HW 4

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ME380

1.a) ....

$G(s) = \frac{(s+2)(s+5)}{s(s+1)(s+3)}$

The poles of $G(s)$ are at 0, -1, and -3. Thus, there is only 1 pole at $s = 0$. Thus, for $\frac{Y(s)}{R(s)}$, $\boxed{\text{the type number is 1}}$.

1.b) ....

$e_{ss} \leq 0.1$

i) Unit step input

Since $G(s)$ is a type 1 system, the error of a step input is 0. Thus, $\boxed{\text{any } K > 0}$ will satisfy the error requirement, since $0 \leq 0.1$ is always true.

ii) Unit ramp input

$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{1}{sKG(s)}$

$= \lim_{s \to 0} \frac{s(s+1)(s+3)}{sK(s+2)(s+5)}$

$= \lim_{s \to 0} \frac{(s+1)(s+3)}{K(s+2)(s+5)}$

$= \frac{(1)(3)}{K(2)(5)}$

$= \frac{3}{10K}$

$\therefore e_{ss} \leq 0.1 \implies \frac{3}{10K} \leq 0.1$

$\therefore \boxed{K \geq 3}$

2.a) ....

$$G(s) = \frac{200}{s^2+20s+200}$$
$$G_R(s) = \frac{Y(s)}{R(s)}$$
$$G_D(s) = \frac{Y(s)}{D(s)}$$
$$Y(s) = \left(\left((R(s) - Y(s)) \cdot \frac{K}{0.1s+1}\right) + D(s)\right) \cdot G(s)$$
$$Y(s) = G(s)(R(s) - Y(s)) \cdot \frac{K}{0.1s+1} + G(s)D(s)$$
$$Y(s) + \frac{G(s)K}{0.1s+1} \cdot Y(s) = \frac{G(s)R(s)K}{0.1s+1} + G(s)D(s)$$
$$Y(s) = \frac{\frac{G(s)R(s)K}{0.1s+1} + G(s)D(s)}{1+\frac{G(s)K}{0.1s+1}}$$
$$Y(s) = \left(\frac{G(s)R(s)K}{0.1s+1} + G(s)D(s)\right) \cdot \frac{0.1s+1}{0.1s+1+G(s)K}$$
$$Y(s) = \frac{G(s)R(s)K}{0.1s+1+G(s)K} + \frac{(0.1s+1)G(s)D(s)}{0.1s+1+G(s)K}$$
$$Y(s) = \frac{K}{\frac{0.1s+1}{\frac{200}{s^2+20s+200}}+K} \cdot R(s) + \frac{0.1s+1}{\frac{0.1s+1}{\frac{200}{s^2+20s+200}}+K} \cdot D(s)$$
$$Y(s) = \frac{K}{\frac{(0.1s+1)(s^2+20s+200)}{200}+K} \cdot R(s) + \frac{0.1s+1}{\frac{(0.1s+1)(s^2+20s+200)}{200}+K} \cdot D(s)$$
$$Y(s) = \frac{200K}{0.1s^3+2s^2+20s+s^2+20s+200+200K} \cdot R(s) + \frac{20s+200}{0.1s^3+2s^2+20s+s^2+20s+200+200K} \cdot D(s)$$
$$Y(s) = \frac{200K}{0.1s^3+3s^2+40s+200+200K} \cdot R(s) + \frac{20s+200}{0.1s^3+3s^2+40s+200+200K} \cdot D(s)$$
$$Y(s) = \frac{2000K}{s^3+30s^2+400s+2000+2000K} \cdot R(s) + \frac{200s+2000}{s^3+30s^2+400s+2000+2000K} \cdot D(s)$$
$$\therefore$$
$$\boxed{G_R(s) = \frac{2000K}{s^3+30s^2+400s+2000+2000K}}$$
$$\boxed{G_D(s) = \frac{200s+2000}{s^3+30s^2+400s+2000+2000K}}$$

2.b) ....

The characteristic equation of $G_R(s)$ is $\boxed{\text{the same}}$ as the characteristic equation of $G_D(s)$

2.c) ....

$$y_{ss} \leq 0.01$$
$$e_{ss} \leq 0.04$$
$$y_{ss} = \lim_{s\to 0} sY(s)$$
$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sR(s) - \lim_{s\to 0} sY(s)$$
$$\therefore e_{ss} = \lim_{s\to 0} sR(s) - y_{ss}$$
$$r_{ss} = \lim_{s\to 0} sR(s) = \lim_{t\to\infty} \mathbf{1}(t) = 1$$
$$\therefore e_{ss} = 1 - y_{ss}$$
$$\therefore 0.04 \geq 1 - y_{ss} \implies 0.96 \leq y_{ss} \implies \perp$$

Thus, there are $\boxed{\text{no values of K}}$ to satisfy both requirements.

3.a) ....

