6.图

（c）广度优先搜索

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算法

❖ 始自顶点s的广度优先搜索 Breadth-First_Search

访问顶点s

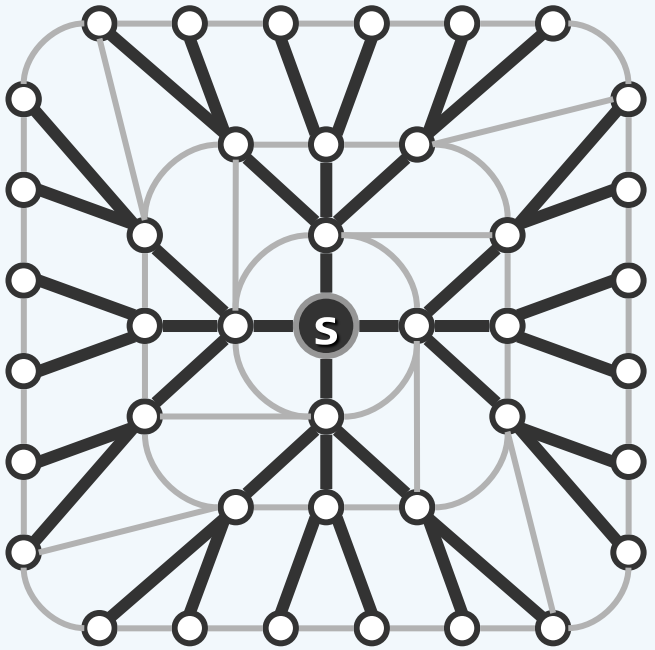
依次访问s所有 尚未访问 的邻接顶点

依次访问它们 尚未访问 的邻接顶点

...

如此反复

直至没有 尚未访问 的邻接顶点



❖ 以上策略及过程完全等同于 树 的 广度优先遍历

❖ 事实上，BFS也的确会构造出原图的一棵 支撑树 （BFS tree）

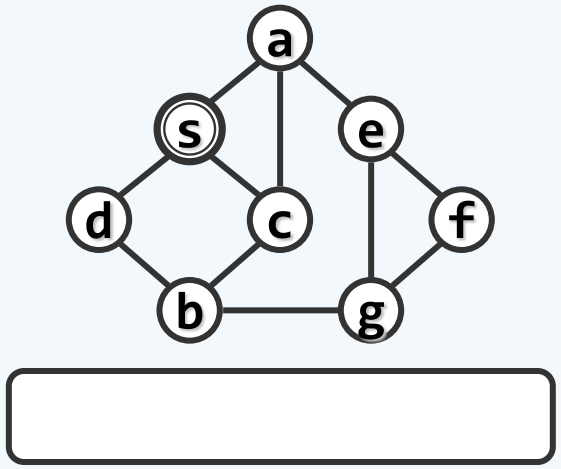
Graph::BFS()

```
template <typename Tv, typename Te> //顶点类型、边类型
void Graph<Tv, Te>::BFS( int v, int & clock ) {
   Queue<int> Q; status(v) = DISCOVERED; Q.enqueue(v); //初始化
   while ( !Q.empty() ) { //反复地
      int v = Q.dequeue();
      dTime(v) = ++clock; //取出队首顶点v，并
      for ( int u = firstNbr(v); -1 < u; u = nextNbr(v, u) ) //考察v的每一邻居u
         /* ... 视u的状态，分别处理 ... */
      status(v) = VISITED; //至此，当前顶点访问完毕
   }
}
```

Graph::BFS()

```
❖   while ( !Q.empty() ) { //反复地
Ⓥ     int v = Q.dequeue(); dTime(v) = ++clock; //取出队首顶点v，并
      for ( int u = firstNbr(v); -1 < u; u = nextNbr(v, u) ) //考察v的每一邻居u
Ⓤ       if ( UNDISCOVERED == status(u) ) { //若u尚未被发现，则
Ⓣ         status(u) = DISCOVERED; Q.enqueue(u); //发现该顶点
          status(v, u) = TREE; parent(u) = v; //引入树边
Ⓤ Ⓤ     } else //若u已被发现（正在队列中），或者甚至已访问完毕（已出队列），则
          status(v, u) = CROSS; //将(v, u)归类于跨边
Ⓥ     status(v) = VISITED; //至此，当前顶点访问完毕
    }
```

