简单随机样本:　1 代表性: X_i 与 X 有相同分布

　　2 独立性: $X_1\ X_2 \cdots X_n$ 相互独立.

联合概率函数: $f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(X_i = x_i)$　离散型

联合概率密度: $f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f(x_i)$　连续型

统计量: 若 g 中不含任何未知参数, 则称 $g(X_1, X_2, \ldots, X_n)$ 为统计量

统计量是随机变量

常用统计量:

- 样本均值 $\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$
- 样本方差 $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$
  $= \frac{1}{n-1}\left[\sum_{i=1}^{n} X_i^2 - n(\overline{X}_n)^2\right]$
- 样本标准差 $S = \sqrt{S}$
- 样本k阶矩 $A_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$
- 样本k阶中心矩 $B_k = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X}_n)^k$

重要结论

- $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
  $\overline{X} \xrightarrow{P} E(X)$
- $A_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$
  $A_k \xrightarrow{P} E(X^k)$.
- $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right]$.
  $S^2 \xrightarrow{P} E(X^2) - E^2(X) = \sigma^2(X)$.

抽样三大分布:

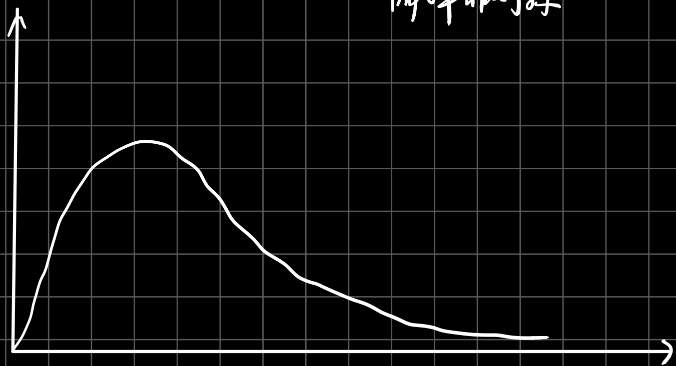
χ^2 分布:

- $X_1, X_2, \ldots, X_n \sim N(0,1)$
  $\chi^2 = X_1^2 + X_2^2 + \cdots + X_n^2$
  $\chi^2 \sim \chi^2(n)$, n为自由度
- $f(y) = \begin{cases} \frac{1}{2^{\frac{n}{2}}\Gamma(n/2)} y^{\frac{n}{2}-1} e^{-\frac{y}{2}}, & y \geq 0 \end{cases}$　$\Gamma(\alpha) = \int^{\infty} x^{\alpha-1} e^{-x} dx$

$, y<0.$

- 图形:

偏峰非对称



- $X_1, X_2 \cdots X_n$ 相互独立.
  - $\chi^2 = \frac{1}{\sigma^2}\sum_{i=1}^{n}(X_i-\mu)^2 \sim \chi^2(n)$
  - $\chi_1^2 \sim \chi^2(n_1)$, $\chi_2^2 \sim \chi^2(n_2)$

    $\chi_1^2 + \chi_2^2 \sim \chi^2(n_1+n_2)$ 可加性.
  - $X \sim \chi^2(n)$

    $E(X)=n$, $D(X)=2n$.
- 上 $-\alpha$ 分位点

  $P\{\chi^2 > \chi_\alpha^2(n)\} = \alpha$. $\chi_\alpha^2(n)$ 为上 $-\alpha$ 分位点,

  $P(\chi^2 \geq \chi_{\frac{\alpha}{2}}^2(n)) + P(\chi^2 < \chi_{1-\frac{\alpha}{2}}^2(n)) = \alpha$.

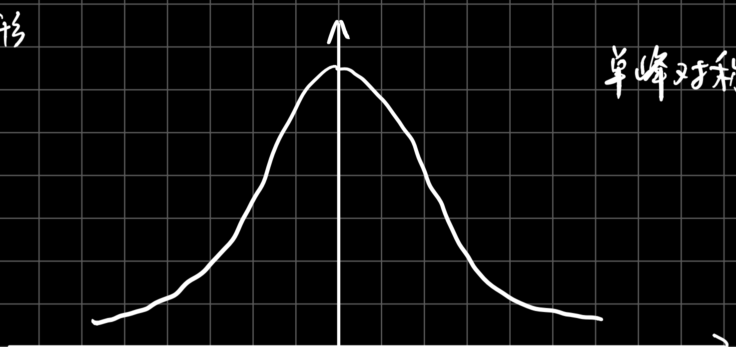
t分布:
(学生氏分布).

- $X \sim N(0,1)$, $Y \sim \chi^2(n)$, X 与 Y 独立.

