FYS3150 - Project 3

Vegard Falmår and Sigurd Sørlie Rustad
Universitetet i Oslo[*]

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Abstract om du vil

I. INTRODUCTION

II. THEORY

A. Newton's laws and the motion of planets

The well-known Newtons second law reads

$$\sum \mathbf{F} = m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}, \tag{1}$$

where $\sum \mathbf{F}$ is the sum of forces acting on an object, m is its mass and $\mathbf{r}$ its position.

Newton's law of gravity says that the total gravitational force acting on an object i from all the other objects in a system is

$$F_{G,i} = -Gm_i\sum_{j\neq i} m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} \tag{2}$$

where $\mathbf{r}_i$ are the positions and m_i are the masses of the objects. G is the gravitational constant.

For the motions of planets, on which the gravitational force is the only force acting, the two equations 1 and 2 combined give us the differential equation that describes the motion of an object:

$$\frac{\mathrm{d}^2\mathbf{r}_i}{\mathrm{d}t^2} = \frac{1}{m_i}F_{G,i} = -G\sum_{j\neq i} m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} \tag{3}$$

B. Velocity Verlet

Classical problem of Newtonian mechanics often involve solvinga set of two coupled first order differential equations, namely

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = a$$

where x is position, v is velocity and a is acceleration. Doing a Taylor expansion around a point in time $t = t_0$ gives

$$\begin{aligned} x(t) &= x(t_0) + (t-t_0)\frac{\mathrm{d}x}{\mathrm{d}t}(t_0) + \frac{1}{2}(t-t_0)^2\frac{\mathrm{d}^2x}{\mathrm{d}t^2}(t_0) + O(h^3) \\ &= x(t_0) + (t-t_0)v(t_0) + \frac{1}{2}(t-t_0)^2a + O(h^3) \end{aligned}$$

[*] vegardfa@uio.no; sigurdsr@gmail.com

An expression can then be found for position at a time $t = t_0 \pm h$:

$$x(t\pm h) = x(t_0) \pm hv(t_0) + \frac{1}{2}h^2a \pm O(h^3) \tag{4}$$

Discretizing equation 4 and letting $x_i = x(t)$, $x_{i+1} = x(t+h)$, $v_i = v(t)$ and $v_{i+1} = v(t+h)$, we get

$$\begin{aligned} x_{i+1} &\approx x_i + hv_i + \frac{h^2}{2}a_i \\ v_{i+1} &\approx v_i + ha_i + \frac{h^2}{2}\dot{a}_i, \end{aligned}$$

where $\dot{a}_i$ is the derivative of the acceleration with respect to time. Similarly to the case of Forward Euler, doing a Taylor expansion of a gives after some manipulation

$$\dot{a}_i \approx \frac{a_{i+1} - a_i}{h} \tag{5}$$

Insering this into the expressions we have for x_{i+1} and v_{i+1} gives us the equations that describe the Velocity Verlet method:

$$\begin{aligned} x_{i+1} &\approx x_i + hv_i + \frac{h^2}{2}a_i \\ v_{i+1} &\approx v_i + \frac{h}{2}(a_{i+1} + a_i) \end{aligned}$$

III. METHODS

A. Discretization of the equations of motion

Equation 3 is the equation of motion that describes the system of planets and stars in motion. We want to solve this equation numerically for all the planets in our solar system. In order to do this we must discretize the equations. We can rewrite the equation as a set of first order differential equations using the velocity:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \mathbf{v}$$
$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -G\sum_{j\neq i} m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

The Verlet is not a self-starting algorithm. Mainly applicable when we are not interested in the velocity Velocity Verlet

[1] Engeland, Hjorth-Jensen, Viefers, Raklev og Flekkøy, 2020, *Kompendium i FYS2140 Kvantefysikk, Versjon 3*, s. 81
[2] Griffiths, D. J, Schroeter, D. F., 2018, *Introduction to Quantum Mechanics, Third edition*, s. 44
[3] Griffiths, D. J, Schroeter, D. F., 2018, *Introduction to Quantum Mechanics, Third edition*, s. 75
[4] Griffiths, D. J, Schroeter, D. F., 2018, *Introduction to Quantum Mechanics, Third edition*, s. 133