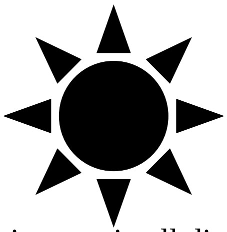
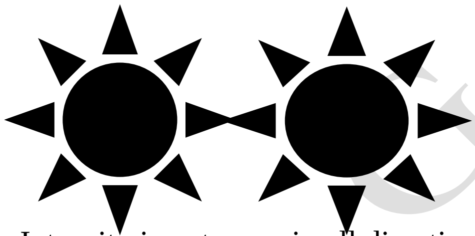
Interference of Light

Introduction

When a single monochromatic light source is placed in a medium, then, the light energy is distributed uniformly in all directions in space around it. Therefore, the intensity of light is same in all directions in the space around a single source.



Intensity is same in all directions

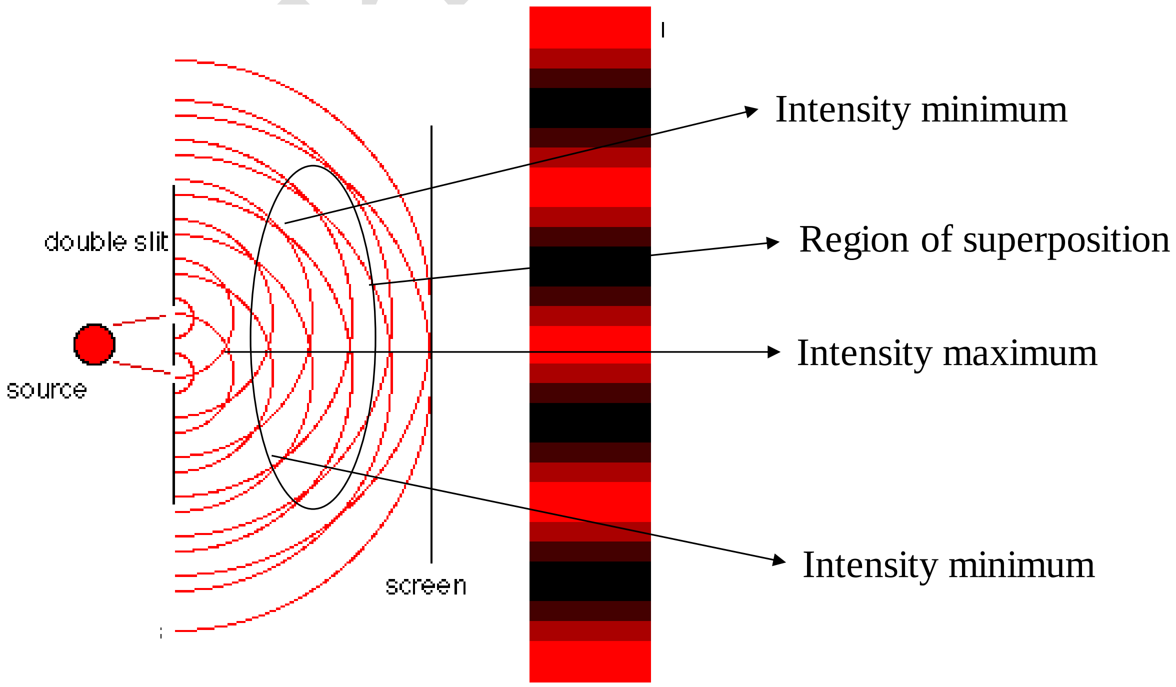


Intensity is not same in all directions

When two or more light source is placed very close to each other in a medium, then, the light energy is not distributed uniformly in all directions in space around it. Therefore, the intensity of light is not same in all directions in the space around two or more sources kept in a medium.

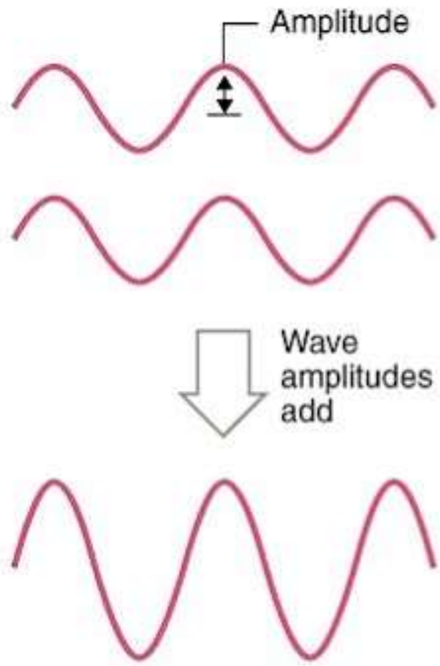
Interference of light:

"The phenomenon of modification in the intensity of light due to redistribution of light energy in the region of superposition of two or more light waves is called as interference of light."



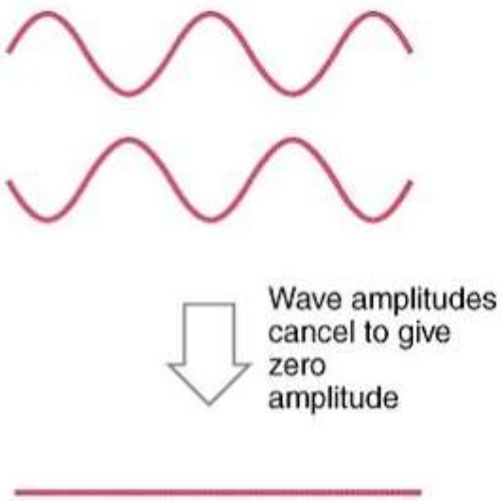
Constructive interference:

At certain points in the region of superposition of two light waves where crest of one wave falls on the crest of another wave or trough of one wave falls on trough of another wave, the resultant amplitude becomes maximum and hence the resultant intensity becomes maximum. At those points constructive interference takes place.

