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Review

# Architectures for distributed and hierarchical Model Predictive Control – A review

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ABSTRACT

The aim of this paper is to review and to propose a classification of a number of decentralized, distributed and hierarchical control architectures for large scale systems. Attention is focused on the design approaches based on Model Predictive Control. For the considered architectures, the underlying rationale, the fields of application, the merits and limitations are discussed, the main references to the literature are reported and some future developments are suggested. Finally, a number of open problems is listed.

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**Contents**



## 1. Introduction

Technological and economical reasons motivate the development of process plants, manufacturing systems and traffic networks with an ever increasing complexity. These large scale systems are often composed by many interacting subsystems and can be difficult to control with a centralized control structure due to the required inherent computational complexity, due to robustness and reliability problems and due to communication bandwidth limitations. For all these reasons, many distributed control structures have been developed and applied over the last forty years. Among them, it is worth mentioning completely decentralized structures, distributed control systems with exchange of information among local regulators and hierarchical structures. Owing to the wide range of the problems considered and of the goals to be achieved, it is not always trivial to properly classify all the proposed solutions and to judge their merits and limitations.

The aim of this paper is to review the main approaches adopted, to propose a classification criterion and to provide a wide list of references focusing the attention on the methods based on Model Predictive Control (MPC). This choice is motivated by the ever increasing popularity of MPC in the process industry, see e.g. the survey papers [80,81] on the industrial applications of linear and nonlinear MPC. Moreover, in recent years many MPC algorithms have been developed to guarantee some fundamental properties, such as the stability of the resulting closed-loop system or its robustness with respect to a wide class of external disturbances

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and/or model uncertainties, see e.g. the survey paper [62]. Therefore, MPC is now recognized as a very powerful approach with well established theoretical foundations and proven capability to handle a large number of industrial control problems.

The rationale underlying MPC is to transform the control problem into an optimization one, so that at any sampling time instant, a sequence of future control values is computed by solving a finite horizon optimal control problem. Then, only the first element of the computed sequence is effectively used and the overall procedure is repeated at the next sampling time according to the so-called receding horizon (or moving horizon) principle. The reformulation of the control problem in terms of an optimization one provides the designer with many degrees of freedom in the choice of the cost function to be minimized, which can reflect different and even conflicting performance requirements. Moreover, it allows one to handle static and dynamic constraints on the plant variables. For an in depth presentation of MPC, the reader is referred to the textbooks [58,12] and the survey papers [30,62].

In the context of distributed and hierarchical control, the MPC approach is potentially very useful. In fact, it is easy for any local regulator designed with MPC to predict its future control actions and the corresponding state trajectories and to transmit them to neighbor local control units. This information is often fundamental to achieve performance comparable to those ideally provided by a centralized control structure.

The paper is organized as follows. In Section 2 completely decentralized control structures are considered and some fundamental references in the field are reported together with a description of the very few results available concerning decentralized MPC with guaranteed stability properties. Section 3 describes the main approaches proposed so far for the design of distributed MPC systems, where information is transmitted among local regulators to achieve global stability and performance results. Section 4 is devoted to introduce a hierarchical control structure where the action of local (decentralized) regulators is coordinated by an algorithm operating at a higher level. The main ideas underlying the design of this coordinator are summarized together with the approaches adopted in the MPC literature. Section 5 deals with hierarchical multilayer systems, i.e. control systems made by a number of control algorithms working at different time scales. Multilayer structures are useful either to control plants characterized by significantly different dynamics or to use different models of the same plant with the aim to optimize a number of criteria. Both these situations are described and the available results are summarized. Distributed MPC algorithms have also been proposed to coordinate totally independent systems in order to achieve a common target and to deal with joint constraints. These cases and the proposed solutions are reviewed in Section 6. Finally, Section 7 briefly describes some open issues and suggests further developments for the research in this field.

## 2. Decentralized control

Most large scale industrial systems are still controlled by decentralized architectures where the control (input $u$) and the controlled (output $y$) variables are grouped into disjoint sets. These sets are then coupled to produce non-overlapping pairs for which local regulators are designed to operate in a completely independent fashion. The local regulators can be single-input single-output or multivariable (locally centralized) depending on the cardinality of the selected input and output groups. An example of a decentralized control structure is reported in Fig. 1, where the system under control is assumed to be composed by two subsystems $S1$ and $S2$, with states, control and output variables $(x_1,u_1,y_1)$ and $(x_2,u_2,y_2)$, respectively, and the interaction between the subsystems is due to the mutual effect of the states $x_1$ and $x_2$. Once the decentralized regulator structure has been defined, the design of the local regulators ($R1$ and $R2$ in Fig. 1) is trivial when the interactions among the inputs and the outputs of different pairs are weak. These interactions can either be direct (input coupling) or caused by the mutual effects of the internal states of the subsystems under control, like in Fig. 1. On the contrary, it is well known that strong interactions can even prevent one from achieving stability and/or performance with decentralized control, see for example [109,17] where the role played by the so-called *fixed modes* in the stabilization problem is highlighted.

