Mathematics-2

Wednesday, 2 December 2021 8:00 PM

Statistics → accuracy / Data (numbers) / $\subset \mathbb{R}$

$P(1,-1,3)$ $\quad \pi: \begin{bmatrix} \\ \\ \end{bmatrix} \quad u$

$2x - 3y + 5z + 8 = 0$

$w = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} \quad u = 8$

$x = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

$d(P, \pi) = \dfrac{w^T x + u}{\|w\|}$

$= \dfrac{\begin{bmatrix} 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + 8}{\sqrt{2^2 + (-3)^2 + 5^2}}$

$= \dfrac{\begin{bmatrix} 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + 8}{\sqrt{4 + 9 + 5^2}}$

$\Rightarrow 6.9$

$\cos\theta = \dfrac{w \cdot u}{\|w\| \|u\|}$

$\Rightarrow \dfrac{2 \times 1 + (-3) \times (-1) + 5 \times 3}{\sqrt{2^2 + (-3)^2 + 5^2} \cdot \sqrt{1^2 + (-1)^2 + 3^2}}$

$\cos\theta \rightarrow \ldots$

$\theta = \cos^{-1}(\ldots)$

dot product

$(u \cdot w)$

np.dot(u,w)

Q2 $\pi_1: 2x + 3y - z = 2$ $\quad \pi_2: 4x + 6y - 2z = 1$

$w_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ $\quad w_2 = \begin{bmatrix} 4 \\ 6 \\ -2 \end{bmatrix}$

$w_1 \cdot w_2 = 0$

$w_1 = -\dfrac{1}{2} w_2$

$\boxed{w_1 = -\dfrac{1}{2} w_2}$

$\boxed{w_1 = 2x + 3y - z = 2}$

$\boxed{w_2 = 4x + 6y - 2z = 1}$

→ same

$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 = 0$

$w_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$w_2 = -\dfrac{1}{2}$

Linear Algebra

(x,y)

(x+h,y+k)

h

k

$\sqrt{h^2 + k^2}$

h = width

k = height

$\sqrt{h^2 + k^2}$ = length

Vectors $\begin{bmatrix} x \\ y \end{bmatrix}$

Dot Prod = $\begin{bmatrix} \text{Indexing} \\ \text{slice} \\ \text{Fancy} \\ \text{etc} \end{bmatrix}$

Strong = $\begin{bmatrix} \text{int} \\ \text{int} \\ \vdots \\ \text{int} \end{bmatrix}$ (numpy)

$\begin{bmatrix} \\ \\ \end{bmatrix}$ $\quad$ $\begin{bmatrix} \\ \\ \end{bmatrix}$

Quick 2D → a vector product

Operations on Vectors

$a + b \Rightarrow \begin{bmatrix} 3+1 \\ -1+4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$a - b \Rightarrow a + (-b)$

$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Scalar Multiplication

a , $\boxed{k.a}$

k

a

$2a$

$-a$ $\quad$ $-a$

Unit Vector

$\dfrac{1}{\sqrt{\ }} \cdot \begin{bmatrix} \\ \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

y

x

* Vector Space:

↳ Linear Combination

$a = 3x + 2y$

$\boxed{\mathbb{R}^2}$

$x \in \mathbb{R}^2$

$a \in \mathbb{R}^2$

$b \in \mathbb{R}^2$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Scalar Dot Product

y

x

$x^2 + y^2$

dot Product:

↳ Cosine-similarity $\boxed{\cos\theta = \dfrac{a \cdot b}{\|a\| \|b\|}}$

$\cos\theta$

plane

Parallel

vector

orthogonal

* Projection of a Vector:

x_1

x_2

θ

θ_1

θ_2

Proj (x_1) on

$x_1 \cdot x_2 \cdot \theta$

$\cos\theta = \dfrac{\theta}{\ }$

{Scalar Component}

$\dfrac{\|x_1\|}{\|x_2\|} = \|x_1\| \cos\theta$

$\text{Proj} = \dfrac{\|x_1\| \cos\theta \times \|x_2\|}{\|x_2\|}$

{Scalar projection}

$\boxed{\text{Proj}_{x_2} x = \dfrac{x_1 \cdot x_2}{\|x_2\|}}$

Vector projection

$\left(x_1 \cos\theta\right) \left(\dfrac{x_2}{\|x_2\|}\right)$

$\Rightarrow \dfrac{x_1 \cdot x_2}{\|x_2\|} \times \dfrac{x_2}{\|x_2\|}$

$\Rightarrow \boxed{\left(\dfrac{x_1 \cdot x_2}{\|x_2\|^2}\right) x_2}$

Matrix:

$M = \begin{bmatrix} a_{11} & & a_{1n} \\ & \ddots & \\ a_{m1} & & a_{mn} \end{bmatrix}$

Transpose → rows ↔ clm

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$(A^T) = A$

$(AB)^T = B^T A^T$

Symmetric: $A^T = A$

$\begin{bmatrix} 1 & 7 & 8 \\ 7 & 2 & 4 \\ 8 & 4 & 3 \end{bmatrix}$

$A_{ij} = A_{ji}$

System of Linear Eq:

$x_1 + x_2 + 3x_3 = 15$

$x_1 + 2x_2 + 4x_3 = 21$

$x_1 + x_2 + 2x_3 = 13$

Solve for $\{x_1, x_2, x_3\}$

$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -15 \\ -21 \\ -13 \end{bmatrix} = 0$

$w^T x + w_0$

$W \cdot X + W_0$

$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 13 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 2 \end{bmatrix}$

A $\quad$ b

$x_1 + x_2 + 3x_3 = 15$

$\boxed{x_3 = 2}$

$\boxed{x_2 = 7}$

$\boxed{x_1 = 2}$

np.linalg.solve(A, b)

$A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 5 \\ 2 & 0 & 0 \end{bmatrix}$

$|A|$ = Determinant

Inverse of a matrix =

$A \cdot A^{-1} = I$

$\begin{bmatrix} \\ \\ \end{bmatrix} \cdot \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A $\quad$ A^{-1}

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A^{-1} $\quad$ A

np.linalg.inv(A)

Singular Matrix → Inverse doesn't exist

$\boxed{A^{-1} = \dfrac{1}{|A|} \text{adj}(A)}$

Adjoint

$|A| = 0$

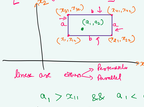
adjoint A

[Gilbert Strang] → Book

Vector MIT lecture

Plane Inverse (Move Forward)

→ np.linalg.pinv(A)



(x_1, y_1)

(x, y)

(x_2, y_2)

(x_3, y_3)

(x_4, y_4)

x_1

Break one $\quad$ Connect $\quad$ Parallelogram

Parallel

$a_{11} > a_{11}$ $\quad$ & $\quad$ $a_{11} < a_{21}$

r

x

c

$\|x - c\|$

$\|0x - c\|$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$\|x - c\|$

$\Rightarrow \left\| \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \right\|$

$\Rightarrow \left\| \begin{bmatrix} x_1 - c_1 \\ x_2 - c_2 \end{bmatrix} \right\|$

$\boxed{r = \sqrt{(x_1 - c_1)^2 + (x_2 - c_2)^2}}$

$r^2 = (x_1 - c_1)^2 + (x_2 - c_2)^2$

if Center origin $\quad c_1 = c_2 = 0$

$\boxed{r^2 = x_1^2 + x_2^2}$

$\boxed{r = \|x - c\|}$

$d = 2$

$d > 3$

$d \geq 4$ Higher

e.g. $\mathbb{R}^n$ of circle is your

Point is given

x_1

x_2

Find Point lies

① Inside circle

② outside circle

③ on circle