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PHYS 512 - Assignment 1

#1 a) $f(x \pm \delta) \sim f_0 \pm f'\delta + \frac{f''}{2}\delta^2 \pm \frac{f'''}{6}\delta^3 + \dots$

$f(x \pm 2\delta) \sim f_0 \pm f'(2\delta) + \frac{f''(2\delta)^2}{2} \pm \frac{f'''(2\delta)^3}{6} + \dots$

From this, we can deduce that

$$f'(x) \sim \frac{f(x+\delta) + f(x+2\delta) - f(x-\delta) - f(x-2\delta)}{6\delta}$$

is a good estimate of $f'(x)$.

Indeed, $\frac{f(x+\delta) + f(x+2\delta) - f(x-\delta) - f(x-2\delta)}{6\delta}$

$$= \frac{\left[2f_0 + 3f'\delta + \frac{5}{2}f''\delta^2 + \frac{3}{2}f'''\delta^3\right] - \left[2f_0 - 3f'\delta + \frac{5}{2}f'''\delta^2 - \frac{3}{2}f'''\delta^3\right]}{6\delta}$$

$$= \frac{6f'\delta + 3f'''\delta^3}{6\delta}$$

$$= f' + \frac{1}{2}f'''\delta^2 \sim f'$$

b) Truncation error $\sim f''' \delta^2 = e_t$

Roundoff error $\sim \varepsilon \left| \frac{f}{\delta} \right| = e_r$

We want to minimize $e_t + e_r$ in terms of δ.

$$\frac{d}{d\delta}(e_t + e_r) = 0$$

$$\frac{d}{d\delta}\left(f''' \delta^2 + \varepsilon \left| \frac{f}{\delta} \right| \right) = 0$$

$$f''' \delta - \frac{\varepsilon |f|}{\delta^2} = 0$$

$$\delta^3 = \frac{\varepsilon f}{f'''}$$

$$\boxed{\delta = \left(\frac{\varepsilon f}{f'''} \right)^{\frac{1}{3}}}$$