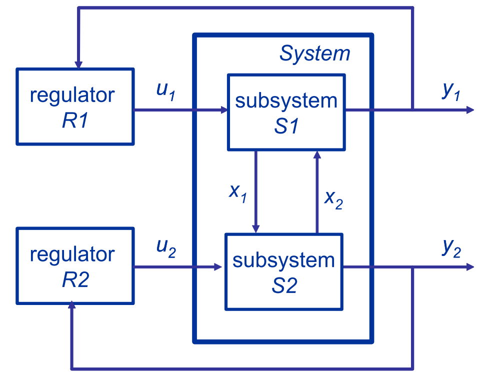


**Fig. 1.** Decentralized control of a two input $(u_1,u_2)$–two output $(y_1,y_2)$ system.

In view of the enormous relevance of decentralized control, starting from the sixties many efforts have been devoted to develop design methods guaranteeing stability and performance. Among them, we recall here those based on vector Lyapunov functions (see e.g. [98]), on sequential design (see e.g. [36]), on optimization (see e.g. [18,95]), and on overlapping decompositions (see e.g. [38,39,37]). Classical textbooks dealing with decentralized control are [98,56], while a milestone paper in the field is [91]. More recently, decentralized control has been considered in the survey papers [99,100,8], which also report an up-to-date list of references.

In principle, the local MPC laws (for example regulators $R1$ for $S1$ and $R2$ for $S2$ in Fig. 1) can be computed with standard MPC algorithms by neglecting the mutual interactions, see e.g. [1] and the application reported in [23]. However, in spite of the great importance of the subject, very few decentralized MPC algorithms with guaranteed properties have been developed so far. This can be motivated by a couple of reasons. First, the intrinsic multivariable nature of MPC allows one to easily commission centralized regulators with many input and output variables, so that often decentralization is not a main issue. Nevertheless, for large scale systems made by many weakly interacting process units, it can be convenient to decompose the overall optimization problem associated to the design of a unique centralized controller into a number of smaller problems, i.e. to resort to a decentralized control structure. A second motivation for the lack of stabilizing decentralized algorithms is because the feedback MPC law is implicit and the control variables are the solution of an optimization procedure rather than computed by an explicit control law. For this reason, the analysis of the closed-loop system with MPC is difficult and the main stability results are obtained by using the optimal cost as a Lyapunov function, see [62]. However, it is not easy to extend this analysis technique to decentralized control structures.

A decentralized state-feedback MPC algorithm for nonlinear discrete-time systems subject to decaying disturbances has been proposed in [60], where the closed-loop stability is obtained with the inclusion of a contraction constraint in the optimization problem to be solved at any time instant. This constraint forces the state trajectories of the subsystems controlled by local MPC regulators to move towards the origin despite the perturbing effect of the mutual interactions and of the disturbances.

By resorting to the recently developed theory of robust MPC, see e.g. [61], in [82] stabilizing decentralized state-feedback regulators for nonlinear discrete-time systems with uncertainty have been derived. In this approach, the plant interactions are treated as disturbances to be rejected. The stabilizing properties of the proposed algorithms are established by resorting to the powerful notion of Input to State Stability (ISS), see e.g. [50].

## 3. Distributed control

In distributed control structures, like the simple example shown in Fig. 2, it is assumed that some information is transmitted among the local regulators ($R1$ and $R2$ in Fig. 2), so that each one of them has some knowledge on the behavior of the others. When the local regulators are designed with MPC, the information transmitted typically consists of the future predicted control or state variables computed locally, so that any local regulator can predict the interaction effects over the considered prediction horizon. With reference to the simple case of Fig. 2, the MPC regulators $R1$ and $R2$ are designed to control the subsystems $S1$ and $S2$, respectively. If the information exchange among the local regulators ($R1$ and $R2$) concerns the predicted evolution of the system states ($x_1$ and $x_2$), any local regulator needs only to know the dynamics of the subsystem directly controlled ($S1$ and $S2$). On the contrary, if the predicted future control actions ($u_1$ and $u_2$) are transmitted, the local regulators must know the model of all the subsystems. In any case, it is apparent that the transmission and the synchronization protocols have a major impact on the achievable performance.

