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Cálculo Integral: Tarea 1.

1.) Demuestra que vale $\lim_{n\to\infty} \frac{1}{n^3}\sum_{K=1}^{n} K^2 = \frac{1}{3}$ con la ayuda de las sumas de Reimann para la Integral... definida $\int_0^1 x^2 dx$

desarrollando...

$\int_0^1 x^2 dx = [0,1]$

$P_n = (x_1, x_2, \dots, x_n)$

$x_K = \frac{K}{n}$

siendo

$K = (1, 2, 3, \dots, n)$

$x_{K-1} \leq t_K \leq x_K \quad \forall K$

$t_K = \frac{x_{K-1} + x_K}{2} = \frac{K - 1/2}{n}$

$S_n = \sum_{K=1}^{n} f(t_K)\underbrace{(x_K - x_{K-1})}_{1/n} \Rightarrow S_n = \sum_{K=1}^{n} \left(\frac{(K-1/2)^2}{n}\right) \cdot \frac{1}{n}$

Si $f(x) = x^2$ encontraremos:

$F(t_K) = t_K^2 = \left(\frac{K - 1/2}{2}\right)^2$ tal que $S_n = \sum_{K=1}^{n} \frac{K^2 - K + 1/4}{n^2} \cdot \frac{1}{n}$

$\therefore \sum_{K=1}^{n} \frac{1}{n^3}(K^2 - K \cdot 1/4) = \frac{1}{n^3}\left\{\sum_{K=1}^{n} K^2 - \sum_{K=1}^{n} K + \sum_{K=1}^{n} 1/4\right\} \dots$

por propiedad...

aplicando el límite:

$\lim_{n\to\infty} \int_0 x^2 dx \Longleftrightarrow \lim_{n\to\infty} \frac{1}{n^3}\sum_{K=1}^{n} K^2 - \frac{1}{n^3}\sum_{K=1}^{n} K + \frac{1}{n^3}\left[\frac{1}{4}n\right]$ tal que $\Rightarrow$

por propiedad del lim.

$\lim_{n\to\infty} \frac{1}{n^3}\sum_{K=1}^{n} K^2 - \underbrace{\lim_{n\to\infty} \frac{1}{n^3}\sum_{K=1}^{n} K}_{0} + \underbrace{\lim_{n\to\infty} \frac{1}{4n^2}}_{0} = \int_0^1 x^2 dx$

Si $\lim_{n\to\infty} \frac{1}{4n^2} = 0$

y

Si $\lim_{n \to \infty} \frac{1}{n^3} = 0 \Rightarrow$

$$\lim_{n \to \infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 - \int x^2 dx = 0 \Rightarrow \int x^2 dx - \frac{k^3}{3} \Big|_0^1 = 0$$

$$\int_0^1 x^2 dx = \frac{1^3}{3} - \frac{0^3}{3} \Rightarrow \int_0^1 x^2 dx = \frac{1}{3}$$

entonces:

$$\lim_{n \to \infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 = \frac{1}{3}$$

Q.ed

2) Calcula las antiderivadas de las Integrales siguientes:

a) $\int [(x+2)^2 - (x-2)^2]\, dx$

$u = (x+2)$ | $w = (x-2)$
$du = dx$ | $dw = dx$

desarrollando...

$\int u^2\, du - \int w^2\, dw \longrightarrow \boxed{\int x^n = \frac{x^{n+1}}{n+1}}$ formula 1a)

$\therefore \frac{u^3}{3} - \frac{w^3}{3} = \frac{(x+2)^3}{3} - \frac{(x-2)^3}{3} = \frac{1}{3}\left[(x+2)^3 - (x-2)^3\right] + C$ //

b) $\int [(x-3)^2 (x+1)]\, dx$

desarrollando...

$\int (x^2 - 6x + 9)(x+1)\, dx = \int [x^3 + x^2 - 6x^2 - 6x + 9x + 9]\, dx$

$\therefore \int [x^3 - 5x^2 + 3x + 9]\, dx = \int x^3 dx - 5\int x^2 dx + 3\int x\, dx + 9\int dx$

$\therefore \frac{x^4}{4} - 5\frac{x^3}{3} + 3\frac{x^2}{2} + 9x + C$ //

c) $\int [(x+1)^3 (x-2)]\, dx$

desarrollando...

