# 质点运动学

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad |\Delta \vec{r}| \neq \Delta r \neq \Delta s$$

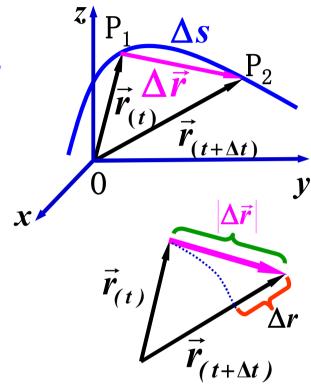
$$\left|\Delta\vec{r}\right| \neq \Delta r \neq \Delta s$$

$$v = |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{d s}{d t}$$

$$\vec{v} = \frac{\mathrm{d}x}{\mathrm{d}t}\vec{i} + \frac{\mathrm{d}y}{\mathrm{d}t}\vec{j} + \frac{\mathrm{d}z}{\mathrm{d}t}\vec{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$



$$\vec{a} = \vec{a}_t + \vec{a}_n$$

$$\vec{a} = \vec{a}_t + \vec{a}_n \qquad a = \sqrt{a_t^2 + a_n^2}$$

$$a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = R\frac{\mathrm{d}\omega}{\mathrm{d}t} = R\beta$$
  $v = \omega R$  线速度

$$v = \omega R$$

$$a_n = \frac{v^2}{R} = R\omega^2$$

$$a_t = 常数$$

$$v = v_0 + a_t t$$

## 动量定理及动量守恒

牛顿运动定律 
$$\vec{F} = \frac{\text{d} \vec{p}}{\text{d}t}$$
  $\vec{F} = m\vec{a}$  ( $m$ —定)   
质点的动量定理  $\int_{t_1}^{t_2} \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$    
 $\vec{F} = \frac{\int_{t_1}^{t_2} \vec{F} dt}{\Delta t} = \frac{m\vec{v}_2 - m\vec{v}_1}{\Delta t}$  恒力  $\vec{F} \Delta t = m\vec{v}_2 - m\vec{v}_1$    
所点系的动量定理  $\vec{I} = \int_{t_1}^{t_2} \vec{F} dt = \vec{p}_2 - \vec{p}_1$    
动量守恒: 当  $\vec{F} = 0$  时  $\vec{p} = 常矢量$    
某一方向: 当  $F_x = 0$  时  $\sum_i mv_i = \vec{n}$ 

### 角动量定理及角动量守恒

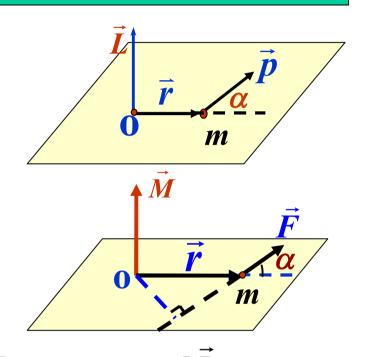
质点角动量 
$$\vec{L} = \vec{r} \times m\vec{v}$$

力矩

$$\vec{M} = \vec{r} \times \vec{F}$$

质点角动量定理

$$\vec{M} = \frac{\mathrm{d}L}{\mathrm{d}t}$$



质点系角动量定理  $\vec{M} = \sum_{i} \vec{r}_{i} \times \vec{F}_{i} = \frac{d}{dt} (\sum_{i} \vec{L}_{i}) = \frac{dL}{dt}$ 

角动量守恒:  $\vec{M}=0$   $\vec{L}_{2}=\vec{L}_{1}$ 

$$\vec{M} = 0$$

$$\vec{L}_2 = \vec{L}_1$$

注意区别:

对某一轴

$$M_z = 0$$

$$\boldsymbol{M}_z = \boldsymbol{0} \qquad \boldsymbol{L}_{z1} = \boldsymbol{L}_{z2}$$

动量守恒与 角动量守恒

## 功和能

$$W_{AB} = \int_{A}^{B} \vec{F} \cdot d\vec{r}$$

保 守力的 功

重力的功

$$W_{AB} = mgh_{A} - mgh_{B}$$

 $W_{\rm GR} = -\Delta E_{\rm p}$ 

弹性力的功 
$$W_{AB} = \frac{1}{2} kx_A^2 - \frac{1}{2} kx_B^2$$

万有引力的功

$$W_{AB} = \left(-\frac{GmM}{r_A}\right) - \left(-\frac{GmM}{r_B}\right)$$

质点动能定理

$$W = \frac{1}{2} m v_{2}^{2} - \frac{1}{2} m v_{1}^{2}$$

质点系动能定理

$$W_{\beta \uparrow} + W_{\beta \downarrow} = E_{kB} - E_{kA}$$

功能原理

$$W_{\text{外}} + W_{\text{非保内}} = E_{B} - E_{A}$$

机械能守恒

$$W_{\text{ph}} + W_{\text{ph}} = 0$$
  $E_{\text{B}} = E_{\text{A}}$ 

# 第2章 刚体的定轴转动

$$a_{n} = r\omega^{2}$$

$$a_{t} = \frac{dv}{dt} = r\beta$$

$$\omega = \omega_{0} + \beta t$$

$$\Delta\theta = \omega_{0} t + \frac{1}{2}\beta t^{2}$$

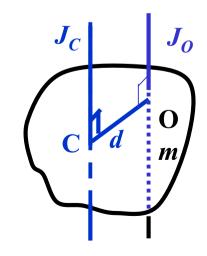
$$\delta\theta = \frac{d\theta}{dt} \beta = \frac{d\omega}{dt}$$

$$\omega^{2} - \omega_{0}^{2} = 2\beta\Delta\theta$$

#### 刚体的定轴转动定律 $M = J\beta$

$$M = J\beta$$

转动惯量 
$$J = \int r^2 dm$$
  $J = \sum_i J_i$  平行轴定理  $J_O = J_C + md^2$ 



#### 定轴转动刚体的角动量定理

$$\int_{t_1}^{t_2} M_z dt = L_2 - L_1 \qquad M_z \Delta t = L_2 - L_1 \qquad (M_z \stackrel{\text{in}}{=} 2)$$

合外力矩对定转动刚体的冲量矩等于该段时间内刚体对 同一轴角动量的增量。

### 角动量守恒定律

$$M_z = 0 \qquad L_{z1} = L_{z2}$$

上述结论对于包含有<mark>刚体、质点、物体</mark>系统也适用, 这里*L*应是系统中所有物体的角动量的和

功能原理 
$$W_{\text{sh}} + W_{\text{sig}} = (E_{k2} + E_{p2}) - (E_{k1} + E_{p1})$$