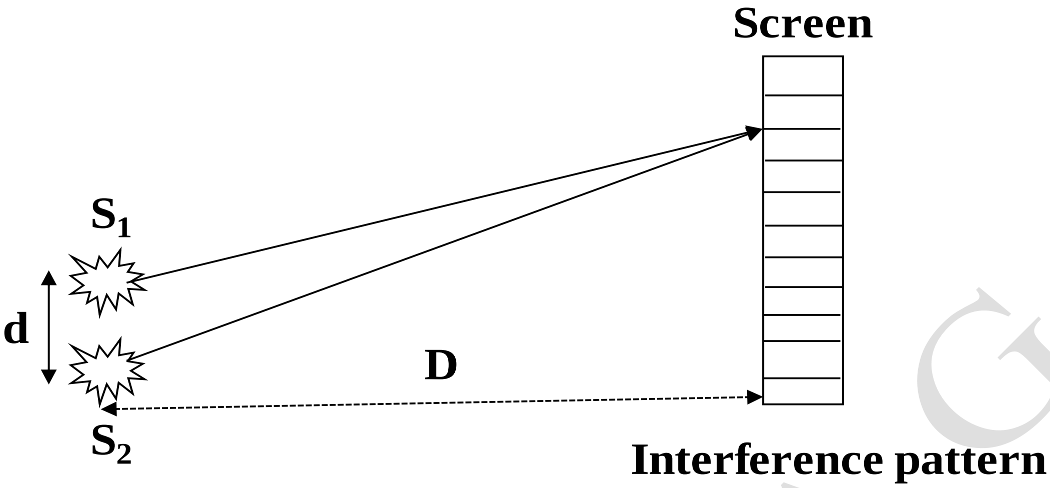


Destructive interference:

At certain points in the region of superposition of two light waves where crest of one wave falls on the trough of another wave or trough of one wave falls on crust of another wave, the resultant amplitude becomes minimum and hence the resultant intensity becomes minimum. At those points destructive interference takes place.



Interference pattern:



When a screen is placed in the region of superposition of light waves perpendicular to the direction of propagation of light, alternate dark and bright bands are formed on the screen. "The collection of alternate dark and bright bands on the screen is called as interference pattern."

Interference fringes:

"The alternate dark and bright bands formed on the screen which is placed in the superposition of light waves are collectively called as interference fringes."

Interference width:

"The width of the dark or bright bands formed on the screen is called fringe width."

or

"The distance between any two consecutive bright or dark fringe is called fringe width."

Conditions for sustained and distinct interference pattern:

- The two light sources must be coherent
- The light emitted by the two source should be monochromatic
- That two source must be very narrow
- The separation between the two sources should be very small i.e., the two source must be very close to each other
- The screen must be placed at the largest distance from the coherent sources
- The amplitudes of the interfering waves must be equal
- The interfering waves should be propagate in almost same direction i.e., the wavefronts should interact with small angles
- If the interfering waves are plane polarised then their plane of polarization should be same

Relation between path difference and phase difference

If the path difference between the two waves is 'λ'then their phase difference is '2π'

Therefore,

$$\text{Phase difference} = \frac{2\pi}{\lambda} x\ path\ difference$$

$$\text{Path difference} = \frac{\lambda}{2\pi} x\ phase\ difference$$

Coherent sources

If the two sources of light is said to be coherent, if they emit light wave of same frequency, same wavelength and same or constant phase difference, then such source of light is called as coherent source.

Methods of producing coherent sources

There are two methods of producing coherent sources they are dependent on the following divisions

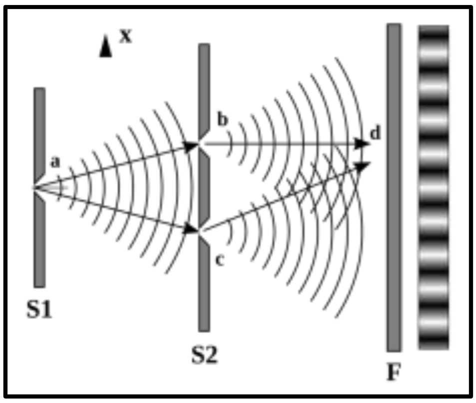
- Division of wave front
- Division of amplitude

<u>Division of wave front</u>

Here in this, a single wave front is splits up into two parts either by reflection or refraction. These two parts are made to travel along different paths so that a constant phase difference is introduced between them.

Example:

- Youngs double slit experiment
- Biprism
- Lloyd's mirror

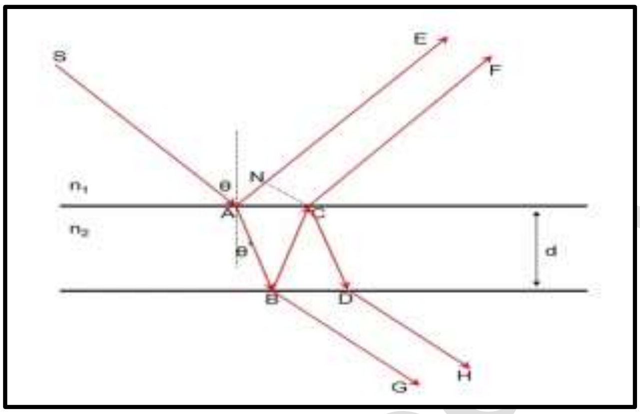


Division of amplitude

Here, the amplitude of incident beam of light is divided into two parts by reflection or refraction or both. Then they are superimposed on each other and produce interference pattern.

Example:

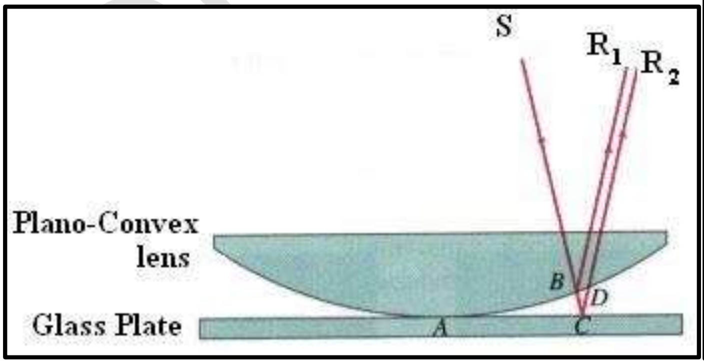
- Colors in thin film
- Air wedge
- Newtons ring

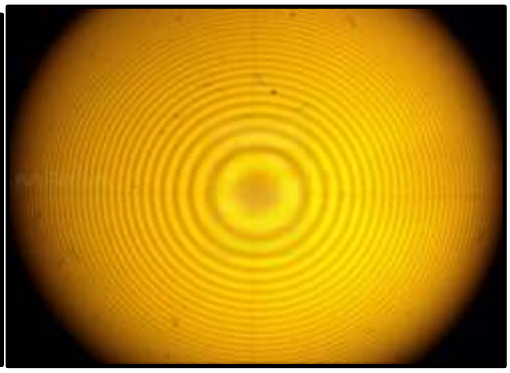


Newton's Ring:

