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PHYS 512 – Assignment 6

**Part a)**

For Part a), I used the simple noise model that was shown in class. In other words, the noise is the square of the absolute value of the data's Fourier transform:

$$N = |FT(S)|^2 \tag{1}$$

In equation 1, N is the noise, S is the data and FT is the Fourier transform.

I smoothened the power spectrum by averaging each point over its respective neighbors. And, I incorporated a window that is flat near the center as to not manipulate the data too much. The aforementioned window is a convolution of $[f(x) = 0.5 + 0.5 \cdot \cos(x)]$ and $[f(x) = 1]$.

Here is an example of the noise yielded by this model for Event #1 in Hanford:

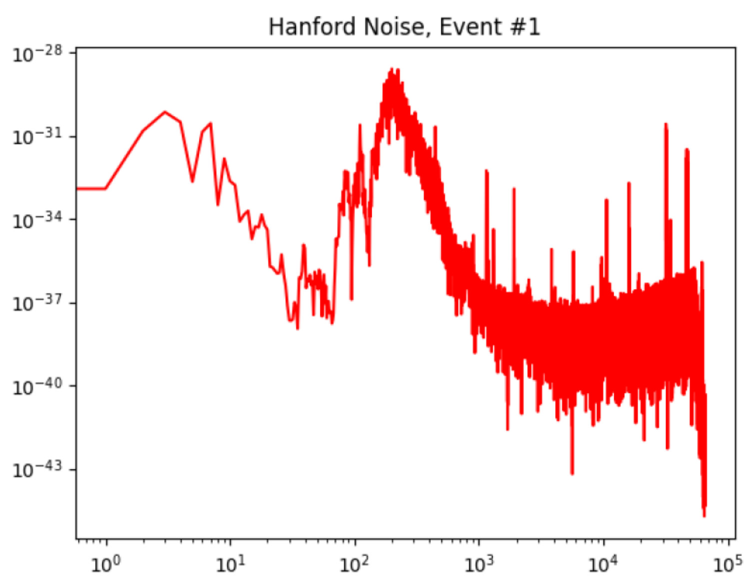



Figure 1: Noise Model for Event #1 in Hanford

**Part b)**

For Part b), I whitened the data and the templates, and then I plotted the matched filters of each event. The code is well commented, and it is probably a better indicator of what I did to complete this part of this assignment.

Here are the plots of the matched filters:

