Fin404 Derivatives

Master in Financial Engineering

Spring 2021

Project 1: Reverse Convertible Notes

The goal of this homework project is to study reverse convertibles notes (RCNs) which constitute one of the most popular classes of structured products. Throughout the project you are asked to produce various deliverables. These are meant as briefing notes for potential clients

Part 1. Documentation

In this part you are asked to provide a text presentation (maximum 5 pages) of reverse convertible notes (RCNs). This part of your report is meant as an introductory briefing for a client and thus should not include equations or technical arguments.

You are free to structure and organize your presentation as you see fit. However, your report should at the minimum adress the following points:

1. Definition of basic products: reverse convertible notes, barrier reverse convertible notes, and their (auto)callable versions.

2. Why do such products exist? What do they allow investors to achieve? What do they allow issuing banks to achieve?

3. Capitalization, Number of issuances/year, Location, Underlying assets, etc...

4. From the point of view of an investor: Benefits and drawnbacks of the barrier version relative to the simple verion of the RCN? Benefits and drawnbacks of the callable versions relative to the non callable versions?

5. From the point of view of an issuing banks : Benefits and drawnbacks of the barrier version relative to the simple version of the RCN? Benefits and drawnbacks of the callable versions relative to the non callable versions?

6. What is the relation between RCNs and Contingent Convertible (CoCo) bonds?

Part 2. Preliminary results

Let S_{kt} be the ex-dividend price of risky asset k and consider a reverse convertible note on an index with ex-dividend price

$$I_t \equiv \sum_{k=1}^{n} S_{kt}.$$

Denote by $(T_j)_{j=1}^N$ with $T_N = T$ the set of cash flow dates of the reverse convertible note, by c its annualized coupon rate and by αI_0 with $\alpha \in [0,1]$ the exercise price of the embedded option so that

$$\sum_{j=1}^{N} \mathbf{1}_{\{t \geq T_j\}} c + \mathbf{1}_{\{t \geq T_N\}} \left[1 - \left(\alpha - \frac{I_T}{I_0} \right)^+ \right] \tag{RCN}$$

represents the cumulative cash flows to the holder of the note until date t.

1. Describe the position of the holder of the note as a portfolio that includes a standard bond and a European derivative to be specified.

Now assume that the note is not a simple RCN but a Barrier–RCN so that the terminal payoff (term in $[\cdot]$ in (RCN)) is given by

$$1 - \mathbf{1}_{\{\tau_\beta \leq T\}} \left(\alpha - \frac{I_T}{I_0} \right)^+$$

for some constant $\beta \in [0,1]$ where

$$\tau_\beta = \inf\{t : I_t \leq \beta I_0\}$$

is the first time that the ex-dividend price of the underlying index drops to or below the barrier level βI_0.

2. Describe the position of the holder of the note as a portfolio that includes a standard bond and a short position in a European derivative to be specified.

3. Show that the Barrier–RCN and the simple RCN coincide for all $\beta \in [\alpha, 1]$.

Consider finally a callable version of the Barrier–RCN in which the issuer of the note has the option to terminate the contract by returning the full principal to the holder at any of the dates $(T_i)_{i=1}^N$ after the payment of the coupon.

4. Describe the position of the holder of the note as a portfolio that includes a standard bond, a short position in a European derivative, and a short position in a Bermudean derivative to be determined. Make sure to specify the underlying asset, the maturity date, and the possible exercize dates of the latter derivative.

5. Assume that markets are arbitrage free and complete. Provide a formula for the price of the callable Barrier-RCN in terms of expectations under the unique EMM and the value function of an optimal stopping problem to be specified.

Part 3. Valuation code

Use your preferred language to write a code that *takes as input* the parameters of a binomial model:

- The annualized interest rate r
- The length Δ of the period
- The initial price of the index I_0
- The annualized dividend yield δ on the index
- The up and down factors U and D

and the charactieristics of the reverse convertible note:

- Simple or Barrier
- Callable by the issuer or non callable
- The payment dates $(T_i)_{i=1}^N$
- The annualized coupon rate c
- The exercize price $\alpha \in [0, 1]$
- The eventual barrier level $\beta \in [0, 1]$

and *which outputs* the arbitrage price of the reverse convertible note at the initial date of the tree and the initial portfolio of the index and the riskless asset that the issuer should hold to replicate the note.

Hint 1. If the note does not include a barrier feature the number of calculations required can be significantly reduced by pricing the different components of the RCN as function of time and the value of the underlying index. This simplification is no longer possible when the note includes a barrier feature because the terminal payoff then depends not only on the value of the index but also on the whole path that led to this value. In that case it is necessary to use the full tree to price the components of the notes as functions of the index price I_t and its running minimum $\min_{s \leq t} I_s$.

Hint 2. To value the Bermudean option embedded in the callable version of the RCNs it is necessary to start by computing the ex-dividend price of the asset that underlies the option at every exercize date.