Within the wide set of distributed MPC algorithms proposed in the literature, a classification can be made depending on the topology of the communication network. Specifically, the following cases can be considered:

- information is transmitted (and received) from any local regulator to all the others (*fully connected* algorithms);
- information is transmitted (and received) from any local regulator to a given subset of the others (*partially connected* algorithms).

A partially connected information structure can be convenient in the case of large scale systems made by a great number of loosely connected subsystems. In these cases, restricting the information exchange among directly interacting subsystems produces a negligible performance deterioration. An interesting discussion on this point is reported in [88], where reference is made to chemical processes composed by subsystems directly interacting only with their neighbors, possibly with additional recirculating flows.

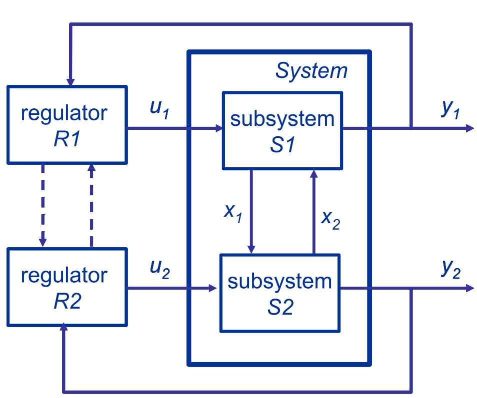


**Fig. 2.** Distributed control of a two input ($u_1$,$u_2$)–two output ($y_1$,$y_2$) system.

The exchange of information among local regulators can be made according to different protocols:

- information is transmitted (and received) by the local regulators only once within each sampling time (*noniterative* algorithms);
- information can be transmitted (and received) by the local regulators many times within the sampling time (*iterative* algorithms).

It is apparent that the amount of information available to the local regulators with iterative algorithms is higher, so that an overall iterative procedure can be set-up to reach a global consensus on the actions to be taken within the sampling interval. To this regard however, a further classification has to be considered:

- distributed algorithms where each local regulator minimizes a local performance index (*independent* algorithms);
- distributed algorithms where each local regulator minimizes a global cost function (*cooperating* algorithms).

As discussed in [107] (see also [108]) by means of game theory considerations (see [9]), it is apparent that in iterative and independent algorithms each local regulator tends to move towards a Nash equilibrium, while iterative and cooperating methods seek to achieve the Pareto optimal solution provided by an ideal centralized control structure. However, Nash equilibria can even be unstable and far from the Pareto optimal solution, so that specific constraints have to be included in the MPC problem formulation to guarantee closed-loop stability.

As for the MPC algorithms published in the literature, the state feedback method described in [13] (see also [42]), for discrete-time linear systems belongs to the set of independent, noniterative algorithms. A stability constraint is included in the problem formulation, although stability can be verified only a-posteriori with an analysis of the resulting closed-loop dynamics. Nash equilibrium solutions are searched in the independent, iterative and fully connected methods developed in [20,54] for discrete-time unconstrained linear systems represented by input–output models. Linear discrete-time systems are also considered in [107], where an iterative, cooperating method with many interesting properties is presented. In particular, the proposed approach guarantees the attainment of the global (Pareto) optimum when the iterative procedure converges, but still ensures closed-loop stability and feasibility if the procedure is stopped at any intermediate iterate.

A partially connected, noniterative and independent MPC algorithm for discrete-time nonlinear systems has been presented in [43]. The approach consists of describing the effect of the interconnections among the subsystems as disturbances acting on the local models. The values of these disturbances can be predicted from the predicted state trajectories broadcasted by the local regulators. Then, a min–max approach aimed at minimizing local cost functions under the worst-case disturbance allows one to compute parametrized distributed control laws. A feasibility property is proven together with convergence to a set. It is believed that the method could be further developed according to many recent results on closed-loop robust MPC, see e.g. [59,61].

An independent, noniterative and partially connected MPC algorithm guaranteeing stability for nonlinear continuous-time systems has been presented in [21], where information is transmitted only among neighboring subsystems. The stabilizing property of the method proposed in [21] heavily relies on the assumption that the mutual dynamic interactions among the subsystems are limited and on a consistency constraint included in the MPC problem formulation forcing the actual input and state sequences to not differ too much from their predicted values. The feasibility and stability proofs are based on the techniques described in [66] and

**Table 1**
Classification of distributed MPC algorithms.

| Distributed MPC | Independent | Cooperating |
|---|---|---|
| Noniterative | [2,3,13,21,42,43] | |
| Iterative | [20,54,64] | [107] |

share many ideas with the robust open-loop MPC algorithms developed in [15,55,61].

