Image Restoration

1. Let's start with a shift-invariant model, meaning that every point in the original image spreads out the same way in forming the blurred image. We model this with convolution:
   $G(u,v) = H(u,v) * F(u,v) + N(u,v)$, where * is 2-D convolution, $H(u,v)$ is the point-spread function (PSF), $F(u,v)$ is the original image, and $N(u,v)$ is noise.

   (a) Model an out-of-blur PSF using `fspecial` function.

   (b) Synthesize a blurred image by implementing circular convolution.

   (c) Apply inverse filtering to obtain the original image.
   RestoredImage = IFFT{FFT(BlurredImage) ÷ FFT(PSF)}

2. (a) Let's add some noise: If we define peak SNR (PSNR) as
   $\text{PSNR} = 10\log_{10}\frac{[gmax-gmin]^2}{\sigma_u^2}$ then the noise scaling will be $\sigma_u = 10^{-\text{PSNR}/20}\times|gmax-gmin|$
   Now, add noise to the blurred image obtained in step:1(b) to get a $40db$ PSNR.

   (b) Attempt inverse filtering on this noisy image and infer the result.

3. (a) Now, attempt pseudo-inverse filtering, $H_I = \frac{H^*}{|H|^2+\delta^2}$ then $H_I \approx \frac{1}{H}$ if $|\delta| << |H|$ and $H_I \approx 0$ if $|\delta| >> |H|$

   (b) Implement Wiener filter,

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

   where

$$S_f(u,v) = |F(u,v)|^2$$

   and

$$S_\eta(u,v) = |N(u,v)|^2$$

4. (a) Blur an image using different types of filters like averaging filter, Gaussian low pass filter, Prewitt filter and Sobel filter. Show them simultaneously.

   (b) Add 'salt & pepper' noise to all the blurred images obtained above. Restore them all and display.

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