Base probabilities:

$$P(smuggler) = \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.99 \\ t & 0.01 \end{array} \right) \end{array}$$

$$P(fever) = \begin{array}{cc} fever & P \\ \left(\begin{array}{cc} f & 0.987 \\ t & 0.013 \end{array} \right) \end{array}$$

$$P(dog\_bark \| smuggler) = \begin{array}{ccc} smuggler & dog\_bark & P \\ \left(\begin{array}{ccc} f & f & 0.95 \\ f & t & 0.05 \\ t & f & 0.2 \\ t & t & 0.8 \end{array} \right) \end{array}$$

$$P(sweat \| smuggler, fever) = \begin{array}{cccc} smuggler & fever & sweat & P \\ \left(\begin{array}{cccc} f & f & f & 1 \\ f & f & t & 0 \\ f & t & f & 0.4 \\ f & t & t & 0.6 \\ t & f & f & 0.6 \\ t & f & t & 0.4 \\ t & t & f & 0.2 \\ t & t & t & 0.8 \end{array} \right) \end{array}$$

---

a) $P(smuggler \| dog\_bark = t) = [P(dog\_bark \| smuggler) \cdot P(smuggler)]_{dog\_bark=t}$

$$= \left[\begin{array}{ccc} smuggler & dog\_bark & P \\ \left(\begin{array}{ccc} f & f & 0.95 \\ f & t & 0.05 \\ t & f & 0.2 \\ t & t & 0.8 \end{array} \right) \end{array} * \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.99 \\ t & 0.01 \end{array} \right) \end{array} \right]_{dog\_bark=t}$$

$$= \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.05 \\ t & 0.8 \end{array} \right) \end{array} * \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.99 \\ t & 0.01 \end{array} \right) \end{array}$$

$$= \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.0495 \\ t & 0.008 \end{array} \right) \end{array}$$

$$\xrightarrow{normalized} \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.8609 \\ t & 0.1391 \end{array} \right) \end{array}$$

---

b) $P(sweat) = \sum_{smuggler} \sum_{fever} [P(sweat \| smuggler, fever) \cdot P(smuggler) \cdot P(fever)]$

$$= \sum_{fever} \sum_{smuggler} \left[\begin{array}{cccc} smuggler & fever & sweat & P \\ \left(\begin{array}{cccc} f & f & f & 0.99 \cdot 0.987 \cdot 1 \\ f & f & t & 0.99 \cdot 0.987 \cdot 0 \\ f & t & f & 0.99 \cdot 0.013 \cdot 0.4 \\ f & t & t & 0.99 \cdot 0.013 \cdot 0.6 \\ t & f & f & 0.01 \cdot 0.987 \cdot 0.6 \\ t & f & t & 0.01 \cdot 0.987 \cdot 0.4 \\ t & t & f & 0.01 \cdot 0.013 \cdot 0.2 \\ t & t & t & 0.01 \cdot 0.013 \cdot 0.8 \end{array} \right) \end{array} \right]$$

$$= \sum_{fever} \left[\begin{array}{ccc} fever & sweat & P \\ \left(\begin{array}{ccc} f & f & 0.99 \cdot 0.987 \cdot 1 + 0.01 \cdot 0.987 \cdot 0.6 \\ f & t & 0.99 \cdot 0.987 \cdot 0 + 0.01 \cdot 0.987 \cdot 0.4 \\ t & f & 0.99 \cdot 0.013 \cdot 0.4 + 0.01 \cdot 0.013 \cdot 0.2 \\ t & t & 0.99 \cdot 0.013 \cdot 0.6 + 0.01 \cdot 0.013 \cdot 0.8 \end{array} \right) \end{array} \right]$$

$$= \begin{array}{cc} sweat & P \\ \left(\begin{array}{cc} f & 0.988 \\ t & 0.012 \end{array} \right) \end{array}$$

---

c)

(Be warned: the order of operations is chosen rather poorly)

