QUIZ # 1: ADVANCE CALCULUS

NO.:
DATE:

2BSCS-2 | Marasigan, Vem Aiensi A.

1.)

a.) $\int 2x(x^2+1)^{23}\,dx = \int (x^2+1)^{23}\,2x\,dx$

$u = x^2+1$

$du = 2x\,dx$

$= \int u^{23}\,du$

$= \dfrac{u^{23+1}}{23+1} + C$

$= \dfrac{u^{24}}{24} + C$

$= \boxed{\dfrac{(x^2+1)^{24}}{24} + C}$

b.) $\int \cos^3 x \sin x\,dx = \int u^3 - du$

$u = \cos x$

$du = -\sin x\,dx$

$= -\int u^3\,du$

$= -\dfrac{u^{3+1}}{3+1} + C$

$= -\dfrac{u^4}{4} + C$

$= \boxed{-\dfrac{\cos^4 x}{4} + C}$

2.) $\displaystyle\int \frac{dx}{(x^2+4)^{3/2}}$

Using trigonometric substition: $\sqrt{u^2+a^2}$

$a = 2 \quad a^2 = 4$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$= \displaystyle\int \frac{2\sec^2\theta\,d\theta}{((2\tan\theta)^2+2^2)^{3/2}}$

$= \displaystyle\int \frac{2\sec^2\theta\,d\theta}{(4\tan^2\theta+4)^{3/2}}$

$= \displaystyle\int \frac{2\sec^2\theta}{\sqrt{(4(\tan^2\theta+1))^3}}\,d\theta$

$= \displaystyle\int \frac{2\sec^2\theta}{\sqrt{(4\sec^2\theta)^3}}\,d\theta$

$= \displaystyle\int \frac{2\sec^2\theta}{4\sec^2\theta\sqrt{4\sec^2\theta}}\,d\theta$

$= \displaystyle\int \frac{2\sec^2\theta}{4\sec^2\theta(2\sec\theta)}\,d\theta$

$= \displaystyle\int \frac{2\cancel{\sec^2\theta}}{2(2\cancel{\sec^2\theta})(2\sec\theta)}\,d\theta$

$= \displaystyle\int \frac{1}{2(2\sec\theta)}\,d\theta$

$= \displaystyle\int \frac{1}{4\sec\theta}\,d\theta$

$= \displaystyle\frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta$

$= \displaystyle\frac{1}{4}\int \cos\theta\,d\theta$

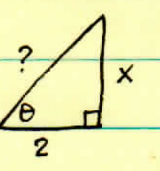
$= \dfrac{1}{4}(\sin\theta) + C$

$= \dfrac{\sin\theta}{4} + C$

*find $\sin\theta$

$x = 2\tan\theta$

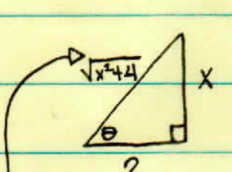
$\tan\theta = \dfrac{x \text{ - opposite}}{2 \text{ - adjacent}}$



*find c or hypotenuse

$c^2 = a^2 + b^2$

$c = \sqrt{x^2+4}$



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$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \dfrac{x}{\sqrt{x^2+4}}$

*meaning

$\dfrac{\sin\theta}{4} + C = \dfrac{\frac{x}{\sqrt{x^2+4}}}{4} + C$

$= \boxed{\dfrac{x}{4\sqrt{x^2+4}} + C}$

DATE:

3.) $\displaystyle\int_5^9 \frac{dx}{x\sqrt{x^2+144}}$ trigonometric substitution: $\sqrt{u^2+a^2}$

$a^2 = 144,\ a = 12,\ x = 12\tan\theta,\ dx = 12\sec^2\theta\,d\theta$

* Integral first

$= \displaystyle\int \frac{12\sec^2\theta\,d\theta}{12\tan\theta\sqrt{(12\tan\theta)^2+12^2}}$

$= \displaystyle\int \frac{12\sec^2\theta\,d\theta}{12\tan\theta\sqrt{144\tan^2\theta+144}}$

$= \displaystyle\int \frac{12\sec^2\theta\,d\theta}{12\tan\theta\sqrt{144(\tan^2\theta+1)}}$

$= \displaystyle\int \frac{12\sec^2\theta\,d\theta}{12\tan\theta\sqrt{144\sec^2\theta}}$

$= \displaystyle\int \frac{12\sec^2\theta\,d\theta}{12\tan\theta(12\sec\theta)}$

$= \displaystyle\int \frac{\sec\theta(12\cancel{\sec\theta})\,d\theta}{12\tan\theta(12\cancel{\sec\theta})}$

$= \displaystyle\int \frac{\sec\theta}{12\tan\theta}\,d\theta$

$= \displaystyle\frac{1}{12}\int \frac{\sec\theta}{\tan\theta}\,d\theta$

$= \displaystyle\frac{1}{12}\int \sec\theta\left(\frac{1}{\tan\theta}\right)d\theta$

