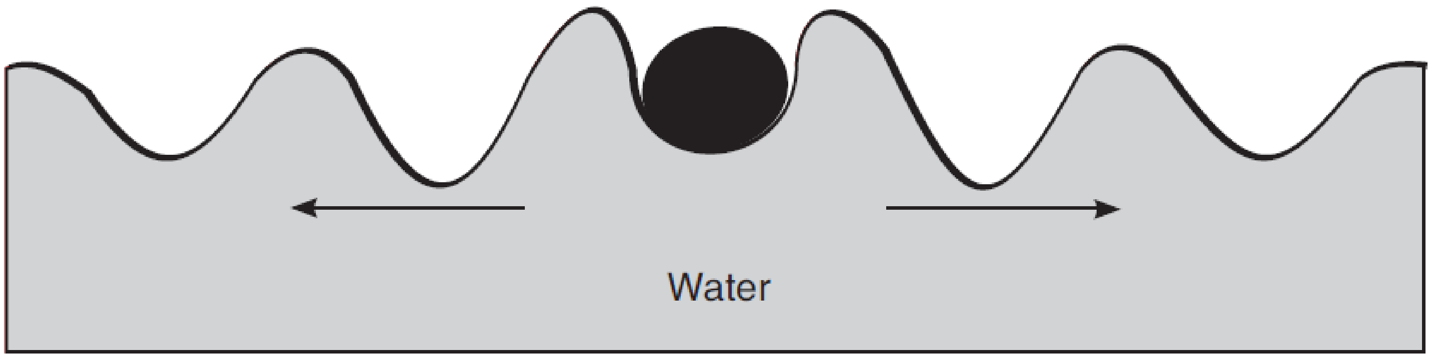
Module-1

Wave



When we observe ripples in a pond, what we see actually is a rearrangement of the surface of water. Without water, there could be no wave. The interesting point here is that a wave transports its energy without transporting matter. The energy supplied by a stone (the agent of disturbance) is transferred from one point to another. Energy is transported through the medium, yet the water molecules are not transported. Therefore, waves are said to be an **energy transport phenomenon**. In conclusion, a wave can be described as *any disturbance, which travels through the medium due to the repeated periodic motion of the particles (of the medium) about their mean position and transporting energy from one location (its source) to another location without transporting matter.*

Form of a wave profile

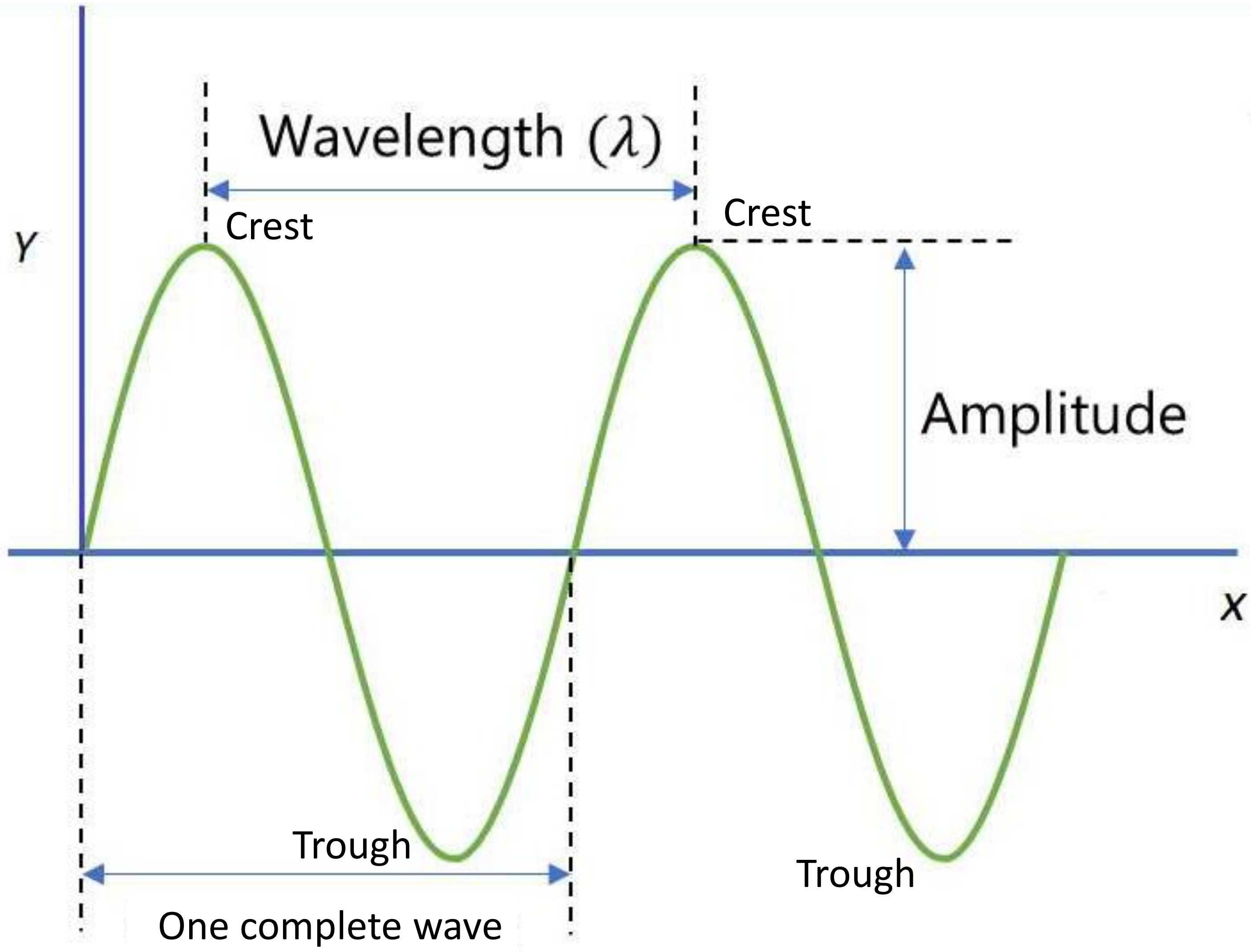


Fig. (1.1)

Any wave is characterized by the following parameters

(a) Time Period, T: When a wave propagates along a particular direction**,** the profile of the wave is seen to repeat at equal intervals of time. This time interval is known as the time period of the wave.

(b) Wavelength, λ: The distance between the corresponding points**,** such as two successive crests or two successive troughs, is called the wavelength. The wavelength is the length of a one complete wave.

(c) Amplitude, A: The maximum displacement in a waveform is known as the amplitude of the wave.

(d) Frequency, ν: The number of complete waves that travels per unit time is called frequency of the wave.

(e) Velocity, *v*: The velocity of the wave is defined by the distance travelled by the wave in unit time. The velocity of the wave can be related to its wavelength and frequency by the following relation:

$$\upsilon = \nu\lambda$$

(f) Phase angle, φ: The displacement of particles in the medium and the direction of their displacement change from point to point along the wave profile. The quantity, which represents the displacement, is called the phase of the vibration, φ. The phase may be expressed in terms of $2\pi \times x/\lambda$ or $2\pi \times t/T$. As an example, in the Fig. (1.1), the phase of the wave at the origin at can be 0, the crest at the next can be $\pi/2$ and the trough at the next can be $3\pi/2$.

