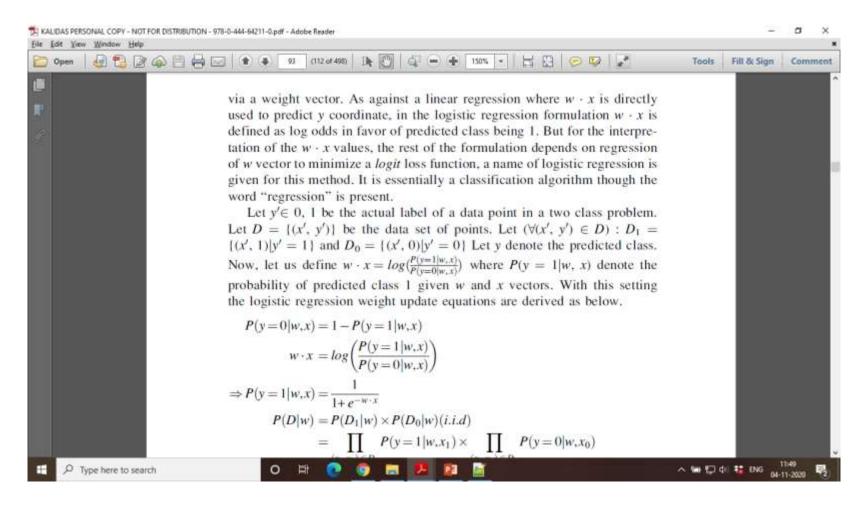
Logistic Regression

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In this class, you will learn about logistic regression

68) key phrase... "Logistic Regression"



Logistic Regression Formulation

- x_i is a $k \times 1$ dimensional vector
- $y_i \in \{-1,1\}$
- Model, $y = f(x) = w \cdot x$
 - w is a $k \times 1$ dimensional vector
- $D = \{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$
- ??? Loss function L(w)

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 - Probability of prediction of +1 given w and x
 - $P(y = +1|w, x_i)$

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Odds in favour of +1

Probability of prediction of +1 given w and x

$$P(y = +1|w, x_i)$$

Ratio of odds in favour of +1 to -1

$$\frac{P(y = +1|w, x_i)}{P(y = -1|w, x_i)}$$

Define the logarithm of the above ratio as $w \cdot x_i$

$$w \cdot x_i = \log(\frac{P(y = +1|w, x_i)}{P(y = -1|w, x_i)})$$

Deriving.. $P(y = y_i | w, x_i) = \frac{1}{1 + e^{-y_i \times (w \cdot x_i)}}$

•
$$\log\left(\frac{P(y=+1)}{P(y=-1)}\right) = w \cdot x$$

$$\bullet \frac{P(y=+1)}{P(y=-1)} = e^{W \cdot \chi}$$

•
$$P(y = +1) = P(y = -1)e^{w \cdot x}$$

•
$$P(y = +1) = (1 - P(y = +1))e^{w \cdot x}$$

•
$$P(y=+1)+P(y=+1)e^{w.x}=e^{w.x}$$

•
$$P(y = +1) = \frac{e^{w \cdot x}}{1 + e^{w \cdot x}} = \frac{1}{1 + e^{-w \cdot x}}$$

• Similarly we can observe,
$$P(y = -1) = 1 - P(y = +1) = \frac{1}{1 + e^{w \cdot x}}$$

• We can simplify and write,
$$P(y = y_i | w, x_i) = \frac{1}{1 + e^{-y_i \times (w \cdot x_i)}}$$

Probability of Data.. Minimize $\sum_{i=1}^{i=N} \log(1 + e^{-y_i \times w \cdot x_i})$

- Data set $D = \{(x_1, y_1), ..., (x_N, y_N)\}$
- Probability of correct predictions
 - on entire data, $P(y = y_1 | x_1) \times \cdots P(y = y_N | x_N)$
 - $\bullet = \prod_{i=1}^{i=N} P(y = y_i | x_i)$
- Entire probability of data set, $P(D|w) = \prod_{i=1}^{i=N} P(y = y_i|x_i)$
- Logarithm of the probability of data set, log(P(D|w))
- $\log(P(D|w)) = \sum_{i=1}^{i=N} \log(P(y=y_i|x_i))$
- $\sum_{i=1}^{i=N} \log(P(y=y_i|x_i)) = \sum_{i=1}^{i=N} \log(\frac{1}{1+e^{-y_i \times w \cdot x_i}})$
- = $-\sum_{i=1}^{i=N} \log(1 + e^{-y_i \times w \cdot x_i})$
- Maximizing $-\sum_{i=1}^{i=N} \log(1 + e^{-y_i \times w \cdot x_i})$ is equivalent to Minimizing $\sum_{i=1}^{i=N} \log(1 + e^{-y_i \times w \cdot x_i})$

Maximizing -f(x) becomes Minimization +f(x)