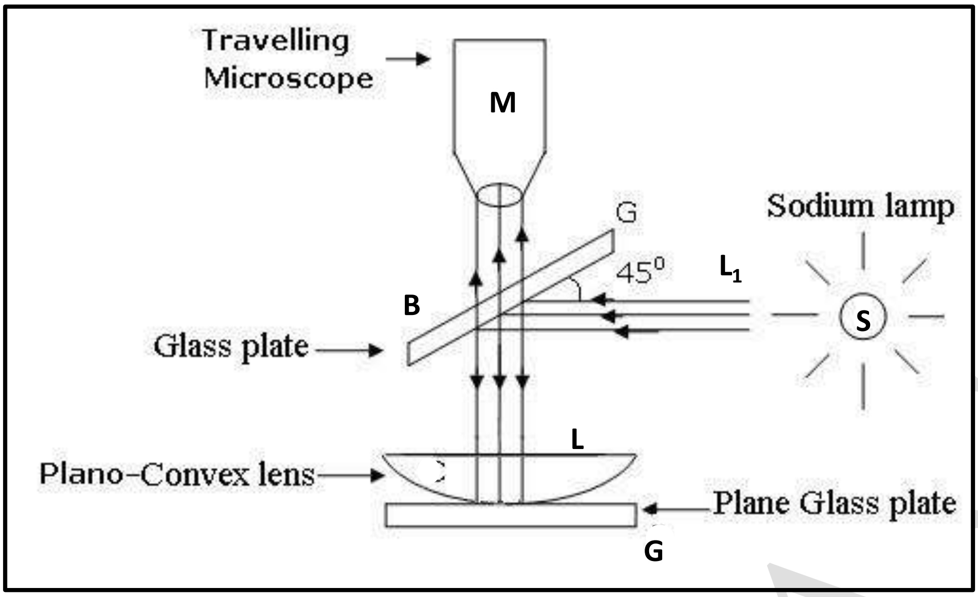
When a Plano-convex lens of large focal length is placed on a glass plate, a thin air film is enclosed between the lower surface of the lens and upper surface of the glass plate. The thickness of the film is very small (zero) at the point of contact and increases gradually from the centre outwards.

When the film is eliminated with monochromatic light and the reflected light is observed, the concentric circular alternate bright and dark fringes are obtained. These circular fringes are called as Newton rings. These fringes are concentric circles with their centre at the point of contact.



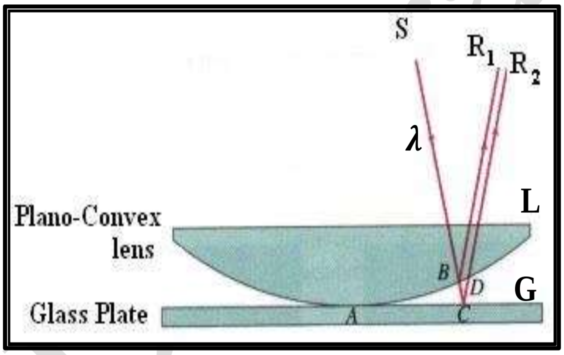


Typical arrangement of Newton's ring experiment and nature of the rings obtained

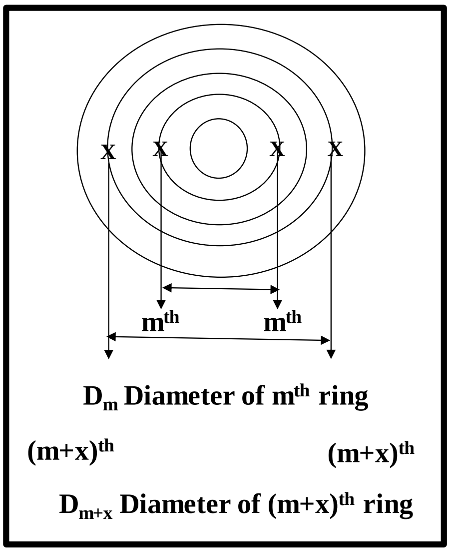


Let ‘S’ be a source of monochromatic light at the principal focus of the lense L_1. The horizontal beam of light falls on the glass plate ‘B’ at an angle of incidence 45^0. The glass plate reflects the path of light towards the air film enclosed between the lens and the glass plate ‘G’. The reflected beam from the air film is viewed with a microscope ‘M’. The interference takes place and alternate bright and dark circular rings with the centre at the point of contact or produced. This is due to interference of reflected light which are reflected from the lower and upper surface of the film.

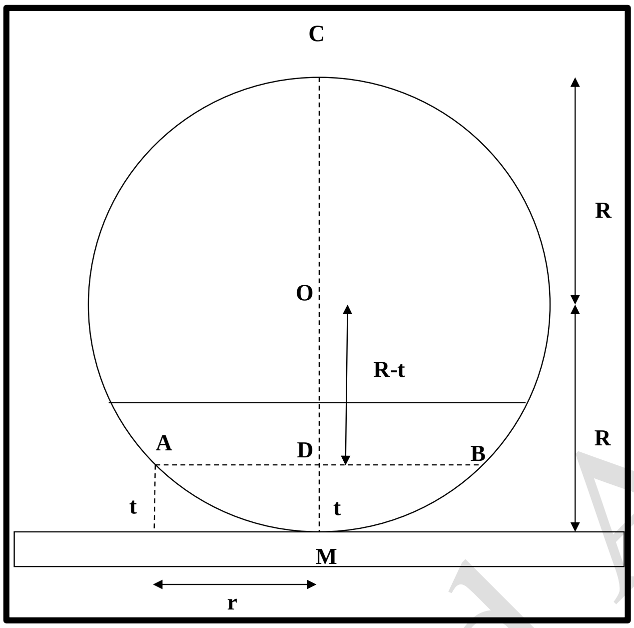
Theory of Newton’s rings by reflected light



‘x’ rings are present in
Newton;s ring



Let us consider a plano-convex lens of radius of curvature ‘R’ is placed over the glass plate and the thin film (air) obtained and illuminated with a monochromatic light of wavelength ‘λ’. If the reflected light is observed through microscope, Newton’s rings are observed. These rings are obtained by the interference of reflected waves from the upper and lower surface of the thin film.



