Linear Regression

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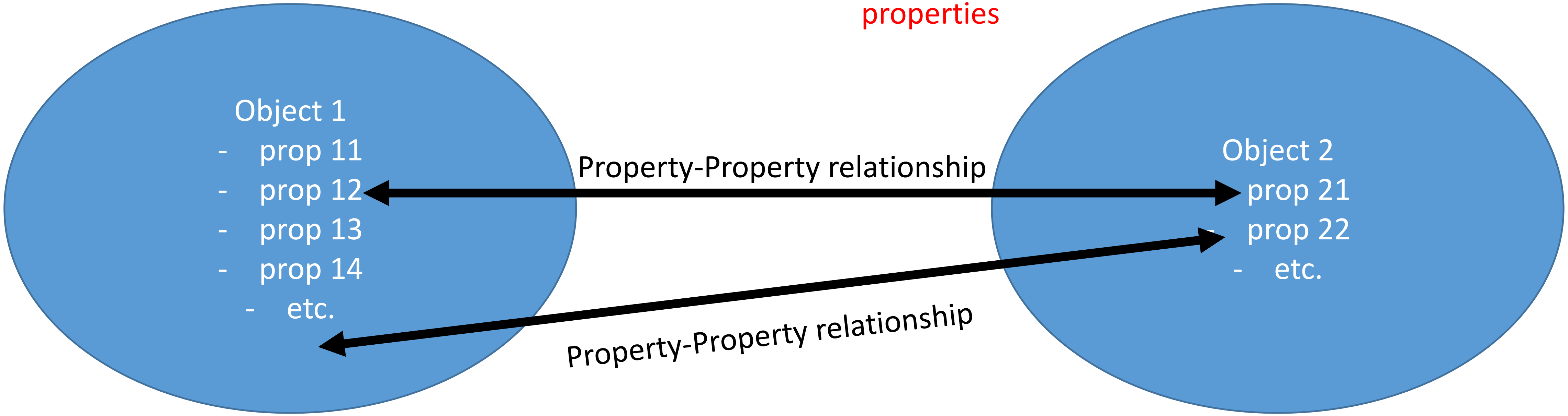
In this lecture you will learn about LOSS Function and SOLVER formulation

Objects can be ANY… literally ANY..! concept in the world!!

(1) Identify objects and properties within each object

(2) A rough understanding of which properties relate to which other properties



(3) Vector representation of objects

Convert <noun/pronoun/etc> → <Noun>objects,
e.g. car -> CarObject, he -> PersonObject etc.

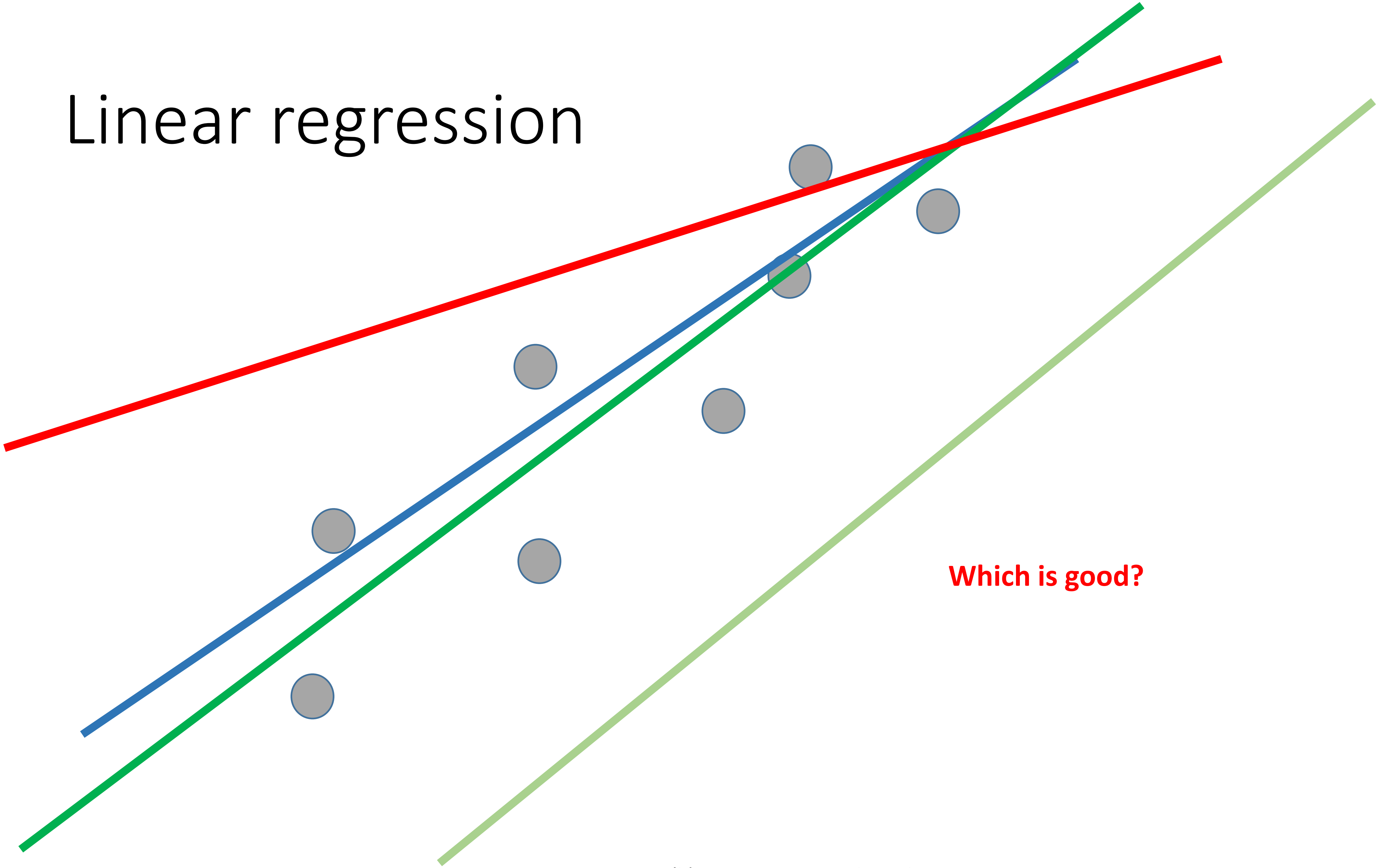
<verb → <Verb>-er, <Verb>-able objects,
e.g. walk -> Walk-er or Walk-able, talk -> Talk-er, Talk-able


AgeObject
- age value
HeightObject
- height value

AudioRecordingTimeDurationObject
- time value

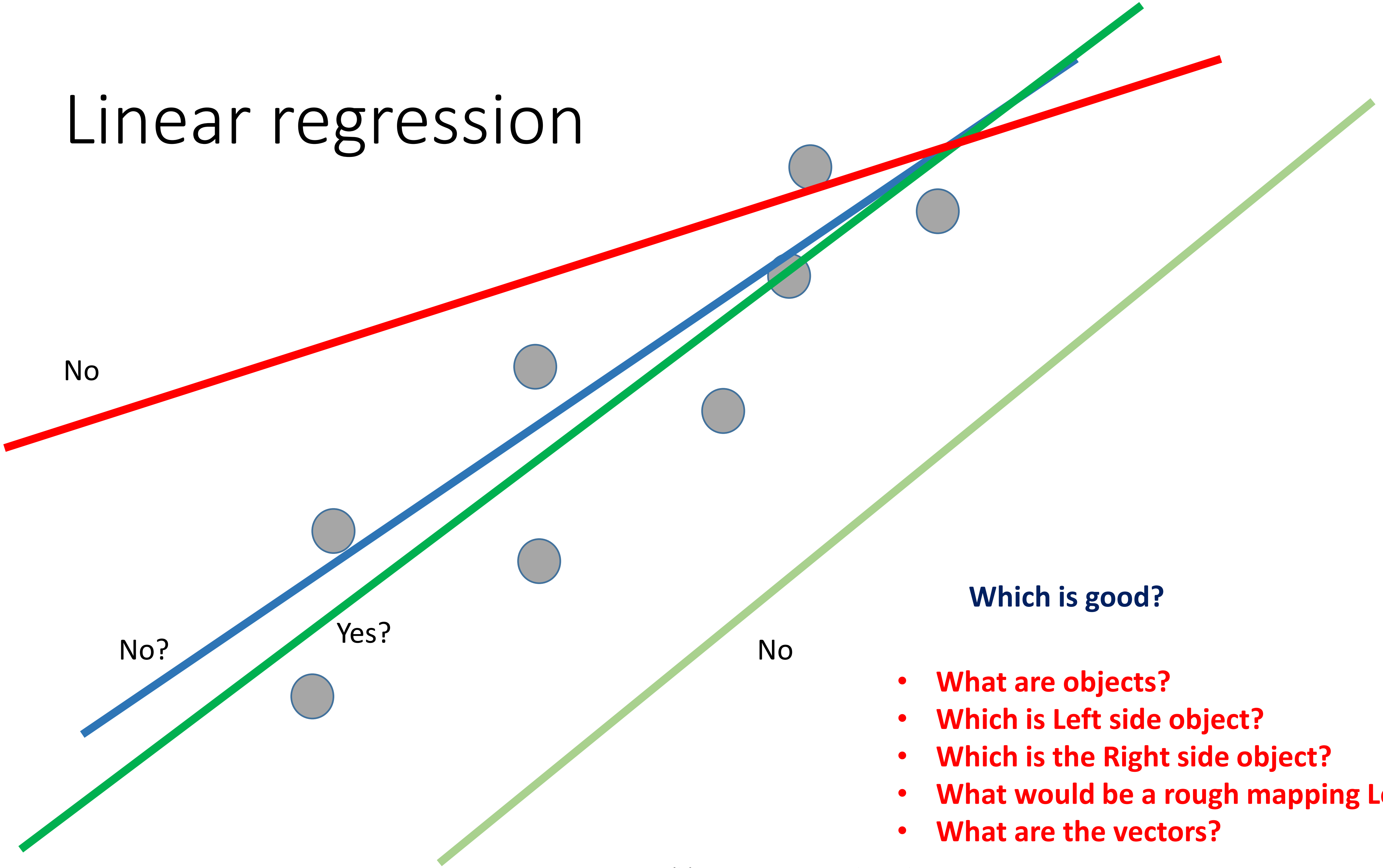
AudioRecordingFileSizeObject
- file size value