Relying on the methods for distributed state estimation and control presented in [70,69], distributed algorithms have been described in [106,105], while an extension of these techniques based on MPC has been described in [64].

Finally, a partially connected, noniterative and independent MPC algorithm for linear discrete-time systems has been described in [2,3], where conditions for the a-posteriori stability analysis are given also in the case of communication failures among the local control units.

A classification in terms of the main attributes of the principal distributed algorithms proposed in the literature is summarized in Table 1. Note that the cell corresponding to noniterative, cooperating methods is empty, since cooperation requires any form of negotiation among the local control laws.

## 4. Hierarchical control for coordination

An alternative to the distributed control schemes described in the previous section consists of considering a two level hierarchical control structure, like the one shown in Fig. 3 for the simple example already considered in the previous sections. In this two level structure, an algorithm at the higher level coordinates the actions of local regulators placed at a lower level and possibly designed with MPC.

The design of the coordinator has been extensively studied over the last forty years, see e.g. the old but still fundamental books [65,28]. The basic idea is to describe the overall system under control as composed by a number of subsystems linked through some interconnecting variables, i.e. the inputs of a given subsystem are the outputs or the states of another one. Then, for any subsystem an optimization problem is solved with MPC by minimizing a suitable local cost function under local state, input and output constraints. If the computed local solutions satisfy the constraints imposed by the interconnecting variables, that is if there is coherence among the values of the interconnecting variables computed by the local regulators, the procedure is concluded. Otherwise, an iterative "price coordination" method is used: the coordinator sets the prices, which coincide with the Lagrange multipliers of the coherence constraints in the global optimization problem, by assuming as given the state, input and output variables defined by the local regulators. In turn, these optimal prices are sent to the low level local optimizers which take them as given and recompute the optimal trajectories of the state, input and output variables over the considered prediction horizon. The iterations are stopped when the interconnecting variables satisfy the required coherence conditions. This conceptual iterative procedure must be specialized to guarantee its convergence as well as some properties of the resulting final solution.

In the context of MPC, coordination schemes for discrete-time systems have been described in [71,74], where also different communication protocols among the local regulators (agents) are considered. The proposed algorithms have been used for control of transportation networks, see [73], and power networks, see [72]. Another two layer structure developed with similar arguments has been presented in [47], which also describes an analogous two-level structure for state estimation.

Finally, it must be noted that similar two-level structures are widely used in the intensive stream of research in computer science/artificial intelligence related to the so-called "autonomous agents". Basically, a number of agents must negotiate their actions through a "negotiator" until a consensus on their actions is attained, see e.g. [4]. The ideas behind this approach have been specialized to the control design problem in [102].

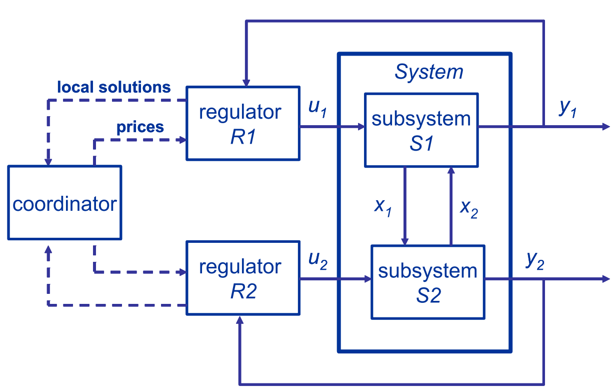


**Fig. 3.** Hierarchical control for coordination of a two input $(u_1,u_2)$–two output $(y_1,y_2)$ system.

## 5. Hierarchical control of multilayer systems

In hierarchical multilayer systems, the control action is performed by a number of regulators working at different time scales. This can be useful at least in two cases: when the overall process under control is characterized by different dynamic behavior, i.e. by slow and fast dynamics, or in plantwide optimization when optimization and control algorithms working at a different rates compute both the optimal targets and the effective control actions to be applied.

