Class room problems on Conic Sections

Lectures 1-2: Circles

1. Find the equation of the circle with centre (–3, 2) and radius 4.
   Ans: $(x + 3)^2 + (y - 2)^2 = 16$
2. Find the equation of the circle with centre (0,2) and radius 2.
   Ans: $x^2 + (y - 2)^2 = 4$
3. Find the centre and radius of the circles.

i) $(x + 5)^2 + (y - 3)^2 = 36$

Ans: centre=$(-5,3)$, radius=6

ii) $x^2 + y^2 - 8x + 10y - 12 = 0$

Ans: centre=$(4, -5)$, radius=$\sqrt{53}$

iii) $2x^2 + 2y^2 - x = 0$

Ans: centre=$(1/4,0)$, radius=1/4

4. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line $4x + y = 16$.
   Ans: $x^2 + y^2 - 6x - 8y + 15 = 0$

5. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).
   Ans: $x^2 + y^2 - 12x + 11 = 0$

6. Find the equation of the circle passing through (0,0) and making intercepts a and $b$on the coordinate axes.
   Ans: $x^2 + y^2 - ax - by = 0$

7. Find the equation of the circle which passes through the points (1, 0), (0,– 6) and (3, 4).
   Ans: $4x^2 + 4y^2 - 142x + 47y + 138 = 0$

8. Find the equation of the circles which touches the axis of x and passes through the points (1,–2) and (3, –4).
   Ans: $x^2 + y^2 - 6x + 4y + 9 = 0$ and $x^2 + y^2 + 10x + 20y + 25 = 0$

Lecture. 3,4: Parabola

1. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

i. $y^2 = 12x$ **2.** $x^2 = 6y$ **3.** $y^2 = -8x$ **4.** $x^2 = -16y$

Ans:

	focus	*axis*	*directrix*	*Length of latus rectum*
1	*(3,0)*	*X axis*	*x=-3*	*12*
2	*(0,3/2)*	*Y axis*	*y=-3/2*	*6*
3	*(-2,0)*	*X axis*	*x=2*	*8*
4	*(0,-4)*	*Y axis*	*y=4*	*16*

2. Find the equation of the parabola with focus (2,0) and directrix $x = -2$.
Ans:$y^2=8x$

3. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).
Ans:$x^2=8y$

4. Find the equation of the parabola that satisfies the following given conditions:

i) Focus (6,0); directrix $x = -6$
Ans:$y^2=24x$

ii) Focus (0,–3); directrix $y = 3$
Ans:$x^2=-12y$

iii) Vertex (0,0), passing through (5,2) and symmetric with respect to *y*-axis.
Ans:$2x^2=25y$

Lecture 5,6: Ellipse

1. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the latus rectum of the ellipse

i. $\frac{x^2}{36}+\frac{y^2}{16}=1$

ii. $4x^2+9y^2=36$.

iii. $\frac{x^2}{4}+\frac{y^2}{25}=1$

iv. $\frac{x^2}{25}+\frac{y^2}{100}=1$

Ans:

	foci	***vertices***	***Length of major axis***	***Length of major axis***	***eccentricity***	***Length of Latus rectum***
1	$(\pm 2\sqrt{5}, 0)$	*(±6,0)*	*12*	*8*	$\frac{\sqrt{5}}{3}$	$\frac{16}{3}$
2	$(\pm\sqrt{5}, 0)$	*(±3,0)*	*6*	*4*	$\frac{\sqrt{5}}{3}$	$\frac{8}{3}$
3	$(0, \pm\sqrt{21})$	$(0, \pm 5)$	*10*	*4*	$\frac{\sqrt{21}}{5}$	$\frac{8}{5}$
4	$(0, \pm 5\sqrt{3})$	$(0, \pm 10)$	*20*	*10*	$\frac{\sqrt{3}}{2}$	5

2. Find the equation of the ellipse,

(a) whose latus rectum is 5 and whose eccentricity is $\frac{2}{3}$.

(b) whose minor axis is equal to the distance between the foci and whose latus rectum is 10.

(c) whose foci are the points (4,0) and (– 4,0) and whose eccentricity is $\frac{1}{3}$.

Ans: (a) $20x^2 + 36y^2 = 405$ (b) $x^2 + 2y^2 = 100$ (c) $8x^2 + 9y^2 = 1152$

3. Find the eccentricity of the ellipse, if its latus rectum be equal to one half its minor axis.

   Ans: $\frac{\sqrt{3}}{2}$

Lecture 7: Hyperbola

1. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbolas.

1. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
2. $\frac{y^2}{9} - \frac{x^2}{27} = 1$
3. $9y^2 - 4x^2 = 36$
4. $16x^2 - 9y^2 = 576$

Ans:

	Foci	***vertices***	***eccentricity***	***Length of Latus rectum***
1	$(\pm 5, 0)$	$(\pm 4,0)$	$\frac{5}{4}$	$\frac{9}{2}$
2	$(0, \pm 6)$	$(0,\pm 3)$	2	18
3	$(0, \pm\sqrt{13})$	$(0, \pm 2)$	$\frac{\sqrt{13}}{2}$	9
4	$(\pm 10, 0)$	$(\pm 6,0)$	$\frac{5}{3}$	$\frac{64}{3}$

2. Find the equations of the hyperbola satisfying the given conditions.

i. Vertices (±2, 0), foci (±3, 0)

Ans: $\frac{x^2}{4} - \frac{y^2}{5} = 1$

ii. Vertices (0, ±5), foci (0, ±8)

Ans: $\frac{y^2}{25} - \frac{x^2}{39} = 1$

iii. Foci (±4, 0), the latus rectum is of length 12

Ans: $\frac{x^2}{4} - \frac{y^2}{12} = 1$

iv. vertices (±7,0), $e = \frac{3}{4}$

Ans: $\frac{x^2}{49} - \frac{9y^2}{343} = 1$

v. Foci (0, $\pm\sqrt{10}$ 10), passing through (2,3) Ans: $\frac{y^2}{5} - \frac{x^2}{5} = 1$