UNIT 2- Integration

Integration using Substitution method

Evaluate the following:

1. $\int \frac{(\log x)^2}{x}\, dx$

Ans. $\frac{(\log x)^3}{3} + C$

2. $\int \sin x \,.\, \sin(\cos x)\ dx$

Ans. $\cos(\cos x) + C$

3. $\int x\sqrt{x+2}\, dx$

Ans. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

4. $\int sec^2(7-4x)\, dx$

Ans. $\frac{-1}{4}\tan(7-4x) + C$

5. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\, dx$

Ans. $e^{\tan^{-1}x} + C$

Integration using Partial Fractions

Evaluate the following:

1. $\int \frac{dx}{(x+1)(x+2)}$

Ans. $\log\left(\frac{x+1}{x+2}\right) + C$

2. $\int \frac{x^2+1}{x^2-5x+6} dx$

Ans. $x - 5\log(x-2) + 10\log(x-3) + C$

3. $\int \frac{x}{(x-1)^2(x+2)}\, dx$

Ans. $\frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$

4. $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\, dx$

Ans. $\frac{3}{5}\log(x+2) + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}x + C$

5. $\int \frac{x^2}{(x^2+1)\,(x^2+4)}\, dx$

Ans. $\frac{-1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\left(\frac{x}{2}\right) + C$

Integration using trigonometric identities

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x+5)$
2. $\sin 3x \cos 4x$
3. $\cos 2x \cos 4x \cos 6x$
4. $\sin^3(2x+1)$
5. $\sin^3 x \cos^3 x$
6. $\sin x \sin 2x \sin 3x$
7. $\sin 4x \sin 8x$
8. $\frac{1-\cos x}{1+\cos x}$
9. $\frac{\cos x}{1+\cos x}$
10. $\sin^4 x$
11. $\cos^4 2x$
12. $\frac{\sin^2 x}{1+\cos x}$
13. $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$
14. $\frac{\cos x - \sin x}{1+\sin 2x}$
15. $\tan^3 2x \sec 2x$
16. $\tan^4 x$
17. $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$
18. $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$
19. $\frac{1}{\sin x \cos^3 x}$
20. $\frac{\cos 2x}{(\cos x + \sin x)^2}$
21. $\sin^{-1}(\cos x)$
22. $\frac{1}{\cos(x-a)\cos(x-b)}$

Integration of some particular functions

Evaluate the following:

1. $\int \frac{dx}{x^2-16}$

Ans. $\frac{1}{8}\log\left(\frac{x-4}{x+4}\right)+C$

2. $\int \frac{dx}{x^2-6x+13}$

Ans. $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$

3. $\int \frac{dx}{3x^2+13x-10}$

Ans. $\frac{1}{17}\log\left(\frac{3x-2}{x+5}\right)+C$

4. $\int \frac{dx}{\sqrt{5x^2-2x}}$

Ans. $\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$

5. $\int \sqrt{x^2-6x}\ dx$

Ans. $\frac{(x-3)\sqrt{x^2-6x}}{2}-\frac{9}{2}\log\left|x+\sqrt{x^2-6x}\right|+C$

6.

(i) $\int \frac{dx}{x^2-16}$ (ii) $\int \frac{dx}{\sqrt{2x-x^2}}$

7.

(i) $\int \frac{dx}{x^2-6x+13}$ (ii) $\int \frac{dx}{3x^2+13x-10}$ (iii) $\int \frac{dx}{\sqrt{5x^2-2x}}$

8.

(i) $\int \frac{x+2}{2x^2+6x+5}dx$ (ii) $\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$

Definite Integrals

Evaluate the following:

1. $\int_2^3 x^2\ dx$

Ans. $\frac{19}{3}$

2. $\int_0^{\frac{\pi}{4}} \sin 2x\ dx$

Ans. $\frac{1}{2}$

3. $\int_2^3 \frac{1}{x}\ dx$

Ans. $\log\left(\frac{3}{2}\right)$

4. $\int_0^1 \frac{dx}{x^2+1}$

Ans. $\frac{\pi}{4}$

5. $\int_{-1}^1 5x^4\sqrt{x^5+1}\ dx$

Ans. $\frac{4\sqrt{2}}{3}$

6.