$P(smuggler \| sweating = t, dog\_bark = t)$

$$= \sum_{fever} [P(dog\_bark \| smuggler) \cdot P(smuggler) \cdot P(sweat \| smuggler, fever) \cdot P(fever)]_{sweating=t, dog\_bark=t}$$

$$= \sum_{fever} \left[\begin{array}{ccc} dog\_bark & smuggler & P \\ \left(\begin{array}{ccc} f & f & 0.95 \cdot 0.99 \\ f & t & 0.2 \cdot 0.01 \\ t & f & 0.05 \cdot 0.99 \\ t & t & 0.8 \cdot 0.01 \end{array} \right) \end{array} \cdot P(sweat \| smuggler, fever) \cdot P(fever) \right]_{sweating=t, dog\_bark=t}$$

$$= \sum_{fever} \left[\begin{array}{ccc} dog\_bark & smuggler & P \\ \left(\begin{array}{ccc} f & f & 0.95 \cdot 0.99 \\ f & t & 0.2 \cdot 0.01 \\ t & f & 0.05 \cdot 0.99 \\ t & t & 0.8 \cdot 0.01 \end{array} \right) \end{array} * \begin{array}{cccc} smuggler & fever & sweat & P \\ \left(\begin{array}{cccc} f & f & f & 1 \cdot 0.987 \\ f & f & t & 0 \cdot 0.987 \\ f & t & f & 0.4 \cdot 0.013 \\ f & t & t & 0.6 \cdot 0.013 \\ t & f & f & 0.6 \cdot 0.987 \\ t & f & t & 0.4 \cdot 0.987 \\ t & t & f & 0.2 \cdot 0.013 \\ t & t & t & 0.8 \cdot 0.013 \end{array} \right) \end{array} \right]_{sweating=t, dog\_bark=t}$$

$$
\begin{aligned}
&= \sum_{fever} \left[\begin{array}{ccc} dog\_bark & smuggler & P \\ \left(\begin{array}{ccc} f & f & 0.95 \cdot 0.99 \\ f & t & 0.2 \cdot 0.01 \\ t & f & 0.05 \cdot 0.99 \\ t & t & 0.8 \cdot 0.01 \end{array} \right) & * & \left(\begin{array}{ccc} smuggler & fever & P \\ f & f & 0 \cdot 0.987 \\ f & t & 0.6 \cdot 0.013 \\ t & f & 0.4 \cdot 0.987 \\ t & t & 0.8 \cdot 0.013 \end{array} \right) \end{array} \right]_{dog\_bark=t} \\
&= \left[\begin{array}{ccc} dog\_bark & smuggler & P \\ \left(\begin{array}{ccc} f & f & 0.95 \cdot 0.99 \\ f & t & 0.2 \cdot 0.01 \\ t & f & 0.05 \cdot 0.99 \\ t & t & 0.8 \cdot 0.01 \end{array} \right) & * & \left(\begin{array}{cc} smuggler & P \\ f & 0 \cdot 0.987 + 0.6 \cdot 0.013 \\ t & 0.4 \cdot 0.987 + 0.8 \cdot 0.013 \end{array} \right) \end{array} \right]_{dog\_bark=t} \\
&= \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.05 \cdot 0.99 \\ t & 0.8 \cdot 0.01 \end{array} \right) & * \end{array} \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0 \cdot 0.987 + 0.6 \cdot 0.013 \\ t & 0.4 \cdot 0.987 + 0.8 \cdot 0.013 \end{array} \right) & \end{array} \\
&= \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.05 \cdot 0.99 \cdot (0 \cdot 0.987 + 0.6 \cdot 0.013) \\ t & 0.8 \cdot 0.01 \cdot (0.4 \cdot 0.987 + 0.8 \cdot 0.013) \end{array} \right) & \end{array} \\
&= \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.0003861 \\ t & 0.0032416 \end{array} \right) & \end{array} \\
&\xrightarrow{normalized} \begin{array}{cc} smuggler & P \\ \left(\begin{array}{cc} f & 0.106 \\ t & 0.894 \end{array} \right) & \end{array}
\end{aligned}
$$