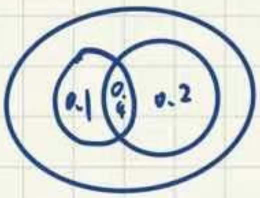
一、1. $P(\bar{A}\cup B)=0.9$



2. A_i 取到i个旧球　　B=取到新旧.

A_2: 2旧1新　$\frac{C_2^2\cdot C_8^1}{C_{10}^3} = \frac{8}{\frac{\cancel{5}10\times9\times8}{3\times2}} = \frac{1}{15}$　　$\frac{1}{15}\times\frac{7}{10}=\frac{7}{150}$

A_1 1旧2新　$\frac{C_2^1\cdot C_8^2}{C_{10}^3} = \frac{2\times\frac{8\times7}{2}}{\frac{10\times9\times8}{3\times2}} = \frac{7}{15}$　　$\frac{7}{15}\times\frac{6}{10}=\frac{42}{150}$

A_0: 0旧3新: $\frac{C_8^3}{C_{10}^3} = \frac{\frac{8\times7\times6}{3\times2}}{\frac{10\times9\times8}{3\times2}} = \frac{7}{15}$　　$\frac{7}{15}\times\frac{5}{10}=\frac{35}{150}$

$P(B)=P(A_1B)+P(A_2B)+P(A_3B)=\frac{7+42+35}{150}=\frac{84}{150}=$

② $P(A_1|B)=\frac{P(A_1B)}{P(B)}=\frac{42}{84}=\frac{1}{2}$

二、

X	0	1	2	3
P	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$

①

$X=0$

$X=1$　$\frac{1}{2}\times\frac{3}{5}=\frac{3}{10}$

$X=2$　$\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}=\frac{3}{20}$

$X=3$　$\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{20}$

$EX=\frac{3}{10}+\frac{6}{20}+\frac{3}{20}$

$=\frac{15}{20}=\frac{3}{4}$

② (1) $EX=\frac{1}{2}$ $\lambda=2$.

$$f_X(x)=\begin{cases}2e^{-2x}, & x\geq 0\\ 0, & x<0\end{cases}$$

$1-y\leq e^{-2x}$

$\ln(1-y)\leq -2x$

(2) $F_Y(y)=P\{Y\leq y\}=P\{1-e^{-2x}\leq y\}=P\{X\leq -\frac{\ln(1-y)}{2}\}$

$0\leq Y<1$ $\quad = 1-e^{\ln(1-y)} = y$

$$F_X(x)=\begin{cases}1-e^{-2x}, & x\geq 0\\ 0, & x<0\end{cases}$$

$$f_Y(y)=\begin{cases}1, & 0\leq y<1\\ 0, & \text{else}.\end{cases}$$

三、 A: ✓ B: ×

A、B 独立分布.

② (1) $f(x,y)=\begin{cases}c, & 0<x\leq y\leq 1\\ 0, & \text{else}.\end{cases}$

$c\int_0^1\int_x^1 1\cdot dy\,dx = c\int_0^1(1-x)\,dx = c\cdot x-\frac{1}{2}x^2\Big|_0^1$

$=c\cdot\frac{1}{2}=1$ $\quad c=2$

(2) $f_X(x)=\int_0^x 2\,dy = 2x = \begin{cases}2x, & 0<x<1\\ 0, & \text{else}.\end{cases}$

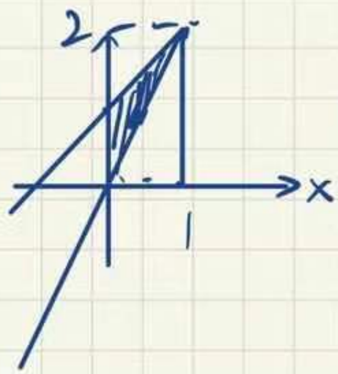
$f_Y(y)=\int_y^1 2\,dy = 2(1-y)$ $\quad 0<y<1$

0 $\quad$ else

(3) $f(x,y) \neq f_X(x)\cdot f_Y(y)$ 不独立.