(g) Intensity, I: The energy transferred on an average by a wave in unit time, through a unit area perpendicular to its propagation direction, is known as the intensity of the wave. It is established that the intensity of a wave is directly proportional to the square of the amplitude of the wave. Thus,

$$I \propto |A|^2$$

Wave Equation

The displacement of a wave can be written in the form of

$$y = f(x, t)$$

A wave in general can be written in the form of a sinusoidal function:

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - \upsilon t)\right] \qquad (1.43)$$

This equation gives the relationship between the space and time dependence of disturbances in a medium. It is seen from the above that the wave is periodic in both space and time.

The equation (1.43) may be rewritten as

$$y = A \sin k (x - \upsilon t)$$

where $$k = \frac{2\pi}{\lambda}.$$

k is known as **propagation constant or wave number**

The equation (1.43) may further be rewritten as

$$y = A \sin (k x - \omega t)$$

ω is called angular frequency

In the most general case, where r is any arbitrary direction, we replace x by r and write

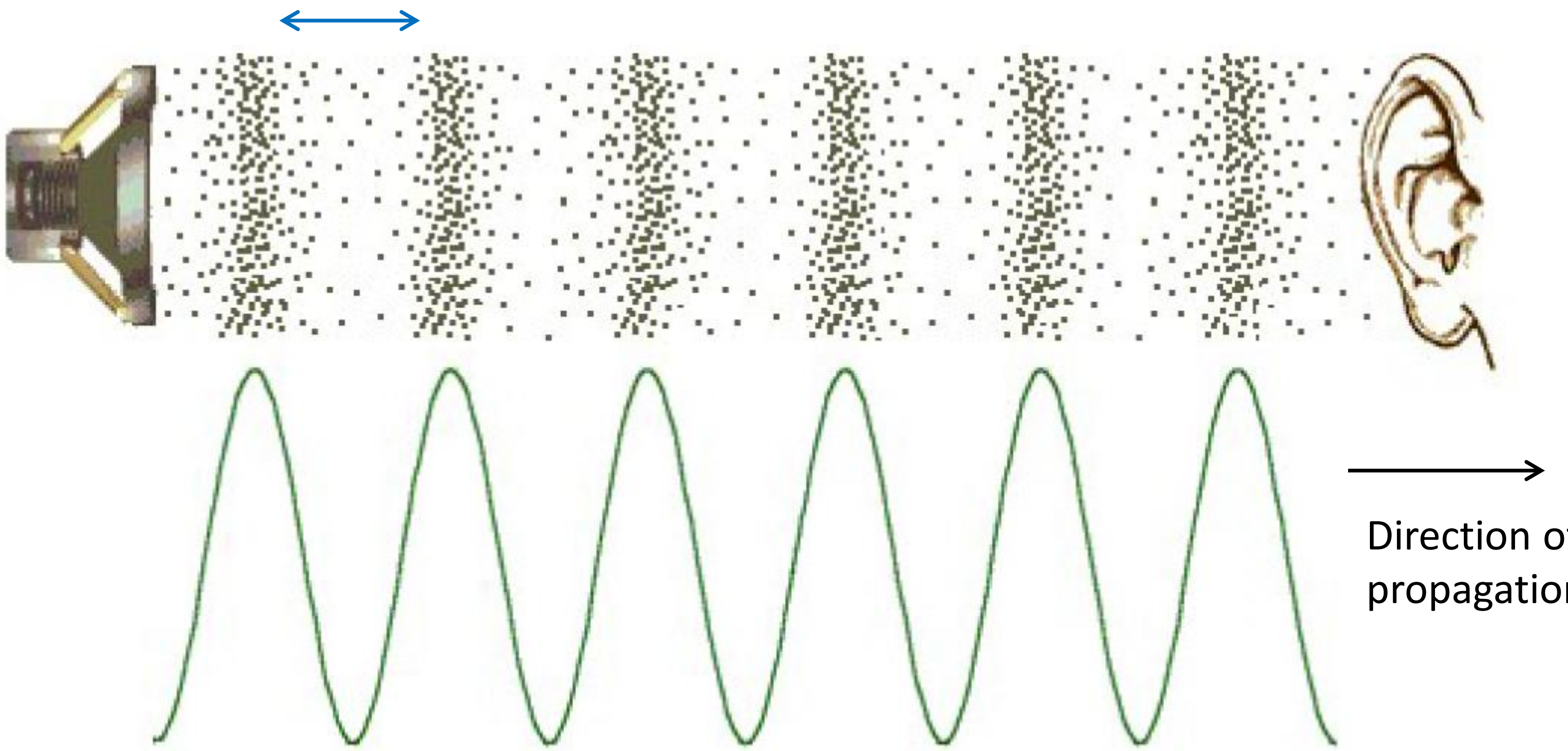
$$y(r, t) = A \sin (\mathbf{k} \cdot \mathbf{r} - \omega t)$$

or

$$y(r, t) = A \sin (\mathbf{k}.\mathbf{r} - \omega t + \underset{\substack{\uparrow \\ \text{phase}}}{\phi})$$

$$|\mathbf{k}| = \frac{2\pi}{\lambda}$$

Longitudinal wave

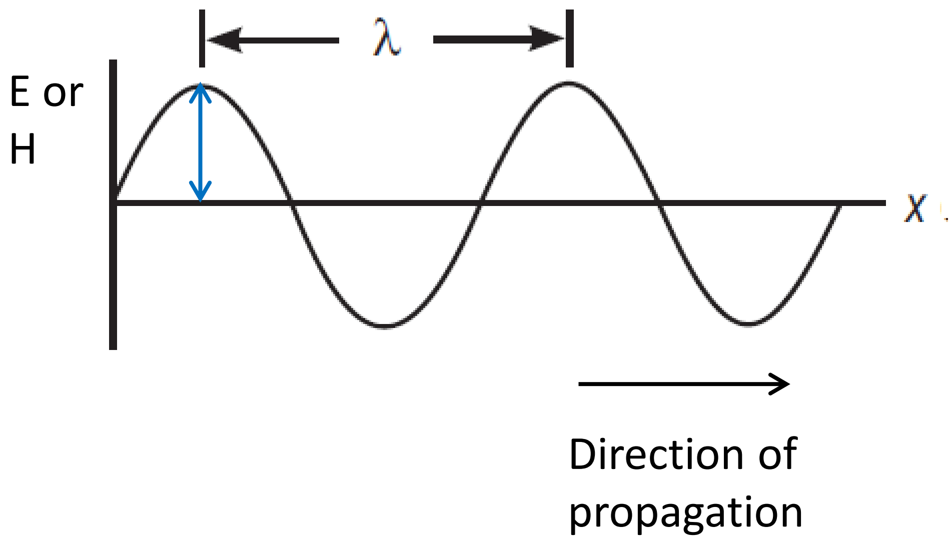


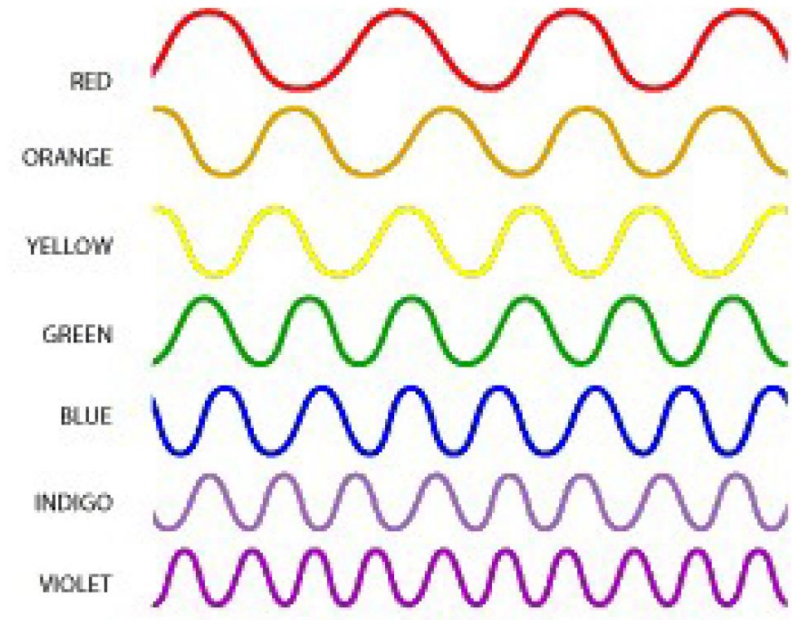
Direction of propagation

A **longitudinal wave** is a wave in which particles of the medium move in a direction *parallel* to the direction, along which the wave moves.

