

**Theorem:** There is an infinite amount of positive primes.

*proof* Assume there is a finite amount of primes  $\mathbf{P} = \{2, 3, 5, \dots, p_n\}$ . Let  $q = 2 \cdot 3 \cdot 5 \cdots p_n + 1$  be the product of all the primes plus 1. By construction,

$$q \equiv 1 \pmod{p_i}$$

for any prime  $p_i \in \mathbf{P}$ . Thus  $q$  is not divisible by any prime, and so it follows from the fundamental theorem of arithmetic that  $q$  must itself be prime. However, clearly  $q > p_i$  for all  $p_i \in \mathbf{P}$ , so  $q \notin \mathbf{P}$ , but  $\mathbf{P}$  is the set of all primes. Thus there cannot be a finite amount of primes.  $\square$