The path difference between the reflected rays R_1 and R_2 from the film is given by

$$P = 2ntcosr + \frac{\lambda}{2}$$

(Since the ray R_2 is reflected from the surface of denser medium, extra PD of $\frac{\lambda}{2}$ is introduced)

But r=0 (normal incidence)

Therefore,

$$P = 2nt + \frac{\lambda}{2} \qquad (1)$$

<u>Case (i) Condition for bright fringe:</u>

Path difference

$$P = 2nt + \frac{\lambda}{2} = m\lambda$$

$$2nt = m\lambda - \frac{\lambda}{2}$$

$$2nt = (2m-1)\frac{\lambda}{2} \qquad (2)$$

If the radius of the convex lens is 'R' and the thickness of the film at point 'A' is 't'. then from the geometry of the circle, the value of 't' can be obtained as follows

$$AD \times DB = MD \times DC$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

$$r^2 = 2Rt$$

Neglecting t^2 because t<<R

Therefore

$$t = \frac{r^2}{2R} \tag{3}$$

Substitute equation (3) in equation (2) we get,

$$\frac{\cancel{2}nr^2}{\cancel{2}R} = (2m-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2m-1)\lambda R}{2n}$$

$$r_m = \sqrt{\frac{(2m-1)\lambda R}{2n}} \tag{4}$$

This is the expression for the radius of the m^{th} bright fringe

$$2r_m = 2\sqrt{\frac{(2m-1)\lambda R}{2n}}$$

$$D_m = \sqrt{\frac{4(2m-1)\lambda R}{2n}}$$

Therefore, this is the expression for the diameter of the m^{th} bright fringe

$$D_m = \sqrt{\frac{2(2m-1)\lambda R}{n}} \tag{5}$$

<u>Case (ii): Condition for dark fringe</u>

$$P = 2nt + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$$

$$2nt = (2m+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2nt = (\cancel{2}m + \cancel{1} - \cancel{1})\frac{\lambda}{\cancel{2}}$$

$$2nt = m\lambda \tag{6}$$

But $t = \frac{r^2}{2R}$

$$\frac{\cancel{2}nr^2}{\cancel{2}R} = m\lambda$$

$$r^2 = \frac{m\lambda R}{n}$$

Therefore,

$$r_m = \sqrt{\frac{m\lambda R}{n}} \tag{7}$$

This is the expression for the radius of the m^{th} dark fringe

$$2r_m = 2\sqrt{\frac{m\lambda R}{n}}$$

Therefore, this is the expression for the diameter of the mth dark fringe

$$D_m = \sqrt{\frac{4m\lambda R}{n}} \tag{8}$$

Similarly, the expression for the diameter of the (m+x)th dark fringe is given by

$$D_{m+x} = \sqrt{\frac{4(m+x)\lambda R}{n}} \tag{9}$$

$$D^2_{(m+x)} - D^2_m = \frac{4(m+x)\lambda R}{n} - \frac{4m\lambda R}{n}$$

$$D^2_{(m+x)} - D^2_m = \frac{4\lambda R}{n}[\cancel{m} + x - \cancel{m}]$$

$$D^2_{(m+x)} - D^2_m = \frac{4\lambda R x}{n}$$

Therefore, the wavelength of the monochromatic light used is given by

$$\lambda = \frac{\left(D^2_{(m+x)} - D^2_m\right)n}{4Rx}$$

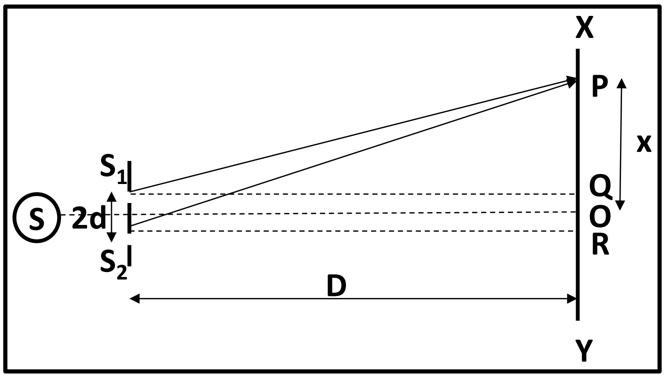
By this relation, we can easily determine the value of the wavelength of the light used

The radius of curvature of the lens can also be determined by using the relation

$$R = \frac{\left(D^2_{(m+x)} - D^2_m\right)n}{4\lambda x}$$

Interference in thin films

Theory of Interference fringes



Consider a narrow monochromatic source 'S' and two pinholes 'S_1' and 'S_2' equidistant from 'S'. The pin holes provide the two point sources vibrating in the same phase. The two sources are separated at a distance '2d'. Let a screen 'XY' (as shown in above figure) be placed at a distance 'D' parallel to 'S_1S_2'. The point 'O' on the screen is equidistant from 'S_1' and 'S_2' and hence the path difference between the two waves reaching at 'O' from 'S_1' and 'S_2' is zero. Therefore, at this point of intensity is maximum. Consider a point 'P' at a distance 'x' from 'O'. We shall now consider the condition for a bright or dark fringe at this point.

From the right angle triangle S_1QP

$(S_1P)^2 = (S_1Q)^2 + (QP)^2$

Or, $(S_1P)^2 = (D)^2 + (x-d)^2$ $\qquad$ [$\because$ (QP) = (x-d)]

Similarly, in the right angle triangle S_2RP

$(S_2P)^2 = (S_2R)^2 + (RP)^2$

Or, $(S_2P)^2 = (D)^2 + (x+d)^2$ $\qquad$ [$\because$ (RP) = (x+d)]

$\therefore$ $(S_2P)^2 - (S_1P)^2 = (x+d)^2 - (x-d)^2 = 4xd$

Or, $(S_2P - S_1P)(S_2P + S_1P) = 4xd$

In Young's experiment, D is very much greater than '2d' or 'x', so that if $(S_2P + S_1P)$ is replaced by '2D', the error is not more than a fraction of one percent.

Hence, $(S_2P - S_1P)\, 2D = 4xd$

Or, $(S_2P - S_1P) = \frac{4xd}{2D} = \frac{2xd}{D}$ $\qquad$ (1)

<u>Case (i): Bright fringes</u>

The point 'P' is bright when the path difference is whole number multiple of wavelength 'λ' i.e.,

$S_2P - S_1P = n\lambda$ [n = 1,2,3…

Substituting the valve of $S_2P - S_1P$ from eqn.1 to the above equation we have

$$\frac{2xd}{D} = n\lambda \; or \; x = \frac{n\lambda D}{2d}$$

The above equation gives the distance of the bright fringe from point 'O'. At 'O', the path difference is zero, hence there is a bright fringe. The next bright fringes are formed when n = 1,2,3,.. and so on

Therefore

When, n = 1, $x_1 = \frac{\lambda D}{2d}$

n = 2, $x_1 = \frac{2\lambda D}{2d}$

n = 3, $x_2 = \frac{3\lambda D}{2d}$

.