Example: Sound wave

Transverse wave





A **transverse wave** is a wave in which particles of the medium move in a direction *perpendicular* to the direction along which the wave moves.

Example: Light wave

Interference

Conditions for observing sustained interference: We may now summarize the conditions that are to be fulfilled in order to observe sustained interference.

(*i*) The waves from the two light sources must be of the same frequency or same wavelength or same colour.

(*ii*) The waves from the two light sources must maintain a constant phase difference.

(*iii*) The waves from the two light sources must be of equal amplitudes.

(*iv*) The two light sources must be close to each other.

Conditions of interference maximums and minimums

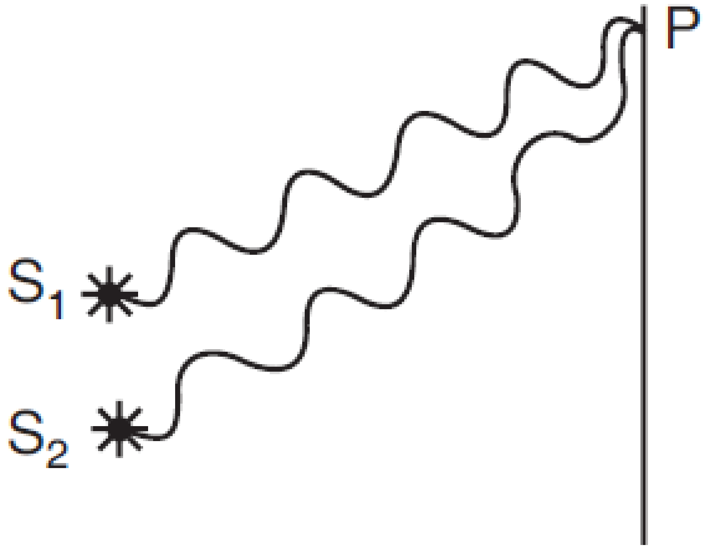


Fig. 5.17: Two light waves overlapping at point P.

When harmonic waves of identical frequency propagating in a medium meet each other, they give rise to the phenomenon of interference. Let us now understand what happens when two or more light waves overlap in some region of space. Let us assume for the sake of simplicity that two sinusoidal waves of the same frequency propagate through different paths *and meet at P in the region of observation, as shown in Fig. (5.17). Let the waves* be represented by

$$E_A = E_1 \sin \omega t \tag{5.61}$$

and

$$E_B = E_2 \sin (\omega t + \delta) \tag{5.62}$$

where δ is the phase difference between them.

According to the Young's principle of superposition, the resultant electric field at a given place due to the simultaneous action of two or more harmonic waves is the algebraic sum of the electric fields of the separate constituent waves

Thus, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$$\begin{aligned} E_R &= E_A + E_B \qquad (5.63) \\ &= E_1 \sin \omega t + E_2 \sin (\omega t + \delta) \\ &= E_1 \sin \omega t + E_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta) \\ &= (E_1 + E_2 \cos \delta) \sin \omega t + E_2 \sin \delta \cos \omega t \qquad (5.64) \end{aligned}$$

Let $E_1 + E_2 \cos \delta = E \cos \phi$ (5.65)

and $E_2 \sin \delta = E \sin \phi$ (5.66)

$$(E_1 + E_2 \cos \delta)^2 + E_2{}^2 \sin^2 \delta = E^2(\cos^2\phi + \sin^2\phi)$$

$$E^2 = E_1{}^2 + E_2{}^2 \cos^2 \delta + 2E_1E_2\cos \delta + E_2{}^2\sin^2\delta$$

$$E^2 = E_1{}^2 + E_2{}^2 + 2E_1E_2 \cos \delta \qquad (5.67)$$

The intensity of a light wave is given by the square of its amplitude.

$$I = \frac{1}{2}\varepsilon_0 c E^2 \propto E^2$$

where ε_0 is the permittivity of free space and c is the velocity of light in vacuum.

Using this relation into (5.67), we get

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta}$$

$$\mathbf{A} = \sqrt{\boldsymbol{A_1^2 + A_2^2 + 2A_1A_2cos}\ \delta}$$

We see that the resultant intensity at P on the screen is not just the sum of the intensities due to the separate waves. The term $2\sqrt{I_1 I_2}\cos\delta$ is known as the **interference term**.

From Eq. (5.68), $$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

When $I_1 = I_2 = I_0$ $$I_{max} = 4I_0$$

Conditions of getting maximums or constructive interference,

Phase difference, $\delta = 0, 2\pi, 4\pi, 6\pi, \ldots\ldots\ldots\ldots$

$= 2n\pi$ with $n = 0, 1, 2, 3, \ldots\ldots$

or

Path difference, $L = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots$

$= n\lambda$ with $n = 0, 1, 2, 3, \ldots\ldots$

Similarly from Eq. (5.68), $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

When $I_1 = I_2 = I_0$ $\qquad$ $I_{min} = 0$

Conditions of getting minimums or destructive interference,

Phase difference, $\delta = \pi, 3\pi, 5\pi, 7\pi, \ldots\ldots\ldots$

$= (2n+1)\pi$ with $n = 0, 1, 2, 3, \ldots\ldots$

or

Path difference, $L = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots\ldots$

$= (n+1/2)\lambda$ with $n = 0, 1, 2, 3, \ldots\ldots$

The stationary pattern of bright and dark bands is called *interference pattern*. The bands are known as **interference fringes** (Fig. 5.19).

