

ADVANCE CALCULUS

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$$1. \int \frac{e^x dx}{\sqrt{1-e^x}} = \int \frac{1}{\sqrt{1-e^x}} e^x dx$$

$$\begin{aligned} u = 1 - e^x &= \int \frac{1}{\sqrt{u}} (-du) \\ du = -e^x dx &= - \int \frac{1}{u^{1/2}} du \\ -du = e^x dx &= - \int u^{-1/2} du \\ &= -(2u^{1/2}) + C \\ &= -2\sqrt{u} + C \\ &= -2\sqrt{1-e^x} + C \end{aligned}$$

$$2. \int \frac{\sec^2 x dx}{\tan x} = \int \frac{1}{\tan x} \sec^2 x dx$$

$$\begin{aligned} u = \tan x &= \int \frac{1}{u} du \\ du = \sec^2 x dx &= \ln(u) + C \\ &= \ln(\tan x) + C \end{aligned}$$

$$3. \int \sqrt{3x+1} dx = \int (3x+1)^{1/2} dx$$

$$\begin{aligned} u = 3x+1 &= \int u^{1/2} \frac{du}{3} \\ du = 3 dx &= \frac{1}{3} \int u^{1/2} du \\ \frac{du}{3} = dx &= \frac{1}{3} \left(\frac{2u^{3/2}}{3} \right) + C \\ &= \frac{2\sqrt{u^3}}{9} + C \\ &= \frac{2\sqrt{(3x+1)^3}}{9} + C \end{aligned}$$

$$4. \int \frac{\ln x}{x} dx = \int \ln x \left(\frac{1}{x} dx \right)$$

$$\begin{aligned} u = \ln x &= \int u du \\ du = \frac{1}{x} dx &= \frac{u^2}{2} + C \\ &= \frac{\ln(x)^2}{2} + C \end{aligned}$$

$$5. \int \left(1 - \frac{1}{x}\right)^2 dx$$

$$= \int \left(1 - \frac{1}{x}\right) \left(1 - \frac{1}{x}\right) dx$$

$$= \int \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx$$

$$= x - 2 \ln x + \left(\frac{x^{-1}}{-1} \right) + C$$

$$= x - 2 \ln x - \frac{1}{x} + C$$