.

n = n $x_n = \frac{n\lambda D}{2D}$

Now, the distance between any two consecutive bright fringes is

$$x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{2d}$$

<u>Case (i): Dark fringes</u>

The point 'P'is dark when the path difference is an odd number multiple of half wavelength

i.e., $S_2P - S_1P = (2n\pm1)\frac{\lambda}{2}$ [n = 1,2,3…

Now,

$$\frac{2xd}{D} = \frac{(2n \pm 1)\lambda}{2}$$

Or

$$x = \frac{(2n \pm)\lambda D}{4d}$$

The above equation gives the distances of the dark fringes from point 'O'. the dark fringes are formed as follows

When, n = 0, $x_0 = \frac{\lambda D}{4d}$

n = 1, $x_1 = \frac{3\lambda D}{4d}$

n = 2, $x_2 = \frac{5\lambda D}{4d}$

.

.

n = n, $x_n = \frac{(2n+1)\lambda D}{4d}$

The distance between any two consecutive dark fringes is

$$x_2 - x_1 = \frac{5\lambda D}{4d} - \frac{3\lambda D}{4d}$$
$$= \frac{2\lambda D}{4d}$$
$$= \frac{\lambda D}{2d}$$

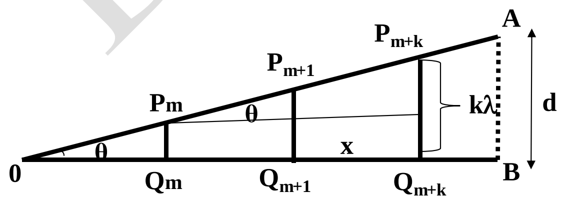
Hence the spacing between any two consecutive dark fringe (maxima or minima) is the same. This is expressed by β. Where, $\beta = \frac{\lambda D}{2d}$ and is known as fringe width.

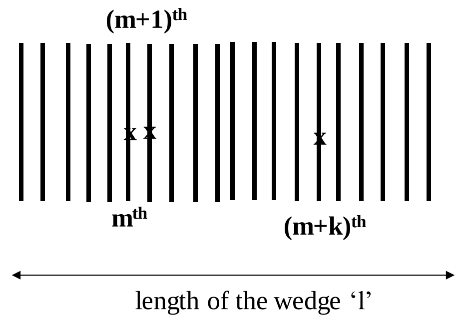
It is obvious that the spacing is directly proportional to ‘D’ and inversely proportional to ‘2d’.

Air wedge

A wedge shaped thin air film which is formed in between the two plane glass surfaces which are inclined at a very small angle is called air-wedge. At the intersection of the 2 surfaces, the thickness of the wedge is zero. The thickness of the wedge increases uniformly as we move from the intersection of the two surfaces.

Theory of Air wedge:





Let us consider an air wedge is formed between the two plane glass plate surfaces OA and OB which are inclined at a very small angle θ as shown in the figure. The thickness of the film is zero at 0 and increases uniformly from 0 to A. Where the air wedge is illuminated with monochromatic light and the reflected light is observed, a system of equidistant interference patterns (fringes) are observed.

These fringes are parallel to the line of intersection of the two plates OA and OB. Let 'l' be the length of the air wedge and 'd' be the thickness of the air wedge at the end of the wedge Let the m^{th} bright fringe occurs at P_m where, the thickness of the film is P_mQ_m. Applying the condition for bright fringe, we get

$$2ntcosr = (2m \pm 1)\frac{\lambda}{2}$$

Where, m=0.,1,2,3……

Here,

n=1 Since refractive index of air is 1

r = 0 the light incident normally on the film and hence, angle of refraction is zero

t = P_mQ_m thickness of the film

Therefore,

$$2P_mQ_m = (2m + 1)\frac{\lambda}{2} \qquad …….. \qquad (1)$$

The next bright fringe i.e., $(m+1)^{th}$ bright fringe occurs at P_{m+1}, where the thickness of the film is $P_{m+1}Q_{m+1}$.

Applying the condition for bright fringe we get,

$$2P_{m+1}Q_{m+1} = [2(m + 1) + 1]\frac{\lambda}{2}$$

$$= [2m + 2 + 1]\frac{\lambda}{2}$$

$$= [2m + 3]\frac{\lambda}{2}$$

Therefore,

$$2P_{m+1}Q_{m+1} = [2m + 3]\frac{\lambda}{2} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Subtract (1) from (2)

$$2P_{m+1}Q_{m+1} - 2P_mQ_m = (2m + 3)\left(\frac{\lambda}{2}\right) - (2m + 1)\left(\frac{\lambda}{2}\right)$$

$$2(P_{m+1}Q_{m+1} - P_mQ_m) = [(2m + 3) - (2m + 1)]\left(\frac{\lambda}{2}\right)$$

$$(P_{m+1}Q_{m+1} - P_mQ_m) = \frac{1}{2}[\cancel{2m} + 3 - \cancel{2m} - 1]\left(\frac{\lambda}{2}\right)$$

$$= \frac{1}{2}[2]\left(\frac{\lambda}{2}\right)$$

$$(P_{m+1}Q_{m+1} - P_mQ_m) = \frac{\lambda}{2} \ldots\ldots\ldots\ldots(3)$$

The next bright fringe will occur at the point where the thickness of the film is increased by λ/2. $(m+k)^{th}$ bright fringe is at P_{m+k} then there will be 'K' bright fringes in between P_m and P_{m+k}.

Therefore,

$$(P_{m+K}\,Q_{m+K} - P_mQ_m) = K\frac{\lambda}{2} \ldots\ldots\ldots\ldots(4)$$

If the distance Q_mQ_{m+K} = x (i.e., width of 'K' fringe), then

$$\tan\theta \approx \theta = \frac{P_{m+K}\,Q_{m+K} - P_mQ_m}{Q_mQ_{m+K}}$$

$$\theta = \frac{K\frac{\lambda}{2}}{x}$$

Therefore,

$$x = \frac{K\lambda}{2\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Therefore, the fringe width is given by

$$\omega = \frac{Width\ of\ K\ fringes}{K}$$

$$\omega = \frac{x}{K}$$

$$\omega = \frac{\frac{\cancel{K}\lambda}{2\theta}}{\cancel{K}}$$

$$\omega = \frac{\lambda}{2\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

But,

$$tan\theta \approx \theta = \frac{d}{l}$$

$$\theta = \frac{d}{l} \ldots\ldots\ldots(7)$$

Where, ‘d’ is thickness of the film at the end of the wedge ‘l’ is the length of the wedge

Substitute (7) in (6) we get,

$$\omega = \frac{\lambda}{2(\frac{d}{l})}$$

$$\omega = \frac{\lambda l}{2d}$$

Therefore, the thickness of the object i.e., wire, paper, hair etc. can be determined by this below eqn.

$$d = \frac{\lambda l}{2\omega}$$

Diffraction

“The phenomenon of bending of light around the edges of the obstacles and hence its encroachment in the region of geometrical shadow is called diffraction”.

