

PHYS 512 - Assignment 2

Question 2

Suppose that I start off with the following set of x values:

$$x = [1, 2, 3, 4, 5]$$

If the error is too large after the first iteration, the interval of x values will be split into the following intervals:

$$x_1 = [1, 1.5, 2, 2.5, 3] \quad x_2 = [3, 3.5, 4, 4.5, 5]$$

One can easily see that the first, third and fifth values of x_1 correspond to the first three values of x , and the first, third and fifth values of x_2 correspond to last three values of x . Since we already computed $f(x)$ for each value in x , we can reuse these five values during the next recursion and so on. Thus, we can easily do the math of how many function-calls were avoided in a given computation.

Suppose that we call the `integrate_adaptive` function and it completes 15 recursions before returning an answer. The initial call computes five values of $f(x)$, and then each recursion computes 10 values of $f(x)$ (5 on the left and 5 on the right). So, we end up calling $5 + 14 \cdot 10 = 145$ functions. However, if we reuse our values of $f(x)$ from the previous iterations, then we compute five values of $f(x)$ on the initial call, and we compute 4 values for each iteration (2 on the left and 2 on the right). This gives us $5 + 14 \cdot 4 = 61$ function calls.

Thus, for a 15-step recursion, we would save ourselves from $145 - 61 = 84$ function calls. This seems to be a large difference.