Arbitrariness & Mathematical Ontology

Introduction

In this paper, I offer a preliminary analysis of arbitrariness-based concerns in motivating alternative ontological conceptions of set theory, particularly regarding our understanding of the "height" of the iterative hierarchy of sets. I focus on four views: standard Minimal Necessitism, Interpretational Potentialism, Priority-Necessitism, and Structural Potentialism. While arbitrariness-based concerns related to the standard minimal necessitist's stipulated "height" of the hierarchy of sets motivated the creation of alternative ontologies (S2), these alternative views seem to also countenance similar positions of explanatory bedrock (S3). However, each view's arbitrariness-charge has *some* meaningful differences from each other (S4), and, as such, an analysis of their differences may be helpful in relatively evaluating each position. Ultimately, I conclude at the following parsimonious stance: distinguishing between the force of arbitrariness worries is an intrinsically arbitrary practice, and one should thus be uncertain of the usefulness of arbitrariness-aversion as a criterion for adjudicating between ontological views (S5). Before turning to this analysis, I first explain technical and philosophical preliminaries related to these views and the interactive hierarchy of sets (S1).

S1: Preliminary set-up

1.1 Russell's Paradox and the Iterative Hierarchy of Sets

The creation of a formalized set theory in the early 20th century helped solve many mathematical problems. David Hilbert famously stated that set theory pushed us into a kind of mathematical paradise.[1] This paradise, however, was not without its problems. Namely, Russell's paradox threatened to dismantle it by way of contradiction. Though undoubtedly familiar to this paper's reader, Russell's paradox briefly goes as follows: first, with unrestricted naïve comprehension, there is a set of things X with a membership condition of not being a member of itself; thus, if x is a member of the set, then by definition x is not a member of itself—but in that case, x satisfies the membership condition and so must be in the set. On the other hand, if x is not a member of the set, it would satisfy the membership condition. Russell's paradox illuminated a need to depart from unrestricted comprehension of sets in favor of a more restricted conception of set theory.

[1] Hilbert (1926)

Ernst Zermelo pioneered the iterative hierarchy conception of set theory. On this view, sets are built in stages, with each stage of sets being composed by the power set of all the sets in the stage below. As Sharon Berry states, "This lets us avoid the appearance that there should be a set of all sets that aren't members of themselves, and hence Russell's paradox." Following standard norms in the literature, when discussing the "height" of the hierarchy of sets, I am referring to the number of stages there are, while the "width" refers to how many sets are introduced at each stage. Importantly, while one can formalize the width of the hierarchy in second-order logic, there is no uniquely specified height of the hierarchy of sets.[2] The mathematical ontological debate over the height of the hierarchy of sets is the subject of this paper.

1.2 Objectualism, Structuralism, and Necessitism

There are two overarching types of positions I am considering in this paper: objectualism and structuralism. Structuralists notably consider "set-like" systems, while objectualists hold that there is some privileged mathematical notion of "set" and "membership", and, for example, there is one object that *is* the empty set. A basic commitment of many objectualists is that our pure mathematical truths are necessarily true, and, as such, some claim that there are some pure necessarily-existent mathematical objects. Like Button[3], I will call those who make this commitment *necessitists*, and they hold the view rebuked by the alternative set-theoretic conceptions I discuss in this paper.

The necessitist's view can be formalized in the following schema for any φ in a language to discuss pure sets:

Empty Box: $\varphi \leftrightarrow \Box\varphi$

Semantically, this connotes the commitment that we necessarily have all of the pure sets that we actually have.

1.3 Formalism

[2] Berry (2022)
[3] Button (2024)

In this paper, I use *italic lowercase* (e.g., x, y) for singular variables and bold lowercase (e.g., **u**, **v**) for plural variables. The expression $x < \mathbf{u}$ is read as "x is among **u**". I use E as an existential quantifier. I write $\{\mathbf{u}\}$ for the set formed from **u**, and $\mathrm{E}\{\mathbf{u}\}$ to express that such a set exists. The box and diamond operators, when discussing modal logic, will not necessarily indicate metaphysical modality in the post-Kripkean sense. They may importantly vary, and, in the case that they do, I will flag it as such.

