Erik Cooper
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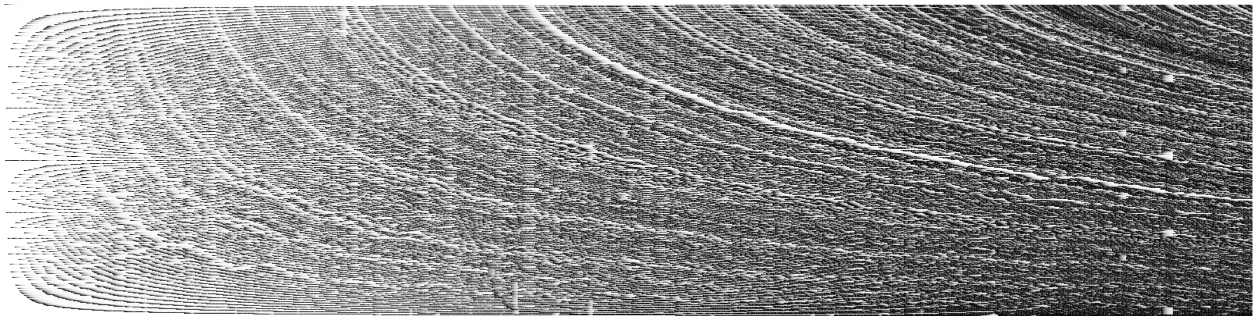
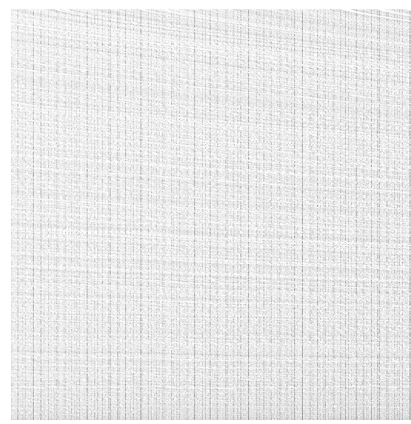
MAT360

A Little Visual of Goldbach's Conjecture



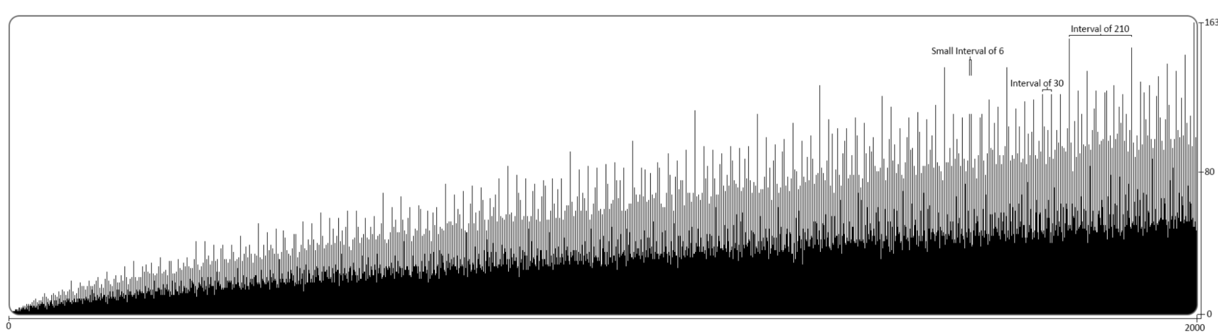
There are a few layers that make up Goldbach's conjecture, but for this demonstration I am only going to be focusing on the one portion of it that states: "Every even integer greater than 2 can be written as the sum of two primes." I chose this because I thought if I played around with organizing the data, we could see some interesting patterns appear. I didn't go into the other portions of the conjecture, however, because it would take some more involved algorithms that I didn't want to spend too much time trying to optimize in order to make it run in real-time.

So, for this, the algorithm is simple enough. I am loading in a table of prime numbers less than 10 million (664k prime numbers) and for each step, I am looping through and finding all the addends that make up an incrementing even number.

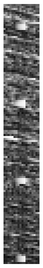


What exactly are we seeing in these images? Each point is showing an addend (value represented on the y-axis) for an even number (value represented on the x-axis). In the majority of my visualizations, I base the y-axis on the index of the prime rather than the value itself because doing the latter would just show empty space where numbers were divisible.