Warning. We are well aware of the availability of many codes that can be used to do precisely what is asked above. *Please do not copy these codes!* Make sure to tell us which language you are using and try to think about (Alexis–)friendliness when preparing your code. A computer science concept that you may find useful to efficiently compute prices in trees is memoization (no typo).

Part 4. Model calibration

Assume that the index underlying the reverse convertible notes is the Swiss market Index (SMI) which is currently worth $S_0 = 11118$ (ex-dividend). Assume the annualized interest rate r is constant and that the index pays dividends continuously at some rate δ. Use a 30/360 day count convention in all your calculations.

1. Use the data in Table 1 (also available in the file `Data-Project1-Fin404.dat`) to estimate the values of r and δ.

2. Use the data in Table 1 and the previous results to calibrate a Binomial model with 12 periods of length one month under the constraint that over a period of one month a price increases is as likely as a price decrease under the equivalent martingale measure.

Maturity: 1 Year		
Strike	Call	Put
12000	381.525	1662.321
11800	420.599	1499.822
11600	493.332	1370.990
11400	571.380	1247.470
11200	649.428	1123.951
11000	727.476	1000.440
10800	805.524	876.917
10600	883.572	753.399
10400	991.344	659.605
10200	1114.862	581.557
10000	1238.381	503.509
9800	1361.900	425.461
9600	1485.422	347.413
9400	1612.790	273.219
9200	1775.280	234.146
9000	1937.781	195.073

Table 1: European SMI options

Part 5. Analysis

1. Use your code and the calibrated model to compute the initial price of i) A simple RCN with maturity one year, monthly payments, annualized coupon rate $c = 10\%$ and strike $\alpha = 1$; ii) A Barrier–RCN with maturity one year, monthly payments, annualized coupon rate $c = 10\%$, strike $\alpha = 1$ and barrier $\beta = 0.8$

2. Use your code to compute the par coupon rate of the simple and Barrier–RCN, that is solve for the values of c such that the initial price of the two notes is equal to par. Illustrate graphically how the par coupon rates of the notes depend on the strike α when $\beta \in \{0.4, 0.6, 0.8, 1\}\alpha$ and how the coupon rates differ between the two notes.

3. Use your code to compute the par$-\alpha$ of the simple RCN, that is solve for the value of α such that the initial price of the note is equal to par. Illustrate graphically how the par$-\alpha$ depends on the coupon rate and interpet the results.

4. Use your code to compute the par$-\alpha$ of the Barrier–RCN, that is solve for the value of the parameter α such that the initial price of the note is equal to par. Illustrate

graphically how the par$-\alpha$ depends on the coupon rate when the barrier level is set at $\beta \in \{0.4, 0.6, 0.8, 1\}\alpha$ and interpet the results

5. Illustrate graphically the difference between the par-α's of the simple RCN and that of the Barrier–RCN. Interpret the results.

6. Use your code and the calibrated model to compute the initial price of i) A callable RCN with maturity one year, monthly payments, annualized coupon rate $c = 10\%$ and strike $\alpha = 1$; ii) A callable Barrier–RCN with maturity one year, monthly payments, annualized coupon rate $c = 10\%$, strike $\alpha = 1$ and barrier $\beta = 0.8$. In case ii) also describe precisely the optimal exercize rule of the issuer.

7. Compute the par coupon rate the callable RCNs, that is solve for the value of the parameter α such that the initial price of the note is equal to par. Illustrate graphically how the par$-$coupon depends on the strike α when the barrier level is set at $\beta \in \{0.4, 0.6, 0.8, 1\}\alpha$. Interpet the results.

8. Compute the par$-\alpha$ of the callable Barrier–RCN as a function of the coupon rate, that is solve for the value of the parameter α such that the initial price of the note is equal to par. Illustrate graphically how the par$-\alpha$ depends on the coupon rate when the barrier level is set at $\beta \in \{0.4, 0.6, 0.8, 1\}\alpha$ and interpet the results.

9. Produce a one page term-sheet that summarizes the conditions your desk can coffer for par-valued 1Y RCNs on the SMI with coupon rates between 5% and 25% . Briefly compare the different possibilities with a focus on the induced risks for investors. This term-sheet is meant to be sent to clients as a complement to the presentation you created in Part 1 and as such should be as non technical as possible.

Guidelines

1. The project is to be done in groups of at least 2 and no more than **4** students without communications between the groups.

2. The name and sciper number of each member of the group should appear clearly on the final report.

3. The final report should be typeset on no more than 15 A4 pages in 12pt font with 1.5cm margins and standard linespacing.

4. The code should be ready to run in one or more standalone files. If there are multiple files please also provide a readme with clear instructions for running the code.

5. The code in Part 4 should be commented with clear explanations for each line. The comments can be either in the code itself or written separately in the report.

6. Your code and the final report should be uploaded on the moodle.

7. Make sure to cite your sources and to not simply copy existing material. The work you submit will automatically be submitted to an antiplagiarism software.

8. Only one final report and one code archive per group should be uploaded. No need to also send your work by email.

9. For the coding parts your are not allowed to use any external libraries or built-in functions for financial engineering tasks. Please develop your own.