### 5.1. Hierarchical control of multi time scale systems

Many systems are characterized by clearly separable slow and fast dynamics, see e.g. [11,35] for a couple of significant industrial examples concerning a waste water treatment plant [11] and a greenhouse control problem [35]. In these cases, the control can be performed at two different time scales. A regulator acting at lower frequencies computes both the control actions $(u_{slow})$ of the manipulated variables which have a long-term effect on the plant, i.e. the "slow" control variables, and the references values of the "fast" control variables, states and outputs $(u^{ref}_{fast}, x^{ref}_{fast}, y^{ref}_{fast})$. A second regulator takes these reference values as inputs and computes the "fast" control variables $u_{\mathrm{fast}}$ solving a tracking problem at a higher rate. A conceptual scheme of this architecture for a two layer structure is reported in Fig. 4.

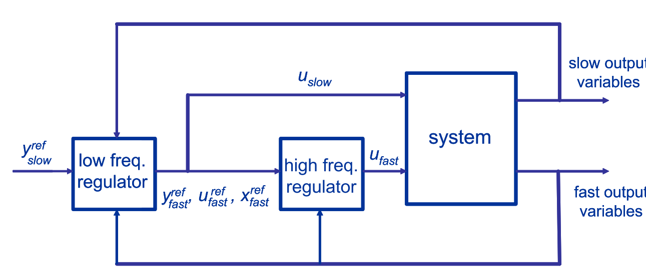


**Fig. 4.** Control of a system with slow and fast dynamics.

Two time scale systems are often referred as singularly perturbed systems, and have been widely studied in the past, see e.g. [52]. However, in the context of MPC, systematic design methods guaranteeing well assessed properties are still lacking and only ad-hoc solutions tailored on some specific industrial problems have been described, see again [11,35]. In the development of new algorithms for these systems, one could take advantage from the multirate MPC methods developed in [92,53,96]. However, in these papers, the multirate nature of the problem usually stems from the adopted output sampling or input updating mechanisms.

### 5.2. Control of systems with hierarchical structure

Many industrial, economical or sociological systems can be described by a hierarchical structure, see e.g. the visionary book [65]. The highest layer of the hierarchy corresponds to a dynamical system with slow dynamics. This system can be controlled by looking at its behavior over a long time scale, and its computed control inputs must be effectively provided by subsystems placed at lower layers of the hierarchy and characterized by faster dynamics. In turn, these subsystems must be controlled at a higher rate and can be placed at an intermediate layer of hierarchy. An example of a three layer structure is reported in Fig. 5.

As a matter of fact, in these structures the regulator at a higher layer computes its desired control inputs, which are the reference signals of the immediately lower layer. As an example, consider an hybrid vehicle with two torque generators, an internal combustion engine and an electric motor. At the higher level, a regulator must compute the torque which must be required to the engines in order to satisfy the driver's load request and to optimize the energy management (production and storage) of the system. At the engines's level, the torque requested must be provided in the prescribed time and under operational constraints.

In these hierarchical structures, in order to guarantee that references computed at the higher layer are feasible for the lower layer dynamics and constraints, as well as to consider the presence of disturbances acting at the lower layer, some additional information has often to be transmitted bottom-up. Moreover, the regulators of the subsystems at the lower layer must guarantee the solution of the corresponding tracking problems with an adequate level of accuracy, so that the mismatch between what is required by the higher level and what is provided by the lower one does not destroy some fundamental properties, such as stability and performance.

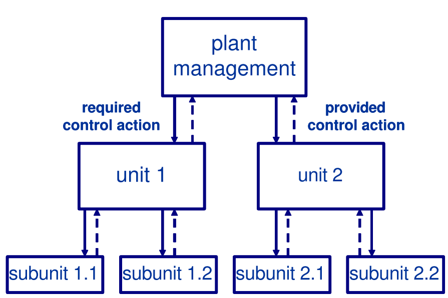


**Fig. 5.** Hierarchical structure of a three layer system.

From a control engineering point of view, this multilayer hierarchical structure corresponds to a classical cascade feedback control system, see e.g. Fig. 6 where again a three layer structure is considered and the inner loops correspond to faster dynamics, while the outer loop corresponds to the control of the system at the highest layer. In industrial control systems, the fastest dynamics is usually associated to the actuators, while the slowest one describes the process under control. The project of cascade control systems is typically made according to a frequency decoupling principle: the dynamics of the feedback loops are so different that in the project of the regulator for a given loop all the other loops can be assumed to be at the steady state. Moreover, no information is transmitted from inner to outer loops (dotted lines in Fig. 6), so that any layer is unaware of the possibility of the lower layers to fulfill its requirements. In the design phase, the inner loops are often closed with standard PI-PID regulators, while MPC is used to design the control algorithm for the slowest system.