$= \displaystyle\frac{1}{12}\int \sec\theta(\cot\theta)\,d\theta$

$= \displaystyle\frac{1}{12}\int \frac{1}{\cancel{\cos\theta}}\left(\frac{\cancel{\cos\theta}}{\sin\theta}\right)d\theta$

$= \displaystyle\frac{1}{12}\int \frac{1}{\sin\theta}\,d\theta$

$= \displaystyle\frac{1}{12}\int \csc\theta\,d\theta$

$\displaystyle\frac{1}{12}\int \csc\theta\,d\theta$

Thinking about the * multiplier: $1 = \dfrac{\csc\theta+\cot\theta}{\csc\theta+\cot\theta}$

$u = \csc\theta$

$du = -\csc\theta\cot\theta$

$u = \cot\theta$

$du = -\csc^2\theta$

So: $u = \csc\theta + \cot\theta$

$du = -\csc\theta\cot\theta - \csc^2\theta$

$= \displaystyle\frac{1}{12}\int \csc\theta\left(\frac{\csc\theta+\cot\theta}{\csc\theta+\cot\theta}\right)d\theta$

$= \displaystyle\frac{1}{12}\int \frac{\csc^2\theta+\csc\theta\cot\theta}{\csc\theta+\cot\theta}\,d\theta$

$u = \csc\theta + \cot\theta$

$du = -\csc\theta\cot\theta - \csc^2\theta\,d\theta$

$= \displaystyle\frac{1}{12}\int (-1)\frac{(-\csc^2\theta - \csc\theta\cot\theta\,d\theta)}{\csc\theta+\cot\theta}$

$= \displaystyle\frac{1}{12}(-1)\int \frac{du}{u}$

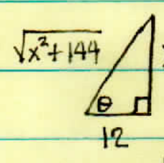
$= \displaystyle-\frac{1}{12}\int \frac{1}{u}\,du$

$= -\dfrac{1}{12}\ln u + C$

$= -\dfrac{1}{12}\ln(\csc\theta + \cot\theta) + C$

*finding $\csc\theta$:

$x = 12\tan\theta \Rightarrow \tan\theta = \dfrac{x}{12}$



$c^2 = a^2 + b^2$

$c = \sqrt{x^2+144}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}}$

$\csc\theta = \dfrac{\sqrt{x^2+144}}{x}$

*finding $\cot\theta$

$\cot\theta = \dfrac{\text{Adjacent}}{\text{opposite}}$

$\cot\theta = \dfrac{12}{x}$

$= -\dfrac{1}{12}\ln\left(\dfrac{\sqrt{x^2+144}}{x} + \dfrac{12}{x}\right) + C$

$= -\dfrac{1}{12}\ln\left(\dfrac{\sqrt{x^2+144}+12}{x}\right) + C$

Definite Integral $= F(b) - F(a)$ $b = 9$ $a = 5$

$\displaystyle\int_5^9 \frac{dx}{x\sqrt{x^2+144}} = -\frac{1}{12}\ln\left(\frac{\sqrt{9^2+144}+12}{9}\right) + \frac{1}{12}\ln\left(\frac{\sqrt{5^2+144}+12}{5}\right)$

$= -\dfrac{1}{12}\ln(3) + \dfrac{1}{12}\ln(5)$

$= -0.09155102405 + 0.13411982603$

$= \boxed{0.04256880198 \text{ or } 0.0426}$

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4.) $\displaystyle\int_{-1}^{0} \frac{1}{2}e^{-2x}\,dx$

*Integral first

$= \displaystyle\int \frac{1}{2}e^u\left(-\frac{1}{2}du\right)$

$u = -2x$

$\dfrac{du}{-2} = \dfrac{-2\,dx}{-2}$

$dx = -\dfrac{du}{2}$ or $-\dfrac{1}{2}du$

$= \displaystyle\int -\frac{e^u}{4}\,du$

$= \displaystyle-\frac{1}{4}\int e^u\,du$

$= -\dfrac{1}{4}e^u + C$

$= -\dfrac{1}{4}e^{-2x} + C$

$= -\dfrac{1}{4e^{2x}} + C$

Definite Integral

$a = -1, \quad b = 0, \quad e = 2.718$

$F(b) - F(a)$

$= -\dfrac{1}{4e^{2(0)}} + \dfrac{1}{4e^{2(-1)}}$

$= -\dfrac{1}{4e^0} + \dfrac{1}{4e^{-2}}$

$= -\dfrac{1}{4} + \dfrac{e^2}{4}$

$= \dfrac{e^2-1}{4}$

$= \dfrac{(2.718)^2-1}{4}$

$= \dfrac{7.3891-1}{4}$

$= \dfrac{6.3891}{4} = \boxed{1.5973}$