S2: Arbitrariness Charge Against Necessitism

The arbitrariness charges against Necessitism begin with the Burali-Forti paradox. In the context of the ordinals, where the Burali-Forti paradox originally came into discussion, one can imagine an ordinal w that signifies the well-order of all ordinals. But by the definition of an ordinal, we can add one to w. Then, we are left with an ordinal (w+1) that is both by definition in the "well-order of all the ordinals" but also greater than w. This worry can be reformulated in the context of set theory, and, as such, we are inclined to think that there is no "set of all sets." What necessitists must deny can be formalized as follows:

1) $\forall z\, (z < \mathbf{s})$ where **s** is stipulated to be the plurality of all sets
2) $\neg\mathrm{E}\{\mathbf{s}\}$

While there exists a plurality of all the sets, a set cannot be made from that plurality. Here, the necessitist will soon reach explanatory bedrock. To answer why 2), they can appeal to the iterative conception of the hierarchy of sets to say the following: a plurality can only form a set if there is a stage that follows that plurality; there is no stage that follows the plurality of all sets **s**, and, as such, no set can be formed. The necessist's issues are not solved but rather shifted (this will soon become a recurring theme). Now, the necessitist must answer the following question: why does no stage come after **s**? In other words, the necessitist, who would like to rely on ambiguities like saying the height of the hierarchy "goes all the way up," seems to countenance some stopping point. Why is it that the hierarchy of sets could not have been extended to a further stage? To the proponents of the alternative viewpoints later discussed, this is an unacceptable arbitrariness worry.[4] This uncomfortability, as Berry puts it, seems to stem from a modal intuition:

[4] See Berry (2022), Linnebo (2013), and Linnebo (2018), for example

> Rather, the problem is that the Actualist takes there to be some plurality of objects (the sets) forming an iterative hierarchy structure, i.e., satisfying the description of the intended width of the hierarchy of sets above. But the following modal intuition seems appealing: for any plurality of objects satisfying the conception of an iterative hierarchy above (i.e., for any model of IHW), it would be in some sense (e.g., conceptually, logically or combinatorically if not metaphysically) possible for there to be a strictly lager model of IHW which, in effect, adds a new stage above all the ordinals within the original structure together with a corresponding layer of classes.

In other words, the hierarchy of sets should be potentially indefinitely extensible to avoid setting an arbitrary height, and we formalize it by taking serious notions of "possibility" and modality.[5]

S3: Arbitrariness & Alternative Set-theoretic Ontologies

I do not intend to defend the Necessist from arbitrariness; instead, I aim to show that the conceptions of set theory made in response to it are simply shifting the explanatory bedrock somewhere else. In this section, I will examine Studds' interpretational modality, Linnebo's modal Priority-Necessitism, and Berry's Structural Potentialism. There are undoubtedly more potentialist conceptions of the iterative hierarchy of sets, but they would be too much to go over in this paper's current form.

3.1 James Studd's Interpretational Possibility

Interpretational possibility is a form of contingentism. A contingentist, in this context, believes that there *could* have been more sets, even if there are not *actually* those sets. Formally, while a necessitist believes:

1) $\forall z\, (z < \mathbf{s})$
2) $\neg E\{\mathbf{s}\}$

A contingentist believes 1) and 2), along with:

[5] It is important to note that, as Berry points out, the modal operators need not be the standard "metaphysical" notion of modality (in fact, if one assumes an S5 logic and plural comprehension, most potentialist constructions of indefinite extensibility collapse to their non-modal analogue). These different modalities, particularly the interpretational modality, will be discussed at length.

3) $\Diamond E\{\mathbf{s}\}$

In other words, it is contingent that **s** is the plurality that represents the actual sets, as it is *possible*, in some sense, for the hierarchy to be extended to then allow for $E\{\mathbf{s}\}$ to hold true. Prima facie, it seems as though the Contingentist still faces the same worry about arbitrariness as their non-modal counterpart. That is, why are all the actual sets actually all the sets? A convincing contingentist account must account for a reason why the actual set-theoretic world is the way that it is. They cannot say, “There couldn’t be more!” as the necessitist does, because they believe there could be more.