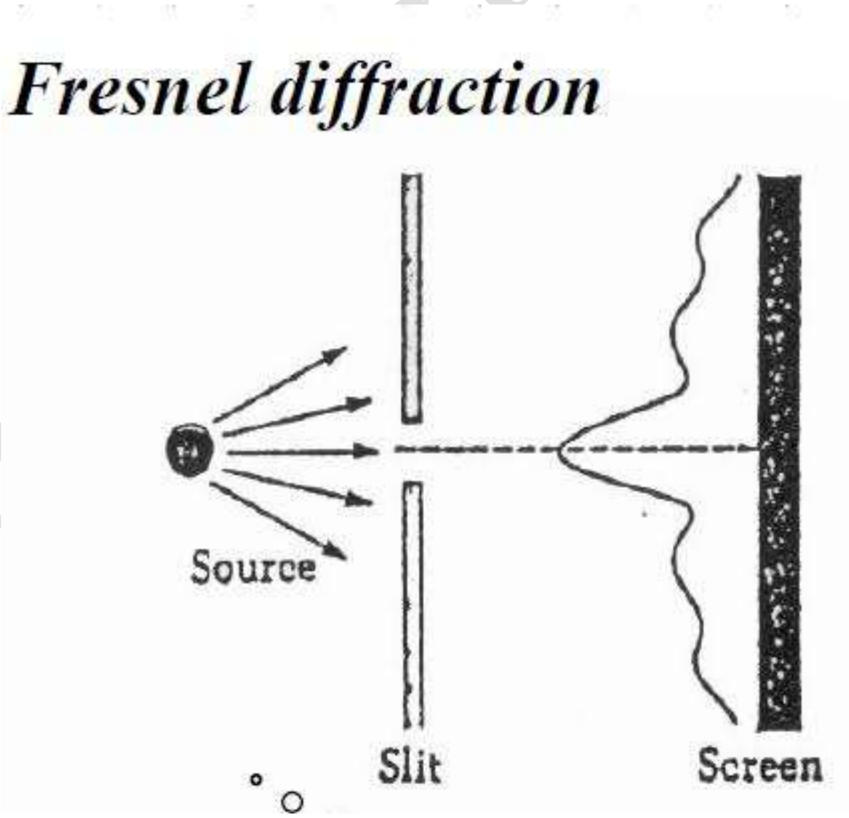
Conditions for diffraction

- When a small obstacle or an aperture is placed in the path of light, some light is found in the geometric shadow.
- If the width of the slit is not very large as compared to the wavelength of light used, then the intensity in the illuminated region is also not uniform.
- Light does not propagate strictly in a straight line when it passes close to the edges of objects, and the patterns produced depend on the shape and size of the object.

Types of diffraction

Fresnel Diffraction

The diffraction pattern obtained on the screen by keeping both source and screen at finite distance from the diffraction slit is called Fresnel diffraction.