When the frequency decoupling principle cannot be assumed, or when also the control of the subsystems at the lower layers of the hierarchy requires a more careful design, MPC can be used at any layer, with the clear advantage to consider the corresponding input, state and output constraints. Although this possibility has many potential advantages, few works have exploited it in depth. In particular, in [93] linear models are used to describe the systems at any layer and information is passed bottom-up to relax the requirements of the higher layer when infeasibility occurs at the lower layer. Overactuated linear systems are analyzed in [94], while [79] deals with the tracking problem for plants at the higher layer described by Wiener models. In all these papers, the regulators at any layer are independently designed by resorting to robust MPC algorithms, so that the design phase turns out to be completely decoupled even when the frequency decoupling principle does not hold.

### 5.3. Hierarchical control for plantwide optimization

In the process industry it is common to design the overall control system according to the hierarchical structure shown in Fig. 7, see e.g. [6,81,97]. At the higher layer, Real Time Optimization (RTO) is performed to compute the optimal operating conditions with respect to a performance index representing an economic criterion. At this stage a detailed, although static, physical nonlinear model of the system is used. At the lower layer a simpler linear dynamic model of the same system, often derived by means of identification experiments, is used to design a regulator with MPC, guaranteeing that the target values transmitted from the higher layer are attained. Also in this case, the lower level can transmit bottom-up information on constraints and performance.

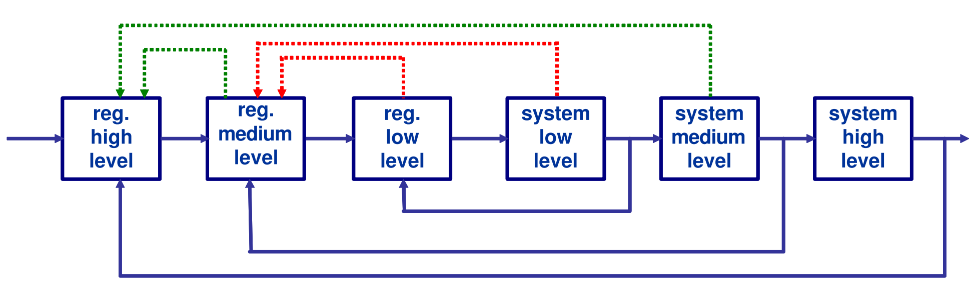


**Fig. 6.** Three layer cascade control structure.

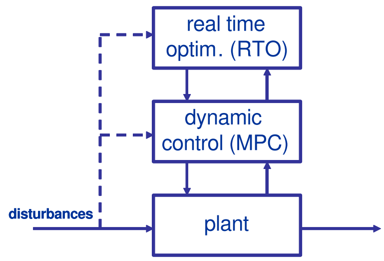


**Fig. 7.** Hierarchical structure for plantwide control and optimization.

Note that, although the previous approach is very popular in process control, in other contexts, such as in air traffic management systems, a somehow dual point of view is taken, see e.g. [78]. In these cases, at the higher level a simpler and more abstract model is considered to predict the long term behavior of the system and to optimize a given cost function over a long time horizon. At the lower level, a more accurate model is used to compute the current control actions by looking at a shorter time horizon.

In process control based on the multilayer structure of Fig. 7, the design of the RTO module plays a fundamental role. In fact, even when it is based on a static model of the process, some main issues must be considered. First, the adopted model has to be periodically updated (adapted) by means of some kind of reconciliation procedure to deal with changing operating conditions due to slow disturbances. Second, coherence must be guaranteed between the model used in the design phase at the upper layer and the model used at the lower layer for the MPC implementation, see e.g. [111]. Third, accurate steady-state target optimization must be done to guarantee that the input and output steady state references computed by RTO are feasible and as close as possible to the desired set-points, see e.g. [85] and the results recently reported in [87,86], to solve this feasibility problem.

Many papers have been published in the MPC literature dealing with the hierarchical structure of Fig. 7. Among them, a recent and interesting survey on the subject is reported in [103], where also a wide list of references is provided. It is also worth recalling [24], where a thorough discussion on the merits, limitation and implementation aspect of RTO is reported. A RTO procedure based on a dynamic model of the process is described in [45]. An attempt to mix the two layers of the hierarchy of Fig. 7, i.e. to integrate (nonlinear) steady-state optimization and (linear) MPC control is described in [112]. Steady-state target calculation for a set of local MPC regulators has been considered in [14] by adopting an approach based on coordination and similar to the one discussed in Section 4, while a high level coordinator maximizing the plant throughput based on the information provided by lower level local MPC regulators has been described in [5].