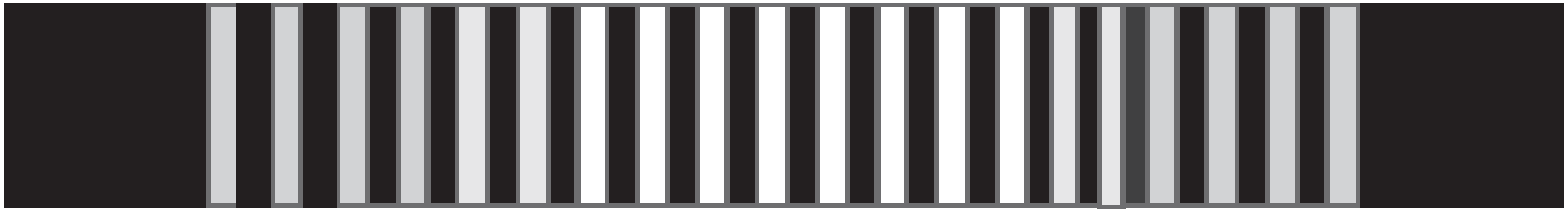
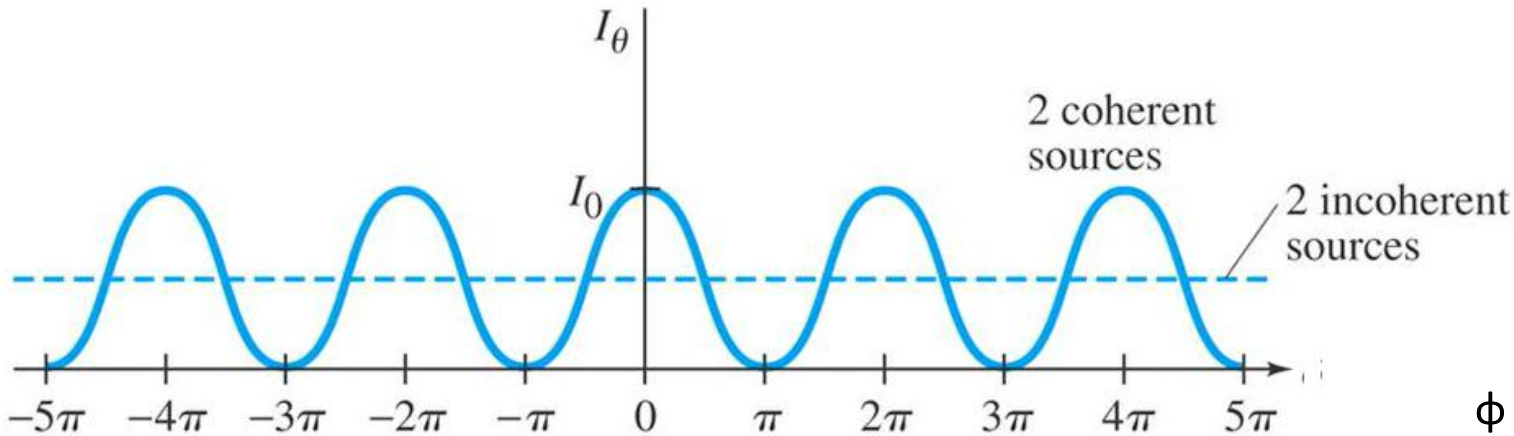


Fig. 5.19: Interference pattern consisting of alternate bright and dark bands.



I_θ
I_0
2 coherent sources
2 incoherent sources
-5π
-4π
-3π
-2π
$-\pi$
0
π
2π
3π
4π
5π
ϕ

Example problems

1) The two waves $y_1 = 5\sin(3\pi t)$ and $y_2 = 5\sin\left(3\pi t + \frac{\pi}{3}\right)$ undergo superposition and form the interference pattern. Find the value of $\left(\frac{I_{\max} - I_\theta}{I_{\max} + I_\theta}\right)$. Where I_θ is the intensity of the resultant wave and $\theta = \frac{\pi}{3}$ is the phase difference between the two waves y_1 and y_2.

2) The path difference between the two waves is 2.5 λ and their intensities are $I_1 = 16\ \frac{W}{m^2}$ and $I_2 = 25\ \frac{W}{m^2}$.

a) Find the resultant intensity at the point ‘p’ on the screen.

b) If the resultant intensity at the point ‘p’ is 41 $\frac{W}{m^2}$, calculate the path difference between these waves.

Young's Double Slit Experiment

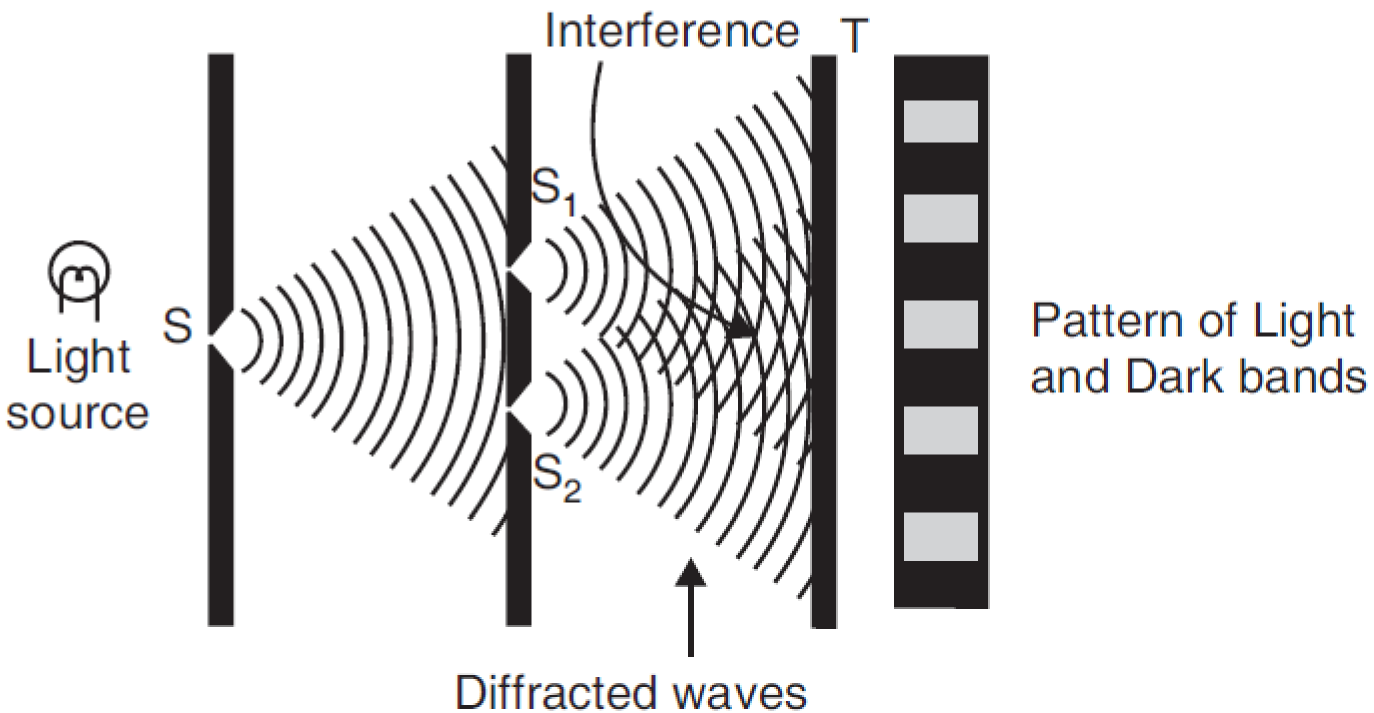


Fig. 5.21: Young's double slit arrangement -The narrow slit *S* acts as a source of cylindrical waves which illuminate the slits S_1 and S_2. S_1 and S_2 behave as coherent sources and produce interference.

Young's Double Slit Experiment

Young gave the first demonstration of the interference of light waves in 1801. Fig. 5.21 shows a plan view of the basic arrangement of his double slit experiment. The primary light source at S is a monochromatic source; it is generally a sodium lamp, which emits yellow light of wavelength at around 5893 Å. The expanding wave front from the primary light source S falls on two narrow closely spaced slits, S_1 and S_2 as shown in Fig. 5.21. The slits at S_1 and S_2 are very narrow and partition the incident wave front. If the slits are equidistant from S, the phase of the wave at S_1 will be the same as the phase at S_2 because parts of the same wave front emerging from S pass through S_1 and S_2. Further, waves leaving S_1 and S_2 have the same frequency as the primary source. Hence, sources S_1 and S_2 act as secondary coherent sources. The waves leaving from S_1 and S_2 interfere and produce alternate bright and dark bands on the screen at T.