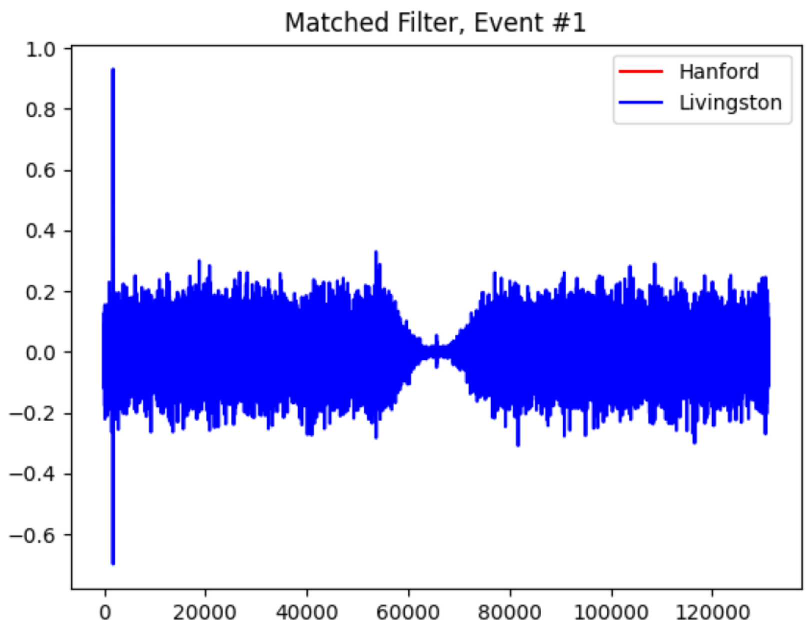



Figure 2: Matched Filters for Event #1

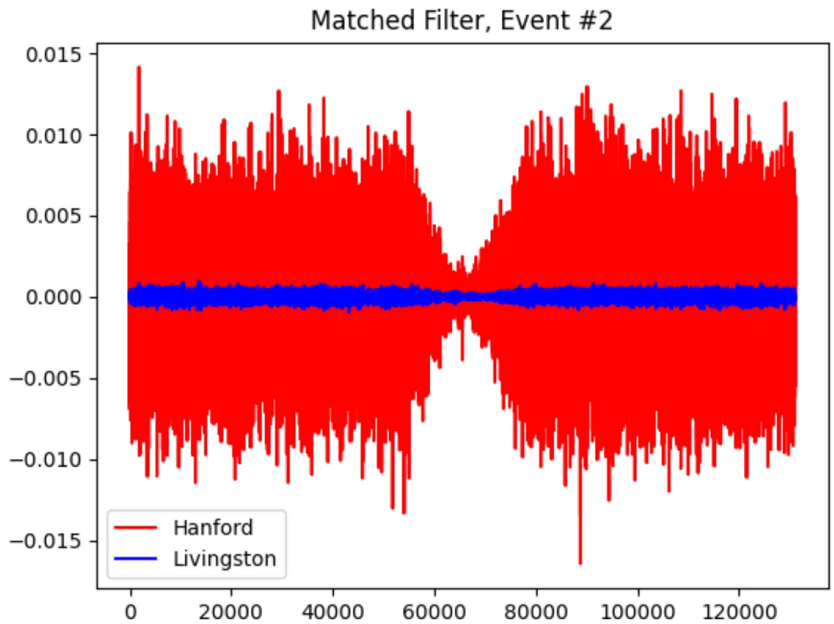



Figure 3: Matched Filters for Event #2

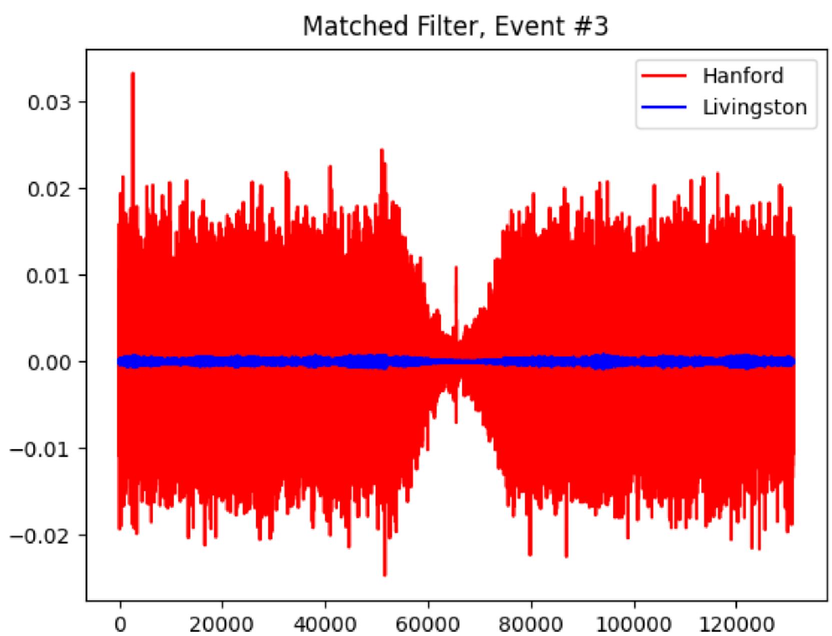



Figure 4: Matched Filters for Event #3

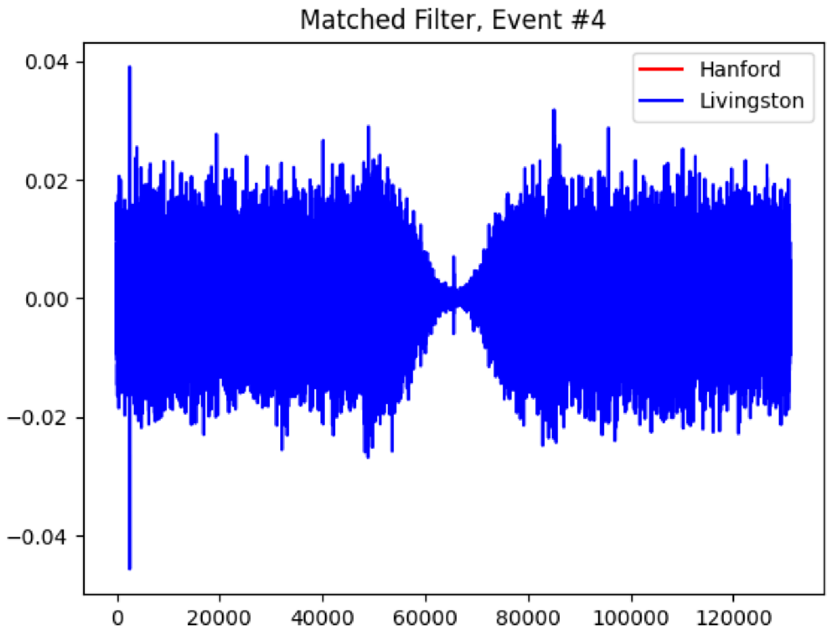



Figure 5: Matched Filters for Event #4

**Part c)**

For Part c), I took the noise to be the standard deviation of the matched filters. This yielded the following:

Table 1: Estimated Noise for Each Event

| | Hanford | Livingston | Combined |
|---|---|---|---|
| Event #1 | 0.004131755 | 0.074880492 | 0.037531912 |
| Event #2 | 0.003264292 | 0.000211189 | 0.001635775 |
| Event #3 | 0.005570704 | 0.000220737 | 0.002786948 |
| Event #3 | 0.000193424 | 0.006805218 | 0.003403997 |

I then computed the signal-to-noise ratio as the maximum of the absolute value of the matched filter, divided by the noise (i.e., the values in Table 1).

This yielded the following signal-to-noise ratios:

Table 2: Estimated Signal-to-noise Ratios for Each Event

| | Hanford | Livingston | Combined |
|---|---|---|---|
| Event #1 | 13.86 | 12.43 | 12.44 |
| Event #2 | 5.04 | 4.50 | 5.00 |
| Event #3 | 5.98 | 3.85 | 5.89 |
| Event #3 | 4.16 | 6.70 | 6.69 |

**Part d)**

For Part d), I took the matched filter of the template with itself, including the noise model, and I took the standard deviation of that. Then, the analytical signal-to-noise ratio to is the maximum of the absolute value of this matched filter, divided by the standard deviation of this matched filter. Here are the results:

Table 3: Analytical Signal-to-noise Ratios for Each Event

| | Hanford | Livingston | Combined |
|---|---|---|---|
| Event #1 | 10.25 | 9.64 | 10.27 |
| Event #2 | 11.13 | 11.22 | 11.12 |
| Event #3 | 12.30 | 11.33 | 10.46 |
| Event #3 | 10.85 | 11.04 | 10.93 |

For event #1, the estimated signal-to-noise ratios (SNRs) are higher than the analytical ones. For events #2 through #4, the analytical SNRs are higher than the estimated ones. While the analytical & estimate values differ significantly for certain events and locations, we can say they somewhat agree. Indeed, they are generally of the same order of magnitude.

**Part e)**

For Part e), I calculated the frequency according to the following equation (as shown in tutorial):

$$\int_0^{f} \widetilde{MF}\ df\ = \frac{1}{2}\int_0^{f_{max}} \widetilde{MF}\ df \qquad (2)$$

Numerically, I took the cumulative integral of the matched filters in Fourier space, and I found the frequency which corresponds to half of the maximum value of the cumulative integral. This yielded the following frequencies:

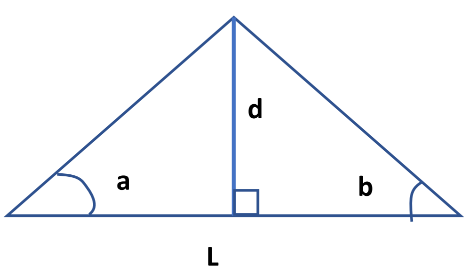
Table 3: Frequency at Which Half the Weight is Above (or Below)

| | Hanford | Livingston |
|---|---|---|
| Event #1 | 1309 | 3564 |
| Event #2 | 876 | 5760 |
| Event #3 | 876 | 15923 |
| Event #3 | 1919 | 667 |

**Part f)**

For Part f), I am not sure that I understand the first question. I assume the width of the spike (i.e. its horizontal shift) represents the uncertainty in time of arrival. For each event, the spike appears to have a width of approximately 10 (or within this order of magnitude). And, as per the LIGO tutorial, the units of the matched filter are $\frac{4096\ Hz}{32\ s}$. According to this information, it seems as though the uncertainty is acceptable, and we can decently localize the time of arrival.

For the second part, I assume that the location in the sky is calculated by triangulation. Since there are only two detectors (i.e. two given points), we can only obtain an arc in which the source can be located. Given the following diagram, we get:



$$d = \frac{L \cdot \sin(a) \sin(b)}{\sin(a+b)} \tag{4}$$

I quickly searched this on the internet, and most sources agree that the positional uncertainty of the gravitational waves is $100 - 150\ degrees^2$. Indeed, this makes sense for two detectors that are a few thousand kilometers apart. This is quite a large region of uncertainty.