I've done a little bit of tweaking and normalizing to make these visualizations look pretty, but I started to notice some patterns. If you look closely, you can see straight lines at regular intervals. I did tweak the opacity of an axis based on the number of addends that fit the even numbers. When I noticed this, I added another mode to my code to graph out the number of addends. And this is what I got:

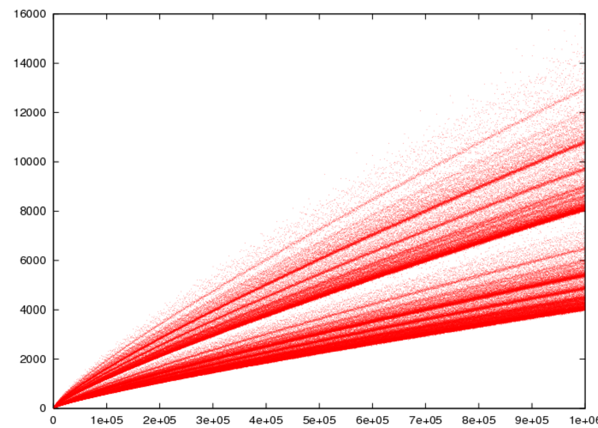


I counted the pixels and realized the spikes are at regular intervals of 210, 30, and 6. Thinking more into it, I realize that this is because they are multiples of the first few prime numbers. $2 * 3 * 5 * 7 = 210$, $2 * 3 * 5 = 30$, $2 * 3 = 6$. I didn't realize at first that when an even number $2n$ has a prime pair such that $p + q = 2n$, the more factors that the number has, the more possibilities for p and q to align modularly as primes. Though I'm still trying to wrap my head around the mechanics going on here.

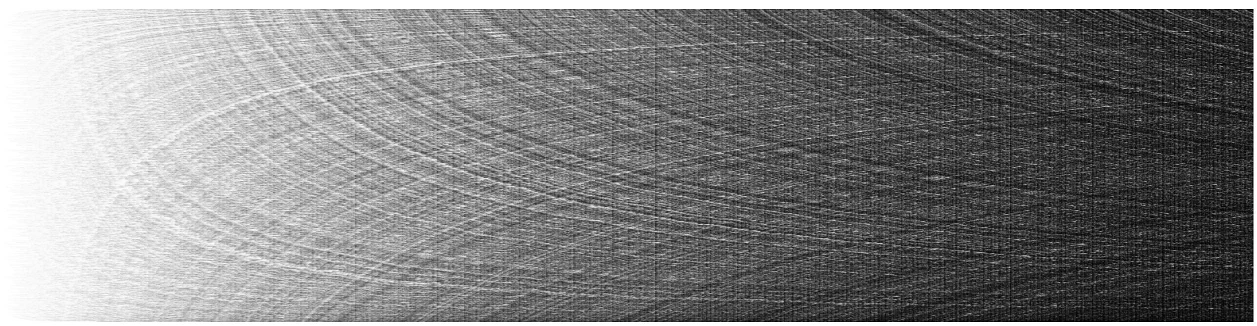


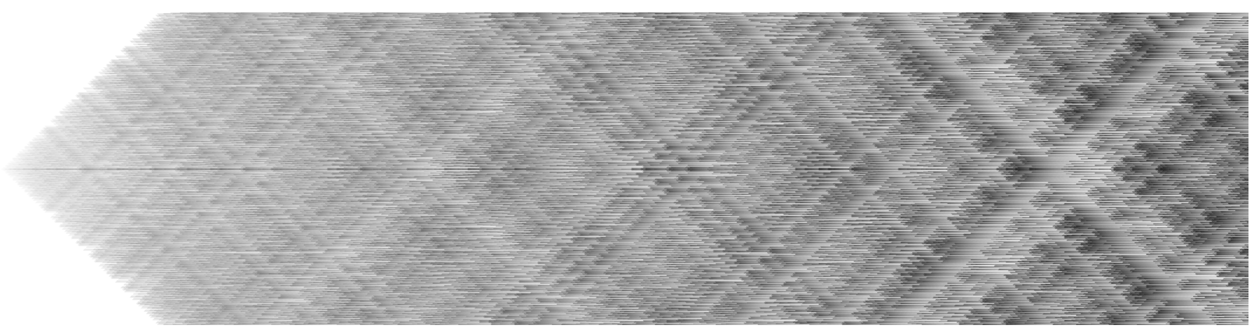
There are some further explorations I would like to get into, but I did run a little low on time for the project. A big curiosity for me is the strange blocky groupings that occur occasionally. It's most visible in the first image on this paper toward the right side, where there are 4 bright spots lined up vertically. In that visual, there is horizontal blurring, so those spots are where there are no addends. So this basically means that there are instances of concurring even numbers that all don't contain several groups of prime numbers.