Linear regression



Which is good?

Linear regression



Which is good?

- **What are objects?**
- **Which is Left side object?**
- **Which is the Right side object?**
- **What would be a rough mapping Left to Right?**
- **What are the vectors?**

Linear regression

Where are objects?

X-coordinate Object
- x coordinate value

Y-coordinate Object
- y coordinate value

Which is good?

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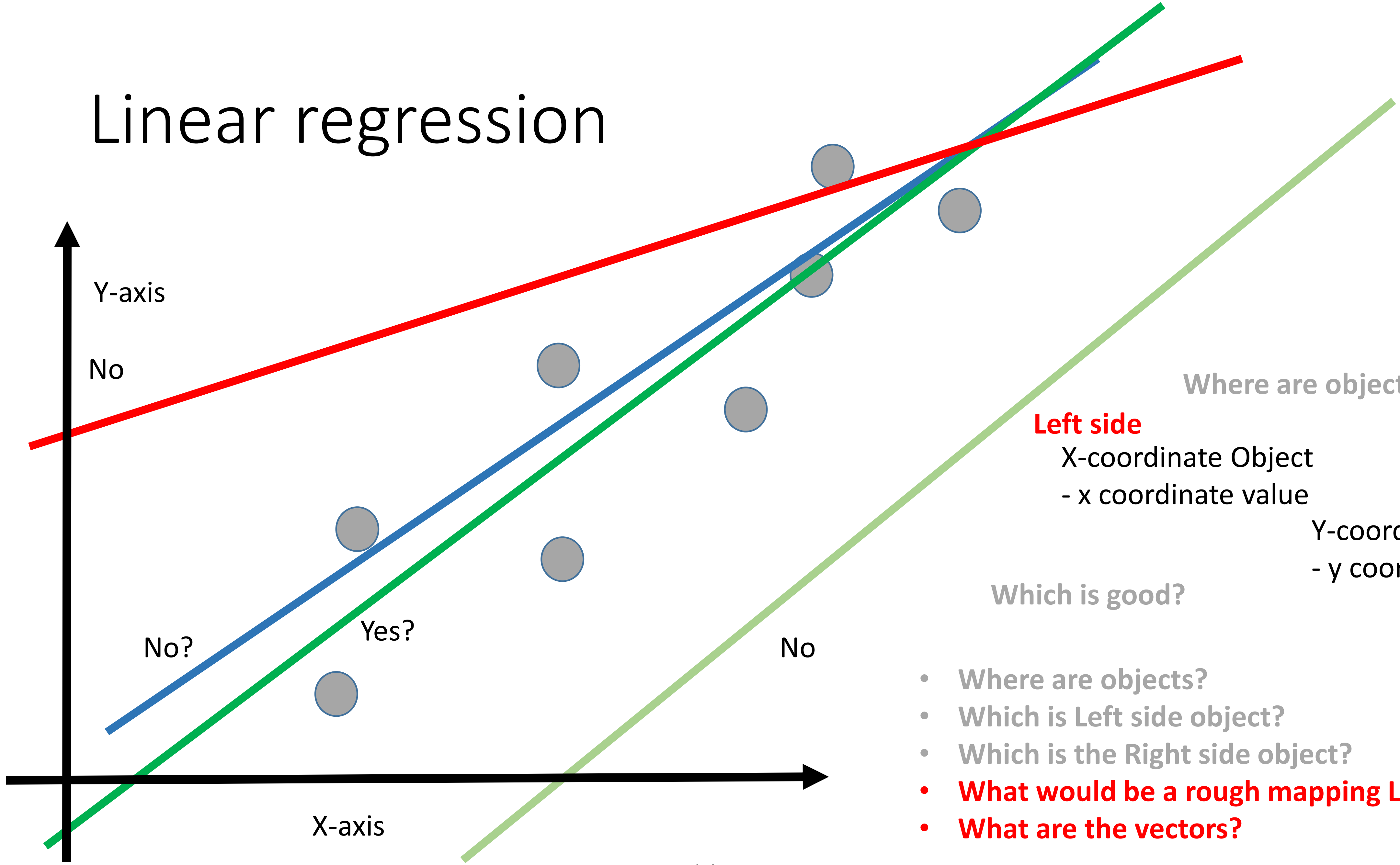
Linear regression



Where are objects?

Left side

X-coordinate Object

- x coordinate value

Right side

Y-coordinate Object

- y coordinate value

Which is good?

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Linear regression

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vector of 1 element

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vector of 1 element

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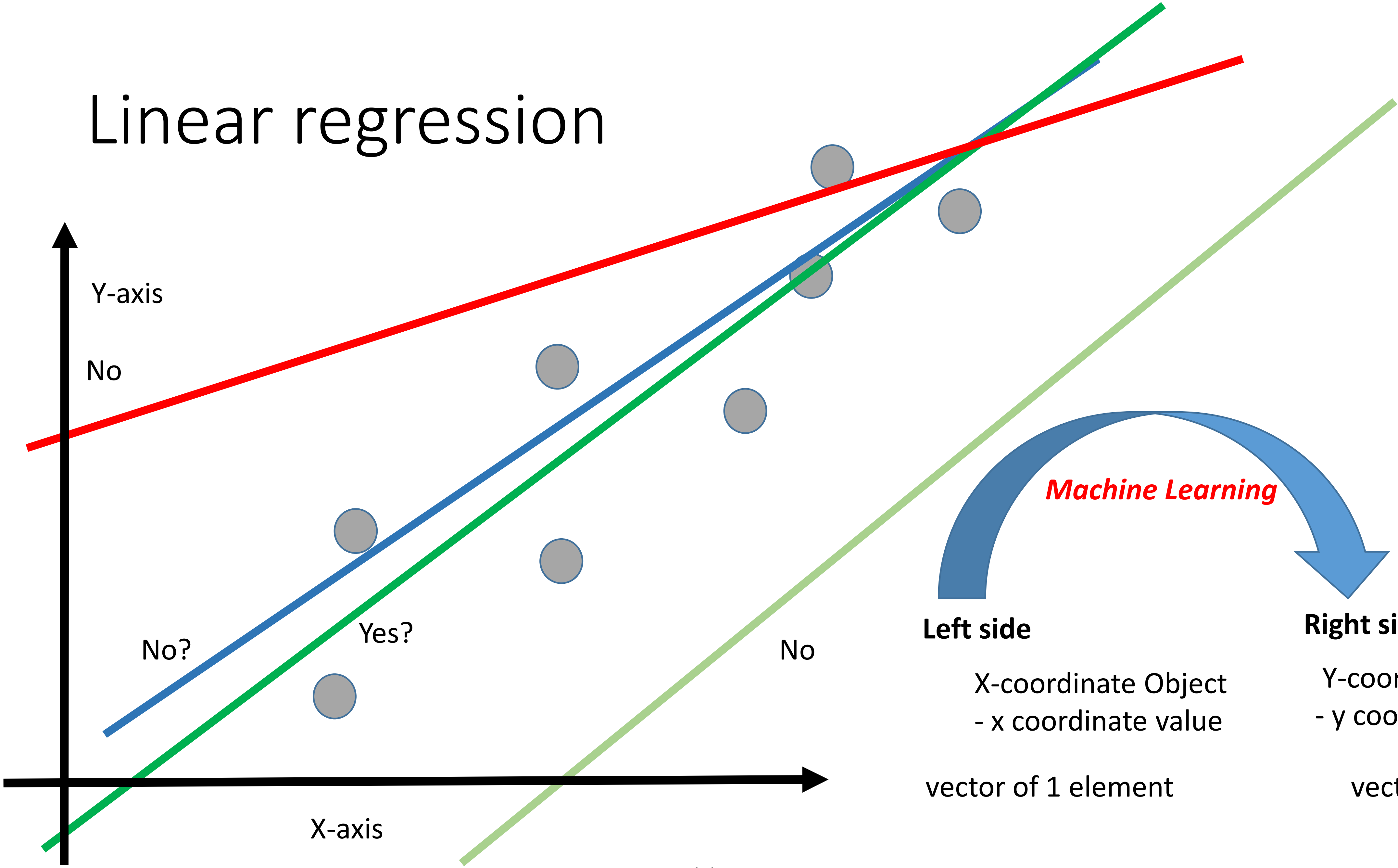
Linear regression