Evaluate $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$

Properties of Definite Integrals

Evaluate the following:

1. $\int_{-\pi/4}^{\pi/4} sin^2x\ dx$

Ans. $\frac{\pi}{4}-\frac{1}{2}$

2. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}dx$

Ans. $\frac{\pi}{4}$

3. $\int_{-5}^5 |x+2|\ dx$

Ans. 29

4. $\int_{-1}^{1} sin^5x.cos^4x\ dx$

Ans. 0

5. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

Ans. $\frac{\pi}{12}$

Additional Problems

I. Integrate the following functions:

(i) $\sin mx$ (ii) $2x \sin (x^2 + 1)$

(iii) $\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$ (iv) $\frac{\sin (\tan^{-1} x)}{1+x^2}$

II. Find the integrals:

(i) $\int \sin^3 x \cos^2 x\, dx$ (ii) $\int \frac{\sin x}{\sin (x+a)} dx$ (iii) $\int \frac{1}{1+\tan x} dx$

18. $\frac{e^{\tan^{-1}x}}{1+x^2}$ 19. $\frac{e^{2x}-1}{e^{2x}+1}$ 20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

21. $\tan^2(2x-3)$ 22. $\sec^2(7-4x)$ 23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

24. $\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$ 25. $\frac{1}{\cos^2 x(1-\tan x)^2}$ 26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

27. $\sqrt{\sin 2x}\cos 2x$ 28. $\frac{\cos x}{\sqrt{1+\sin x}}$ 29. $\cot x \log \sin x$

30. $\frac{\sin x}{1+\cos x}$ 31. $\frac{\sin x}{(1+\cos x)^2}$ 32. $\frac{1}{1+\cot x}$

1. $\frac{2x}{1+x^2}$ 2. $\frac{(\log x)^2}{x}$ 3. $\frac{1}{x+x\log x}$

4. $\sin x \sin(\cos x)$ 5. $\sin(ax+b)\cos(ax+b)$

6. $\sqrt{ax+b}$ 7. $x\sqrt{x+2}$ 8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$ 10. $\frac{1}{x-\sqrt{x}}$ 11. $\frac{x}{\sqrt{x+4}}, x>0$

12. $(x^3-1)^{\frac{1}{3}}x^5$ 13. $\frac{x^2}{(2+3x^3)^3}$ 14. $\frac{1}{x(\log x)^m}, x>0,\ m\neq 1$

15. $\frac{x}{9-4x^2}$ 16. e^{2x+3} 17. $\frac{x}{e^{x^2}}$

Partial Fraction method

Find the integrals of the following functions:

1. $\frac{x}{(x+1)(x+2)}$ 2. $\frac{1}{x^2-9}$ 3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

4. $\frac{x}{(x-1)(x-2)(x-3)}$ 5. $\frac{2x}{x^2+3x+2}$ 6. $\frac{1-x^2}{x(1-2x)}$

7. $\frac{x}{(x^2+1)(x-1)}$ 8. $\frac{x}{(x-1)^2(x+2)}$ 9. $\frac{3x+5}{x^3-x^2-x+1}$

10. $\frac{2x-3}{(x^2-1)(2x+3)}$ 11. $\frac{5x}{(x+1)(x^2-4)}$ 12. $\frac{x^3+x+1}{x^2-1}$

13. $\frac{2}{(1-x)(1+x^2)}$ 14. $\frac{3x-1}{(x+2)^2}$ 15. $\frac{1}{x^4-1}$

Integrate the functions in Exercises 1 to 9.

1. $\sqrt{4-x^2}$
2. $\sqrt{1-4x^2}$
3. $\sqrt{x^2+4x+6}$
4. $\sqrt{x^2+4x+1}$
5. $\sqrt{1-4x-x^2}$
6. $\sqrt{x^2+4x-5}$
7. $\sqrt{1+3x-x^2}$
8. $\sqrt{x^2+3x}$
9. $\sqrt{1+\frac{x^2}{9}}$

Evaluate the following definite integrals as limit of sums.

1. $\int_a^b x\,dx$
2. $\int_0^5 (x+1)\,dx$
3. $\int_2^3 x^2\,dx$
4. $\int_1^4 (x^2-x)\,dx$
5. $\int_{-1}^1 e^x\,dx$
6. $\int_0^4 (x+e^{2x})\,dx$

Evaluate the definite integrals in Exercises 1 to 20.