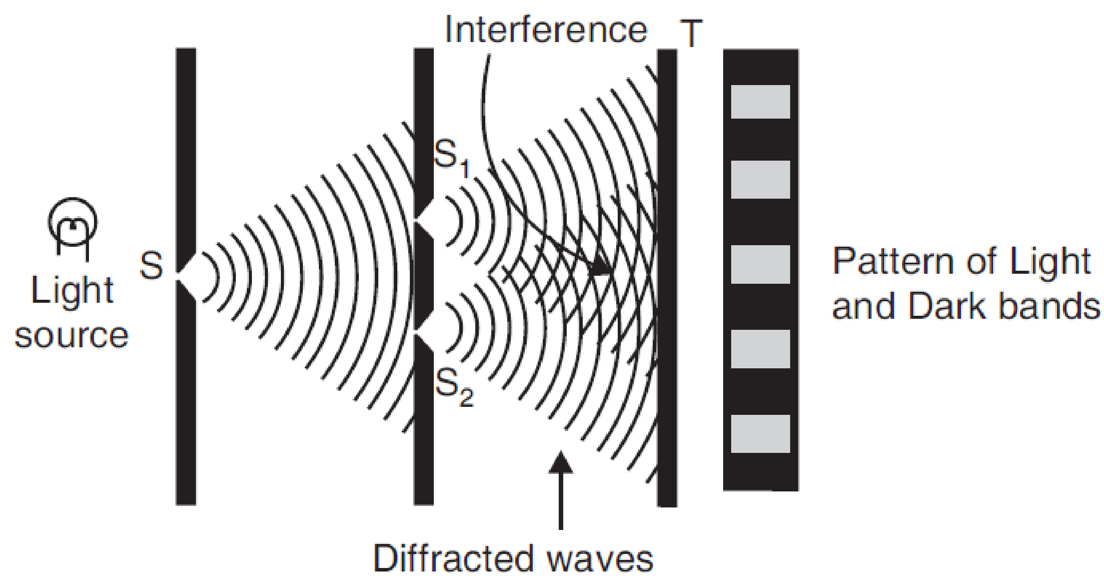
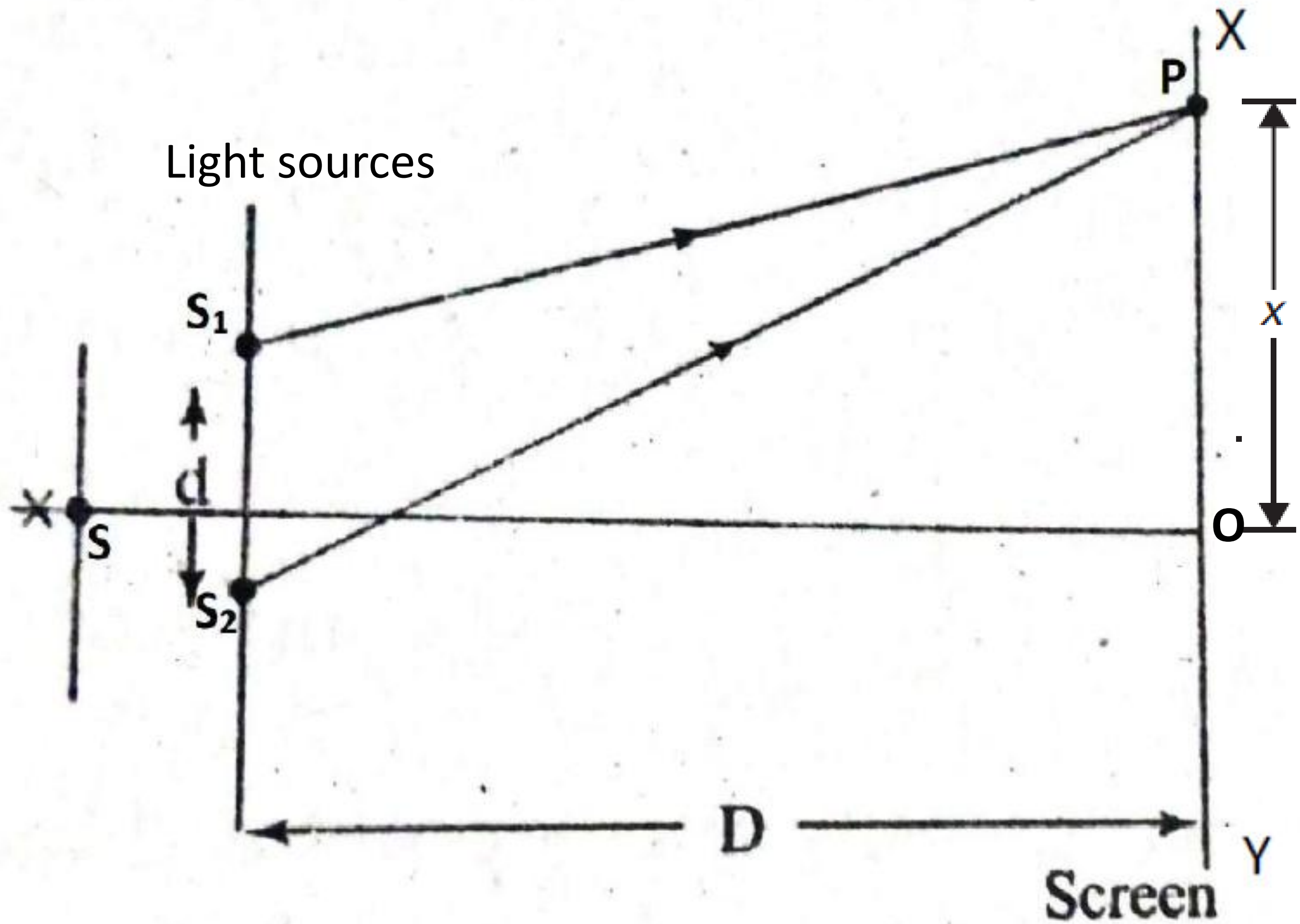


Fig. 5.21: Young's double slit arrangement -The narrow slit S acts as a source of cylindrical waves which illuminate the slits S_1 and S_2. S_1 and S_2 behave as coherent sources and produce interference.



$$\text{Path difference} = S_2P - S_1P = \frac{xd}{D}$$

Position of bright fringes:

The condition for finding a bright fringe at P is that

$$S_2P - S_1P = m\lambda$$

$$\frac{xd}{D} = m\lambda$$

where m is called the **order of the fringe**. The bright fringe at O, corresponding to $m = 0$, is called the *zero-order* fringe. The first-order bright fringe from the axis corresponds to $m = 1$ and the second order bright fringe to $m = 2$ and so on.