  $t = \frac{X}{\sqrt{Y/n}}$, n 为自由度

  $t \sim t(n)$.
- $f(t) = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{n}{2})\sqrt{n\pi}}\left(1+\frac{t^2}{n}\right)^{-\frac{n+1}{2}}$, $-\infty < t < +\infty$
- 图形

单峰对称分布



n 充分大时 t分布近似 $N(0,1)$ $\lim f(t) = \frac{1}{\ }e^{-\frac{t^2}{2}}$

- $E(t)=0$, $D(t)=\frac{n}{n-2}$ （对 $n>2$）
- 上-α分位点，

  $P\{t>t_\alpha(n)\}=\alpha$，则 $t_\alpha(n)$ 为上-α分位点。

  $P(|T_n|\geq t_{\frac{\alpha}{2}}(n))=\alpha$，$t_{1-\alpha}(n)=-t_\alpha(n)$

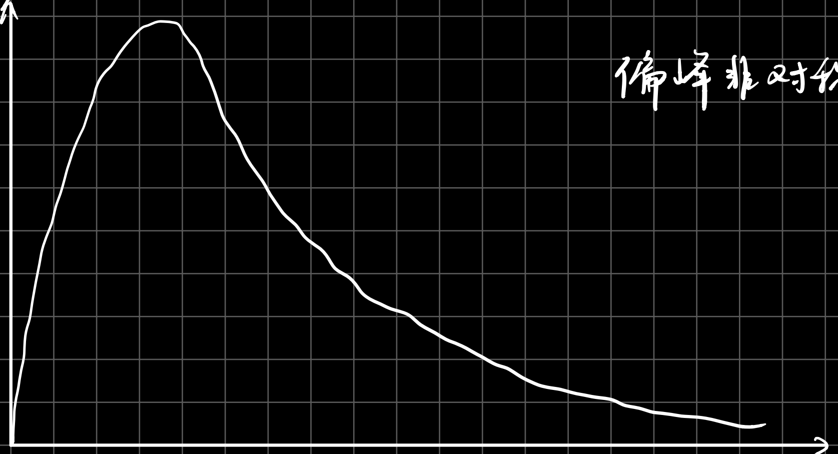
F分布：

- $U\sim\chi^2(n_1)$，$V\sim\chi^2(n_2)$，U与V独立

  $F=\frac{U/n_1}{V/n_2}\sim F(n_1,n_2)$

- $$f(y)=\begin{cases}\dfrac{\Gamma(\frac{n_1+n_2}{2})(\frac{n_1}{n_2})^{\frac{n_1}{2}}y^{\frac{n_1}{2}-1}}{\Gamma(\frac{n_1}{2})\Gamma(\frac{n_2}{2})(1+\frac{n_1}{n_2}y)^{\frac{n_1+n_2}{2}}}, & y>0\\ 0, & y\leq 0\end{cases}$$

- 图形



偏峰非对称

- $F\sim F(n_1,n_2)$

  $\frac{1}{F}\sim F(n_2,n_1)$

- 上-α分位点。

  $P\{F>F_\alpha(n_1,n_2)\}=\alpha$，则 $F_\alpha(n_1,n_2)$ 为上-α分位点，

  $P(F_{n,m}\geq F_{\frac{\alpha}{2}}(n,m))+P(F_{n,m}\leq F_{1-\frac{\alpha}{2}}(n,m))=\alpha$

  $F_{1-\alpha}(n_1,n_2)=\frac{1}{F_\alpha(n_2,n_1)}$

  $\hookrightarrow$ $P(F>F_{1-\alpha}(n_1,n_2))=1-\alpha$

  $P(F\leq F_{1-\alpha}(n_1,n_2))=\alpha$.

  $P(\frac{1}{F}\geq\frac{1}{F_{1-\alpha}(n_1,n_2)})=\alpha$.

  $\because \frac{1}{F}\sim F(n_2,n_1)$ $\therefore F_\alpha(n_2,n_1)=\frac{1}{F_{1-\alpha}(n_1,n_2)}$

一个正态总体：

- $X_1, X_2, \cdots, X_n \sim N(\mu, \sigma^2)$.

  (1) $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

  (2) $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

  (3) $\bar{X}$ 与 S^2 相互独立.

- $\dfrac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1) \longleftarrow \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$, $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

两个正态总体：

- $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X Y 相互独立

  $$\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

  $\hookrightarrow$ $\bar{X} \sim N(\mu_1, \frac{\sigma_1^2}{n_1})$, $\bar{Y} \sim N(\mu_2, \frac{\sigma_2^2}{n_2})$

  $\bar{X}-\bar{Y} \sim N(\mu_1-\mu_2, \frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2})$

- $X \sim N(\mu_1, \sigma^2)$, $Y \sim N(\mu_2, \sigma^2)$, X Y 相互独立

  $$\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

  其中 $S_w^2 = \dfrac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$, $S_w = \sqrt{S_w^2}$

  $\hookrightarrow$ $\bar{X}-\bar{Y} \sim N(\mu_1-\mu_2, \frac{\sigma^2}{n_1}+\frac{\sigma^2}{n_2})$

  $$U = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim N(0,1)$$

  $$\frac{(n_1-1)S_1^2}{\sigma^2} \sim \chi^2(n_1-1) \qquad \frac{(n_2-1)S_2^2}{\sigma^2} \sim \chi^2(n_2-1)$$

  $$V = \frac{(n_1-1)S_1^2}{\sigma^2} + \frac{(n_2-1)S_2^2}{\sigma^2} \sim \chi^2(n_1+n_2-2)$$

  $$\frac{U}{\sqrt{V/(n_1+n_2-2)}} = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t(n_1+n_2-2)$$

- $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, X Y 相互独立

$$\frac{S_1^2 / S_2^2}{\sigma_1^2 / \sigma_2^2} \sim F(n_1 - 1, n_2 - 1)$$