Given that we are discussing pure sets, the Contingentist cannot appeal to facts about the world to derive their response to this question. As Button puts it, “Only one conceivable source of contingency remains: *us, we, ourselves*.” That is, that there is some contingent fact about human mathematical behavior that has caused **s** to be the way it is and not another way. This has primarily been conceptualized as a sort of “interpretational modality,” wherein $\Diamond\varphi$ reads as “the interpretation of mathematical language could have varied so that φ”. Necessitists, in response, treat interpretations that lead us to our mathematically necessary truth as metaphysically necessarily-existent objects. So, they reinterpret the interpretational operator as a quantification over actual interpretations. This is a real disagreement. The Interpretationalist posits that there is no complete interpretation of all the sets:

1) Given any **u**, $\Diamond_i(E\mathbf{u} \wedge Ez\, z \not\prec \mathbf{u})$

In response, the necessitist says:

2) Given any **u**, there *is* an interpretation such that $(E\mathbf{u} \wedge Ez\, z \not\prec u)$

But, as Button shows, this commits the necessitist to:

3) Given 2), no interpretation includes everything

This seems problematic for a necessitist, as there then seems no way to conceive of “everything” without falling into talks about some sort of richer interpretation. Thus, while the necessitist and Interpretationalist are seemingly using modalities that could coexist, there is real underlying disagreement between the two. With their incompatibility established, we can now evaluate whether the interpretationalist explains contingency in a meaningfully better way than the necessitist’s brute explanation. Button explains their attempt as follows:

> The interpretationally-actual sets are the sets which we *happen* to be talking about. Exactly which sets these are is settled by specific historical contingencies: if mathematicians had done otherwise, we might have talked about other sets than **s**. As it is, our contingent behaviour makes it interpretationally-actually the case that $\forall z\, z < \mathbf{s}$. Contingency explained!

While the Interpretationalist is able to explain the contingency of **s**, one can shift one of the questions asked to the neccessitist up to the modal-level and ask it to the interpretationalist: Why can't there be a set of all possible interpretations? Note, just as the necessitist has to deny the E{**s**}, the Interpretationalist must deny $(\Box_i \forall z\, z < \mathbf{u}) \rightarrow \Diamond_i E\{u\}$. Here, the Interpretationalist reaches explanatory bedrock. They can claim the following:

1) Anything that can form a set must be able to coexist
2) **p**—the plurality of all possible sets—cannot coexist
3) Thus, $-\Diamond_i E\{\mathbf{p}\}$

This does not feel meaningfully less arbitrary than the necessitist's appeal to the iterative hierarchy of sets to explain why **s** cannot form a set. The Interpretationalist must be able to explain why $\Box_i \forall z\, z < \mathbf{p}$; that is, why is **p**, and not something larger than **p**, the totality of all possible sets under admissible interpretations. Obviously, there seems to be no non-arbitrary answer to this question. Studd, then, advocates the following idea: there *could* be an interpretation of mathematical language that strictly extends the first interpretational possibility operator $\Diamond_{i\text{-}1}$ to $\Diamond_{i\text{-}2}$ and so on.[6] Thus, **p** does not coexist and cannot form a set at the interpretation that uses $\Diamond_{i\text{-}1}$, but it can coexist at the first stage of $\Diamond_{i\text{-}2}$. But while we now have a way to indefinitely extend modal space through reinterpreting possibility, there seems to be a new arbitrariness worry: why is the cutoff between $\Diamond_{i\text{-}1}$ and $\Diamond_{i\text{-}2}$ where it is? This is meaningfully similar to the question asked of the necessitist, so it feels fair to posit it. Given that we have no way to make sense of any "different interpretations" of the interpretational modality, there seems to be no way to answer this question. This sort of demand can go on for every

[6] Studd (2019)

interpretation of the interpretational modality that Studd posits. As such, there seems to be an infinite regress of meaningful arbitrariness worries regarding the Interpretationalist's stance.

3.2 Oystein Linnebo's Priority-Necessitism

First, it is important to further clarify the difference between contigentism and prioritism. The Prioritist holds that for any **u**, **u** is always prior to {**u**}. In other words, the plurality that forms a set always comes prior to the set itself, in some sort of hyperintensional way. This notion of priority is prima facie similar to a "time-like" relation.[7] For example, the Prioritist can state: $\Diamond(\mathrm{E}a \land \neg \mathrm{E}b)$. Critically, the Prioritist does not depart from the "actual" world, in the sense of positing alternative domains or varying interpretations. Instead, they remain within a fixed ontology—the same base domain of objects—and merely assert that some objects (e.g., certain sets) are ontologically dependent on others (e.g., their members), even if they don't always exist. Their modal claims reflect the asymmetric priority structure between pluralities and the sets they can form, not shifts between possible worlds with different total domains. Linnebo (2013) explicitly countenances this idea of priority: "6.2… I now turn to the second aspect: the principle that the elements of a set are prior to the set itself."