Left side

X-coordinate Object
- x coordinate value

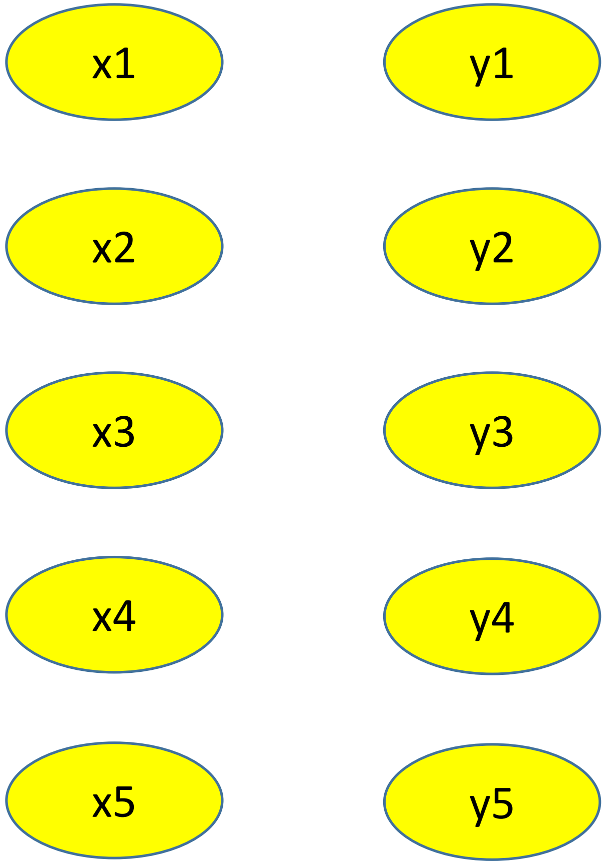
vector of 1 element

Right side

Y-coordinate Object
- y coordinate value

vector of 1 element

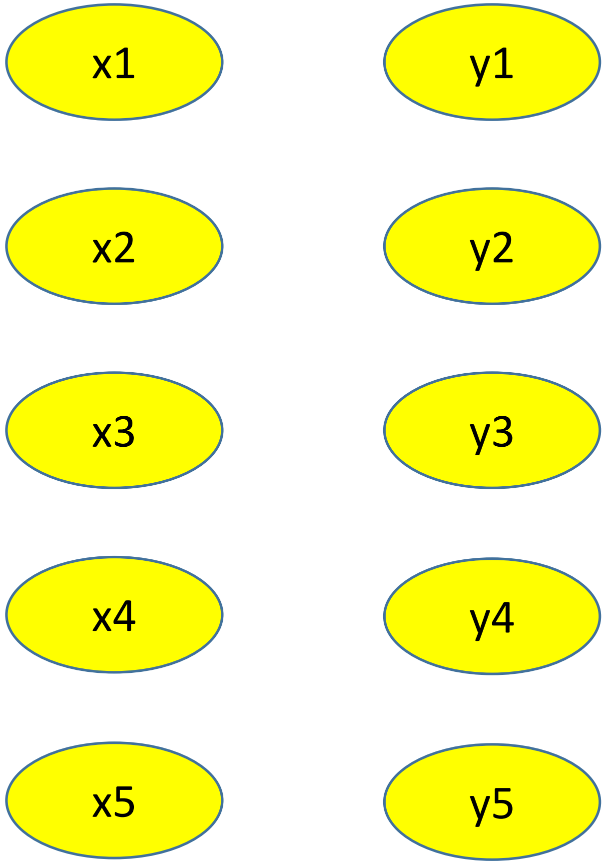
Representation



Mapping

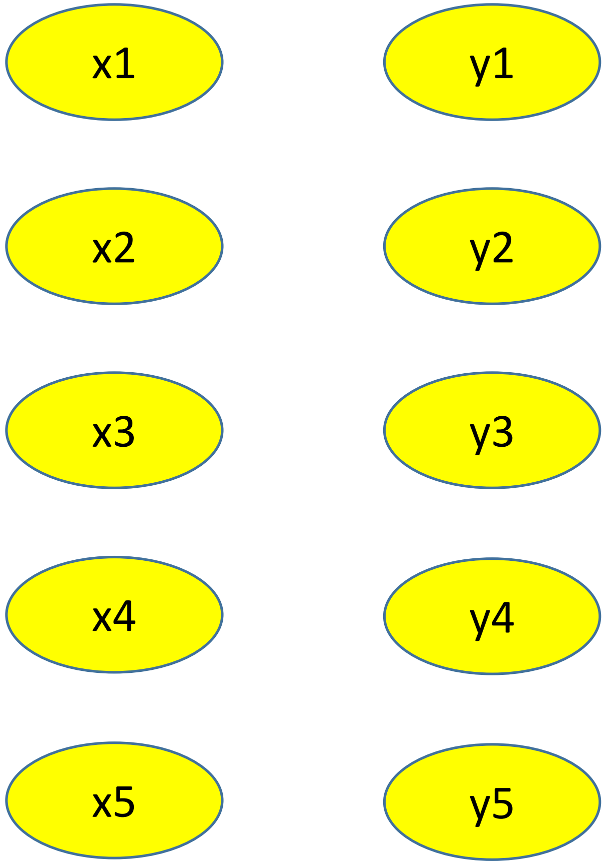
x_i vector to y_i vector

Representation



X	**Y** **Target or Label**
x1	y1
x2	y2
x3	y3
x4	y4
x5	y5

Representation



Mapping

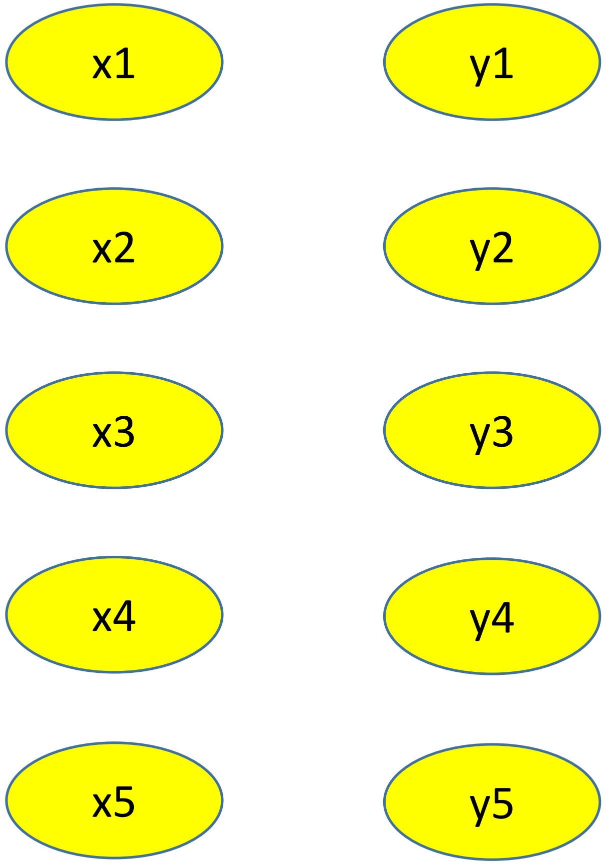
x_i vector to y_i vector

List notation

$$X = [x_i]_{i \in [1..5]}$$

$$Y = [y_i]_{i \in [1..5]}$$

Representation



Mapping

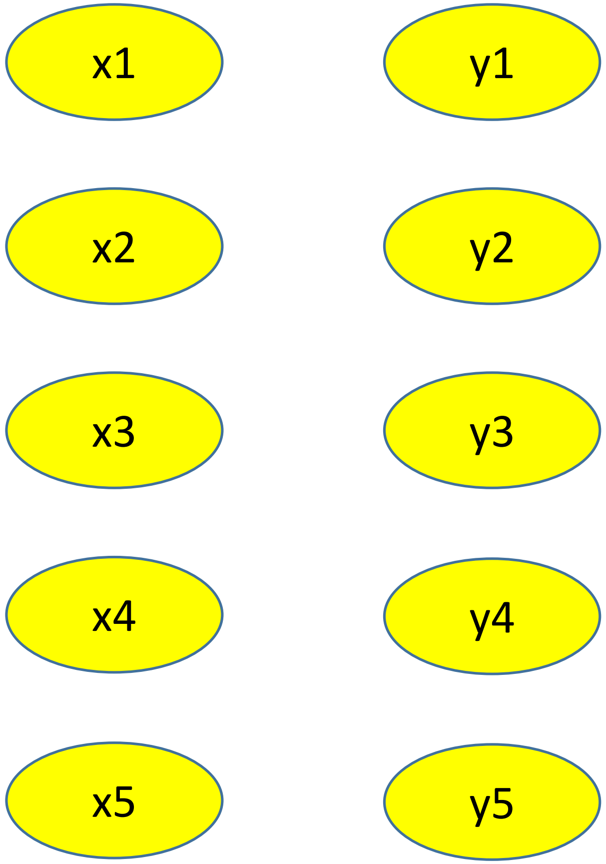
x_i vector to y_i vector

List notation

X

Y

Representation



