Варіант 47

① $\vec{r} = \{3a\cos t, 3a\sin t, 4at\}$

1) $r'(t) = \{-3a\sin t, 3a\cos t, 4a\}$

2) $|r'(t)| = \sqrt{9a^2\sin^2 t + 9a^2\cos^2 t + 16a^2} =$

$= \sqrt{25a^2} = 5a$

3) $s(t) = \int_0^t |r'|\,dt = \int_0^t 5a\,dt = 5at \Rightarrow t = \frac{s}{5a}$

Отримаємо:

$r = \{3a\cos\frac{s}{5a}, 3a\sin\frac{s}{5a}, \frac{4}{5}s\}$

4) Одиничний дотичний вектор:

$t = \frac{dr}{ds} = \{-\frac{3}{5}\sin\frac{s}{5a}, \frac{3}{5}\cos\frac{s}{5a}, \frac{4}{5}\}$

5) Вектор нормалі:

$n = \frac{t'}{|t'|} = \{-\cos\frac{s}{5a}, -\sin\frac{s}{5a}, 0\}$

$t' = \{-\frac{3}{25a}\cos\frac{s}{5a}, -\frac{3}{25}\sin\frac{s}{5a}, 0\}$

$|t'| = \sqrt{\frac{9}{625a^2}(\cos^2\frac{s}{5a} + \sin^2\frac{s}{5a})} = \frac{3}{25a}$

6) Кривизна:

$K = |t'| = \frac{3}{25a}$

7) Вектор бінормалі:

$$b = t \times n = \begin{vmatrix} i & j & k \\ -\frac{3}{5}\sin\frac{s}{5a} & \frac{3}{5}\cos\frac{s}{5a} & \frac{4}{5} \\ -\cos\frac{s}{5a} & -\sin\frac{s}{5a} & 0 \end{vmatrix} =$$

$$= \frac{4}{5}\sin\frac{s}{5a}\,i - \frac{4}{5}\cos\frac{s}{5a} + \frac{3}{5}k =$$

$= \left\{ \frac{4}{5}\sin\frac{5}{5}a ; -\frac{4}{5}\cos\frac{5}{5}a ; \frac{3}{5} \right\}$

2) Скрут:

$$\tau = \frac{(r' r'' r''')}{r'' \cdot r''} = \frac{36a^3}{9a^2} = 4a$$

$r' = \{-3a\sin t, 3a\cos t, 4a\}$

$r'' = \{-3a\cos t, -3a\sin t, 0\}$

$r''' = \{3a\sin t, -3a\cos t, 0\}$

$$(r' r'' r''') = \begin{vmatrix} -3a\sin t & 3a\cos t & 4a \\ -3a\cos t & -3a\sin t & 0 \\ 3a\sin t & -3a\cos t & 0 \end{vmatrix} =$$

$= -3a\sin t \cdot 0 + 0 + (9a^2\cos^2 t + 9a^2\sin t) \cdot 4a$

$= 36a^3$

$r'' \cdot r'' = 9a^2\cos^2 t + 9a^2\sin t + 0 = 9a^2$

② $\vec{r} = \{u\cos v, u\sin v, av\}$

$\bar{r}_u = \{\cos v, \sin v, 0\}$

$\bar{r}_v = \{-u\sin v, u\cos v, a\}$

$$[\bar{r}_u, r_v] = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & a \end{vmatrix} =$$

$= \sin v \cdot a \cdot i - \cos v \cdot a \cdot j + u \cdot k =$

$= \{a\sin v, -a\cos v, u\} \neq 0$

$|[\bar{r}_u, r_v]| = \sqrt{a^2 + u^2}$

1)

$$\bar{R}(\lambda,\phi) = \bar{r}(u_0,v_0) + \lambda\bar{r}_u + \phi\bar{r}_v =$$
$$= \{u_0\cos v_0, u_0\sin v_0, av_0\} +$$
$$+\lambda\{\cos v_0, \sin v_0, 0\} + \phi\{-u_0\sin v_0, u_0\cos v_0, a\}$$

Запишемо у вигляді системи:

$$\begin{cases} x = u_0\cos v_0 + \lambda\cos v_0 - u\sin v_0\cdot\phi \\ y = u_0\sin v_0 + \lambda\sin v_0 + \phi u_0\cos v_0 \\ z = av_0 + \phi a \end{cases}$$

рівняння дотичної площини

2) Вектор нормалі:

$$N = \frac{a\sin v, -a\cos v, u}{\sqrt{a^2+u^2}}$$

3) $E = r_u \cdot r_u = 1$

$G = u^2\sin^2 v + u^2\cos^2 v + a^2 = u^2 + a^2$

$F = r_u \cdot r_v = -u\cos v\sin v + u\cos v\sin v + 0 = 0$

$R_{uu} = \{0,0,0\}$

$R_{uv} = \{-\sin v, \cos v, 0\}$

$R_{vv} = \{-u\cos v, -u\sin v, 0\}$

$L = (R_{uu}, n) = \{0,0,0\} = 0$

$$M = \left(\frac{a\sin v}{\sqrt{a^2+u^2}}, \frac{-a\cos v}{\sqrt{a^2+u^2}}, \frac{u}{\sqrt{a^2+u^2}}\right)\cdot(-\sin v, \cos v, 0) =$$

$$= \frac{-a^2\sin^2 v - a^2\cos^2 v}{\sqrt{a^2+u^2}} = \frac{-a^2}{\sqrt{a^2+u^2}}$$

$$N=\left(\frac{a\sin v}{\sqrt{a^2+u^2}}, \frac{-a\cos v}{\sqrt{a^2+u^2}}, \frac{u}{\sqrt{a^2+u^2}}\right)\cdot$$

$$\cdot(-u\cos v, -u\sin v, 0)=$$

$$=\frac{-a\sin v\cdot u\cos v + ua\sin v\cos v}{\sqrt{a^2+u^2}}=0$$

$$k_H=\frac{L+2M\lambda+N\lambda^2}{E+2F\lambda+G\lambda^2}=\frac{0+\frac{-2a^2\cdot\lambda}{\sqrt{a^2+u^2}}+0}{1+(u^2+a^2)\lambda^2}$$

$$=\frac{\left(-\frac{2a^2\cdot\lambda}{\sqrt{a^2+u^2}}\right)(1+u^2\lambda^2+a^2\lambda^2)}{}$$

4)

$$K=\frac{-a^4(a^2+u^2)}{a^2+u^2}=-a^4$$ — Гаусова кривина

$H=0$ — середня кривина

5)

$k_1=-a^2 \quad k_2=a^2$