The necessitist reading (rather than contingentist) of Linnebo comes from his stated goal of "[defining] a translation from the nonmodal language to the modal one and to show that, under some plausible assumptions, this translation preserves relations of proof-theoretic (and thus also semantic) entailment." **…** In fact, Linnebo even states that the modalized version "provide[s] powerful instruments for studying the same subject matter" as the non-modal version.[8] Thus, it seems clear that we are operating within some sense of "actuality," but we are using modal operators to refine our study.

A priority necessitist can still be asked the following question: why, given $\forall z\, z < \mathrm{s}$, can we not form a set {s}? Linnebo's strategy is to reject Plural Comprehension.[9] Given that, sets are by definition their members, and that "given some objects u, we can make good mathematical and philosophical sense of the associated set {**u**}," and thus, {**u**} exists.[10] Notably, **u** was arbitrary, so Linnebo lands on the following generalization:

[7] Button (2024)
[8] Linnebo (2013)
[9] Button (2024)
[10] Linnebo (2018)

Collapse: $\forall \mathbf{u}\ E\{\mathbf{u}\}$

Given Russell's paradox, Collapse is inconsistent with Plural Comprehension, and, accordingly, Linnebo argues that we should reject unrestricted Plural Comprehension. Prima facie, this allows Linnebo to escape the following troubling question: why, given $\forall z\ z \prec \mathbf{s}$, can we not form a set $\{\mathbf{s}\}$? Because Plural Comprehension is restricted, we cannot speak about the plurality **s**, and, as such, there is nothing to explain.

Two arbitrariness problems arise from Linnebo's move. First, the explanatory bedrock seems to shift from set formation to plurality formation. In other words, one can ask, why $\neg E\mathbf{s}$ such that $\forall z\ z \prec \mathbf{s}$? Presumably, Linnebo must accept some stopping point for the formation of pluralities; that feels as arbitrary as the necessitist's acceptance of some stopping point for the formation of sets. For example, undoubtedly one can understand a plurality of *some* sets, so, there then must be some last set of sets that can form a plurality according to Linnebo.

Secondly, the motivation to reject Plural Comprehension feels arbitrary as well. Linnebo's endorsement of Collapse relies on a notion of sets being "thin." That is, a set, though it makes "demands on the world that go beyond" its members, "the former demands do not substantially exceed the latter".[11] As such, Linnebo believes that when one has a plurality, one has the set corresponding to that plurality. The problem is that one can imagine other necessitists accepting a "thin" conception of sets while rejecting unrestricted Collapse. Linnebo argues that "we can make good mathematical and philosophical sense of… $\{\mathbf{u}\}$," but, similarly, one can argue that, for some condition ϕ, we can make good mathematical and philosophical sense of **q** such that $\forall z(z \prec \mathbf{q} \leftrightarrow \phi)$. The person who argues this will undoubtedly want to restrict notions of Collapse instead of Plural Comprehension. As such, the same notion of permissible definitions can be argued to mandate a restriction to Collapse or a restriction to Plural Comprehension.

The axiomatic decision, then, feels arbitrary. Linnebo attempts to argue that restricting Plural Comprehension is necessary for "mathematical freedom," but this argument can go both ways. Button (2024) explains:

> Restricting Collapse certainly seems to stifle our mathematical freedom to form sets. But…restricting [Plural Comprehension] also seems to stifle our freedom to represent

[11] Linnebo (2018)

> things *de rebus*. We learned from Russell that we must restrict some initially attractive principle. So Linnebo needs to give us an argument that Collapse is more important to mathematical freedom than [Plural Comprehension]. Unfortunately, Linnebo gives us no such argument. Moreover, none is possible. What Collapse says is that any things, *de rebus*, could form a set. But when does ordinary mathematics clearly present us with sets *de rebus*?

Thus, it seems that explanatory bedrock is not averted but rather shifted for the priority necessitist: why is it that a notion of "thinness," which can reasonably support either a restriction of Collapse or a restriction of Plural Comprehension, countenance either axiom?