Despite the large amount of results on RTO, it is believed that much work has still to be done to extend many theoretical results (stability, performance, robustness) nowadays available for standard MPC implementations to the considered hierarchical structure, see e.g. [62,61].

Finally, a couple of remarks are in order. First, it can be noted that the conceptual scheme of Fig. 7 can be given the equivalent and more "control oriented" representation of Fig. 8, where a two layer structure is considered and each layer uses a different plant model ($S_a$ and $S_b$) in the design of the corresponding regulator. Second, it is worth pointing out that the conceptual classification adopted here, which distinguishes between the schemes of Figs. 5 and 7 is not always clear in the technical literature. In fact, a very popular picture is the one depicted in Fig. 9 where the regulators (PI-PID) at the lowest layer control the actuators, so that they make reference to the actuators' models (as in Section 5.1), while conceptually, the two higher levels make reference to the plantwide optimization problem described in this subsection.

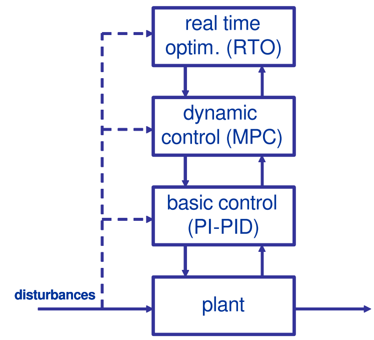


**Fig. 9.** A popular representation of plantwide control.

## 6. Coordinated control of independent systems

So far, large scale systems made by interacting subsystems have been considered. Another significant scenario is related to the problem of coordinating a number of decoupled systems (agents) which must cooperate to achieve a given goal, i.e. to globally minimize a cost function subject to joint constraints. Also in these cases, instead of solving a unique centralized control problem, in principle it is possible to solve a number of local optimization problems and to coordinate the local actions of the agents by means of a suitable exchange of information. It is apparent that the design of local, but coordinated control algorithms shares many features with the problem considered in Section 3, so that the more detailed classifications proposed there will not be replicated here.

In the context of MPC, the coordination of independent nonlinear discrete-time systems with delayed intercommunication has been considered in [29], which also provides stability results obtained by resorting to Input to State Stability (ISS) concepts. In [48,49], any agent (a node of a graph) is described as a discrete-time nonlinear system which knows the state of its neighborhoods without delay. The local performance indices weight the state and the inputs of the neighborhoods and the local future control values are computed over the considered horizon to predict the transient of the local state. Stability is achieved with a zero terminal constraint, as usual in MPC, see [62]. The problem considered in [89] consists of controlling a number of disturbed subsystems described by discrete-time linear models with independent dynamics but with coupling constraints. The proposed solution is based on a noniterative procedure where any regulator solves its

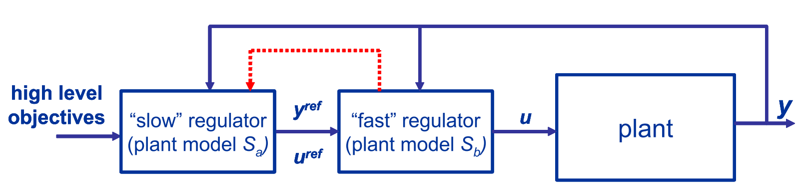


**Fig. 8.** Hierarchical control structure for plantwide control and optimization.

own optimization problem (with local performance indices) but knows the most recent or the predicted plans for the other subsystems. Constraint satisfaction is guaranteed by a constraint tightening procedure similar to the one already used in [15] and in [55]; feasibility and convergence are guaranteed. An extension of this approach based on recent results on robust MPC design (see [63]) is reported in [104]. The formation control of vehicles with independent second order nonlinear continuous-time dynamics and coupling cost functions has been solved with the Receding Horizon approach also in [22], while in [32] a similar problem (independent cost functions) is solved for double integrators by looking to a Nash equilibrium solution.

While in all the above mentioned approaches the final goal is specified a-priori, typically the equilibrium point to be achieved is known a-priori, in another class of coordinated control problems the subsystems must negotiate on-line their final outcome starting from a partial knowledge of the overall system, for example the state of their neighbors. These are usually called "consensus problems" and have a wide number of potential applications, such as flocking, rendez-vous, formation control and alignment problems or coordination of sensor networks. Among the ever increasing number of publications on this subject, it is possible to recall here that the problem has been formulated and solved in [40] for systems described by a single integrator, while fundamental contributions have been given in [67]. In the context of MPC, preliminary results have been reported in [44], while the consensus problem for single and double integrator dynamics has been solved with stabilizing MPC approaches in [25,27,26].