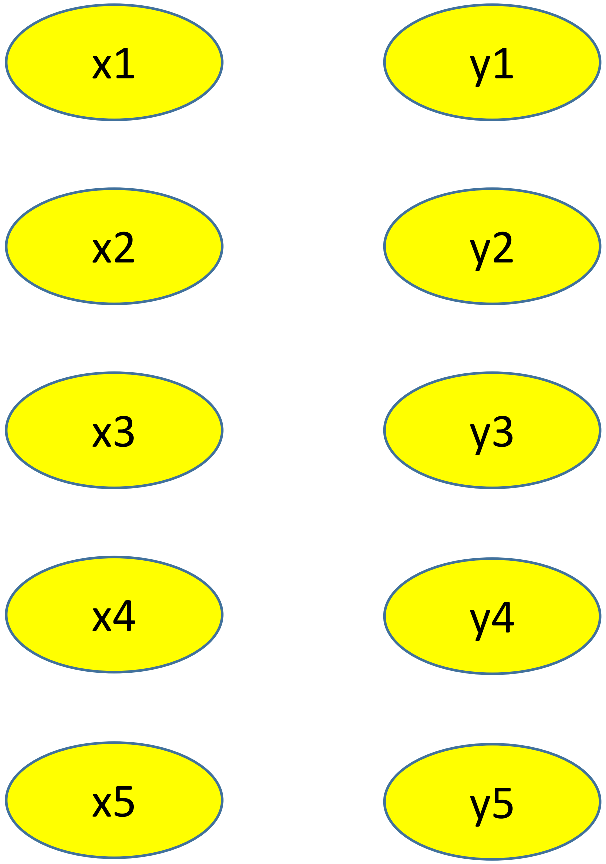
Mapping

x_i vector to y_i vector

List notation

$$Y = f(X)$$

Representation



Mapping

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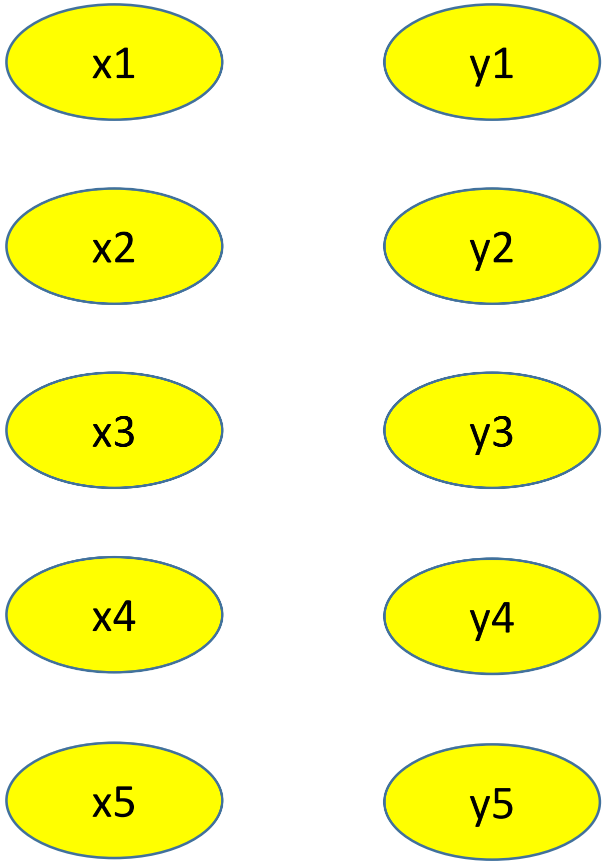
$Y = f(X)$

f() is called model

Representation



Mapping

x_i vector to y_i vector

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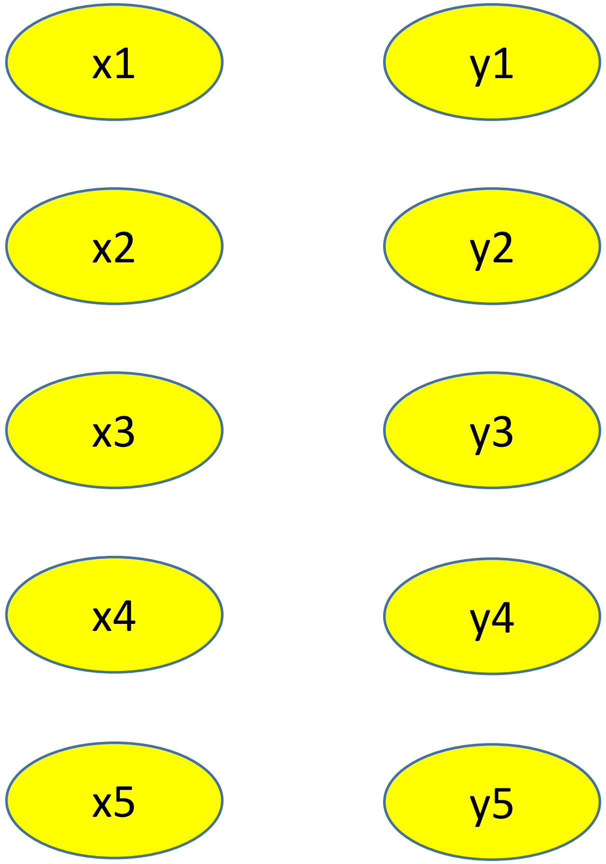
f() is called model

model's fit(X,Y)

Representation



Mapping

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List notation

$Y = f(X)$

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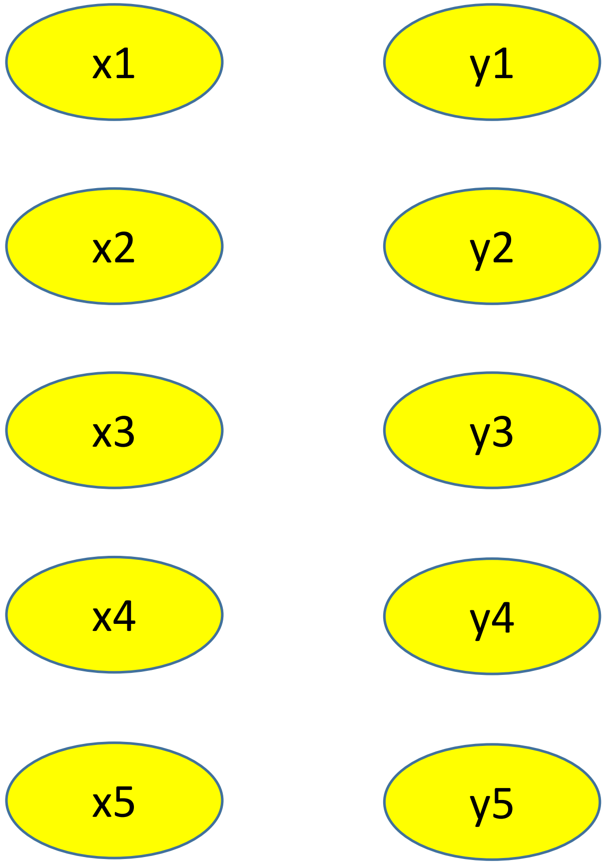
model's fit(X,Y)

model's predict($\mathbf{X}^{\mathbf{new}}$)

