**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676 (**Correct**)
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.

**Ans.** The probability of employee age between 38 and 44 is 34.13 %

The probability of employee age more than 44 is 15.87 %

Therefore, the statement that “More employees at the processing centre are older than 44 than between 38 and 44” is TRUE.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans.**

Probability of employees less than age of 30 is 9.12%

So the number of employees with probability 0.912 of them being under age 30 is 0.0912\*400=36.48( or 36 employees).

Therefore, the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans.

As we know that if *X*1 ∼ N(µ1, σ1^2 ), and *X*1 ∼ N(µ2, σ2^2 ) are two independent random variables

Then

*X*+ Y∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and

X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if

Z = aX + bY ,

where X and Y are as defined above, i.e Z is linear combination of X and Y.

then

Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5 **(Correct)**
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans.**

Rupee Ranges from 2.2 to 21.8 million in Dollars

Rupee Ranges from 9.91 to 98.1 Crore in Rupees

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans.**

The 5th percentile of Profit for the company is 3.78 Million Dollars

The 5th percentile of Profit for the company is 17.0 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.**

The Probability of Division #1 making a loss is 4.78 %

The Probability of Division #2 making a loss is 4.01 %

The Division 1 has a larger Probability of making a loss