3.3 Sharon Berry's Structural Potentialism

As noted earlier, structuralists, unlike objectualists, do not refer to a privileged notion of a "set" or "membership" but rather talk about some "set-like" structure. Structural potentialists hold that, for any possible set-like structure, $\boldsymbol{H}$, there could be another structure with an isomorphic copy of $\boldsymbol{H}$ as a proper initial segment.[12] This is predicated on the intuition that for any plurality of things $\mathbf{v}$, we could always find more things, and as such arrange a taller hierarchy. In any case, structural potentialists face very similar arbitrariness concerns as their counterparts. For example, we can reformulate the earlier questions in the context of Structural Potentialism to ask: If $\Box \forall z\, z < \mathbf{p}$, can $\mathbf{p}$ form a set? It seems unreasonable to deny that there can be some $\mathbf{p}$ such that $\Box \forall z\, z < \mathbf{p}$, and, as such, the structural potentialist has the same issues with $\mathbf{p}$ as the necessitist has with the non-modal $\mathbf{s}$.

S4: Difference in Arbitrariness

In the previous section, I analyzed four views: Minimal Necessitism, Interpretational Potentialism, Priority Necessitism, and Structural Potentialism. Each of these views seem to reach an explanatory bedrock. This can be shown in the form of questions to the proponents of each view:

> *Minimal Necessitism*: If $\forall z\, z < \mathbf{s}$, why is $\mathbf{s}$ all there *is*?

[12] Berry (2022)

Interpretational Potentialism: If we accept that $\Diamond_{i-2}$ strictly extends $\Diamond_{i-1}$ (and so on), why is it that the boundary between $\Diamond_{i-2}$ and $\Diamond_{i-1}$ is where it is?

Priority Necessitism: What significant explanatory reason is there to reject Plural Comprehension while accepting Collapse? & Where is the stopping point for the formation of pluralities?

Structural Potentialism: If $\Box\forall z\, z \prec \mathbf{p}$, why is **p** all there *could be*?

The next reasonable step, then, should be analyzing whether some types of arbitrariness are *more* intolerable than others. In the above four accounts, I find two types of arbitrariness:

1) *Boundary arbitrariness*: the arbitrariness of drawing a line—i.e., positing a stopping point in a hierarchy or modal space without a deeper justification for why the boundary is here and not somewhere else.

2) *Axiomatic arbitrariness*: the arbitrariness of rule selection or restriction—i.e., positing or rejecting a formal principle (like Plural Comprehension or Collapse) in a way that feels *ad hoc* or ungrounded

All of the views face some sort of *Boundary Arbitrariness* issue. In fact, the necessitist and structural potentialist face almost analogous problems: the former cannot explain why **s** is all there is, while the latter cannot explain why **p** is all there could be. Similarly, the priority necessitist must explain why there is a maximal plurality that we can consider. Interestingly, the Interpretational necessitist seems to be burdened with facing this question more times than his counterparts. At every posited limit stage of each $\Diamond_{i-n}$, the interpretational potentialist must explain why the limit is there rather than somewhere else. In any other regard, though, the necessitist, interpretational potentialist, structural potentialist, and priority necessitist face the same arbitrariness worry here.

Priority Necessitism also has a problem of *Axiomatic Arbitrariness*. As explained earlier, the priority necessitist's commitment to notions of "thinness" and hyperintensional priority lead to an acceptance of Collapse. As Collapse is inconsistent with unrestricted Plural

Comprehension, the priority necessitist rejects Plural Comprehension. But one can seemingly be a priority necessitist, accept a notion of "thinness," while disagreeing with the idea of unrestricted Collapse. As stated by Button, there is no compelling *de rebus* reason to accept Collapse while restricting Plural Comprehension. As such, it feels like the priority necessitist is making an arbitrary move in their reasoning to accept one axiom while ruling out another. The priority necessitist does, however, avoid the *Boundary Arbitrariness* of set formation, as there is no way to form the question of "why is **s** the final stage of the hierarchy?"—since in their view, the rejection of unrestricted Plural Comprehension means asking about **s** is unfounded.

Both types of arbitrariness seem to stem from an issue with choosing one option over another. While the subject of the choice stemming from *Axiomatic Arbitrariness* and *Boundary Arbitrariness* is different, both types of arbitrariness charges can be adapted from the same principle: epistemic parity.

