Steepest Descent

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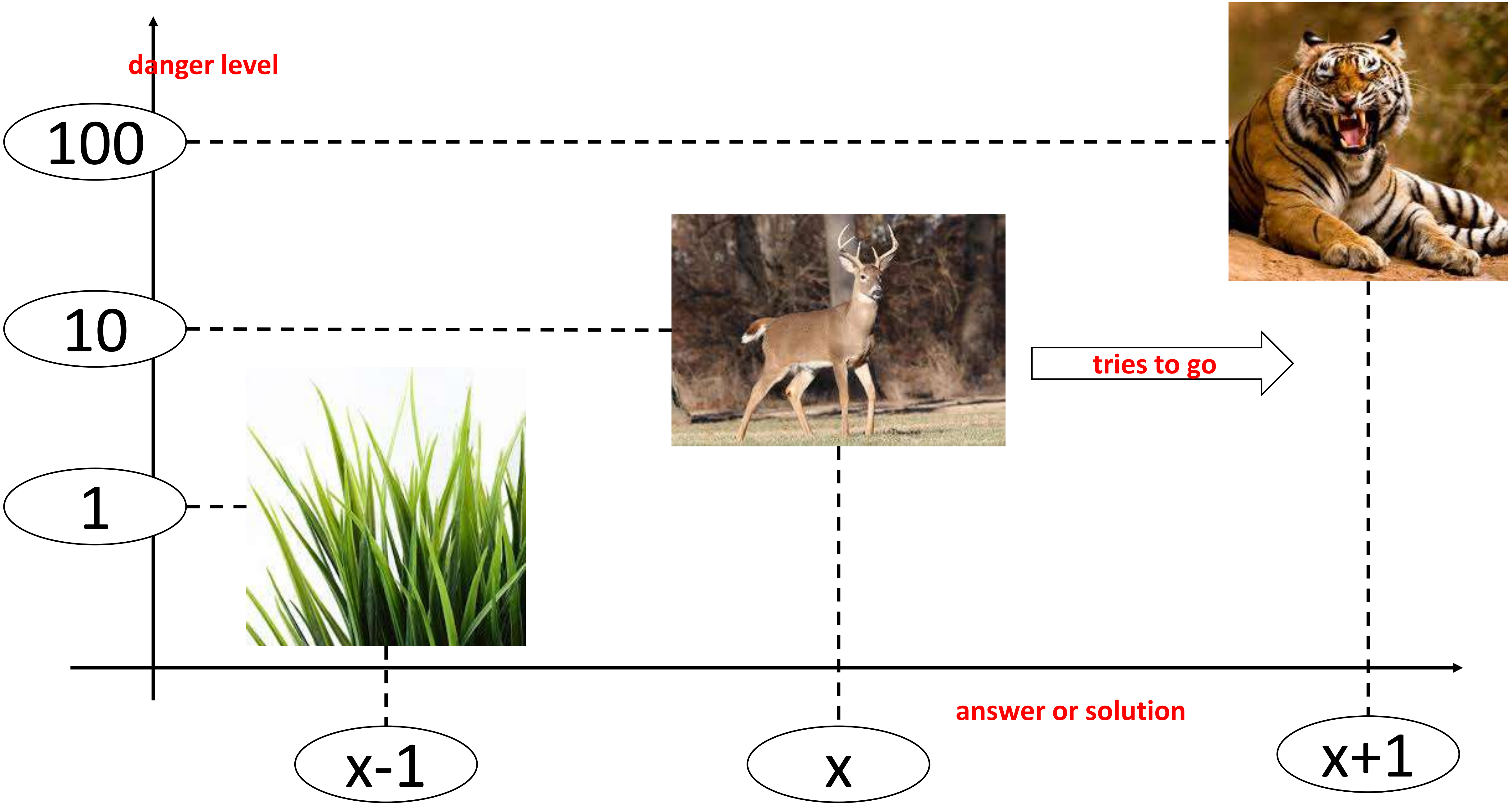
In this lecture you will understand intuition behind steepest descent