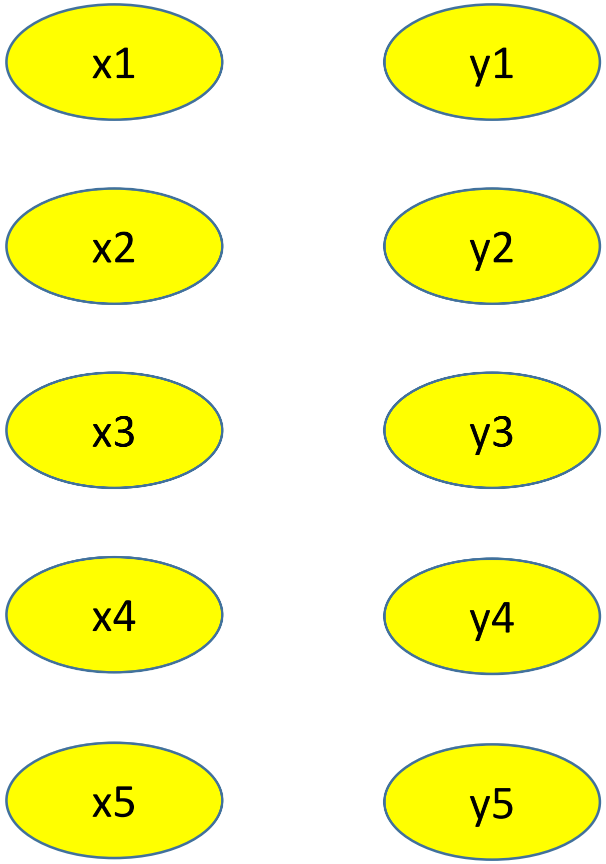
Left side (X) → Machine Learning → Right side (Y)



PREDICT..

Predict Y (right side), Given X (left side)

Left side (X) → Machine Learning → Right side (Y)



$y = x^2$? or..

$y = x^3$? or..

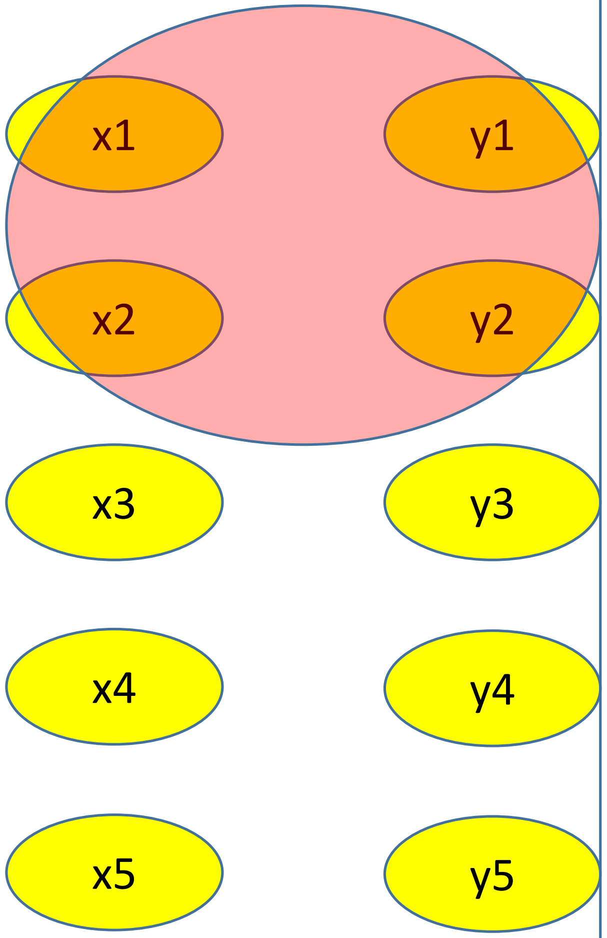
$y = 3\,x + 34$? or..

$y = -67\,x^2 + 45$? or..

????

????

Left side (X) → Machine Learning → Right side (Y)



Line…

passing through two points (x1,y1) and (x2,y2)

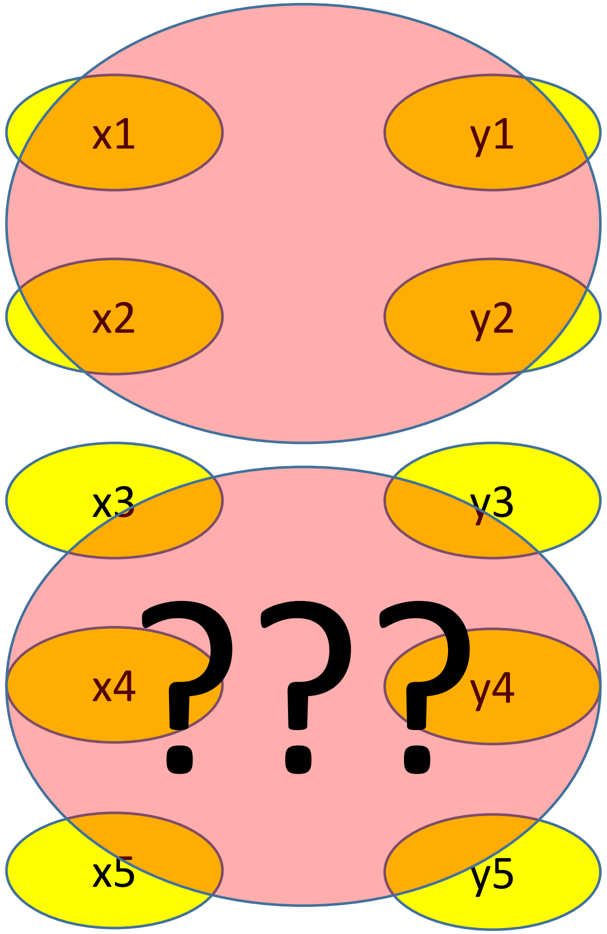
$$y = m\,x + c$$

$$\frac{y - y1}{x - x1} = \frac{y2 - y1}{x2 - x1}$$

$$y = \frac{y2 - y1}{x2 - x1} * \mathrm{x} + \left(\mathrm{y1} - \mathrm{x1} * \frac{y2 - y1}{x2 - x1}\right)$$

Linear Regression

Left side (X) → Machine Learning → Right side (Y)



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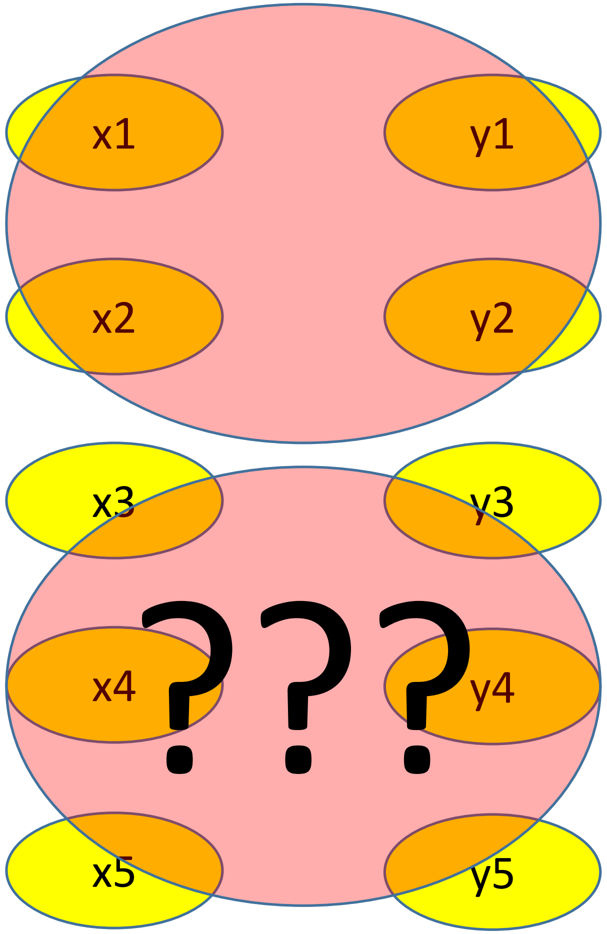

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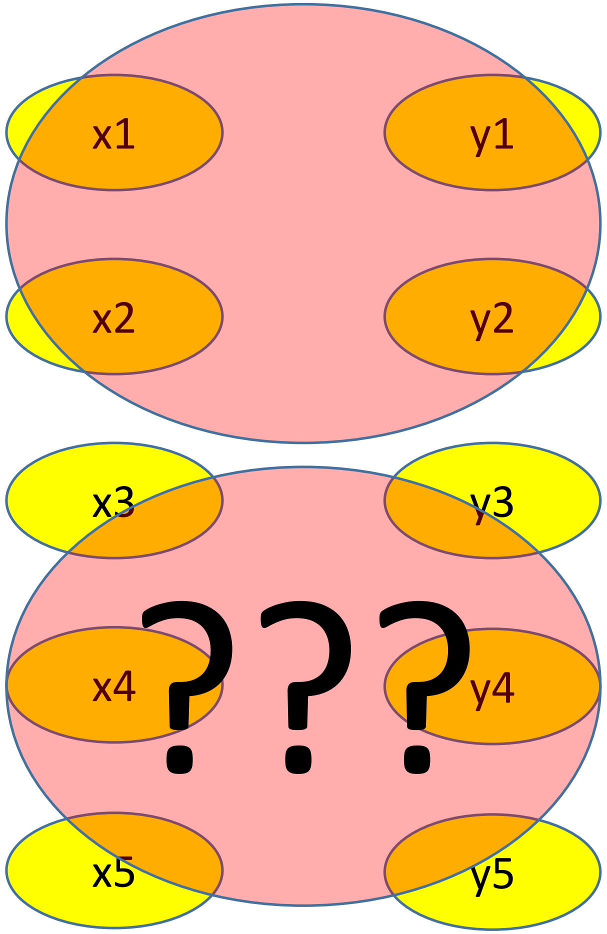
Substitute (x,y) = (x3,y3)

Substitute (x,y) = (x4,y4)

…and then do what!!??

