INDEFINITE INTEGRATION FORMULAE:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

2. $\int dx = x + c$

3. $\int \frac{1}{x} dx = \log|x| + c$

4. $\int a^x dx = \frac{a^x}{\log a} + c$

5. $\int e^x dx = e^x + c$

6. $\int \sin x dx = -\cos x + c$

7. $\int \cos x dx = \sin x + c$

8. $\int \sec^2 x \, dx = \tan x + c$

9. $\int \cos ec^2 x$ dx = $-$cotx +c

10. $\int \sec x \tan x dx = \sec x + c$

11. $\int \cos ecx \cot x dx = -\cos ecx + c$

12. $\int \tan x dx = -\log|\cos x| + c$ **or** $\log|\sec x| + c$

13. $\int \cot x dx = \log|\sin x| + c$

14. $\int \sec x dx = \log|\sec x + \tan x| + c$ or $\log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

15. $\int \cos ecx dx = \log|\cos ecx - \cot x| + c$ **or** $\log\left|\tan\frac{x}{2}\right| + c$

16. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x$ + c

17. $\int -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1} x$ + c

18. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

19. $\int -\frac{1}{1+x^2}\,dx = \cot^{-1}x + c$

20. $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + c$

21. $\int -\frac{1}{x\sqrt{x^2-1}}dx = \cos ec^{-1}x + c$

ALGEBRA OF INTEGRATION

1. $\int \left[f_1(x) \pm f_2(x) \right] dx = \int f_1(x)\,dx \pm \int f_2(x) dx$

2. $\int k.f(x) dx = k\int f(x)\,dx$

INTEGRATION BY PARTS:

Integration by parts is used in integrating functions of the type $f(x).g(x)$ *or* $u.v$ **as follows.**

$$\int \left(I^{st}\ function \times II^{nd}\ function \right) dx = I^{st}\ function \int \left(II^{nd}\ function \right) dx - \int \left(\frac{d}{dx}\left(I^{st}\ function \right) \times \int \left(II^{nd}\ function \right) dx \right) dx$$

Where the Ist and IInd functions are decided in the order of LIATE;

L: Logarithmic function

I: Inverse trigonometric function

T: Trigonometric functions

A: Algebraic functions

E: Exponential Functions

INTEGRATION BY PARTIAL FRACTIONS:

GIVEN FUNCTION	**CORRESPONDING PARTIAL FRACTION**
(Linear factors) $\frac{f(x)}{(ax+b).(\mathrm{cx+d})}$	$\frac{A}{ax+b} + \frac{B}{cx+d}$
Repeated linear factor	

(i) $\dfrac{f(x)}{(ax+b)^2}$ **(ii)** $\dfrac{f(x)}{(ax+b)^n}$	$\dfrac{A}{ax+b}+\dfrac{B}{(ax+b)^2}$ $\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\dfrac{A_3}{(ax+b)^3}+...+\dfrac{A_n}{(ax+b)^n}$
Quadratic factor $\dfrac{f(x)}{ax^2+bx+c}$	$\dfrac{Ax+B}{ax^2+bx+c}$
Repeated quadratic factor **(i)** $\dfrac{f(x)}{(ax^2+bx+c)^2}$ **(ii)** $\dfrac{f(x)}{(ax^2+bx+c)^n}$	**(i)** $\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}$ **(ii)** $\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\dfrac{A_3x+B_3}{(ax^2+bx+c)^3}+...+\dfrac{A_nx+B_n}{(ax^2+bx+c)^n}$

NOTE: Where A, B and A_i's, B_i's are real numbers and are to be calculated by an appropriate method

NOTE: If in an integration of the type $\dfrac{p(x)}{q(x)}$ **(i.e.) a rational expression** $\deg\left(p(x)\right)\geq\deg\left(q(x)\right)$ **then we first divide** $p(x)$ **by** $q(x)$ **and write** $\dfrac{p(x)}{q(x)}$ **as**

$\dfrac{p(x)}{q(x)}=quotient+\dfrac{remainder}{divisor}$ **And then proceed with partial fraction.**

INTEGRATION BY SUBSTITUTION

Integration of the type $\int\left[f(x)\right]^n.f'(x)dx;\int\dfrac{f'(x)}{\left[f(x)\right]^n}dx$; $\int\dfrac{f'(x)}{f(x)}dx$; $\int g\left(f(x)\right).f'(x)dx$

METHOD: Put f(x) = t and f ′(x) dx = dt and proceed.

NOTE: $\int\dfrac{f'(x)}{f(x)}dx=\log\left|f(x)\right|+c$

SOME SPECIAL INTEGRALS

1. $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c$

2. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

3. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

4. $\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

5. $\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

6. $\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

7. $\int e^x[f(x)+f'(x)]dx = e^x.f(x)+C$

DEFINITE INTEGRAL

If $\int f(x)dx = F(x)$, then $\int_a^b f(x)dx = \left[F(x)\right]_a^b = F(b)-\mathrm{F(a)}$

PROPERTIES OF DEFINITE INTEGRALS

1. $\int_a^b f(x)dx = \int_a^b f(\mathrm{t})dt$

2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3. $\int_a^a f(x)dx = 0$

4. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

6. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

7. $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a-x)dx$

8. $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx, if \quad f(2a-x) = f(x)$

$= 0 \qquad ,\text{if} \quad f(2a-x) = -f(x)$

9. i) $\int_{-a}^{a} f(x)dx = 2\int_0^{a} f(x)dx$, if f(x) is an even function.

ii) $\int_{-a}^{a} f(x)dx = 0$, if f(x) is an odd function.

AREA BY USING INTEGRALS

1. Area of the region under the curve $y = f(x)$ between the points a and b on x-axis is given by

$$Area = \int_a^b f(x)dx$$

2. Area of the region under the curve $x = g(y)$ between the points c and d on y-axis is given by

$$Area = \int_c^d g(y)dy$$