)$ that is two unknown variables. Since the approximation is similar to a simple circuit representation (Norton) of independent current source in parallel with a resistor as shown below.

$$I_{eq} = \frac{2C}{\Delta t}v(t) + i(t)$$



$$G_{eq} = \frac{2C}{\Delta t}$$

The current source and the resistor are stamped into the Y matrix as discussed in Linear DC analysis. LU decomposition and substitution methods are used to compute the node voltages for every time step. Similar companion model is realized for evaluating the inductor behaviour. The following steps are implemented to perform linear transient analysis:

1. The companion models are stamped into the Y matrix during linear DC analysis using the initial user-specified capacitance voltages and inductance currents.
2. Time loop with user-specified time-step to compute the current and conductance values in the companion models
3. DC analysis for every time step to compute the node voltages
4. Return to step 1 with the current time step voltage and current values to compute the current and voltage values of the next time step.

C. *Nonlinear DC analysis*

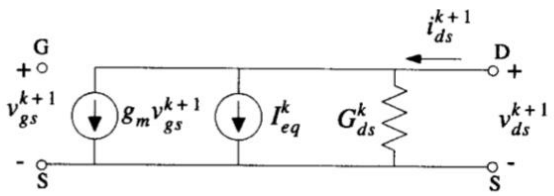
Nonlinear DC analysis is performed to obtain the quiescent operating point for nonlinear elements like MOS transistors. Theoretically, this operating point is obtained by means of load line, but computationally the operating point is obtained iteratively. Newton Raphson algorithm seeks the solution of a nonlinear equation by successively solving the linearized approximations of nonlinear elements. The iteration is repeated until the change in nodal voltages is less than the tolerance value which is 1μV for this simulator. For multidimensional Newton Raphson algorithm, Jacobian matrix is constructed with the linearized terms of individual nonlinear elements. Only MOS transistors are considered in this simulator. Similar to the capacitors and inductors, MOS transistors can be linearized with a companion model. A simple NMOS can be realized from the following set of current equations depending on the regime of operation:

$$i_{ds} = \beta\left[(v_{gs}-v_T)v_{ds} - \frac{v_{ds}^2}{2}\right] \quad \text{for} \quad v_{ds} < v_{gs} - v_T \quad \text{(linear)}$$

$$i_{ds} = \frac{\beta}{2}(v_{gs}-v_T)^2 \quad \text{for} \quad (v_{ds} > v_{gs} - v_T) \quad \text{(saturation)}$$

$$i_{ds} = 0 \quad \text{for} \quad v_{gs} < v_T \quad \text{(cutoff)}$$

The Jacobian matrix is constructed with the derivates of drain current with respect to gate and drain voltages. From the above equations the companion model of NMOS can be developed. The MOS transistor can be linearized with a simple circuit representation as shown below containing a voltage controlled current source, independent current source and a resistor. g_m is the small signal transconductance and G_{ds} is the small signal drain to source conductance.



$$i_{ds}^{k+1} = i_{ds}^k + g_m(v_{gs}^{k+1} - v_{gs}^k) + G_{ds}^k(v_{ds}^{k+1} - v_{ds}^k)$$

$$I_{eq}^k = i_{ds}^k + g_m(-v_{gs}^k) + G_{ds}^k(-v_{ds}^k)$$

After linearizing the MOS transistor, the following sequence of steps are implemented for non-linear DC analysis.

1. The initial values of circuit voltages are guessed.
2. The companion models of the nonlinear elements (with the guessed values) are stamped into the Y matrix
3. Nodal voltages are computed
4. When the change in nodal voltages of consecutive iterations is less than the tolerance value, convergence is achieved. If the changes is greater than tolerance the analysis is repeated from step 1 using the new voltage and current values in place of the initial guessed values

Sometimes the initial guess values may be far away from the solution values, in such cases the Newton Raphson algorithm can result in an infinite loop or low converge rate.

To avoid such cases, damped Newton Raphson algorithm is included. When the absolute value of change in nodal voltages is greater than a certain limit, which in this simulator is 1V, the nodal voltage changes is reset to a smaller value (0.1V).

D. *Nonlinear transient analysis*

This analysis is used to determine the large signal time domain behaviour of the nonlinear elements in a circuit. In this simulator, the nonlinear energy storage elements like the parasitic capacitances of a MOS transistor were not considered. However, transient analysis for an ideal MOS transistor is implemented. For each time step, multidimensional Newton Raphson algorithm was implemented. The nonlinear DC analysis performed for each time step.

III. Challenges and Results

This simulator is a very crude version of circuit analysis. There is immense scope of improvement. Features like round-off correction, finding good initial guess values, consideration of parasitic capacitances and dynamic step control can be added to improve the accuracy and efficiency of the simulator significantly. No measures were implemented to enhance the storage of Y matrix, which is a sparse matrix. For larger circuits like RC mesh, the number of non-zero fill ins are significantly lesser than the zero elements. When properly implemented, the combination of multidimensional Newton Raphson algorithm, Trapezoidal approximation and Modified nodal analysis can compute a nonlinear transient simulation with good accuracy.

A. *Linear DC Analysis*

The simulator can successfully perform linear DC analysis. The accuracy of the node voltages can be improved by accounting for the round-off errors due to finite machine precision. The round-off errors are relatively small when the Y matrix is far from being singular. The LU decomposition can also be improvised for singular matrix by incorporating row or column pivoting. In, this simulator the Y matrix is assumed to be non-singular and the conditions for singular-matrix are not considered.

B. *Linear transient analysis*

This simulator can successfully perform linear transient analysis for RLC circuits. Only the trapezoidal approximation model is considered for computing the equivalent current and conductance in the companion models. The accuracy of the node voltages is heavily dependent on the time step used during transient analysis. Dynamic time step control (not implemented in this simulator) can be implemented to choose appropriate time step based on stability, accuracy and efficiency considerations. Other methods like Forward Euler and Backward Euler can also be implemented to reduce the time computation time when accuracy is not a priority.

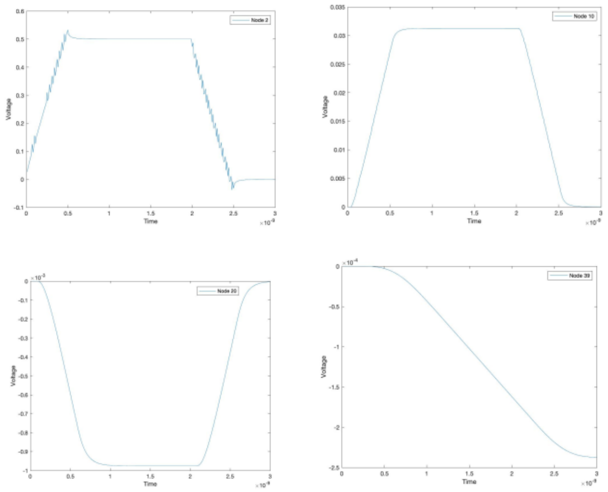


Figure 1: Voltages responses of Nodes 2, 10, 20 and 39 for rlc_line ciruit

C. *Nonlinear DC analysis*

Finding the initial guess values closest to the solution value is the biggest challenged. Fixed values are assigned to the gate and drain voltages in this simulator. If the fixed value is far away from the solution, the time taken for Newton Raphson convergence significantly increases, thereby reducing the efficiency of the simulator.

D. *Nonlinear transient analysis*

Nonlinear transient analysis of the storage elements that is parasitic capacitances in the MOS transistor is not implemented. The Jacobian stamping is performed for every element for every Newton Raphson iteration. Hence the convergence rate determines the overall speed of the simulator.

References

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