Left side (X) → Machine Learning → Right side (Y)



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$$y = \frac{y2 - y1}{x2 - x1} * \mathrm{x} + \left(\mathrm{y1} - \mathrm{x1} * \frac{y2 - y1}{x2 - x1}\right)$$

~~Substitute (x,y) = (x3,y3)~~
~~Substitute (x,y) = (x4,y4)~~

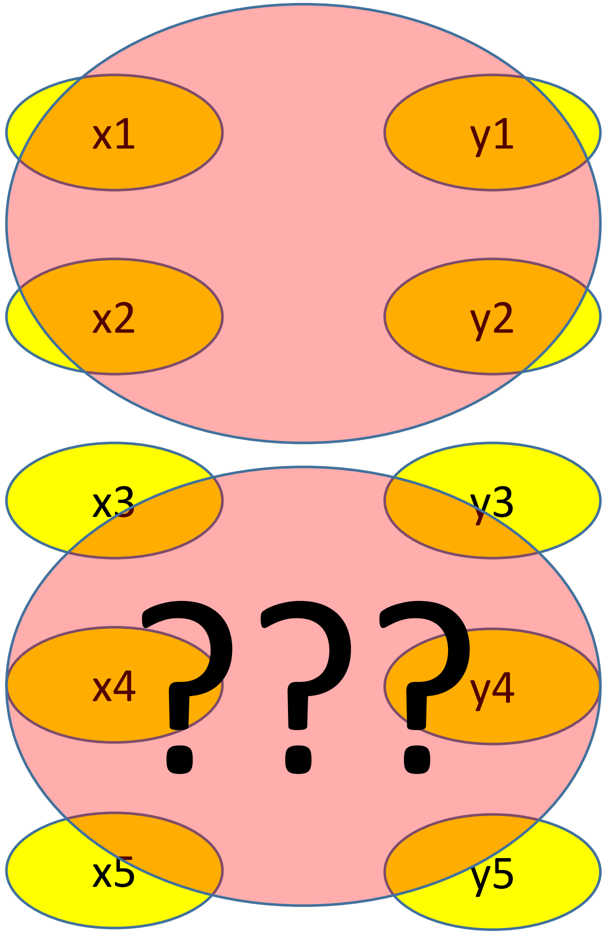
Need goodness measure!

Need badness measure!

Need quality measure!

Linear Regression

Left side (X) → Machine Learning → Right side (Y)

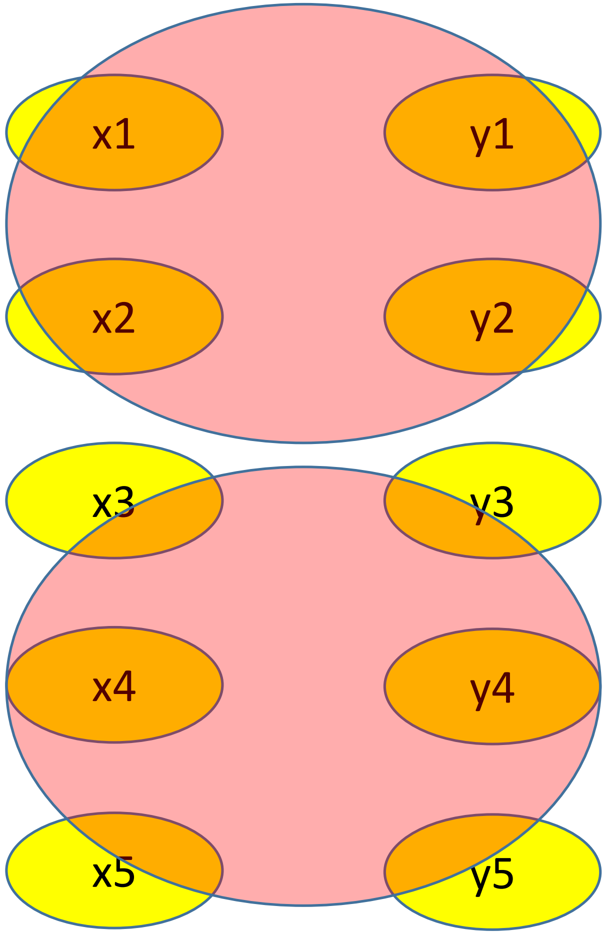


Line passing ~~through two points (x1,y1) and (x2,y2)~~

$$\cancel{y = \frac{y2 - y1}{x2 - x1} * \mathrm{x} + \left(\mathrm{y1} - \mathrm{x1} * \frac{y2 - y1}{x2 - x1}\right)}$$

Linear Regression

Left side (X) → Machine Learning → Right side (Y)

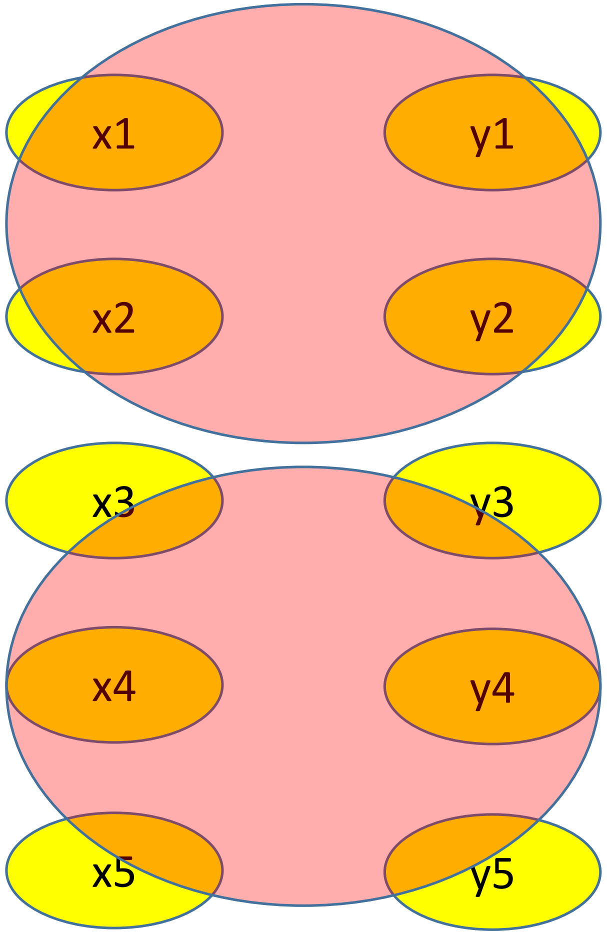


Equation of Line :- $y = m * x + c$

Need goodness or badness measure!

Linear Regression

Left side (X) → Machine Learning → Right side (Y)

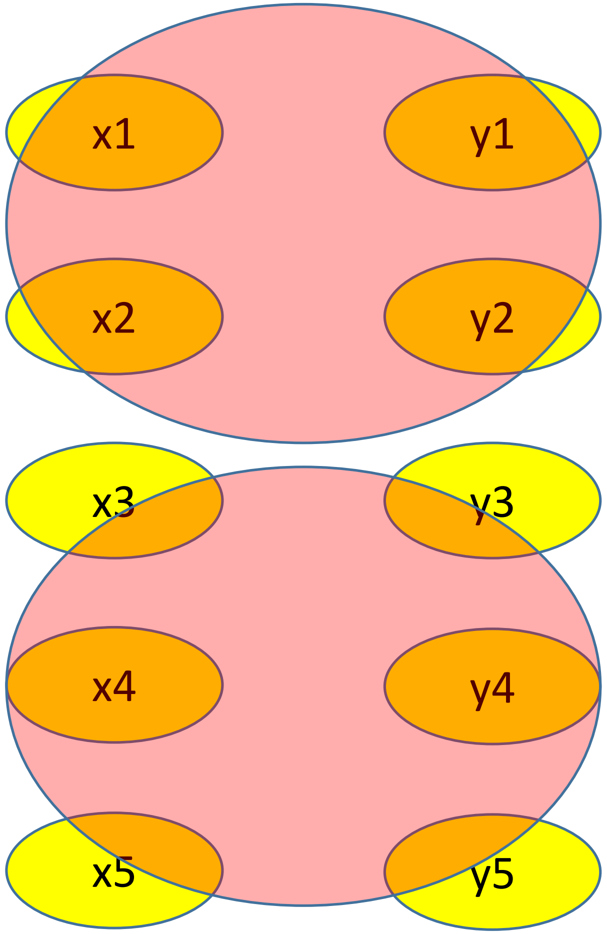


Equation of Line :- $\mathrm{y} = \mathrm{m} * x + c$

$$y1 = m * x1 + c\ ?$$
$$y2 = m * x2 + c\ ?$$
$$\ldots$$
$$y5 = m * x5 + c\ ?$$

???
Left side = Exactly same as = Right side ???
Exactly can they be same?
For any value of m and c
What is learning here?

