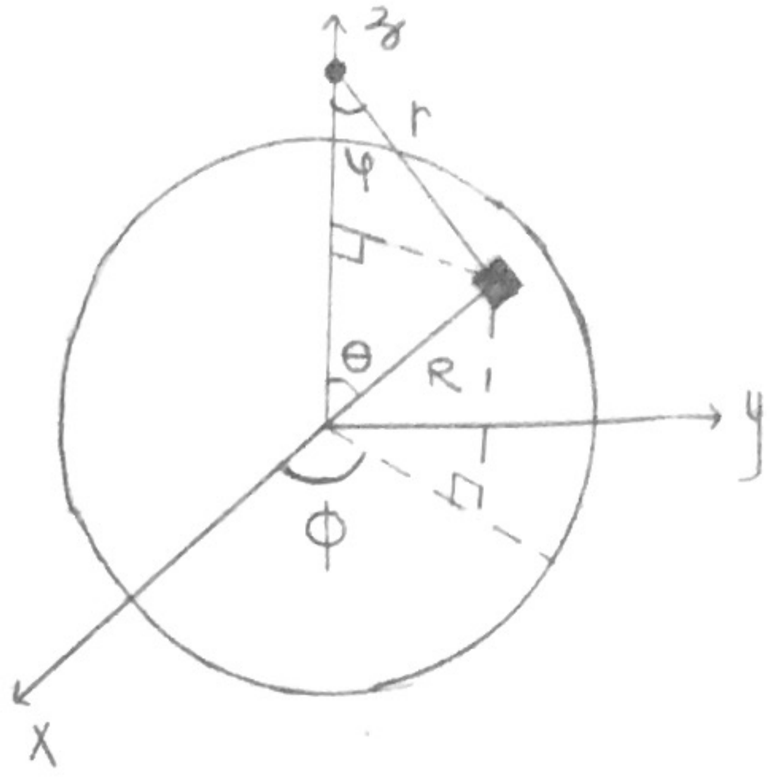
Vega Hitti
260 381 396
PHYS 512 - Assignment 2

#1] could work along any axis, but as per the Griffiths solution manual,] will set up the integral to find the electric field along the z-axis.



$dq = \sigma\, da$ where $da = R^2 \sin\theta\, d\theta\, d\phi$

So $dq = \sigma R^2 \sin\theta\, d\theta\, d\phi$

And $r^2 = R^2 + z^2 - 2Rz\cos\theta$

And $\cos\psi = \dfrac{z - R\cos\theta}{r}$

So $F = \dfrac{1}{4\pi\varepsilon_0} \int \dfrac{\cos\psi}{r^2}\, dq$

$$F = \frac{1}{4\pi\varepsilon_0} \int \left[\frac{z - R\cos\theta}{(R^2 + z^2 - 2Rz\cos\theta)^{\frac{1}{2}}} \right] \frac{\sigma R^2 \sin\theta\, d\theta\, d\phi}{R^2 + z^2 - 2Rz\cos\theta}$$

$$F = \frac{1}{4\pi\varepsilon_0} \sigma R^2 \int_0^{\pi}\int_0^{2\pi} \frac{(z - R\cos\theta)\sin\theta\, d\theta\, d\phi}{(R^2 + z^2 - 2Rz\cos\theta)^{\frac{3}{2}}}$$ ⟶ this is what we want to compute

