DDP Thesis Structure

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  - Innovation Needed (Respectively)
- Extrapolated Boundary Conditions
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  - Singular Perturbation Problems
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- Experiment on EBC Design
  - Program Algorithm to allow various choices
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PINNS

Theory behind PINNS

- Neural Networks
- Boundary Value Problems
- Boundary Conditions

Made PPT on this

Programming PINNs

Flowchart

Algorithm

Made PPT on this

Examples of PINNs

1-D problem

Square Plate Problem

3D problems

Already well-documented problems

Shortcomings in PINNs

1. NN Solutions need not be admissible wrt Essential Boundary condition
2. Easy for solver to converge to a PDE-compliant but BC - ignorant solution
3. Requires monitoring of the weightage being given to all of the terms
4. Slow and computationally expensive wrt Finite Methods.
    a. In Finite Methods, The Boundary conditions are incorporated in the system of linear Equations
    b. In PINNs, The loss functions for PDE and BC are only added so no influence is felt in the Interior. Makes it take a lot of iterations and solutions until the boundary solution approaches the right solution only after which convergence of boundary condition loss begins
    c. In short, PDE Loss converges and after a lot of iterations only does the boundary solution converge as well. This can be solved if Boundary condition compliance can be linked to the Interior strongly.

Explanation and evidences will be presented with upto two paragraphs each

Innovation Needed (Respectively)

- Stronger way of describing boundary conditions
- Make convergence on wrong boundary solution difficult
- Lighten need of scaling the weights
- Link boundary compliance influence onto the interior as well.

Single para explanation each

Extrapolated Boundary Conditions

Basic Theory of Extrapolated Boundary Condition

- Theory already in a PPT made

Variation in Extrapolated Boundary Condition

- Weighted or Dataset - 2 choices
- Basis Function Wise - 4 choices
- Initialiser or Always - 2 choices

Programming EBCs

Flowchart

Algorithm

Examples of EBCs

Class of Problems

Standard One-Dimensional BVP

$$P_2(x)u''(x) + P_1(x)u'(x) + P_0(x)u(x) + Q(x) = 0$$

Singular Perturbation Problems

$$Ly \equiv \varepsilon y'' + a(x)y' + b(x)y = f(x),$$
$$y(0) = \alpha_0, \qquad y(1) = \alpha_1, \qquad \alpha_0, \alpha_1 \in R,$$

Higher Order Eigenvalue Problems

$$EI \frac{\mathrm{d}^4 \hat{w}}{\mathrm{d}x^4} - \mu\omega^2 \hat{w} = 0\,.$$

Example: Cantilevered beam [edit]

The boundary conditions for a cantilevered beam of length L (fixed at $x = 0$) are

$$\hat{w}_n = 0\,, \quad \frac{d\hat{w}_n}{dx} = 0 \quad \text{at} \;\; x = 0$$
$$\frac{d^2\hat{w}_n}{dx^2} = 0\,, \quad \frac{d^3\hat{w}_n}{dx^3} = 0 \quad \text{at} \;\; x = L\,.$$

Two-Dimensional Regular *

Two-Dimensional Irregular *

Three-Dimensional Regular *

Experiment on EBC Design

In One-Dimensional Problems, we saw three types of Classes.

We explore the EBC conditions over these three.

Hence, Total Number of Experiments

$3 \times (1 + 2 \times 4 \times 2) = 51$

Program Algorithm to allow various choices

Results

Conclusions