实例（无向图）


a
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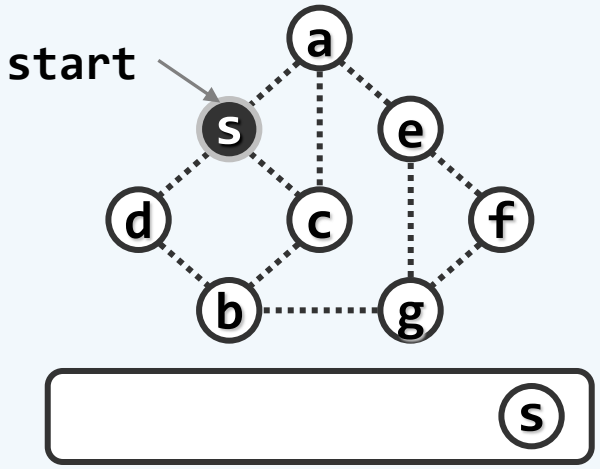
start
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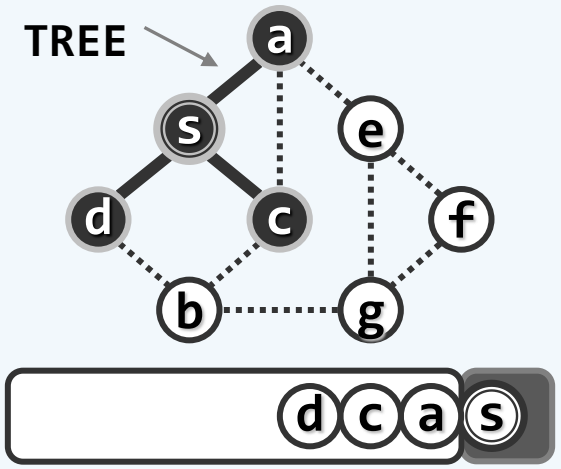
TREE
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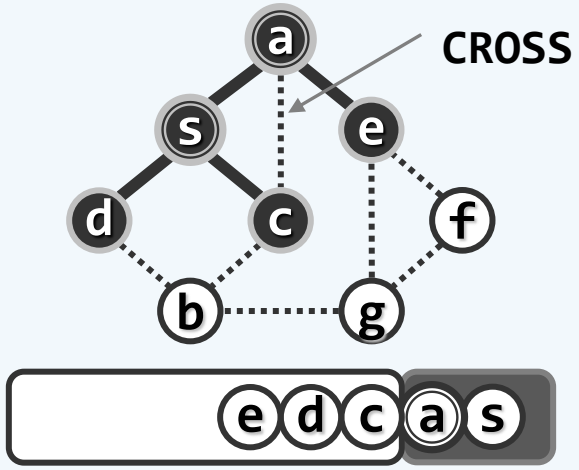
CROSS
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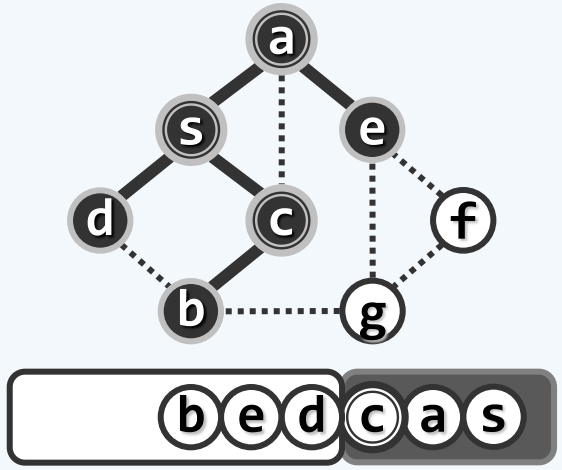