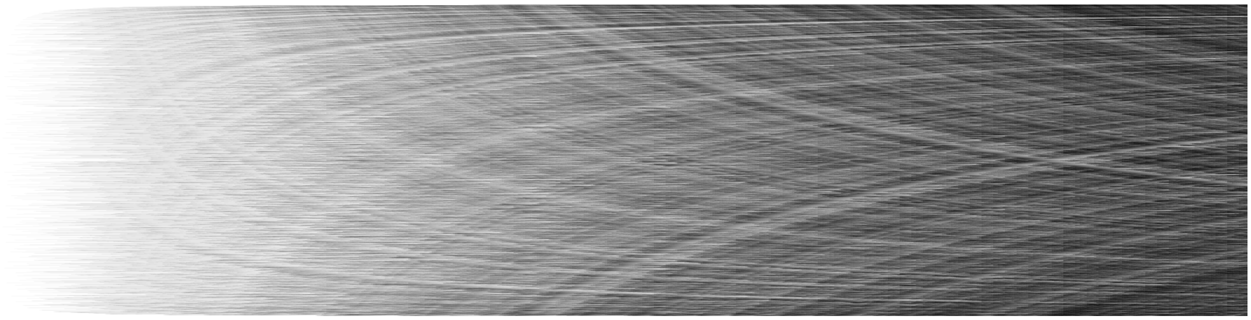
After completing this project I saw the graph on the wikipedia page for Goldbach's conjecture that is basically displaying the same information but with bigger numbers, so I felt I should include it here.

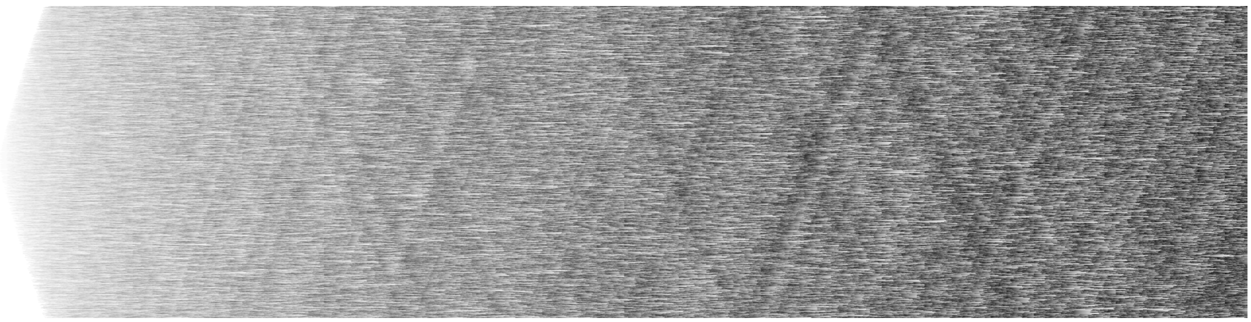


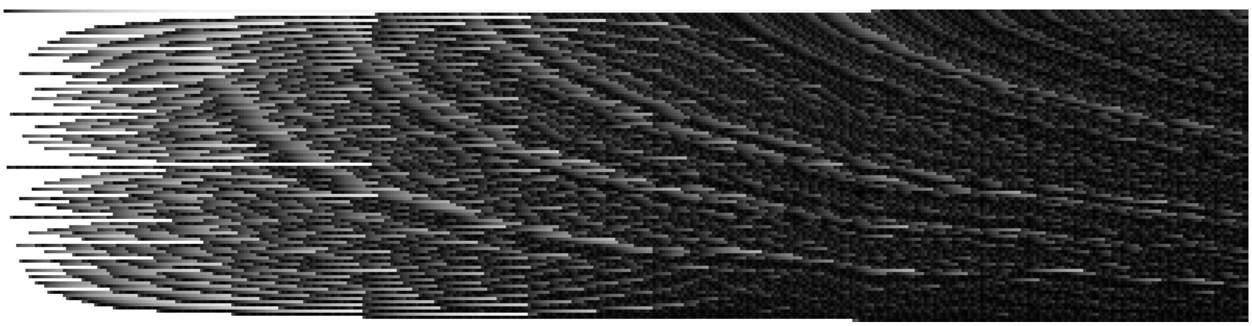
Anyways, the program is rather flexible with a bunch of parameters to play with, so here are some decent frames I managed to create.











This project was created using Java in Processing 4.3. To run code, install Processing 4.3 from the processing website, open `MAT360Project.pde`, adjust parameters to preference and press play.

Processing 4.3:

`https://processing.org/download`

Prime numbers dataset:

`https://www.kaggle.com/datasets/patricklford/prime-numbers?resource=download&select=prime_numbers_to_10000000.csv`

Goldbach's Conjecture Wikipedia

`https://en.wikipedia.org/wiki/Goldbach%27s_conjecture`

Goldbach's Conjecture Britannica

`https://www.britannica.com/science/Goldbach-conjecture`

Dirichlet's Theorem on Arithmetic Progressions Wikipedia

`https://en.wikipedia.org/wiki/Dirichlet%27s_theorem_on_arithmetic_progressions`

Geeks for Geeks Facts About Prime Numbers

`https://www.geeksforgeeks.org/amazing-facts-about-prime-numbers/`