> **Epistemic Parity:** For theories T and T', if we have no reasons to favor T over T', and T and T' are incompatible, we should not believe T and we should not believe T'.[13]

This principle can be adapted to formulate a principle against the arbitrariness of boundaries in the context of this paper. Take the following example: When setting the height of the hierarchy of sets at A rather than B, and we have no reason to favor A over B, we should not choose A over B. This is a problem facing the necessitist, priority necessitist, and structural potentialist. Further, the interpretational potentialist faces the following iteration of the epistemic parity principle: For boundary points A and B between $\Diamond_{i-n+1}$ and $\Diamond_{i-n}$, when we have no reason to favor A over B, we should not choose A over B. The principle can also be adapted to formulate about arbitrariness regarding the choice of axioms: If a motivation, *X*, can reasonably lead one to accept either axioms α or β, we have no reason to favor α over β, and α and β are incompatible, we should not accept α over β. In the context of the priority necessitist, the motivation is "thinness," and the axioms are Collapse and Plural Comprehension.

S5: Evaluating the Force of Arbitrariness Concerns

In the above section, I created an initial typology of the arbitrariness problems plaguing the

[13] Fairchild (2022)

variations of conceptions of the hierarchy of sets. I further explained the principles that illustrate what it means for these theories to be arbitrary (namely, iterations of Epistemic Parity). In this section, I argue that no form of arbitrariness can be non-arbitrarily "worse" when using an adapted Epistemic Parity standard.

First, let's reformulate the Epistemic Parity standard to help evaluate arbitrariness concerns:

> **Arbitrariness Parity:** For arbitrariness concerns A and B, if there is no significant reason to condemn A over B, we should not treat A as a decisive objection while excusing B.

Notably, every proponent of the alternative conceptions of the hierarchy of sets discussed, who accept Epistemic Parity, should similarly be committed to this principle. Unfortunately, the acceptance of this principle creates a paralysis wherein no arbitrariness concern can be treated as a decisive objection while others are not.

First, there seems to be no significant reason to say that the arbitrariness of the boundary concern facing the necessitist is more problematic than the boundary concern facing the structural potentialist or priority necessitist. All are accepting some sort of unexplained stopping point in their ontological space. The fact that the structural potentialist's restriction happens at the modal level or the priority necessitist's happens with pluralities rather than sets does not seem to meet the standard for explaining why their boundary is any less arbitrary or more principled than the necessitist's. I can illustrate this reasoning as follows: the necessitist's stopping point at **s** is an instantiation of *Boundary Arbitrariness*; the structural potentialist's stopping point at **p** is an instantiation of *Boundary Arbitrariness*; shifting the boundary to the modal level changes nothing about the actual arbitrariness concern; thus, if the structural potentialist finds the necessitist's stopping point to be intolerably arbitrary, so is the structural potentialist's stopping point. This same reasoning can be easily reformulated to replace the structural potentialist with the priority necessitist.

Further, while the interpretational potentialist faces the *Boundary Arbitrariness* question more times than his counterparts, the structural potentialist and necessitist are not able to claim to be meaningfully "less" arbitrary. In order to claim to be tolerably "less" arbitrary, the structural potentialist and necessitist would have to claim something like the following:

> There is a particular number of arbitrariness concerns that are tolerable for a mathematical ontology; anything more than that number crosses a threshold into intolerability.

Unfortunately, this argument seems to fall victim to a *Boundary Arbitrariness* concern, as there seems to be no significant reason as to why one arbitrariness problem is tolerable while two, or three, and so on, are not. Thus, between Minimal Necessitism, Structural Potentialism, and Interpretational Potentialism, there is no clear non-arbitrary difference between the force of each view's arbitrariness concern.

Finally, there is also no clear non-arbitrary difference between the force of the priority necessitist's *Axiomatic Arbitrariness* and his counterparts' *Boundary Arbitrariness*. Both types of arbitrariness reduce to an iteration of the Epistemic Parity principle; thus, there does not seem to be a "significant reason" that one violation of an iteration of the Epistemic Parity principle is tolerable while the other is not.

Conclusion

Throughout this paper, I illustrated the arbitrariness problem facing necessitists that motivated alternative mathematical ontologies. I then showed that the main alternative conceptions all face similar arbitrariness concerns. Then, I created a typology of the arbitrariness concerns to help evaluate whether one type of concern is intolerable while the others are not. Finally, I showed that, given the standard of the principles that motivate arbitrariness-aversion, there is no non-arbitrary way to distinguish between the force of an arbitrariness concern regarding it being a decisive objection. As such, I hoped to show that, given our current set of mathematical ontologies, arbitrariness-aversion does very little to resolve the debate regarding the height of the hierarchy of sets.

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