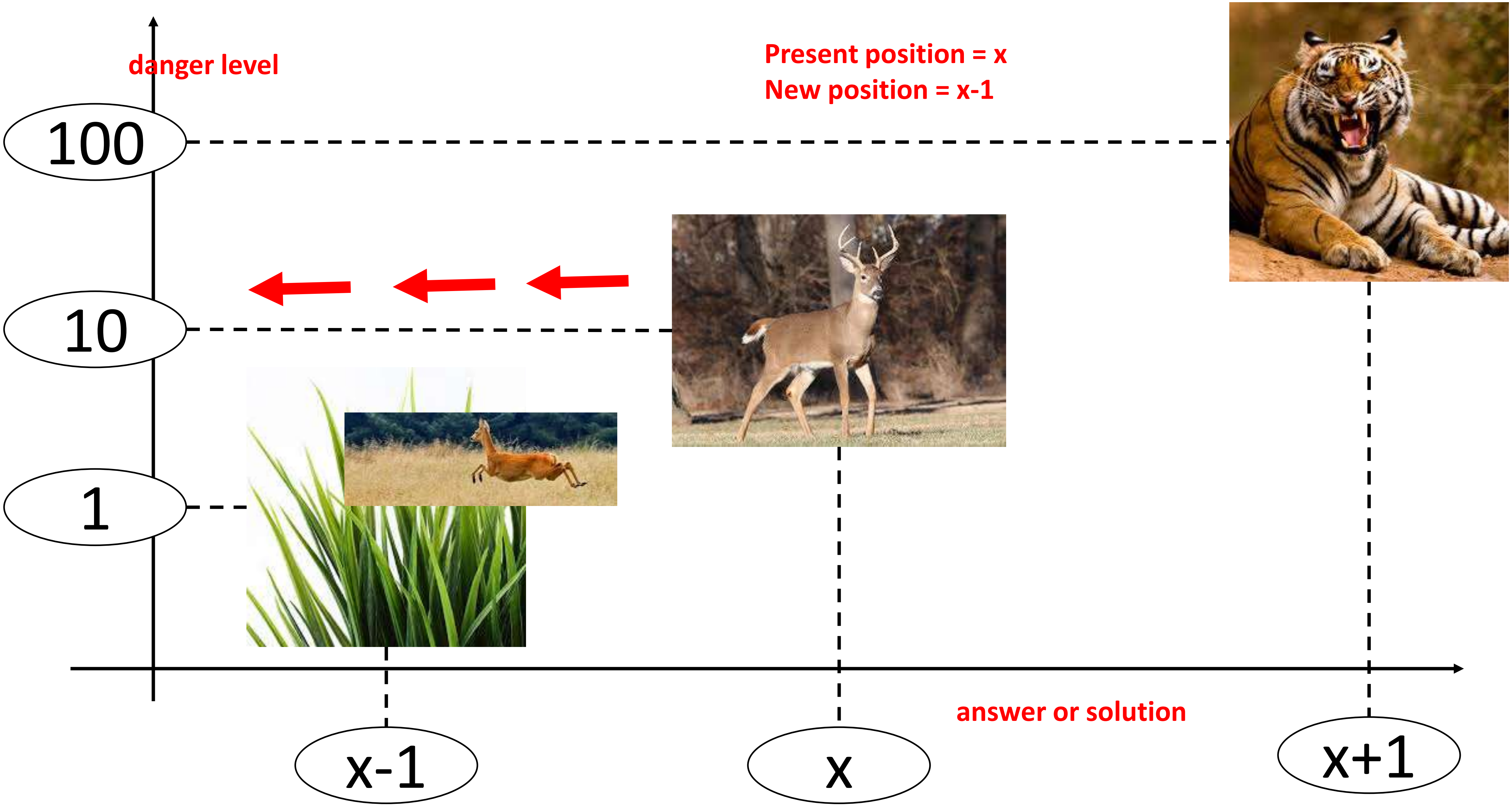
Method of **Steepest Descent**

- $f(x - a + a) = f^{(0)}(a)\frac{(x-a)^0}{0!} + f^{(1)}(a)\frac{(x-a)^1}{1!}$
- $x\ is\ a$
- $x - a\ is\ \delta$
- $\boldsymbol{f(x + \delta) = f(x) + f'(x) \times \delta}$

