Assume we have $T^1_{A_i}$ and want $T^1_{C_i}$ but we can only get $T^{C_i}_{A_i}$

then $T^1_{C_i} = T^1_{A_i} \left(T^{C_i}_{A_i}\right)^{-1}$

Assume we have $T^1_{C_i}$ and want $T^1_{A_i}$ but we can only get $T^{C_i}_{A_i}$

$$T^1_{A_i} = T^1_{C_i} T^{C_i}_{A_i}$$

Projection:

Let p^1 be a corner in the base frame

$\left(p^1 \text{ is actually } T^1_{A_i} \begin{bmatrix} \pm S/2 \\ \pm S/2 \\ 0 \end{bmatrix} \right)$

$$p^1 = T^1_{A_i} \begin{bmatrix} \pm S/2 \\ \pm S/2 \\ 0 \end{bmatrix} = a(T^1_{A_i})$$

$$p^{C_i} = \left(T^1_{C_i}\right)^{-1} p^1 = h(p^1, T^1_{C_i})$$

$$p^{C_i}_n = \frac{p^{C_i}}{p^{C_i}_z} = \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} = g(p^{C_i})$$

$$\begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix} = K \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} = b(p^{C_i}_n)$$

$$\begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix} = b(g(h(a(T^1_{A_i}), T^1_{C_i})))$$

For each feature, there are 12 variables that effect it and there are 4N features when N is the # of tags

I have an abuse of notation. Both image index and tag index are denoted i

From now on, image index is i and tag id is t

$p^S = b(g(h(a(T^1_{A_t}), T^1_{C_i})))$ depends on

$$\underbrace{\psi^{A_t}, \theta^{A_t}, \phi^{A_t}}_{q^{A_t}_R}, \underbrace{x^{A_t}, y^{A_t}, z^{A_t}}_{q^{A_t}_t}, \underbrace{\psi^{C_i}, \theta^{C_i}, \phi^{C_i}}_{q^{C_i}_R}, \underbrace{x^{C_i}, y^{C_i}, z^{C_i}}_{q^{C_i}_t}$$

$$T^1_{At}\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} = R^1_{At}\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} + t^1_{At} = p^1 \qquad R^1_{At} = R_z(\psi^{At})\,R_y(\theta^{At})\,R_x(\phi^{At})$$

$$(T^1_{C_i})^{-1}p^1 = R^{1T}_{C_i}\left(p^1 - t^1_{C_i}\right) = p^{C_i} = R^{1T}_{C_i}\left(R^1_{At}\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} + t^1_{At} - t^1_{C_i}\right)$$

$$y = K p^{C_i}$$

$$x_{it} = \frac{y}{y_z} = g(y) \Rightarrow x_{it} = g\left(K R^{1T}_{C_i}\left[R^1_{At}\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} + t^1_{At} - t^1_{C_i}\right]\right)$$

$$q^{At}_R$$

$$\frac{\partial x_{it}}{\partial \psi^{At}} = \frac{\partial x_{it}}{\partial y}\frac{\partial y}{\partial p^{C_i}}\frac{\partial p^{C_i}}{\partial p^1}\frac{\partial p^1}{\partial \psi^{At}}$$

$$\frac{\partial x_{it}}{\partial y} = \frac{\partial}{\partial y}\begin{bmatrix}\frac{y_x}{y_z}\\ \frac{y_y}{y_z}\\ 1\end{bmatrix} = \begin{bmatrix}\frac{1}{y_z} & 0 & -\frac{y_x}{y_z^2}\\ 0 & \frac{1}{y_z} & -\frac{y_y}{y_z^2}\\ 0 & 0 & 0\end{bmatrix}$$

$$\frac{\partial y}{\partial p^{C_i}} = K$$

$$\frac{\partial p^{C_i}}{\partial p^1} = R^{1T}_{C_i}$$

$$\frac{\partial p^1}{\partial \psi^{At}} = \frac{\partial}{\partial \psi^{At}}\left(R^1_{At}\right)\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} = \frac{\partial}{\partial \psi^{At}}\left(R_z(\psi^{At})\right)R_y(\theta^{At})R_x(\phi^{At})\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} = S\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)R_z(\psi^{At})R_y(\theta^{At})R_x(\phi^{At})\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix}$$

$$q^{At}_t$$

$$\frac{\partial x_{it}}{\partial q^{At}_t} = \underbrace{\frac{\partial x_{it}}{\partial y}\frac{\partial y}{\partial p^{C_i}}\frac{\partial p^{C_i}}{\partial p^1}}_{\text{known}}\frac{\partial p^1}{\partial q^{At}_t}$$

$$\frac{\partial p^1}{\partial q^{At}_t} = \frac{\partial}{\partial t^1_{At}}\left(R^1_{At}\begin{bmatrix}\pm S/2\\ \pm S/2\\ 0\end{bmatrix} + t^1_{At}\right) = \underline{1}$$

$$q^{C_i}_R$$

$$\frac{\partial x_{it}}{\partial \psi^{C_i}} = \underbrace{\frac{\partial x_{it}}{\partial y}\frac{\partial y}{\partial p^{C_i}}}_{\text{known}}\frac{\partial p^{C_i}}{\partial \psi^{C_i}}$$

$$\frac{\partial p^{C_i}}{\partial \psi^{C_i}} = \frac{\partial}{\partial \psi^{C_i}}\left(R^{1T}_{C_i}\left(p^1 - t^1_{C_i}\right)\right) = -R_x(-\phi^{C_i})\,R_y(-\theta^{C_i})\,S\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)R_z(\psi^{C_i})\,\cancel{p^1}\;\left(p^1 - t^1_{C_i}\right)$$

$$q^{C_i}_t$$

$$\frac{\partial x_{it}}{\partial q^{C_i}_t} = \underbrace{\frac{\partial x_{it}}{\partial y}\frac{\partial y}{\partial p^{C_i}}}_{\text{known}}\frac{\partial p^{C_i}}{\partial q^{C_i}_t}$$

$$\frac{\partial p^{C_i}}{\partial q^{C_i}_t} = \frac{\partial}{\partial t^{C_i}}\left(R^{1T}_{C_i}\left(p^1 - t^1_{C_i}\right)\right) = -R^{T}_{C_i}$$