Left side (X) → Machine Learning → Right side (Y)



Equation of Line :- $y = m * x + c$

$$y1 = m * x1 + c\ ?$$
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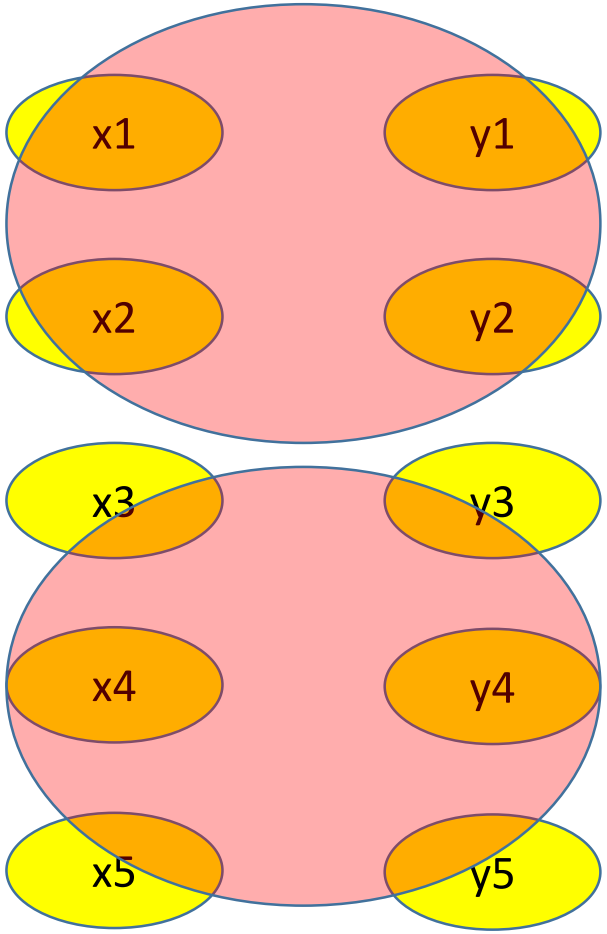
$$\ldots$$
$$y5 = m * x5 + c\ ?$$

What could be a badness measure?...

Linear Regression

Left side (X) → Machine Learning → Right side (Y)



Equation of Line :- $y = m * x + c$

Badness for (x1,y1) :- $(y1 - (m * x1 + c))^2$

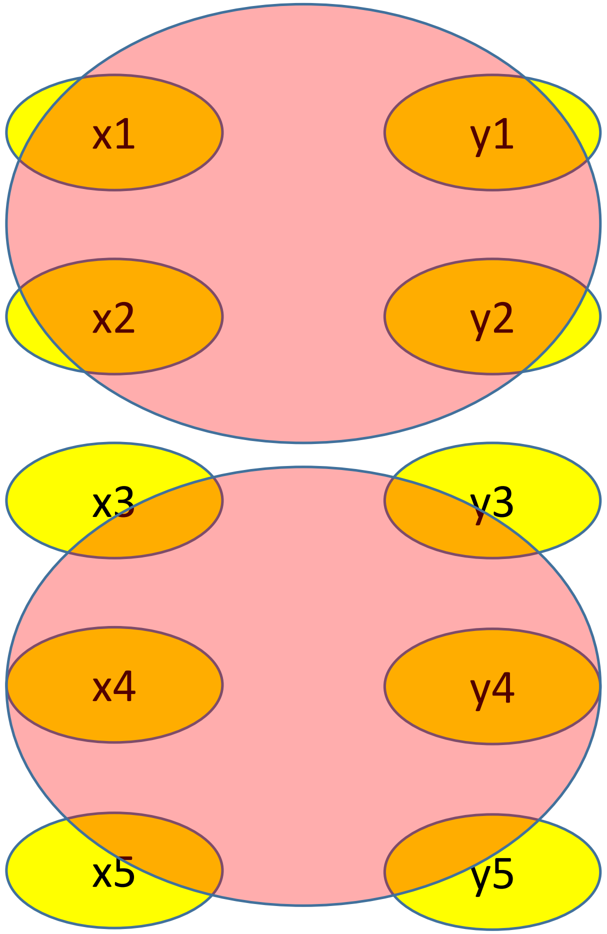
Badness for (x2,y2) :- $(y2 - (m * x2 + c))^2$

...

Badness for (x5,y5) :- $(y5 - (m * x5 + c))^2$

Linear Regression

Left side (X) → Machine Learning → Right side (Y)



Equation of Line :- $\mathrm{y} = \mathrm{m} \; * x + c$

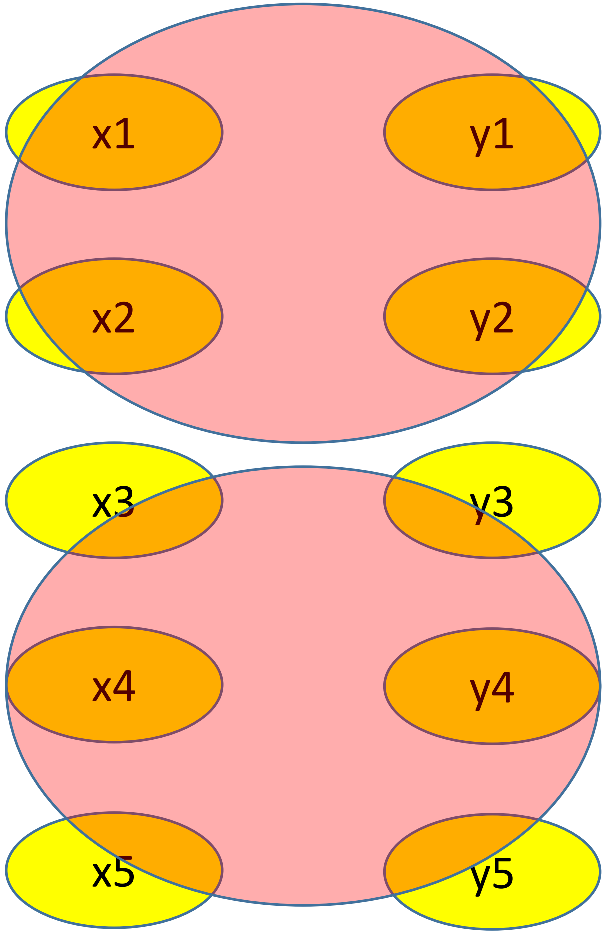
$$\mathrm{L}(\mathrm{m}, \mathrm{c}, \mathrm{X}, \mathrm{Y}) = \sum_{i=1}^{i=5} (y_i - (m * x_i + c))^2$$

LOSS FUNCTION

ERROR FUNCTION

Linear Regression

Left side (X) → Machine Learning → Right side (Y)



Equation of Line :- $y = \mathrm{m} * x + c$

Actual.. Ground truth..

