PHYS 512 – Assignment 2

Question 3

I found question 3 a bit unclear, and so my code might unclear as well. Consequently, I will explain my process here.

I created a set of x values between 0.5 and 1, and I evaluated said values with numpy.log2(). These are the true values. I then fitted the results with numpy.polynomial.chebyshev.chebfit() and, I tested this fit with numpy.polynomial.chebyshev.chebval(). I found that a Chebyshev fit of at least order 6 yields an error of less than 10^{-6}.

To create mylog2(), I first used np.frexp() to separate any real number into its mantissa and an exponent of 2. For example:

$$x = np.frexp(x) = a * 2^b$$

And if we take the log base 2 of x, we obtain:

$$\log_2 x = \log_2 a * 2^b = \log_2 a + \log_2 2^b$$

My Chebyshev fit somewhat holds for $\log_2 a$ because $-1 < a < 1$. As for $\log_2 2^b$, it is known that:

$$\log_2 2^b = b$$

So, the log base 2 of $x = a * 2^b$ would be $\log_2 a$ + b. To find the natural logarithm, we can use this information along with the change-of-base formula:

$$\log_b a = \frac{\log_d a}{\log_d b}$$

$$\ln(a) = \log_e a = \frac{\log_2 a}{\log_2 e}$$

Where each log base 2 is calculated as explained above. Indeed, the Chebyshev fit seems to be a good estimate.

Figure 1: A Chebyshev Approximation of Ln vs. the True Values of Ln