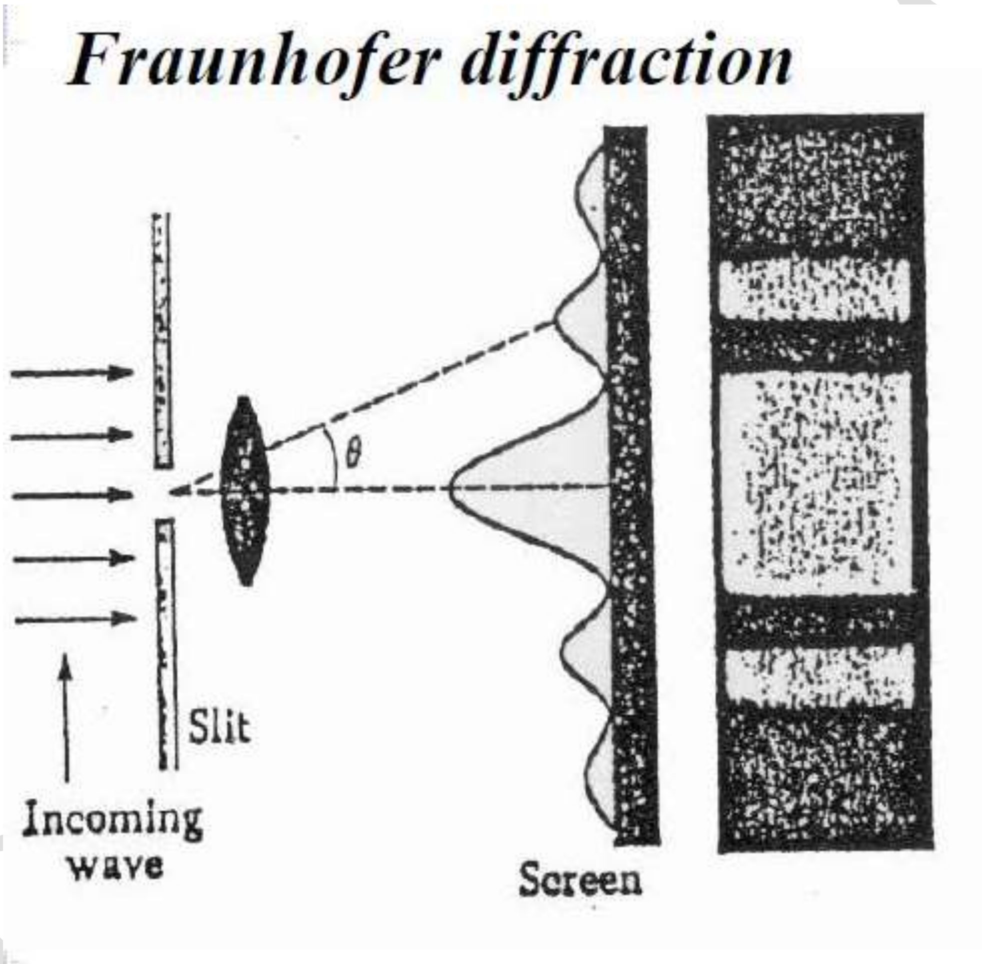


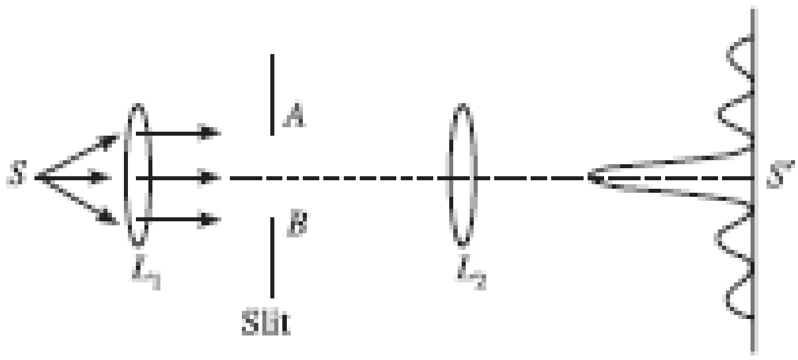
A spherical wave front emitted by the source is incident on the slit, the light waves emitted by the secondary sources of light bends through edges of the slit and enclosed in the region of geometrical shadow.

The light waves superpose on one another to produce alternate maxima and minima of decreasing intensity on both sides of central bright maxima, this is called Fresnel diffraction.

Fraunhofer Diffraction

The diffraction pattern obtained by keeping both the source and the screen at infinite distances from the diffraction slits this is called Fraunhofer diffraction.





A source of light is kept at focal point of convex lens L_1 therefore a sharp intense beam of light is produced, hence a plan wave front is incident on the slit. The light waves emitted by the secondary source of light bends through the edges of slit and enclosed at the region of geometrical shadow. These light waves are focused by the lens L_2 on the screen, the light wave superpose on one another to produce a diffraction pattern this is called Fraunhofer diffraction.

Difference between Fresnel Diffraction Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
If the source of light and screen are at finite distance from the obstacles, then the diffraction is referred to as Fresnel diffraction	If the source of light and screen are at infinite distance from the obstacles, then the diffraction is referred to as Fraunhofer diffraction
Fresnel diffraction pattern occur on flat surface	Fraunhofer diffraction pattern occur on spherical surface
To obtain Fresnel diffraction, zone plates are used	To obtain Fraunhofer diffraction, plan diffraction grating are used
Shape and intensity of diffraction pattern changes as the wave propagates downstream of the scattering source	Shape and intensity of Fraunhofer diffraction pattern remains constant
Diffraction pattern move along the corresponding shift in the object	Diffraction pattern remains in fixed position
In Fresnel diffraction, source and screen are not far away from each other	In Fraunhofer diffraction, source and screen are far away from each other
In Fresnel diffraction, incident wave fronts are spherical	In Fraunhofer diffraction, the incident wave fronts are in plan

In Fresnel diffraction, the convex lens is not required to converge the spherical wave fronts	In Fraunhofer diffraction, the plane wavefronts are converged by convex lens to produce diffraction pattern

Diffraction grating

It is a device which produces clear-cut diffraction patterns, such devices are called as diffraction grating

Types of diffraction grating

1. **Transmission grating**
2. **Reflection grating**

Transmission grating

A plane glass plate of length 2.5 cm taken a thin transparent film is coated. 6000 or 8000 or 15000 opaque lines are drawn on it using a diamond. The space between two opaque line transmits light, the opaque line obstruct light. The light waves while passing through narrow slits get diffracted. The diffracted light waves superpose one another to produce clear cut diffraction patterns, this is called transmission grating.

Let **a** is the width of space between two opaque lines

b is the width of opaque line

then $a + b = \frac{2.5}{15000}$

The grating constant is also defined as the reciprocal of number of lines ruled per centimetre on the grating

Therefore, $a + b = \frac{1}{\frac{15000}{2.5}} = \frac{1}{N}$

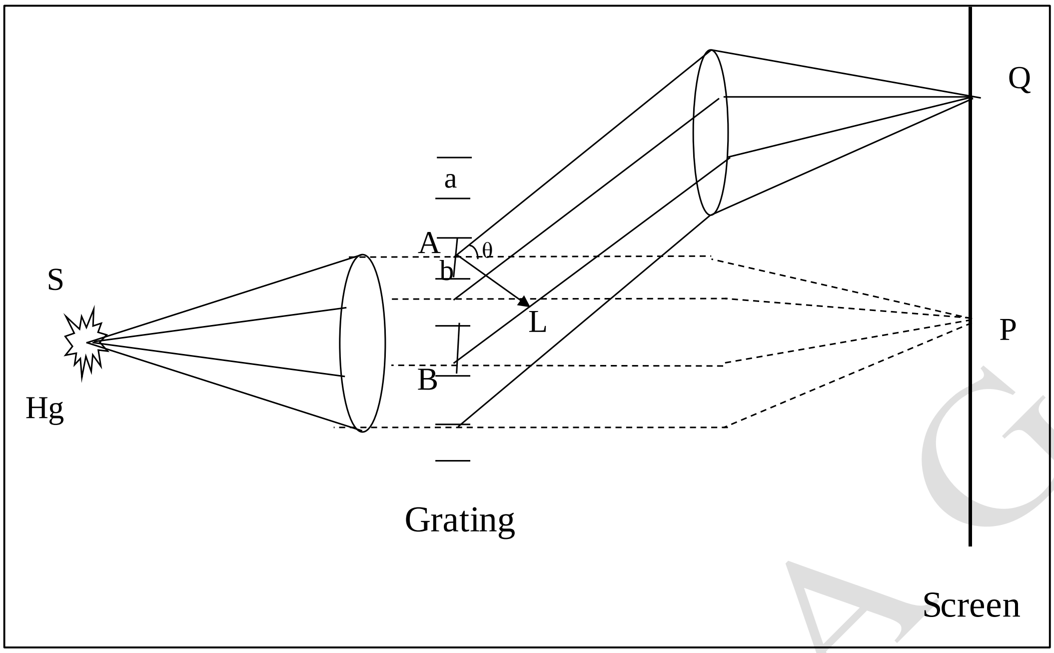
Reflection grating

A plane glass plate of length 2.5 cm taken a thin film of silver is coated on it 6000 or 8000 or 15000 opaque lines are drawn on it, when it is kept in front of source of light. The space between opaque lines reflects light but opaque line obstruct light.

The reflected light waves get diffracted and produce clear cut diffraction pattern, this is called reflection grating.