Prediction

$$\mathrm{L}(\mathrm{m}, \mathrm{c}) = \sum_{i=1}^{i=5} \left(y_i - (m * x_i + c)\right)^2$$

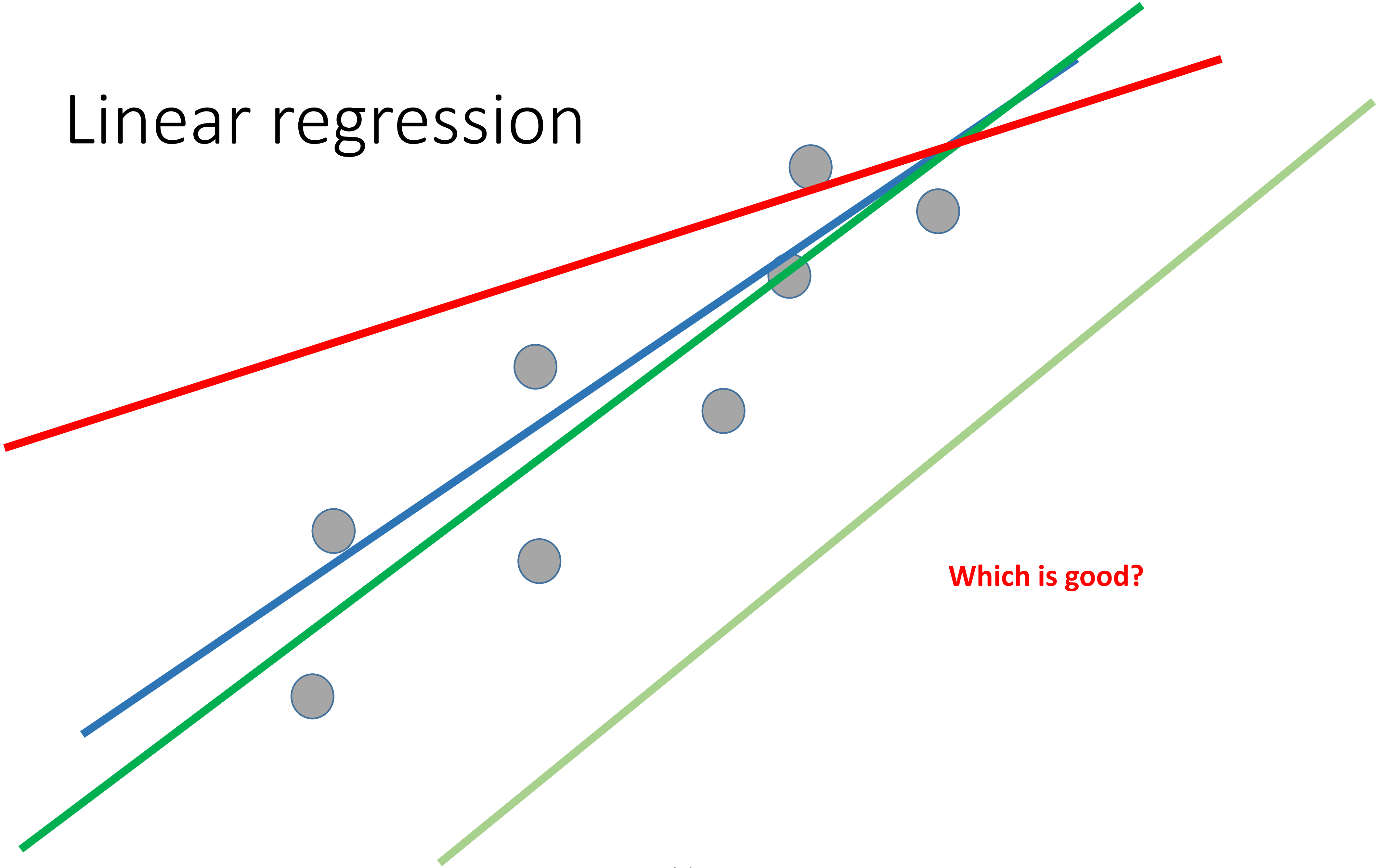
REGRESS..

prediction to return to.. actual

LOSS FUNCTION

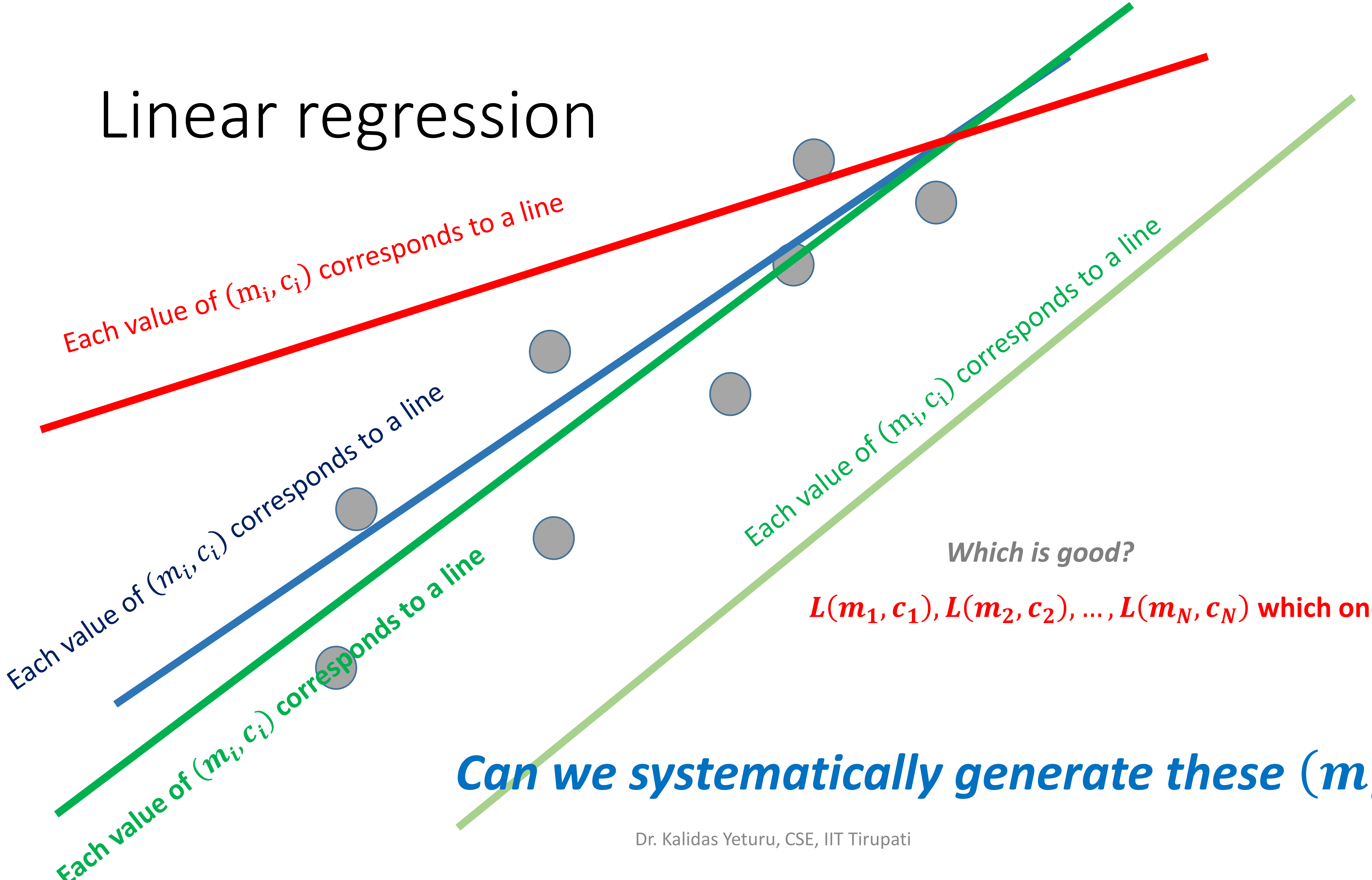
PARAMETERS

Linear regression



Which is good?

Linear regression



Which is good?

$L(m_1, c_1), L(m_2, c_2), \ldots, L(m_N, c_N)$ **which one is better?**

Can we systematically generate these (m_i, c_i)?

What are the different ways?

2nd argument is *the list of Parameters*

BruteForceSolver(L, [m1,c1])

1st argument is the *Loss function*

$\boldsymbol{minval = +10000.0}$

FOR $\mathbf{m} \in [\mathbf{-100, \dots, +100}]$

FOR $\mathbf{c} \in [\mathbf{-100, \dots, +100}]$

Compute $\boldsymbol{v = L(m, c)}$

IF $\boldsymbol{v < minval}$**:**

$\boldsymbol{v = minval}$

$\boldsymbol{m1 = m,}$

$\boldsymbol{c1 = c}$

Questions…

- *-100 to +100, who gave the range?*
- *What is the step size?*
- *What if the solution is highly fine, (3.451, -89.1123)*
- *Can we speed up?*

What are the different ways?

2nd argument is *the* ***list of Parameters***

RandomSolver(L, [m1,c1])

1st argument is the Loss function

$\boldsymbol{minval = +10000.0}$

FOR ITER= 1:100000

FOR $\mathbf{m} = \boldsymbol{RAND(-100, 100)}$

FOR $\boldsymbol{c = RAND(-100, 100)}$

Compute $\boldsymbol{v = L(m, c)}$

IF $\boldsymbol{v < minval}$**:**
$\boldsymbol{v = minval}$
$\boldsymbol{m1 = m,}$
$\boldsymbol{c1 = c}$

Questions…

- *-100 to +100, who gave the range?*
- *What is the step size?*
- *What if the solution is highly fine, (3.451, -89.1123)*
- *Can we speed up?*

What are the different ways?

2nd argument is *the list of Parameters*

GradiantSolver(L, [m1,c1], gradL)

1st argument is the Loss function

3rd argument is the gradient function