CMB Temperature From FIRAS Data

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I. TASK

Analyse the FIRAS data set. It is the average spectrum of the sky as measured by the FIRAS experiment, which is considered to be a near-perfect blackbody, given by the Planck blackbody law.

Estimate the temperature of the blackbody and the uncertainty of the temperature. One important thing to note is the non-standard frequency units, reported in inverse-centimetres.

We also ask that you perform the analysis using as few external packages as possible, for example avoiding the use of `scipy.minimize` or other "black box" routines.

II. DATA SET

The data set contains five sets of data points, each with 43 samples.

1: Frequency

The frequency is given in units of inverse-centimetres (cm^{-1}). If desired this can be converted to Hertz through multiplying with $100c$, where $c \approx 3 \cdot 10^8$ m/s is the speed of light.

2: Spectrum

Intensity, as obtained from Planck's blackbody law, given in units of MegaJanskys per steradian (MJy / sr).

4: Spectrum uncertainty

The 1-σ uncertainty in the measurement of the monopole spectrum, given in units of kiloJanskys per steradian (kJy / sr).

III. PLANCK'S LAW OF BLACKBODY RADIATION

It reads

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1}. \tag{1}$$

In SI units it is expressed as

$$\text{J m}^{-2}\ \text{sr}^{-1} = \text{W m}^{-2}\ \text{sr}^{-1}\ \text{Hz}^{-1}. \tag{2}$$

To convert into MJy / sr multiply with

- 10^{26} for the Janskys ($1\text{Jy} = 10^{-26}\text{W m}^{-2}\ \text{Hz}^{-1}$) and
- 10^{-6} for the Mega.

IV. TEMPERATURE ESTIMATE

To estimate the temperature of the blackbody radiation we would like to fit (1) to the observed spectrum by finding the optimal value of T. To quantify the deviance I have used

$$\chi^2(T) = \sum_i \frac{(\tilde{B}_i - B(\nu_i, T))^2}{\sigma_i^2}, \tag{3}$$

where $\tilde{B}_i$ is the observed spectrum at frequency ν_i with uncertainty σ_i.

To find the optimal value of T I have done a simple 'grid search' to find the minimum value of $\chi^2(T)$. The value obtained is $T_{\text{opt}} = 2.72502$ K with a corresponding χ^2-value of 45.09834.

To quantify the uncertainty of T I have used a simple heuristic for the 95 % confidence interval of finding the values of T which lie in the region $\chi^2(T) < \chi^2(T_{\text{opt}}) + 4$. The obtained confidence interval is $[2.72500\text{ K}, 2.72503\text{ K}]$, a quite short interval which seems to me to be too optimistic. I was unable to set aside the time to do a more rigorous analysis, in particular I would like to derive a more exact number than four in the condition on χ^2 in the confidence interval.