The m^{th} order bright fringe occurs when

$$x_m = \frac{m\lambda D}{d}$$

Position of dark fringes:

The first dark fringe occurs when ($S_2P - S_1P$) is equal to $\lambda / 2$. The waves are now in opposite phase at P. The second dark fringe occurs when ($S_2P - S_1P$) equals $3\lambda / 2$. The m^{th} dark fringe occurs when

$$(S_2P - S_1P) = (2m + 1)\, \lambda / 2$$

The condition for finding a dark fringe is

$$\frac{xd}{D} = (2m+1)\frac{\lambda}{2}$$

The '*m*'th order dark fringe occurs at

$$x_m = \frac{(m + 1/2)\lambda D}{d}$$

The m^{th} order bright fringe occurs when

$$x_m = \frac{m\lambda D}{d}$$

and the $(m + 1)^{th}$ order bright fringe occurs when

$$x_{m+1} = \frac{(m+1)\lambda D}{d}$$

The bright fringe separation, β is given by

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d}$$

The same result will be obtained for dark fringes. Thus, neighbouring bright and dark fringes are separated by the same amount everywhere on the screen. The separation β is called the ***fringe width***.

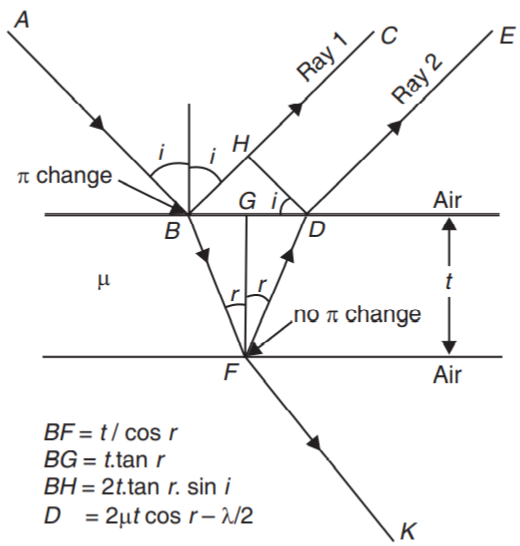
The width of the dark or bright fringe is given

$$\beta = \frac{\lambda D}{d}$$

Example problem

The distance between the coherent sources is 0.2 mm and the screen is 80 mm from the sources. The second dark band is 0.3 mm from the central bright fringe. Find the wavelength and the distance of the fourth order dark fringe from the central fringe.

Interference in thin films:



(i) Geometrical Path Difference:

Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal paths. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD.

The geometric path difference between the two rays is

BF + FD – BH

(*ii*) **Optical Path Difference:**

Optical path difference $\Delta_a = \mu L$

$$\therefore \quad \Delta_a = \mu(BF + FD) - 1(BH) \qquad (1)$$

In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad BF + FD = \frac{2t}{\cos r} \qquad (2)$$

Also,

$$BG = GD$$

$$\therefore \quad BD = 2BG$$

$$BG = FG \tan r = t \tan r$$

$$\therefore \quad BD = 2t \tan r$$

In the Δ^{le} BHD

$$\angle HBD = (90 - i)$$

$$\angle BHD = 90°$$

$$\therefore \quad \angle BDH = i$$

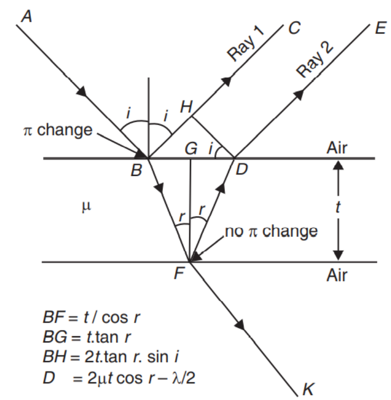
$$\therefore \quad BH = BD \sin i = 2\, t \tan r \sin i$$

From Snell's law,

$$\sin i = \mu \sin r$$

$$\therefore \quad BH = 2t \tan r\,(\mu \sin r) = \frac{2\mu t \sin^2 r}{\cos r} \qquad (3)$$

Put (2) and (3) in (1)



$$\Delta_a = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu t \sin^2 r}{\cos r}\right]$$

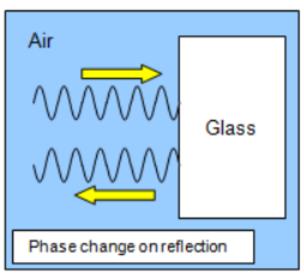
$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

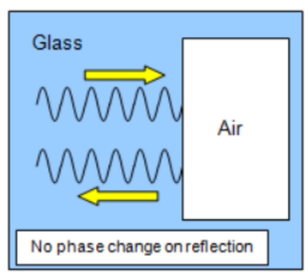
$$= \frac{2\mu t}{\cos r}\cos^2 r$$

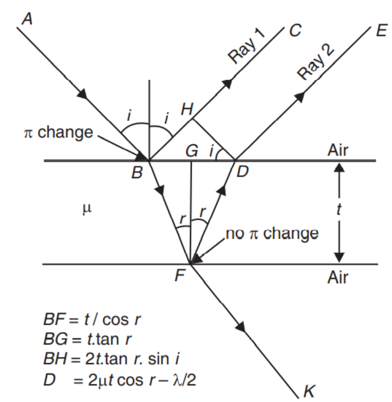
$$\Delta_a = 2\mu\, t \cos r \qquad (4)$$

(*iii*) **Correction on account of phase change at reflection:** When a ray is reflected at the boundary of a rarer to denser medium, a path-change of $\lambda/2$ occurs for the ray BC (see Fig.   ). There is no path difference due to transmission at D. Including the change in path difference due to reflection in eqn. (4), the true path difference is given by

$$\Delta_t = 2\mu\, t \cos r - \lambda/2$$







Conditions for Maxima (Brightness) and Minima (Darkness)

Maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, when

$$2\mu\, t \cos r - \frac{\lambda}{2} = m\lambda$$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting.

$$2\mu\, t \cos r = m\lambda + \lambda/2$$

or $$2\mu\, t \cos r = (2m + 1)\lambda/2$$ **Condition for Brightness**

Minima occur when the optical path difference is $\Delta = (2m + 1)\,\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. Thus, when

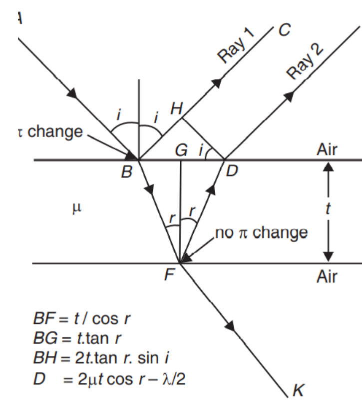
$$2\mu\, t \cos r - \lambda/2 = (2m + 1)\lambda/2$$

the reflected rays undergo destructive interference to produce darkness. Equ.(~~6.23~~) may be rewritten as

$$2\mu\, t \cos r = (m + 1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore $(m + 1)\lambda$ can be as well replaced by $m\lambda$ for simplicity in expression. Thus,

$$2\mu\, t \cos r = m\lambda$$ **Condition for Darkness**



Example Problem - 1

A soap film of refractive index 1.33 is illuminated by the light of wavelength 5890 x 10^{-10} m at an angle of 45 degrees. There is complete destructive interference. Then, find the thickness of the film

$$i = 45^0 \quad \mu = \frac{4}{3} \quad \lambda = 5890 \times 10^{-10}\, m \qquad \mu = \frac{\sin i}{\sin r} \Longrightarrow \sin r = \frac{\sin i}{\mu}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45}{1.33} = \frac{1/\sqrt{2}}{1.33} = 0.5317$$

