$$\text{or}, \quad d \cos\theta \; d\theta = \lambda \,. \qquad \text{...... (1)}$$

From Fig. 2 ,

$$\omega = OP \times d\theta \,,$$

$$= OP \times \frac{\lambda}{d \cos\theta} \,, \qquad [\text{ From Eq(1)}].$$

Since $\theta$ is small, $\cos\theta \approx 1$, and $OP \approx D$.

$$\therefore \; \omega = \left(\frac{D}{d}\right) \lambda \,,$$

which is the expression for distance between successive bright spots ( or successive dark spots). If $S_1$ and $S_2$ are slits instead of pin holes, then there will be bright and dark lines at which time, $\omega$ becomes the fringe width.

On the basis of foregoing discussion, the condition for interference of light waves can be listed as follows.

**Conditions for Interference of Light.**

1. The light waves participating in the interference must originate from two identical coherent sources, so that the two waves will have same wavelength, and constant or zero phase difference.

2. The two waves must be of equal amplitudes without which the regions where destructive interference occurs will also have illumination depending upon the difference in amplitudes.

3. The two coherent sources must be closely located to avoid overlapping of bright & dark points at the place of interference. This is because, the separation between bright & dark points is inversely proportional to the distance between the two sources.

4. The light waves must be continuously emitted by the two coherent sources, since the interference pattern vanishes during the time when one of the sources fails to emit light.

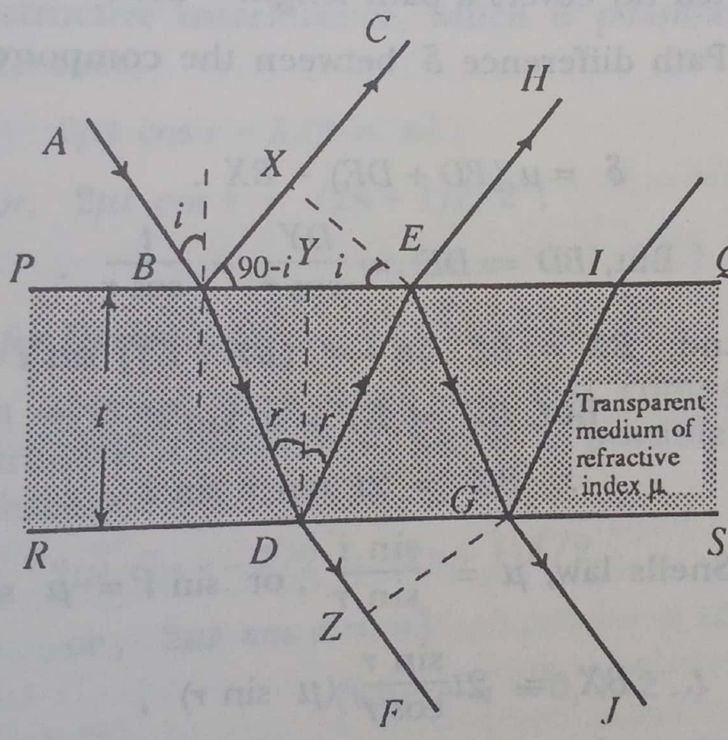
5. The two coherent sources must be extremely small in size (unless it is a laser source), since, for coherent sources of larger dimensions, interference may occur from light coming from different parts of same source which is superimposed on the

interference of light from the two sources, thus affecting the overall clarity of the interference pattern.

6. Constructive interference occurs for path difference of $n\lambda$ between the two waves, and the destructive interference occurs for a path difference of $(2n+1)\lambda/2$, where $n$ is an integer.

## INTERFERENCE AT THIN FILMS

Consider a thin film of a transparent medium of refractive index $\mu$. Let it be of uniform thickness $t$ whose upper surface is $PQ$, and the lower one is $RS$.




INTERFERENCE AT A THIN FILM

Fig. 3

Let a ray of light $AB$ of a monochromatic light of wavelength $\lambda$ travelling in air be incident on the surface $PQ$ at an angle of incidence $i$. Due to change of medium, the ray suffers partial reflection and transmission (which is known as division of amplitude) at $B$. The reflected part travels along $BC$ in air and the transmitted part is refracted along $BD$ into the medium at an angle of incidence $r$. Further it encounters again a change of medium at $D$ at which point it is partly reflected along $DE$ within the medium, and partly refracted along $DF$ emerging into air. In a similar way, partial reflection and transmission occur at $E, G, I,$ ...... etc.

**a) Interference Effects observed in Reflected System of Rays :**

Let $EX$ and $DY$ be the perpendiculars drawn to $BC$, and $BE$ respectively. Now, we can think that beyond $EX$, the component ray pair $XC$ and $EH$ will cover same path length upto the point of focus, such as retina of the observer's eye. Earlier, since they were incorporated in the ray $AB$, they can be assumed to have covered same path length upto $B$. Between these two stages, the transmitted ray covers an optical path length $= \mu(BD + DE)$, and the reflected ray covers a path length $= BX$.