1. $\int_{-1}^1 (x+1)\,dx$
2. $\int_2^3 \frac{1}{x}\,dx$
3. $\int_1^2 (4x^3-5x^2+6x+9)\,dx$
4. $\int_0^{\frac{\pi}{4}} \sin 2x\,dx$
5. $\int_0^{\frac{\pi}{2}} \cos 2x\,dx$
6. $\int_4^5 e^x dx$
7. $\int_0^{\frac{\pi}{4}} \tan x\,dx$
8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}\, x\,dx$
9. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
10. $\int_0^1 \frac{dx}{1+x^2}$
11. $\int_2^3 \frac{dx}{x^2-1}$
12. $\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$
13. $\int_2^3 \frac{x\,dx}{x^2+1}$
14. $\int_0^1 \frac{2x+3}{5x^2+1}dx$
15. $\int_0^1 x\,e^{x^2}dx$
16. $\int_1^2 \frac{5x^2}{x^2+4x+3}$
17. $\int_0^{\frac{\pi}{4}} (2\sec^2 x+x^3+2)\,dx$
18. $\int_0^{\pi} (\sin^2\frac{x}{2}-\cos^2\frac{x}{2})\,dx$
19. $\int_0^2 \frac{6x+3}{x^2+4}dx$
20. $\int_0^1 (x\,e^x+\sin\frac{\pi x}{4})\,dx$

Particular functions

Integrate the functions in Exercises 1 to 23.

1. $\frac{3x^2}{x^6+1}$
2. $\frac{1}{\sqrt{1+4x^2}}$
3. $\frac{1}{\sqrt{(2-x)^2+1}}$
4. $\frac{1}{\sqrt{9-25x^2}}$
5. $\frac{3x}{1+2x^4}$
6. $\frac{x^2}{1-x^6}$
7. $\frac{x-1}{\sqrt{x^2-1}}$
8. $\frac{x^2}{\sqrt{x^6+a^6}}$
9. $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

10. $\frac{1}{\sqrt{x^2+2x+2}}$ 11. $\frac{1}{9x^2+6x+5}$ 12. $\frac{1}{\sqrt{7-6x-x^2}}$

13. $\frac{1}{\sqrt{(x-1)(x-2)}}$ 14. $\frac{1}{\sqrt{8+3x-x^2}}$ 15. $\frac{1}{\sqrt{(x-a)(x-b)}}$

16. $\frac{4x+1}{\sqrt{2x^2+x-3}}$ 17. $\frac{x+2}{\sqrt{x^2-1}}$ 18. $\frac{5x-2}{1+2x+3x^2}$

19. $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$ 20. $\frac{x+2}{\sqrt{4x-x^2}}$ 21. $\frac{x+2}{\sqrt{x^2+2x+3}}$

22. $\frac{x+3}{x^2-2x-5}$ 23. $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Definite Integrals

Evaluate the definite integrals in Exercises 1 to 20.

1. $\int_{-1}^{1}(x+1)\,dx$ 2. $\int_{2}^{3}\frac{1}{x}\,dx$ 3. $\int_{1}^{2}(4x^3-5x^2+6x+9)\,dx$

4. $\int_{0}^{\frac{\pi}{4}}\sin 2x\,dx$ 5. $\int_{0}^{\frac{\pi}{2}}\cos 2x\,dx$ 6. $\int_{4}^{5}e^x dx$ 7. $\int_{0}^{\frac{\pi}{4}}\tan x\,dx$

8. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\text{cosec}\,x\,dx$ 9. $\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$ 10. $\int_{0}^{1}\frac{dx}{1+x^2}$ 11. $\int_{2}^{3}\frac{dx}{x^2-1}$

12. $\int_{0}^{\frac{\pi}{2}}\cos^2 x\,dx$ 13. $\int_{2}^{3}\frac{x\,dx}{x^2+1}$ 14. $\int_{0}^{1}\frac{2x+3}{5x^2+1}dx$ 15. $\int_{0}^{1}x\,e^{x^2}dx$

16. $\int_{1}^{2}\frac{5x^2}{x^2+4x+3}$ 17. $\int_{0}^{\frac{\pi}{4}}(2\sec^2 x+x^3+2)\,dx$ 18. $\int_{0}^{\pi}(\sin^2\frac{x}{2}-\cos^2\frac{x}{2})\,dx$

19. $\int_{0}^{2}\frac{6x+3}{x^2+4}dx$ 20. $\int_{0}^{1}(x\,e^x+\sin\frac{\pi x}{4})\,dx$

Evaluate the integrals in Exercises 1 to 8 using substitution.

1. $\int_0^1 \frac{x}{x^2+1}dx$

2. $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi$

3. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

4. $\int_0^2 x\sqrt{x+2}$ (Put $x+2=t^2$)

5. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

6. $\int_0^2 \frac{dx}{x+4-x^2}$

7. $\int_{-1}^1 \frac{dx}{x^2+2x+5}$

8. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$