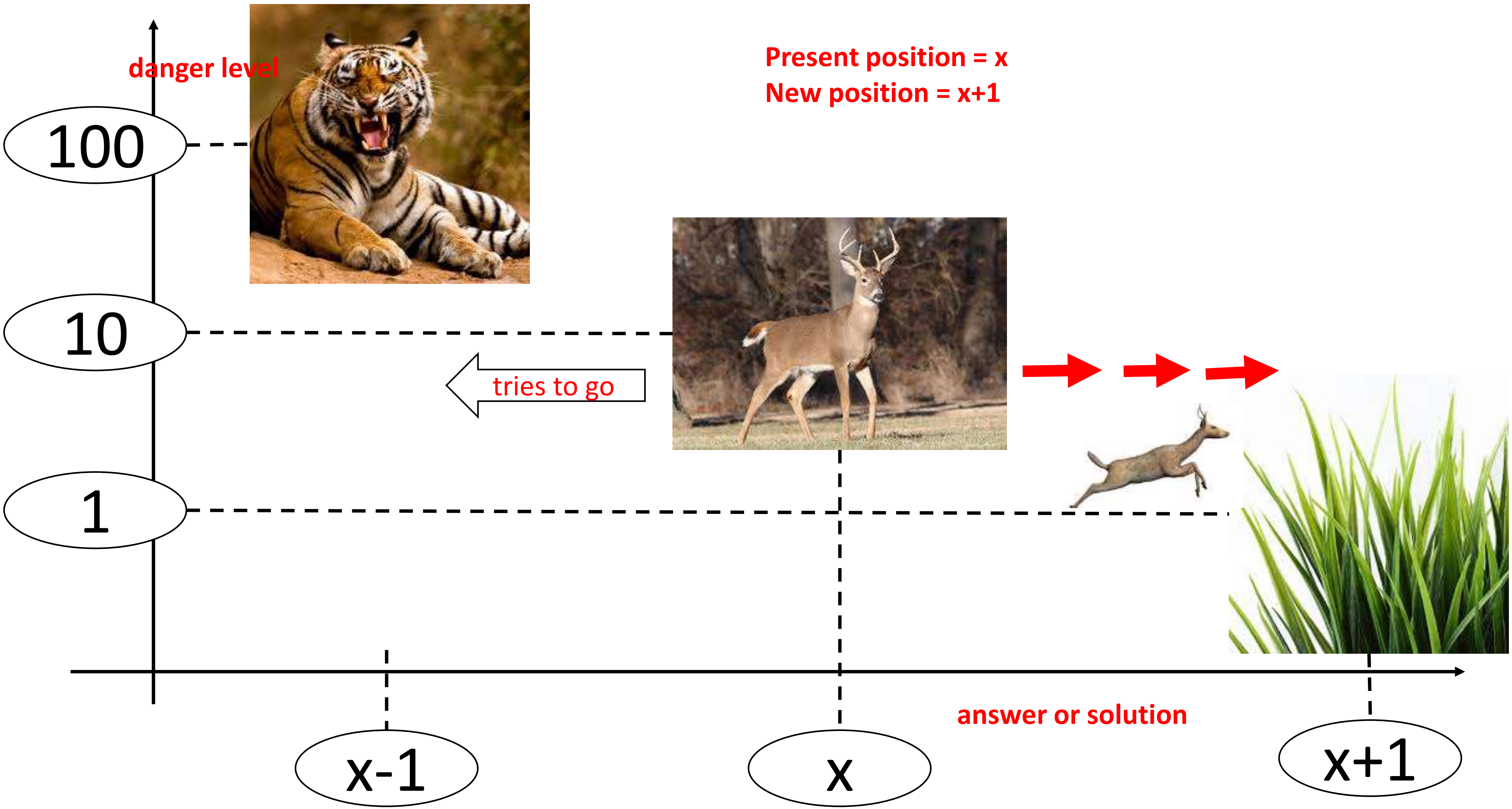
- When is $f(x + \delta) < f(x)$?
  - $\delta = -\eta\, f'(x)\ (\exists \eta > 0)$
  - $f(x + \delta) = f(x) - \eta\, f'(x)^2$
  - $f(x + \delta) < f(x)$
- $x_{(new)}\ is\ x + \delta$

$$\boldsymbol{x_{(new)} = x_{(old)} - \eta\, f'\left(x_{(old)}\right)}$$


danger level
100
10
1
tries to go
answer or solution
x-1
x
x+1



danger level
Present position = x
New position = x-1
100
10
1
answer or solution
x-1
x
x+1


danger level
100
10
1
Present position = x
New position = x+1
tries to go
answer or solution
x-1
x
x+1

Present position = x
New position = x + 1 or x -1 ???

Present position = x

New position = x + 1

IF danger is on *the Left*

Present position = x
New position = x -1
IF danger is on *the Right*

Present position = x

New position = …

$x - \text{sign}(\text{danger}(x+1) - \text{danger}(x))$

Steepest Descent Interpretation

- Left, Current, Right values: $L(x-\delta), L(x), L(x+\delta)$
- STEP: δ
- Imagine locally it is a line, $(x \in [x-\delta, x+\delta])$ i.e. $L(x) = m\,x + c$
- Slope: $\mathrm{m} = \frac{L(x+\delta)-L(x)}{\delta}$
- Positive slope (m>0)
  - LOSS INCREASES if you go along $+\delta$
  - LOSS DECREASES if you go along $-\delta$
- Negative slope (m<0)
  - LOSS INCREASES if you go along $-\delta$
  - LOSS DECREASES if you go along $+\delta$

- m and δ are inversely related
- Loss decreases
  - $x_{(new)} = x_{(old)} + \delta$
  - $x_{(new)} = x_{(old)} - m$

$$x_{(new)} = x_{(old)} - \eta\, L'(x)$$