$\therefore$ Path difference $\delta$ between the component rays $XC$, & $EH$ is,

$$\delta = \mu\,(BD + DE) - BX\ . \qquad \text{......(1)}$$

$$\text{But, } BD = DE = \frac{DY}{\cos r} = \frac{t}{\cos r}\ . \qquad \text{......(2)}$$

$$\text{And, } BX = BE \sin i = (BY + EY) \sin i\ .$$

$$\text{But, } BY = EY = t \tan r\ .$$

$$\therefore\ BX = 2t \tan r \sin i\ .$$

$$\text{By, Snells law, } \mu = \frac{\sin i}{\sin r}\text{, or } \sin i = \mu \sin r\ .$$

$$\therefore\ BX = 2t\frac{\sin r}{\cos r}(\mu \sin r)\ ,$$

$$\text{or, } BX = 2\mu t\,\frac{\sin^2 r}{\cos r}\ . \qquad \text{......(3)}$$

Using Eqs. (2) and (3), Eq.(1) can be written as,

$$\delta = \mu\,\frac{2t}{\cos r} - \mu\,\frac{2t\sin^2 r}{\cos r}$$

$$= 2\mu t\,\frac{1-\sin^2 r}{\cos r}\ ,$$

$$= 2\mu t\,\frac{\cos^2 r}{\cos r}\ ,$$

$$\therefore\ \delta = 2\mu t \cos r\ . \qquad \text{......(4)}$$

Since the part of the ray reflected along $BC$ is due to reflection at a denser medium, an additional path difference of $\lambda/2$ must be considered for $BX$ in Eq.(1).

$$\therefore \text{ Total path difference } = \mu(BD + DE) - (BX + \lambda/2) ,$$

$$= \delta - \lambda/2 , \quad \text{[From Eq(1)]}$$

$$\therefore \text{ Total path difference } = 2\mu t \cos r - \lambda/2 , \quad \text{......(5)}$$

$$\text{[From Eq(4)]} .$$

*i) Condition for Brightness :*

The film appears bright when the two component rays undergo constructive interference, which is possible when the total path difference,

$$2\mu t \cos r - \lambda/2 = n\lambda ,$$

$$\text{or, } 2\mu t \cos r = (2n + 1)\lambda/2 , \quad \text{...... (6)}$$

$$\text{(where } n = 0, 1, 2 \text{ ...)} .$$

*(ii) Condition for Darkness :*

The film appears dark when the two component rays undergo destructive interference, which is possible when the total path difference,

$$2\mu t \cos r - \lambda/2 = (2n + 1)\lambda/2,$$

$$\text{or , } 2\mu t \cos r = n\lambda , \quad \text{......(7)}$$

$$\text{(where } n = 0, 1, 2 \text{ ..)} .$$

Also, if the film is exceedingly thin so that $t << \lambda$, then the factor $2\mu t \cos r$ becomes negligible. Then, the path difference between the two rays $\approx \lambda/2$ in Eq.(5), because of which the two rays interfere destructively to produce darkness.

**b) Interference Effects observed in Transmitted System of Rays :**

Here, we must consider the interference between the transmitted rays $DF$ and $GJ$. If $GZ$ is drawn normal to the ray $DF$, then the path difference $\delta$ between the component rays $ZF$ and $GJ$ can be written in similar way as,

$$\delta = \mu\,(DE + EG) - DZ = 2\mu t \cos r \quad . \qquad \text{......(8)}$$

Since, the part of the ray refracted along $DF$ is due to transmission no additional path difference of $\lambda/2$ be considered for $DZ$.

*(i) Condition for Brightness :*

The film appears bright if the path difference,

$$2\mu t \cos r = n\lambda \quad . \qquad \text{......(9)}$$

Also, if the film thickness $t << \lambda$, then ,

$$2\mu t \cos r \approx 0 \quad .$$

$$\text{Or ,} \quad \delta \approx 0 \quad , \qquad \text{[From Eq.(9)]}$$

i.e., the path difference is zero. Hence the film appears bright.

*(ii) Condition for Darkness :*

The film appears dark if the path difference,

$$\delta = 2\mu t \cos r = (2n+1)\lambda/2 \quad . \qquad \text{......(10)}$$

**c) The Complementary Nature of Reflected and Transmitted System :**

By comparing Eqs.(6) and (10), we realize that, what condition is there for brightness in reflected system, the same one holds good for darkness in transmitted system. Just the vice versa condition is applicable in case of Eqs.(7) and (9). Therefore if a light beam comprising of rays of different wavelengths is incident on a thin film, then the rays of those wavelengths which appear bright in the reflected system will undergo extinction in the transmitted system, and vice versa. Thus, the two systems consists of lights of wavelengths which if present in one are absent in the other & vice versa. But, on the whole they mutually complete lights of all wavelengths present in the incident light. Hence, **the reflected and the transmitted system are said to be complementary to each other.**