(4) $Y = Z - X$　　$X < Z - X < 1$

$2X < Z < X+1$



$0 < Z < 1$　　$\int_0^{\frac{z}{2}} 2\,dx = \underline{z}$

$1 < Z < 2$　　$\int_{z-1}^{\frac{z}{2}} 2\,dx = z - 2z + 2 = \underline{2 - z}$

四、(1) 线性相关关系　$|\rho| < 1$　　$|\rho| = 0$

(2) A ✓　　B ×　　$X \sim U(-\frac{1}{2}, \frac{1}{2})$　　$EX = 0$

$Y \sim X^2$　$E(XY) = 0$　　$DX = 1$

$= EX\cdot EY$. X, Y不相关

(3) ① $E(X - 2Y) = EX - 2EY$　　$DY = 4$

$Y \sim \chi^2(2)$　　$= 0 - 2\times 2 = -4$　　$EY = 2$

$D(X - 2Y) = DX + 4DY = 1 + 16 = 17$

② $E(XY) = EX\cdot EY = 2\times 0 = 0$

$D(XY) = E(X^2Y^2) - (E(XY))^2$

$= EX^2\cdot EY^2 = (DX - (EX)^2)(DY + (EY)^2)$

$= 1\times 8 = 8$

③ $\mathrm{Cov}(U,V) = \mathrm{Cov}(X+Y, X-Y)$

$= \mathrm{Cov}(X+Y, X) - \mathrm{Cov}(X+Y, Y)$

$= DX - DY = -3$ $\quad DU = DX + DY = 5$

$DV = DX + DY = 5.$

$\rho = \text{———} = -\frac{3}{5}$

五、

$$\frac{\sum X_i - n\mu}{\sqrt{n}\cdot\sigma} \sim N(0,1)$$

$$\frac{\sum X_i - 100\times 0.5}{10\times 0.1} \geq \frac{51-50}{1} = \Phi(1)$$

$= 1 - \Phi(1) = 1 - 0.8413$

六、 $\bar{X} \sim N(\mu, \frac{\sigma^2}{5})$ $\qquad \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$X_6 - \bar{X} \sim N(0, \frac{6}{5}\sigma^2)$ $\qquad n-1=4$

$$\frac{\dfrac{X_6-\bar{X}}{\sqrt{\frac{6}{5}}\,\sigma}}{\sqrt{\dfrac{\frac{4S^2}{\sigma^2}}{4}}} \sim t(4) \qquad a = \sqrt{\frac{6}{5}}$$

$6+2+5+2+6=12+4+5=21$

t ① $\hat{a}=\bar{x}=\frac{1}{12}(21)=\frac{7}{4}$

② $L(\theta)=\prod_{i=1}^{n}\theta x_i^{\theta-1}=\theta^n\cdot(x_1\cdots x_n)^{\theta-1}$

$\ln L(\theta)=n\ln\theta+(\theta-1)\sum_{i=1}^{n}\ln(x_i)$

$\frac{\partial\ln L(\theta)}{\partial\theta}=\frac{n}{\theta}+\sum_{i=1}^{n}\ln(x_i)=0.$

$\hat{\theta}=-\frac{n}{\sum_{i=1}^{n}\ln x_i}.$

$\hat{\beta}=e^{-\frac{1}{\hat{\theta}}}$

③ 1) $E\mu_1=\frac{1}{3}\mu+\frac{1}{3}\mu+\frac{1}{3}\mu=\mu.$

$E\mu_2=\frac{2}{3}-\frac{5}{9}+\frac{8}{9}=\mu.$

(2) $D\mu_1=\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1}{3}\sigma^2$

$D\mu_2=\frac{4}{9}+\frac{25}{81}+\frac{64}{81}>D\mu_1.$

$\hat{\mu}_1$ 更有效

八、1) 二 一

2) $H_0: \mu = \mu_0 = 500$ $H_1: \mu \neq \mu_0 = 500$

$\sigma = 1.2$

$$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \overset{\text{在}H_0}{\sim} N(0,1)$$

$\geq z_{\frac{\alpha}{2}}$

$$\left|\frac{499.5-500}{\frac{1.2}{3}}\right| = \frac{0.5}{0.4} = 1.25 < 1.96.$$

接收 合格