$$\cos r = \sqrt{1 - sin^2 r} = \sqrt{1 - (0.5317)^2} = 0.8496$$

For destructive interference

$$2\, \mu\, t \cos r = n\, \lambda$$

$$t = \frac{n\lambda}{2\, \mu \cos r}$$

Consider the first fringe n = 1

$$t = \frac{1 \times 5890 \times 10^{-10}}{2 \times 1.33 \times 0.8496}$$

$$\boxed{t = 2.614 \times 10^{-7}\, m \quad t = 2.614 \times 10^{-4}\, mm}$$

Example Problem - 2

A thin film of soap solution is illuminated by white light at an angle of incidence i = $\sin^{-1}$ (4/5). In the reflected light, two dark consecutive overlapping fringes are observed corresponding to wavelengths 6.1×10^{-7} m and 6.0×10^{-7} m. The refractive index for soap solution is 4/3. Calculate the thickness of the film

$$i = sin^{-1}\left(\frac{4}{5}\right) \quad \lambda_1 = 6.1 \times 10^{-7} m$$

$$\mu = 4/3 \quad \lambda_2 = 6.0 \times 10^{-7} m$$

$$\sin i = \frac{4}{5} \qquad \mu = \frac{\sin i}{\sin r} \Longrightarrow \sin r = \frac{\sin i}{\mu}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = \frac{3}{5} = \boxed{0.6}$$

$$\cos r = \sqrt{1 - sin^2 r} = \sqrt{1 - (0.6)^2}$$

$$\cos r = \sqrt{1 - 0.36} = \sqrt{0.64} = \boxed{0.8}$$

Two dark consecutive fringes are overlapping

$$n\,\lambda_1 = (n+1)\,\lambda_2$$

$$n\,\lambda_1 = n\,\lambda_2 + \lambda_2$$

$$n\,(\lambda_1 - \lambda_2) = \lambda_2$$

$$n = \lambda_2 / (\lambda_1 - \lambda_2)$$

$$n = \frac{6.0 \times 10^{-7}}{6.1 \times 10^{-7} - 6.0 \times 10^{-7}} \qquad \boxed{n = 60}$$

$$2\,\mu\,t \cos r = n\,\lambda_1$$

$$t = \frac{n\lambda_1}{2\,\mu \cos r}$$

$$t = \frac{60 \times 6.1 \times 10^{-7}}{2 \times 4/3 \times 0.8}$$

$$\boxed{t = 1.72 \times 10^{-7}\ m}$$

Example Problem - 3

A soap film of refractive index 1.33 is illuminated by white light incident at an angle 30º. The reflected light is examined by a spectroscope in which dark band corresponding to the wavelength 6000 Å. Calculate the smallest thickness of the film.

$\mu = 1.33$ $\quad i = 30^o$ $\quad \lambda = 6000\ \text{Å} = 6000 \times 10^{-10}$ m

$n = 1$ (smallest thickness) $\quad t = ?$

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 30^0}{1.33} = \frac{0.5}{1.33} = 0.3759$$

$$\cos r = \sqrt{1 - sin^2 r} = \sqrt{1 - (0.3759)^2} = 0.9267$$

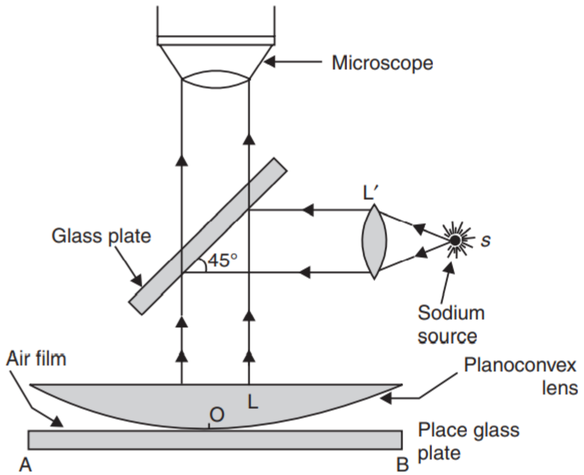
Dark Band

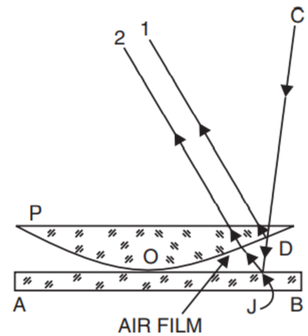
$$2\,\mu\, t \cos r = n\,\lambda$$

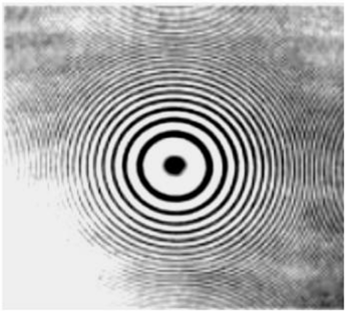
$$t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 6000 \times 10^{-10}}{2 \times 1.33 \times 0.9267} = 2.434 \times 10^{-7}\ \text{m}$$

$$\boxed{t = 2.434 \times 10^{-7}\ \text{m}}$$

NEWTON'S RINGS



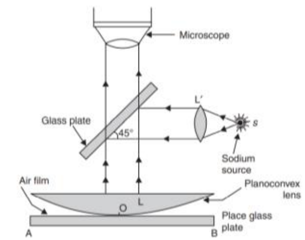




Condition for Bright and Dark Rings

The optical path difference between the rays is given by $\Delta = 2\mu t \cos r - \lambda/2$. Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$$\Delta = 2t - \lambda/2$$

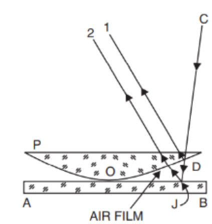


Intensity maxima occur when the optical path difference $\Delta = m\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, if $2t - \lambda/2 = m\lambda$

$$2t = (2m + 1)\lambda/2$$

bright fringe is obtained.

Intensity minima occur when the optical path difference is $\Delta = (2m + 1)\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave fall on the troughs of the other and *the waves interfere destructively.*



Hence, if

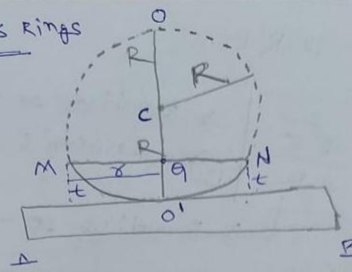
$$2t - \lambda/2 = (2m + 1)\lambda/2$$

$$2t = m\lambda$$

and dark fringe is produced.

Newton’s Rings

Newton's Rings (1)



$t \rightarrow$ thickness of the air film

let me take r is the radius of the r^{th} ring

↓ last ring

$$MQ \times QN = QO \times QO'$$

$$r \times r = (2R - t)\,t$$

$$\boxed{r^2 = 2Rt - t^2} \quad \text{—— (1)}$$

thickness of air film is very small $\therefore$ neglect t^2

$r^2 = 2Rt$

$$\boxed{t = \frac{r^2}{2R}} \quad (2)$$

$r \rightarrow$ radius of the Newton's ring

$R \rightarrow$ Radius of the curvature of the lense

Bright fringe condition:-

$$2t = (2m+1)\frac{\lambda}{2}$$

$$2\left(\frac{r_m^2}{2R}\right) = (2m+1)\cdot\frac{\lambda}{2}$$

$\hookrightarrow$ from (2)

$$4r_m^2 = 2(2m+1)\lambda R$$

$$(2r_m)^2 = 2(2m+1)\lambda R$$

(3)

$$D_m^2 = 2(2m+1)\lambda R$$

$$\boxed{D_m = \sqrt{2(2m+1)\lambda R}} \longrightarrow (3)$$

Bright fringe Diameter

Dark fringe condition:-

$$2t = m\lambda$$

$$2\left(\frac{r_m^2}{2R}\right) = m\lambda$$

$$(2r_m)^2 = m\lambda 2R \times 2$$

$$D_m^2 = 4m\lambda R$$

Diameter of the Dark fringe $\boxed{D_m = \sqrt{4m\lambda R}} \longrightarrow (4)$