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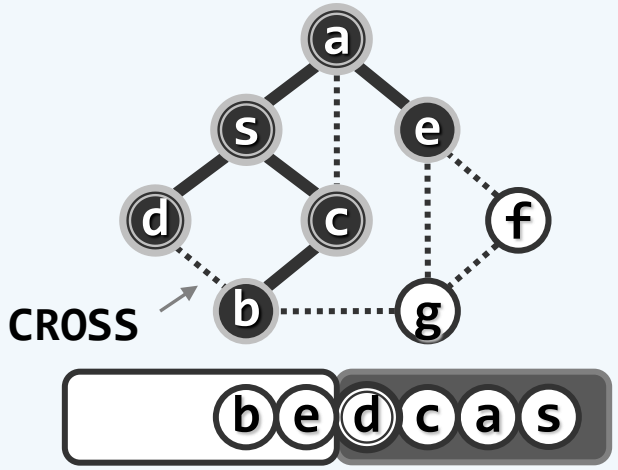
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CROSS
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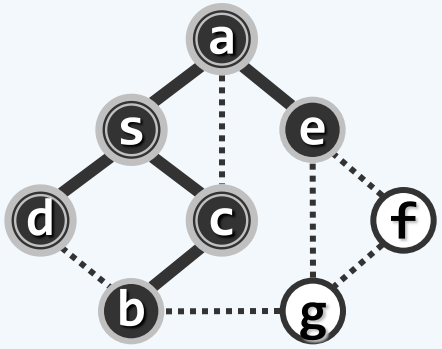
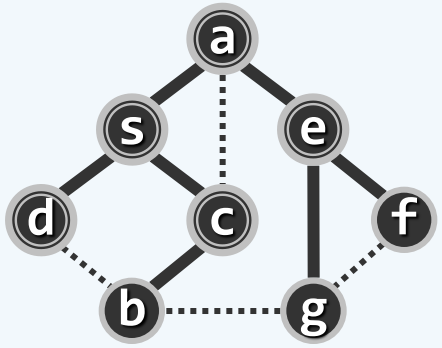
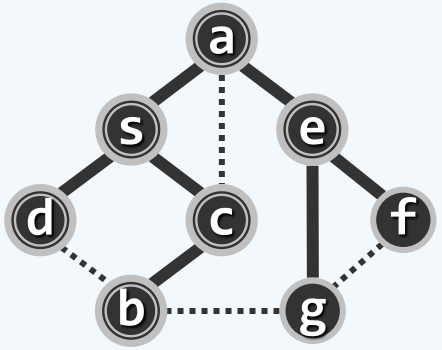
实例（无向图）