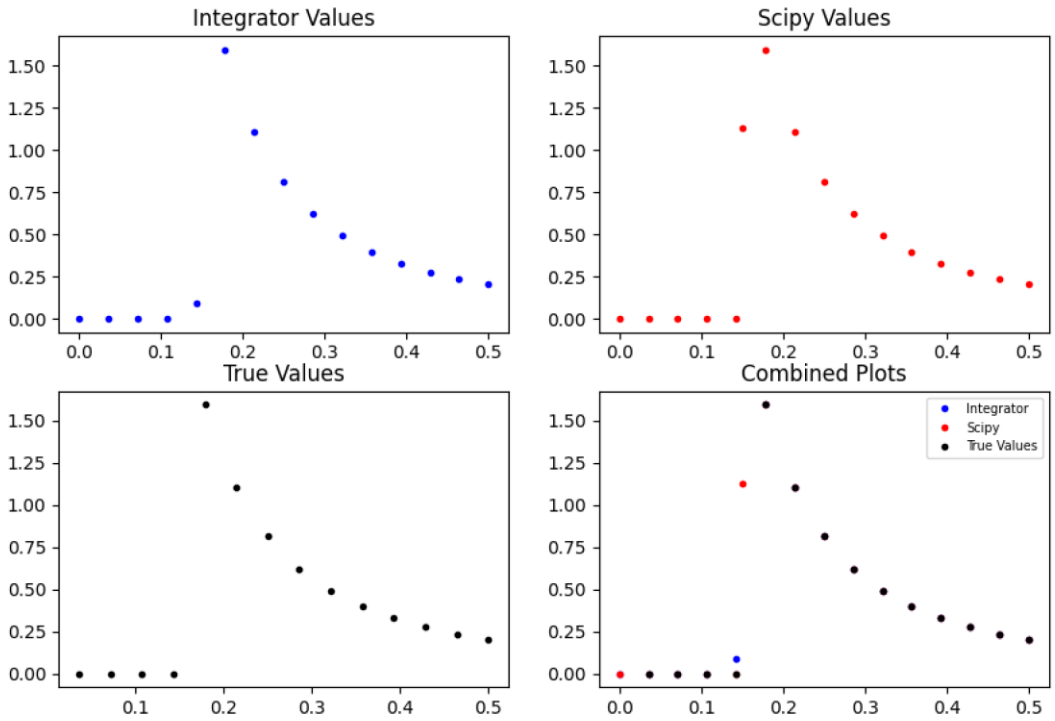
It should give $F = \dfrac{R^2\sigma}{2\varepsilon_0 z^2}\left[\dfrac{(z-R)}{|z-R|} - \dfrac{(-z-R)}{|z+R|} \right]$

So $$F = \begin{cases} \dfrac{R^2\sigma}{\varepsilon_0 z^2}, & z > R \\ 0, & z < R \end{cases}$$

PHYS 512 – Assignment 2

Question 1

Figure 1: The Electric Field About a Spherical Shell



As seen in Figure 1, my integrator and SciPy's integrator are a good estimate of the true values of the electric field. Indeed, in the 4th subplot, the data points from all 3 methods were included, and almost all the points overlap.

As per the derivation above, there should be a singularity at z = R. Indeed, at this point, one of the resulting terms is divided by zero. In the figure below, you will find the value of the electric field at various points as calculated by the three prescribed methods. The sixth value in each array is that of z = R = 0.15. My integrator and the "true value" returned a value of Nan, but SciPy returned a value of about 1.129. This seems a bit suspicious.

```
My integrator's values are: [-5.697974838917979e-17, 4.16347317120154e-10, 1.1544179643768045e-07, 5.8425083110607566e-05
, 0.09277503678215358, nan, 1.5939554908686902, 1.1068420929814238, 0.8131940572104619, 0.6226000469652926, 0.49193081232
2319, 0.39846396288713365, 0.32930906407810184, 0.2767110899891614, 0.2357774972968416, 0.2032979442808674]

 Scipy's values are: [-8.045731097933886e-17, 0.0, 4.996003610813204e-16, 2.7200464103316335e-15, 6.811218256075335e-14,
1.129433024621715, 1.5938558843460557, 1.1068443641292076, 0.813191777727581, 0.6225999548226777, 0.49193082850186987, 0.
39846397108651466, 0.3293090670136484, 0.27671109103230185, 0.23577749768432818, 0.2032979444318945]

 The true values are: [nan, 0.0, 0.0, 0.0, 0.0, nan, 1.5938558843460584, 1.1068443641292072, 0.8131917777275808, 0.622599
954822679, 0.4919308285018699, 0.3984639710865146, 0.3293090670136484, 0.2767110910323018, 0.23577749768432818, 0.2032979
444318952]
```