④

Note :-

$$D_4 = 2\sqrt{m\lambda R}$$
$$= 2\sqrt{4\lambda R}$$

4^{th} dark ring $= 4\sqrt{\lambda R}$

$$D_1 = 2\sqrt{\lambda R}$$

~~first~~ dark ring

$$\boxed{D_4 - D_1 = 2\sqrt{\lambda R}}$$

Space b/w rings ⇒ 4 Rings

|||ly

$$D_{16} - D_9 = 8\sqrt{\lambda R} - 6\sqrt{\lambda R}$$

$$\boxed{D_{16} - D_9 = 2\sqrt{\lambda R}}$$

8 Rings

(5)

Radius of curvature :—

m^{th} dark ring

$$D_m^2 = 4 m \lambda R$$

$$D_n^2 = 4 n \lambda R$$

n^{th} dark ring

$$D_m^2 - D_n^2 = 4 (m-n) \lambda R$$

$$\boxed{R = \frac{D_m^2 - D_n^2}{4 (m-n) \lambda}} \quad \text{— (5)}$$

Refractive index :—

$$D_m^2 = 4 m \lambda R$$

$$D_n^2 = 4 n \lambda R$$

$$D_m^2 - D_n^2 = 4 (m-n) \lambda R \quad \text{—} (6)$$

↓ with air

$$\boxed{D_m'^2 - D_n'^2 = \frac{4(m-n) \lambda R}{\mu}} \quad \text{—} (7)$$

↓ in medium

$$(6)/(7) \quad \frac{D_m'^2 - D_n'^2}{D_m^2 - D_n^2} = \frac{\cancel{4(m-n)\lambda R}}{\mu \cdot \cancel{4(m-n)\lambda R}} \Rightarrow (7)/(6)$$

$$\frac{D'^2_m - D'^2_n}{D^2_m - D^2_n} = \frac{1}{\mu}$$

$$\boxed{\mu = \frac{D^2_m - D^2_n}{D'^2_m - D'^2_n}}$$

Example Problem

A plano-convex lens of radius 3 m is placed on an optically flat glass plate and is illuminated by monochromatic light. The radius of the 8th dark ring is 3.6 mm. Calculate the wavelength of light used.

$R = 3\ \mathrm{m}$ $\quad n = 8$ $\quad r_8 = 3.6\ \mathrm{mm} = 3.6 \times 10^{-3}\ \mathrm{m}$ $\quad \lambda = ?$

$$r_n = \sqrt{n R \lambda}$$

$$r_n^2 = n R \lambda$$

$$\lambda = \frac{r_n^2}{n R}$$

$$\lambda = \frac{(3.6 \times 10^{-3})^2}{8 \times 3}$$

$$\lambda = 5400 \times 10^{-10}\ \mathrm{m}$$

$$\lambda = 5400\ \text{Å}$$

$$\boxed{\begin{gathered}\lambda = 5400 \times 10^{-10}\ \mathrm{m} \\ \lambda = 5400\ \text{Å}\end{gathered}}$$

Example Problem

In Newton's ring experiment, the diameter of certain order dark ring is measured to be double that of second ring. What is the order of the ring?

Radius of the n^{th} dark ring

$$r_n = \sqrt{n R \lambda}$$

Diameter of the n^{th} dark ring

$$d_n = 2\sqrt{n R \lambda} = \sqrt{4 n R \lambda}$$

$$d_n^2 = 4 n R \lambda$$

$$d_2^2 = 4 \times 2 \times R \lambda$$

$$d_2^2 = 8 R \lambda$$

Given data

$$d_n = 2 d_2 \Rightarrow d_n^2 = 4 d_2^2$$

$$n = ?$$

$$\frac{d_n^2}{d_2^2} = \frac{4 n R \lambda}{8 R \lambda} \Rightarrow \frac{d_n^2}{d_2^2} = \frac{n}{2}$$

$$\frac{4 \cancel{d_2^2}}{\cancel{d_2^2}} = \frac{n}{2} \Rightarrow \frac{n}{2} = 4 \Rightarrow n = 8$$

$$\boxed{n = 8}$$

Example Problem

In a Newton's rings experiment the diameter of 15^{th} ring was found to be 0.59 cm and that of 5^{th} ring is 0.336 cm. If the radius of curvature of lens is 100 cm find the wavelength of the light. (June 2005, Set No. 2)

Solution

Formula $\lambda = \dfrac{D_{n+m}^2 - D_n^2}{4mR}$

Given $D_5 = 0.336\,\text{cm} = 0.336 \times 10^{-2}\,\text{m}$

$D_{5+10} = 0.59\,\text{cm} = 0.59 \times 10^{-2}\,\text{m}$

Therefore $m = 10$

$R = 100\,\text{cm} = 1\,\text{m}$

Hence $\lambda = \dfrac{(0.59 \times 10^{-2})^2 - (0.336 \times 10^{-2})^2}{4 \times 10 \times 1}$

$= \dfrac{(0.59^2 - 0.336^2)}{40} \times 10^{-4}$

$= \dfrac{0.926 \times 0.254 \times 10^{-4}}{40}$

$= 0.588 \times 10^{-6}\,\text{m}$

$= 588\,\text{nm}$ (Answer)

Example Problem

Newton's rings are formed with reflected light of wavelength 5.895×10^{-5} cm with a liquid between the plane and the curved surface. The diameter of the 5th dark ring is 0.3 cm and the radius of curvature of the curved surface is 100 cm. Calculate the refractive index of the liquid.

Solution

Formula

For nth dark ring $\dfrac{D_n^2}{4R} = \dfrac{n\lambda}{\mu}$

Given $\lambda = 5.895 \times 10^{-5}\,\text{cm} = 5.895 \times 10^{-7}\,\text{m}$

$$D_n = 0.3\,\text{cm} = 0.3 \times 10^{-2}\,\text{m}$$

$$R = 100\,\text{cm} = 1\,\text{m}$$

$$\mu = \frac{4R}{D_n^2} \times n\lambda$$

$$= \frac{4 \times 1 \times 5 \times 5.895 \times 10^{-7}}{(0.3 \times 10^{-2})^2}$$

$$= 1.310 \qquad \text{(Answer)}$$

Example Problem

In a Newton's ring experiment the diameter of the 10^{th} ring changes from 1.40 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

(June 2006, Set No. 3, May 2007, Set No. 4)

Solution

Diameter of n^{th} dark ring is given by

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

or

$$D_n = \left(\frac{4nR\lambda}{\mu}\right)^{1/2}$$

$$D = 1.40 \times 10^{-2} = \left(\frac{4 \times 10 \times R \times \lambda}{1}\right)^{1/2} \qquad (\mu = 1 \text{ for air})$$

$$D' = 1.27 \times 10^{-2} = \left(\frac{4 \times 10 \times R \times \lambda}{\mu}\right)^{1/2}$$

Hence

$$\frac{1.4}{1.27} = \mu^{1/2}$$

or

$$\mu = \left(\frac{1.4}{1.27}\right)^2 = 1.215 \qquad \text{(Answer)}$$