$\int [(x^3 + 3x^2 + 3x + 3)(x-2)]\, dx = \int [x^4 + 3x^3 + 3x^2 + 3x - 2x^3 - 6x^2 - 6x - 6]\, dx$

$$\therefore \int \left[x^4 + x^3 - 3x^2 - 3x - 6\right] dx$$

$$\therefore \int x^4 dx + \int x^3 dx - 3\int x^2 dx - 3\int x\, dx - 6\int dx$$

$$\therefore \frac{x^5}{5} + \frac{x^4}{4} - \frac{3x^3}{3} - 3\frac{x^2}{2} - 6x = \frac{x^5}{5} + \frac{x^4}{4} - x^3 - \frac{3x^2}{2} - 6x + C \;//$$

3) Calcula las Integrales Indefinidas usando la tabla de Integrales básicas del Manuscrito y demuestra que la antiderivada es correcta haciendo la derivación:

a) $$\int \left[-\operatorname{sen}(x) + \frac{1}{2}\cos(x) - \frac{\tan(x)}{\cos(x)}\right] dx = F(x)$$

desarrollando...

Utilizamos formulas: 3a), 3b), 5c) del manuscrito:

$$-\int \operatorname{sen}(x)dx + \frac{1}{2}\int \cos(x)dx - \int \frac{\tan(x)}{\cos(x)} dx$$

$$\therefore -\left[-\cos(x)\right] + \frac{1}{2}\left[\operatorname{sen}(x)\right] - \left[\frac{1}{\cos(x)}\right]$$

$$\therefore \cos(x) + \frac{\operatorname{sen}(x)}{2} - \sec(x) + C \;//$$

demostrando...

$$\frac{d\cos(x)}{dx} + \frac{d\operatorname{sen}(x)}{2dx} - \frac{d\sec(x)}{dx} + \frac{dC}{dx}$$

de la forma

$$\therefore -\operatorname{sen}(x) + \frac{1}{2}\cos(x) - \frac{d\,1}{d\cos(x)} + 0 \qquad \left| \frac{c}{F(x)} = \frac{-c}{F(x)^2} \cdot F'(x) \right|$$

$$\therefore F'(x) = -\operatorname{sen}(x) + \frac{1}{2}\cos(x) - \frac{1 \cdot \operatorname{sen}(x)}{[\cos(x)]^2} \; ; \; -\left[\frac{\operatorname{sen}(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}\right]$$

$$\therefore F'(x) = -\operatorname{sen}(x) + \frac{1}{2}\cos(x) - \frac{\tan(x)}{\cos(x)}$$ //

Q. ed

b) $\int \left[\frac{1}{\cos^2(x)} + \frac{\cot(x)}{\operatorname{sen}(x)}\right] dx = F(x)$

desarrollando...

$$\int \frac{1}{\cos^2(x)}\,dx + \int \frac{\cot(x)}{\operatorname{sen}(x)}\,dx$$

Utilizando formulas 5b), 5d) del manuscrito

$$\therefore \tan(x) - \frac{1}{\operatorname{sen}(x)} + C$$ //

de la forma

$$\left| \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \right|$$

demostrando...

$$\frac{d\tan(x)}{dx} - \frac{d\,1}{d\operatorname{sen}(x)} + \frac{dC}{dx} = \frac{d\operatorname{sen}(x)}{dx\cos(x)} - \frac{d\,1}{dx\operatorname{sen}(x)} + 0$$

$$\therefore \frac{\cos(x)\cos(x) - \operatorname{sen}(x) \cdot -\operatorname{sen}(x)}{[\cos(x)]^2} - \left[\frac{-1}{\operatorname{sen}^2(x)} \cdot \cos(x)\right]$$

$$\therefore \frac{\cos^2(x) + \operatorname{sen}^2(x)}{\cos^2(x)} + \cos(x) \cdot \frac{1}{\operatorname{sen}^2(x)} = \frac{1}{\cos^2(x)} + \frac{\cos(x)}{\operatorname{sen}(x)} \cdot \frac{1}{\operatorname{sen}(x)}$$

por Identidad
$|\cos^2(x) + \operatorname{sen}^2(x) = 1|$

$$\therefore F'(x) = \frac{1}{\cos^2(x)} + \frac{\cot(x)}{\operatorname{sen}(x)}$$ //

Q. ed