Machine Learning I: Foundations
Exercise Sheet 0

Prof. Marius Kloft TA: Billy Joe Franks

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1)
- Please organize yourself in groups of up to 3 people.
- You must hand in LaTeX compiled pdfs or your .ipynb file or a zip of both.

2) a) The scalar projection, $\Pi_{\mathbf{w}}(\mathbf{x})$, of a vector $\mathbf{x}$ onto another vector $\mathbf{w}$ is defined as the coordinate of the orthonormal projection of $\mathbf{x}$ onto a line parallel to $\mathbf{w}$, or $span\{\mathbf{w}\}$. Assume $\mathbf{b} := \mathbf{x} - a_1 \frac{\mathbf{w}}{\|\mathbf{w}\|}$ and $\mathbf{b}^T\mathbf{w} = 0$ then $\Pi_{\mathbf{w}}(\mathbf{x}) := a_1$. Show that the scalar projection of vector $\mathbf{x}$ onto $\mathbf{w}$ is equal to:

$$\Pi_{\mathbf{w}}(\mathbf{x}) := \frac{1}{\|\mathbf{w}\|}\mathbf{w}^T\mathbf{x}.$$

b) Let $f(\mathbf{x}) := \mathbf{w}^T\mathbf{x} + b$, with $\mathbf{w}, \mathbf{x} \in \mathbb{R}^d$, $b \in \mathbb{R}$. Let H be a hyperplane defined as $H := \{\mathbf{x} \in \mathbb{R}^d | f(\mathbf{x}) = 0\}$ and let $\tilde{\mathbf{x}} \in H$. The signed distance of $\mathbf{x} \in \mathbb{R}^d$ to H is $d(\mathbf{x}, H) := \Pi_{\mathbf{w}}(\mathbf{x} - \tilde{\mathbf{x}})$. Show that the signed distance can be equivalently defined as:

$$d(\mathbf{x}, H) := \frac{\mathbf{w}^T\mathbf{x} + b}{\|\mathbf{w}\|}.$$

3) In this question we will have you do some exploration of the applications of machine learning. One of the goals of this lecture is for you to be able to identify learning problems on your own. To this end, describe three learning problems that have not been mentioned in the lecture. For each problem adress the following:

- What is the data? (What are the inputs; what are the labels?)
- What is the goal? Are humans good at solving this task? Why, or why not?

4) Make sure to set python notebook up on your computer, you can use the following link to do so `https://jupyter.readthedocs.io/en/latest/install.html`. Then solve programming task 0.