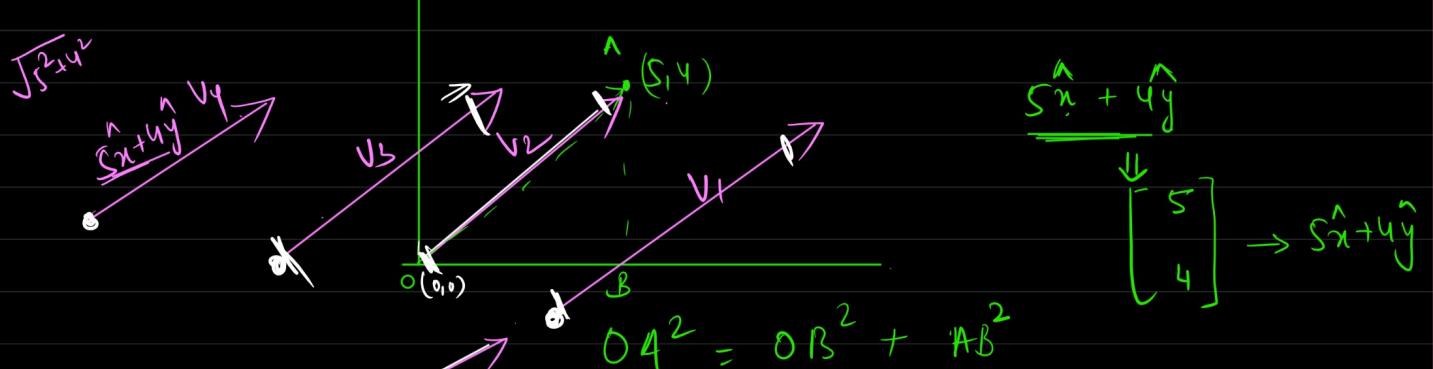


Vectors



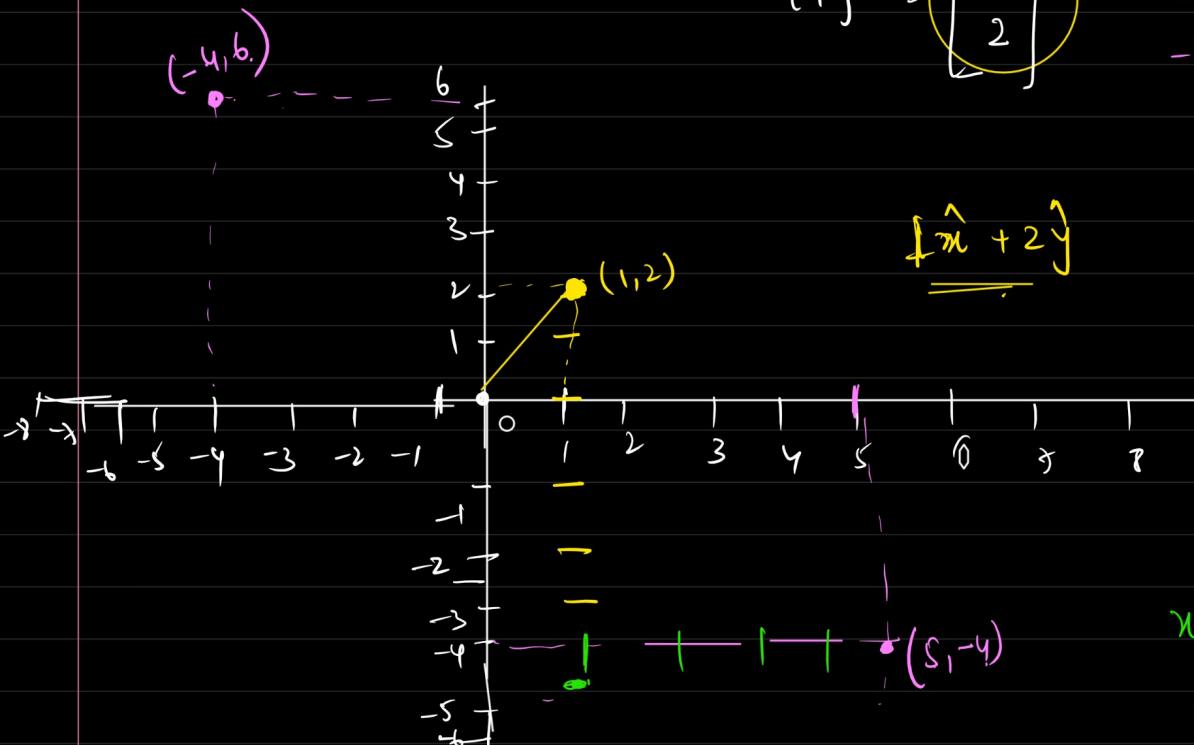
* There can be multiple vectors in the space of same magnitude & direction; the origin is different.