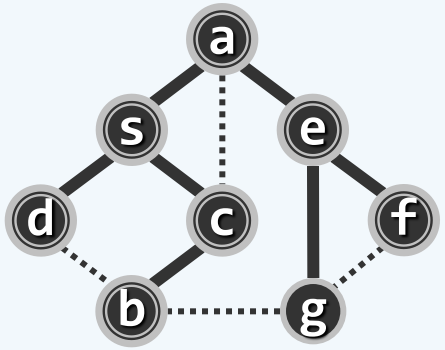
b e d c a s



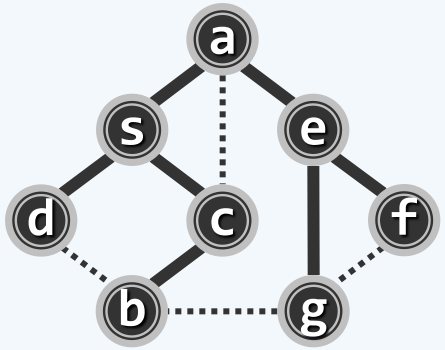
g f b e d c a s



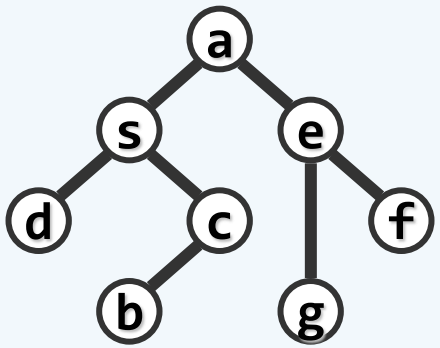
g f b e d c a s



g f b e d c a s



g f b e d c a s

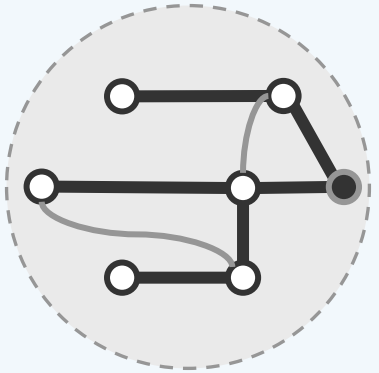


连通分量与可达分量

- 问题

  给定**无向图**，找出其中任一顶点s所在的**连通分量**

  给定**有向图**，找出源自其中任一顶点s的**可达分量**

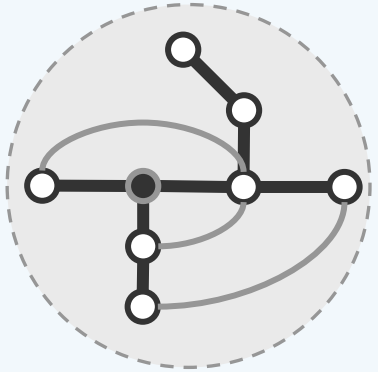


- 算法

  从s出发做BFS

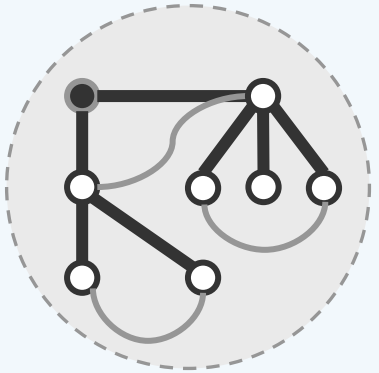
  输出所有**被发现**的顶点

  队列为空后立即终止，无需考虑其它顶点



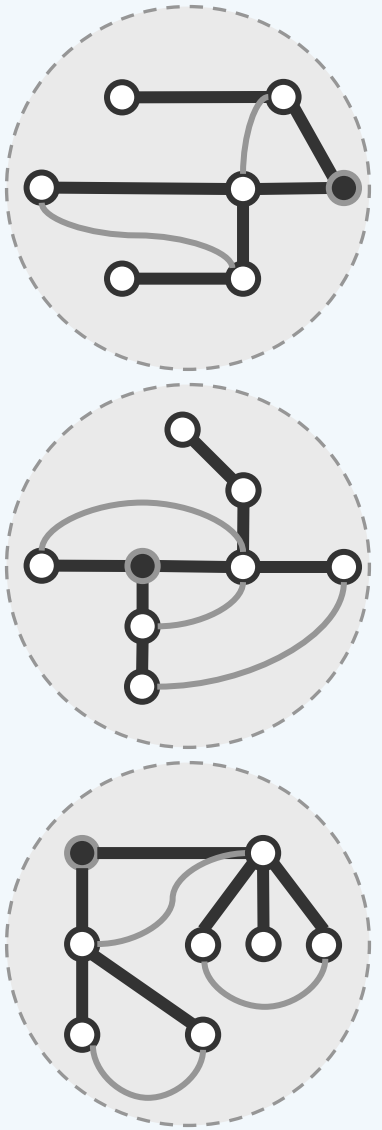
- 若图中包含**多个**连通/可达分量

  又该如何保证对**全图**的遍历呢？



Graph::bfs()

```
template <typename Tv, typename Te> //顶点类型、边类型
void Graph<Tv, Te>::bfs( int s ) { //s为起始顶点
    reset(); int clock = 0; int v = s; //初始化Θ(n + e)
    do //逐一检查所有顶点，一旦遇到尚未发现的顶点
        if ( UNDISCOVERED == status(v) ) //累计Θ(n)
            BFS( v, clock ); //即从该顶点出发启动一次BFS
    while ( s != ( v = ( ++v % n ) ) );
        //按序号访问，故不漏不重
} //无论共有多少连通/可达分量...
```



- bfs()均可遍历它们，而且自身累计仅需线性时间...