## 7. Related design problems, open issues and conclusions

This paper has reviewed a number of architectures for control of large scale systems with MPC. Concerning a widespread industrial application of the proposed distributed and hierarchical solutions, many fundamental problems have still to be solved. Among them, the following are considered of paramount importance.

- *New algorithms with guaranteed properties*. Many theoretical contributions are required to develop efficient algorithms with guaranteed properties, such as stability and performance. In particular, this is true for decentralized MPC, where very few results are available, and for the design of hierarchical MPC regulators for multilayer systems. It is believed that in all the considered cases, one could take advantage of recent results on robust MPC, see e.g. [61], and on the analysis of interconnected systems with a "small gain" approach, see [16]. However, robust MPC and small gain properties naturally lead to very conservative results, with performance not acceptable in real world applications.
- *Selection of the control structure*. Criteria must be developed for the selection of the proper control structure based on the relative improvements achievable by increasing the complexity. For example, it is apparent that a distributed controller can stabilize systems which cannot be stabilized by a decentralized one because of the presence of fixed modes. However, if stability can be provided by both the schemes, it is still to be evaluated in term of performance whether it is worth considering a more complex structure, which requires more information to be transmitted among local control units. A second issue concerns the comparison of the performance provided by distributed regulators (see Section 2) with respect to those achievable with the hierarchical approach for coordination described in Section 4.
- *Reconfigurable control structures and hybrid systems.* With reference to the hierarchical structures described in Section 5.2, one should explore the possibility to reconfigure the system, for example by adding or removing actuators and sensors ("plug and play control", see [51]). This could be useful to consider time varying performance requirements and to control systems described by a hybrid model. For example, consider the problem of the optimal management of the start-up of a thermal power plant. During this phase, the control configuration and the control objectives are usually very different from those to be considered during standard operating conditions. Preliminary work on hierarchical control of hybrid systems has been described in [57], where however non-predictive approaches have been taken. Finally, a flexible control configuration can better cope with the requirement of a high tolerance to faults.
- *Optimization algorithms*. Many optimization algorithms have been developed to solve efficiently the minimization problems related to linear and nonlinear centralized MPC, see e.g. [10,19]. On the contrary, optimization methods for distributed and hierarchical MPC are still lacking. This is an important and critical point where significant improvements are expected.
- *Distributed state estimation*. Distributed control algorithms call for the availability of distributed state estimators guaranteeing the asymptotic convergence of the local state estimates. Preliminary results have been reported in e.g. [77,76,46], where sensor networks have been considered. However, further developments are required to include in the state estimation problem the knowledge on state and noise constraints, as well as to link the convergence properties of the local estimates to local or global observability properties. A predictive approach can be taken also for this problem by resorting to the ideas underlying the moving horizon estimators described in [66,83,84].
- *System partitioning*. In the design of decentralized and distributed control systems (Sections 2 and 3), eventually coordinated as described in Section 4, the process under control must be partitioned a-priori into subsystems properly defined to reduce the dynamic couplings and to facilitate the control design. In some cases partitioning is natural in view of the process layout, see for example [88] where chemical plants are considered. In other cases, the partitioning can be made by means of an input–output analysis based on the Relative Gain Array and related indices, see [101,31,75,33,33,34], or on a state-space analysis based on gramians, see e.g. [110,90]. Temporal decomposition and model reduction, useful for the design of the hierarchical control systems described in Section 5, can be performed with singular value decomposition, see again [101]. In spite of the many methods available, it is believed that new methods tailored for the design of distributed and hierarchical control systems with MPC have still to be developed. To this regard, some preliminary results have been reported in [41,68].
- *Synchronization and communication protocols.* Whenever the adopted control structure requires an exchange of information among local regulators, at the same or at different layers of a hierarchical structure, the achievable performance strongly depends on the adopted implementation (see e.g. the discussion in [74]) and communication protocols. Moreover, some fundamental problems related to these aspects must be considered, such as low transmission frequency or loss of information. The interested reader is referred to [7] for an insightful discussion on these aspects.

The story of centralized MPC has shown that efficient and reliable algorithms have been developed well before than a solid theory was established, see the fundamental paper [62]. In the author's opinion, a similar development can be predicted for distributed and hierarchical MPC. In fact, many efficient algorithms are nowadays available, but strong theoretical results and a unifying picture are still partially lacking. Then, much work has to be done to develop new tools and methods.

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