Operations on Vectors

$$(\text{addition}) \quad \mathbb{R}^2 \xrightarrow{\text{2d Space}} \vec{i} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}, \quad \vec{j} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

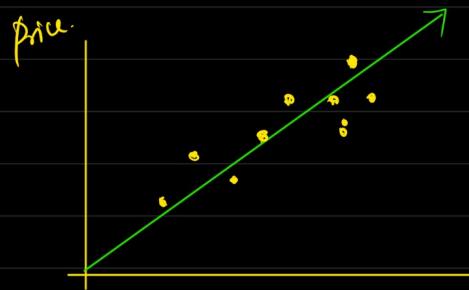
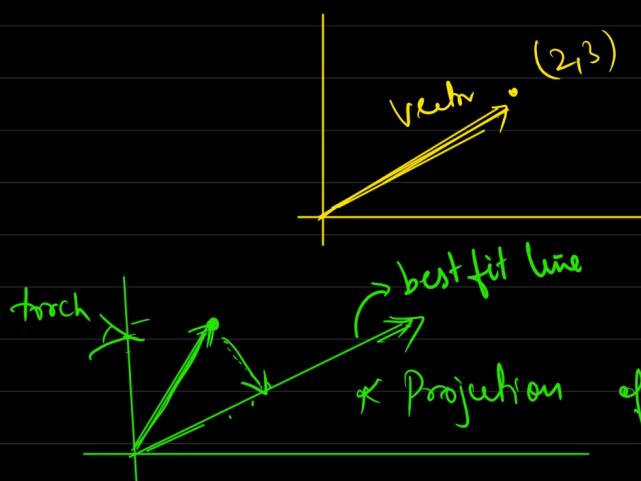
$$i, j \in \mathbb{R}^2$$

$$\vec{i} + \vec{j} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{array}{l} 5 + (-4) \Rightarrow 1 \\ -4 + (6) \Rightarrow 2 \end{array}$$



x-direction $(5) + (-4)$
y-direction $-4 \rightarrow 6$ on top

Projection (Shadow)



Why?

8 dgs \rightarrow 16 coordinates

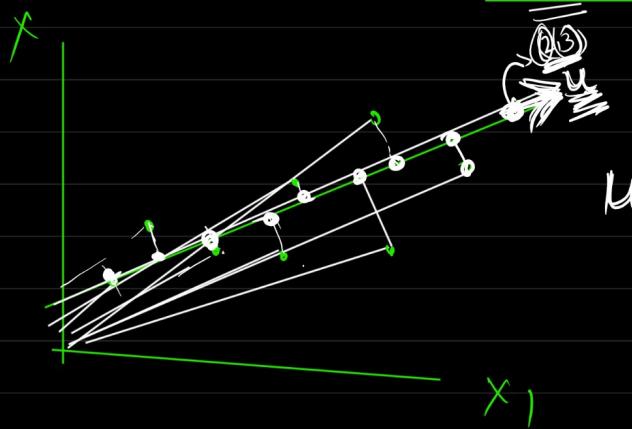
Same direction

No of rooms

then best fit line lies in

2 columns \rightarrow 16 coordinates

10 millions



$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

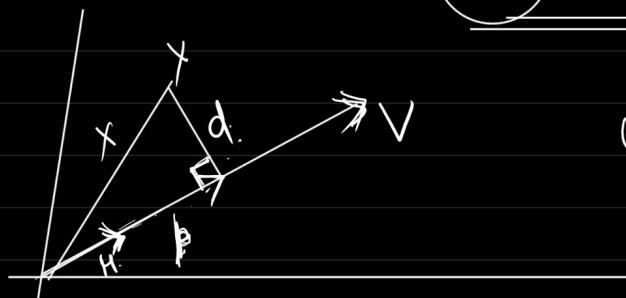
& 8 magnitude
 k_1, k_2, \dots, k_8 .