复杂度

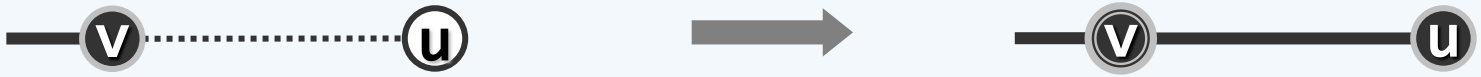
- 考查无向图...
- bfs()的初始化（reset()） $O(n + e)$
- BFS()的迭代 $O(n + 2e)$
  - 外循环（while (!Q.empty())），每个顶点只进入1次，累计n次 $O(n)$
  - 内循环（枚举v的每一邻居），每个邻居至多进入1次，累计deg(v)次
    - 采用邻接矩阵 $O(n)$
    - 采用邻接表 $O(1 + deg(v))$
  - 总共 = $O(\sum_v (1 + deg(v))) = O(n + 2e)$
- 整个算法：$O(n + e) + O(n + 2e) = O(n + e)$
- 有向图呢？亦是如此！

边分类

❖ 经BFS后，所有边将确定方向，且被分为两类　　//有向图有何不同？

tree edges + cross edges

❖ 树边(v, u)



联边之前：v DISCOVERED && u UNDISCOVERED

联边之后：v == parent(u)

❖ 跨边(v, u)

无向图：只可能v和u均为DISCOVERED

有向图：还可能v是DISCOVERED、u是VISITED

BFS树/森林

- 对于每一连通/可达分量，bfs()进入BFS(v)恰好1次（v为该分量的起始顶点）
- 进入BFS(v)时，队列为空

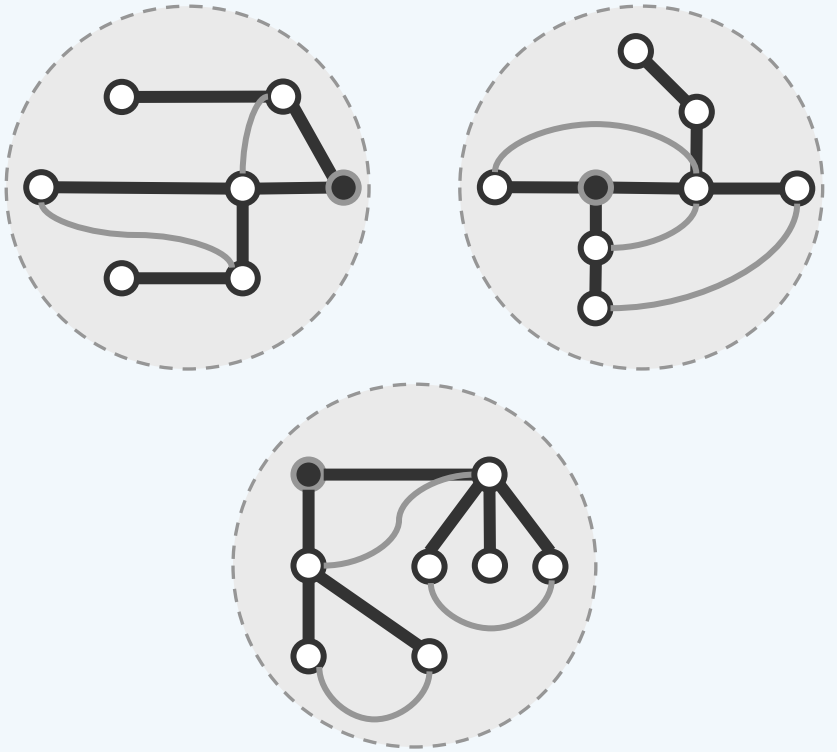
  v所属分量内的每个顶点

    迟早会以UNDISCOVERED状态进队1次

    进队后随即转为DISCOVERED状态，并生成一条树边

    迟早会出队并转为VISITED状态

  退出BFS(v)时，队列为空



- BFS(v)以v为根，生成一棵BFS树
- bfs()生成一个BFS森林包含

    c棵树、n - c条树边和e - n + c条跨边

最短路径

- 从无向图中顶点s出发，到任一顶点v的所有路径中
  - 长度最短者π(s, v) = ?
  - （最短）距离 = dist(v) = |π(s, v)| = ?
  - 退化情况：可能有多条 //任取其一
- 在BFS过程中的任一时刻
  - 队列中顶点按dist()单调排列
  - 队列中相邻顶点，dist()相差不超过1
  - 队首、队末顶点，dist()相差不超过1
  - 由树边联接的顶点，dist()恰好相差1
  - 由跨边联接的顶点，dist()至多相差1 //至少相差2的情况呢？
- BFS树中从s到v的路径，即是π(s, v)