Determination of wavelength by using a Plan transmission diffraction grating

(i) Normal Incidence



A monochromatic beam of light is kept in front of focal point of convex lens. A sharp intense parallel beam of light is produced. A plan wave front is incident on the diffraction grating, the light waves travel along initial horizontal direction and passes through narrow slit and focus by the lens at centre.

Case I: The light waves from the source will passes through narrow slit get diffracted through all angle θ the diffracted light waves superpose one another at point P to produce n^{th} maxima or minima depending on the path difference.

The path difference between two superposing diffracted light waves is given by

$$P.D. = BP - AP$$

From triangle ALB,

$$Sin\theta = \frac{BL}{AB}$$

$$BL = (a+b)sin\theta$$

If the incident plane wave front between A and B it consists of odd number of Fresnel zones, the light waves from odd number of Fresnel zones superimpose on one another at point P to produce n^{th} maxima

Therefore, $(a+b)sin\theta = n\lambda$

Where, n is the order of diffraction

(a+b) is the grating element

Θ is angle of diffraction

Case II: If the plane wave front A and B if it consists of even numbered Fresnel zones the light waves from even number of Fresnel zones superpose on one another at the point P to produce nth maxima

$$(a+b)sin\theta = (2n+1)\frac{\lambda}{2}$$

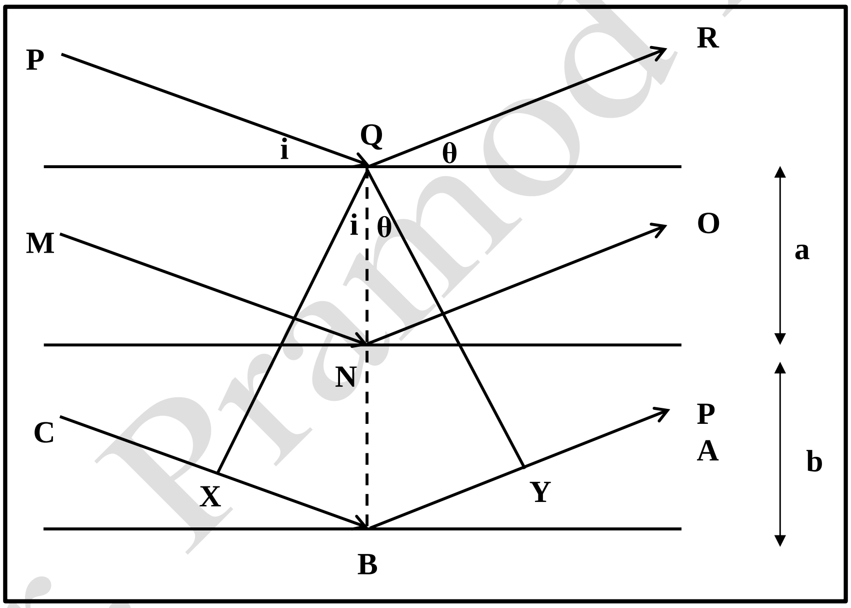
Therefore, we are getting 1st order maxima and its 1st order maxima and 2nd order maxima of its 2nd order maxima on both side of central bright maxima.

The light waves get diffracted through an angle θ+dθ and superpose on one another at point P to produce nth minima after nth maxima.

(ii) Oblique Incidence

Suppose a beam of light is incident along PQ at an angle of incidence 'i' on the diffraction grating, the beam of light get diffracted along QR at an angle θ.

Another beam of light is incident along CB, get diffracted along BD.



The two diffracted beams of light superpose on one another to produce nth maxima or nth minima depending upon the path difference. The path difference between the two superposing diffracted beams of light is given by

$$P.D = XB + BY \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ } (1)$$

From the triangle QXB

$$\sin(i) = \frac{XB}{QB}$$

$$XB = QBsin(i)$$

$$XB = (a+b)\sin(i) \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ } (2)$$

From the triangle QYB

$$\sin(\theta) = \frac{BY}{QB}$$

$$BY = QBsin(\theta)$$

$$BY = (a+b)\sin(\theta) \qquad (3)$$

Subustuting the value of XB and BY from the equations (2) and (3) in the equation (1), we have

$$P.D. = XB + BY$$

$$= (a+b)\sin(i) + (a+b)\sin(\theta)$$

$$= (a+b)[Sin(i) + \sin(\theta)]$$

$$= 2(a+b)\left\lfloor sin\frac{\theta+i}{2}\cos\frac{\theta-i}{2}\right\rfloor \qquad (4)$$

Case I: If the path difference between two superposing beams of light is nλ, then the light superpose on one another to produce n^{th} maxima after central bright maxima

i.e.,

$$2(a+b)\left\lfloor sin\frac{\theta+i}{2}\cos\frac{\theta-i}{2}\right\rfloor = n\lambda$$

therefore,

$$\sin\left(\frac{\theta+i}{2}\right) = \frac{n\lambda}{2(a+b)\cos(\frac{\theta-i}{2})}$$

Where,

D = $\theta + i$ deviation produced in the beam of light .

Therefore, the minimum deviation of light through a diffraction grating

$$Sin\left(\frac{\theta+i}{2}\right) \text{ is minimum}$$

$$Cos\left(\frac{\theta-i}{2}\right) \text{ is maximum}$$

$\frac{\theta-i}{2}$ should be 0

$$\frac{\theta-i}{2} = 0$$

$$\theta = 2i$$

Condition for minimum deviation of light through a diffraction grating

$$D_m = i + i = \theta + \theta$$

$$D_m = 2i = 2\theta$$

$$D_m = \frac{\theta}{2} \; or \; i = \frac{D_m}{2}$$

Therefore, the condition for the n^{th} maxima after central bright maxima is

$$2(a+b)sin\theta = n\lambda$$

$$2c\sin\left(\frac{D_m}{2}\right) = n\lambda$$

n→Order of diffraction

n→1 first order diffraction

Case II: the path difference between two superposing beams of light is $2(n+1)\frac{\lambda}{2}$, then the light waves superpose on one another to produce nth minima after central bright maxima

Therefore,

$$2(a+b)Sin\left(\frac{\theta+i}{2}\right)\cos\left(\frac{\theta-i}{2}\right) = 2(n+1)\frac{\lambda}{2}$$

$$2c\sin\left(\frac{D_m}{2}\right) = 2(n+1)\frac{\lambda}{2}$$

Therefore, first order spectra, second order spectra of decreasing intensity are obtained on both side of central bright maxima.