$8+2 \Rightarrow 10$ coordinates

10 vs 16 coordinates

$$U = \frac{V}{\|V\|}$$

Unit vector



$$U = \frac{V}{\|V\|}$$

magnitude of V

$$X = p + q$$

$$d = \underline{x} - \underline{p}$$

$$p = kv$$

$$d = \underline{x} - kv$$

then dot product will be 0

$$\begin{aligned} a \cdot b &= ab \cos \theta \\ &= \downarrow \cos 90^\circ = 0 \end{aligned}$$

$$= 0$$

$$p \cdot d = 0$$

$$= k u \times (x - k u) = 0$$

$$= k x u - k^2 u \cdot u = 0$$

$$k(x u - k u \cdot u) = 0$$

$k=0$

$$p = k u$$

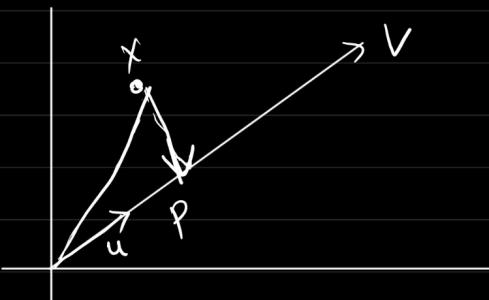
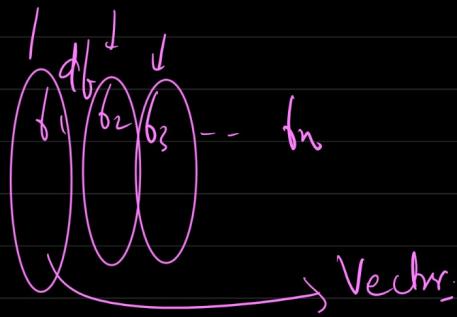
$$p = (x \cdot u) \frac{u}{u} = \frac{v}{\|v\|}$$

$$x u - k u \cdot u = 0$$

$$k = x u$$

(magnitude of $u=1$)
 $u \cdot u = 1$

$$p = \frac{x \cdot v \cdot u}{\|v\|}$$



* Linear Algebra

$$\begin{cases} 3x + 2y = 8 \\ 5x + 4y = 18 \end{cases}$$

↓

$$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 18 \end{pmatrix}$$

matrix addition

$$\begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \Rightarrow$$

Matrix multiplication

$$A = 3 \times 4$$

$$B = 4 \times 3$$

$m \times n$ $n \times p$

$A @ B$ (no doubt)

$$\begin{pmatrix} 2 & 2 \\ 4 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 2 \times 2 + 2 \times 1 & 2 \times 3 + 2 \times 4 \\ 4 \times 2 + 1 \times 1 & 4 \times 3 + 1 \times 4 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 14 \\ 9 & 16 \end{pmatrix}$$

Transpose

$$A = \begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix}$$

* Determinant of a Matrix

$\det(A)$

$$A = \begin{bmatrix} a & b & c \\ -d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(A) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= aei + bfg + cdh - afh - bdi -cef.$$

$$a(ei - fh) \Rightarrow ae^i - afh - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} - b(di - fg) - bdi + bfg$$

* Inverse matrix

np. inv

$$\begin{matrix} 2 & \rightarrow \frac{1}{2} \\ 3 & - \frac{1}{3} \end{matrix}$$

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = \frac{1}{\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}}$$

Inverse of matr

$$A \cdot A^{-1} = 0$$



for 2×2 (sq matrix — 2×2 , 3×3 , 4×4)

$$\downarrow \quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\text{Inv}(A) = A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \xrightarrow{\text{Adjoint } A}$$

for order 3×3

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\text{inv}(A) = \frac{1}{\det A} \begin{vmatrix} \begin{matrix} a & b \\ d & g \end{matrix} & \begin{matrix} b & c \\ g & i \end{matrix} & \begin{matrix} a & c \\ d & e \end{matrix} \\ \begin{matrix} c & f \\ h & i \end{matrix} & \begin{matrix} d & f \\ g & i \end{matrix} & \begin{matrix} d & e \\ g & h \end{matrix} \end{vmatrix}$$

$$\left| \begin{array}{c|cc|c} bc & ac & ab \\ hi & gi & gh \\ \hline ef & dg & de \end{array} \right|$$

Q $A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} = A^{-1} = \frac{1}{4 \times 6 - 7 \times 2} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$

$$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

$A \cdot A^{-1} = I$ → order matters.

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Vectors → Matrix

$$v_1 = 3x + 4y \Rightarrow \begin{bmatrix} 3 & 4 \end{bmatrix}$$

$$v_2 = x + 3y \Rightarrow \begin{bmatrix} 1 & 3 \end{bmatrix}$$

Z-Score → Standardisation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$



Rotation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

* Vector — matrix

$$\begin{array}{l} 2x + y - z = 2 \\ x + 3y + 2z = 1 \\ x + y + z = 2 \end{array} \rightarrow \left(\begin{array}{cccc} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{x=2} \begin{array}{l} \\ \\ \end{array}$$

$x = 2$
 $y = -1$
 $z = 1$

→ Row echelon method
or
column echelon method
→ inverse method