$E(s) = R(s) - Y(s)$

$Y(s) = (KR(s) - KY(s) - BY(s)) \cdot A \cdot \frac{1}{s}$

$Y(s) = A\frac{1}{s}KR(s) - A\frac{1}{s}KY(s) - A\frac{1}{s}BY(s)$

$Y(s)\left(1 + \frac{AK}{s} + \frac{AB}{s}\right) = \frac{AK}{s}R(s)$

$Y(s) = \frac{AK}{s+AK+AB}R(s)$

$\therefore E(s) = R(s) - \frac{AK}{s+AK+AB}R(s)$

$= R(s)\left(1 - \frac{AK}{s+AK+AB}\right)$

$= R(s)\frac{s+AB}{s+AK+AB}$

$\therefore \boxed{G_E(s) = \frac{s+AB}{s+AK+AB}}$

3.b) ....

$E(s) = G_E(s)R(s)$

$r(t) = 2 \cdot \mathbf{1}(t)$

$R(s) = \frac{2}{s}$

$E(s) = \frac{s+AB}{s+AK+AB} \cdot \frac{2}{s}$

$e_{ss} = \lim_{s\to 0} sE(s)$

$= \lim_{s\to 0} \frac{s+AB}{s+AK+AB} \cdot 2$

$\therefore \boxed{e_{ss} = \frac{2B}{K+B}}$

3.c) ....

$r(t) = 2t \cdot \mathbf{1}(t)$

$R(s) = \frac{2}{s^2}$

$E(s) = \frac{s+AB}{s+AK+AB} \cdot \frac{2}{s^2}$

$e_{ss} = \lim_{s\to 0} \frac{s+AB}{s+AK+AB} \cdot \frac{2}{s}$

$= \lim_{s\to 0} \frac{2s+2AB}{s^2+AKs+ABs}$

$= \frac{2AB}{0}$

$\therefore \boxed{e_{ss} = \infty}$

3.d) ....

$r(t) = 2t^2 \cdot \mathbf{1}(t)$

$R(s) = \frac{4}{s^3}$

$E(s) = \frac{s+AB}{s+AK+AB} \cdot \frac{4}{s^3}$

$e_{ss} = \lim_{s\to 0} \frac{s+AB}{s+AK+AB} \cdot \frac{4}{s^2}$

$= \lim_{s\to 0} \frac{4s+4AB}{s^3+AKs^2+ABs^2}$

$= \frac{4AB}{0}$

$\therefore \boxed{e_{ss} = \infty}$

4.a) ....

$G(s) = \frac{1}{s+2}$
$G_c(s) = \frac{K}{s+10}$

i)
The open-loop transfer function $G(s)\,G_c(s)$ is $\frac{K}{(s+2)(s+10)}$ which has no poles at $s = 0$, thus the system is $\boxed{\text{type number } 0}$.

ii)
$e_{ss} \leq 0.2$
Since the system is type 0, the response to a step input is:

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+G(s)G_c(s)}$$
$$= \lim_{s\to 0} \frac{1}{1+\frac{K}{(s+2)(s+10)}}$$
$$= \lim_{s\to 0} \frac{(s+2)(s+10)}{(s+2)(s+10)+K}$$
$$= \frac{(2)(10)}{(2)(10)+K}$$
$$= \frac{20}{20+K}$$
$$\therefore 0.2 \geq \frac{20}{20+K} \implies \boxed{80 \leq K}$$

4.b) ....

$G_c(s) = \frac{K}{s(s+10)}$

i)
The open-loop transfer function $G(s)\,G_c(s)$ is $\frac{K}{s(s+2)(s+10)}$ which has one poles at $s = 0$, thus the system is $\boxed{\text{type number } 1}$.

ii)
$e_{ss} \leq 0.2$
Since the system is type 1, the response to a step input is 0. Thus, since $0 \leq 0.2$ is always true, $\boxed{\text{any } K > 0}$ will satisfy the requirement.

4.c) ....

i)
For $G_c(s) = \frac{K}{s+10}$:

$$\frac{Y(s)}{R(s)} = \frac{G(s)G_c(s)}{1+G(s)G_c(s)}$$
$$= \frac{\frac{K}{(s+2)(s+10)}}{1+\frac{K}{(s+2)(s+10)}}$$
$$= \frac{1}{\frac{(s+2)(s+10)}{K}+1}$$
$$= \frac{K}{(s+2)(s+10)+K}$$
$$= \boxed{\frac{K}{s^2+12s+20+K}}$$

For $G_c(s) = \frac{K}{s(s+10)}$:

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s(s+2)(s+10)}}{1+\frac{K}{s(s+2)(s+10)}}$$
$$= \frac{1}{\frac{s(s+2)(s+10)}{K}+1}$$
$$= \frac{K}{s(s+2)(s+10)+K}$$
$$= \boxed{\frac{K}{s^3+12s^2+20s+K}}$$

ii)

Attached at end.

4.d) ....

The system with $G_c(s) = \frac{K}{s(s+10)}$ has better steady-state performance, since the output settles to 1, meaning the error is 0, while the system with $G_c(s) = \frac{K}{s+10}$ always has an offset in the output. This matches the steady-state error results from (b).