EXERCISE #11

ADVANCE CALCULUS : 2BSCS-2

MARASIGAN, VEM AIENSI

Integration by parts

1.) $\int x \ln x \, dx$

$u = \ln(x) \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$

$$\int u\,dv = uv - \int v\,du$$

$$= \ln(x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\left(\frac{1}{x}dx\right)$$

$$= \frac{x^2\ln(x)}{2} - \frac{1}{2}\int \frac{x^2}{x}dx$$

$$= \frac{x^2\ln(x)}{2} - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2\ln(x)}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) + C$$

$$= \frac{x^2\ln(x)}{2} - \frac{x^2}{4} + C$$

$$= \frac{2x^2\ln(x) - x^2}{4} + C$$

$$= \frac{x^2}{4}(2\ln(x) - 1) + C$$

$$= \boxed{\frac{x^2}{4}(\ln(x^2) - 1) + C}$$

2.) $\int 2x^2 e^{x^2} dx$

if $u = e^{x^2}$; applying the chain rule

$f(x) = e^x \qquad g(x) = x^2$

we'll get $du = f'(g(x)) \cdot g'(x)dx$

$du = e^{x^2} 2x \, dx$

or can be rewritten as $2x e^{x^2} dx$

looking back at original problem:

$\int 2x^2 e^{x^2} dx$ can be $\int \underset{u}{x}(\underset{dv}{2xe^{x^2}dx})$

$u = x \qquad v = e^{x^2}$

$du = dx \qquad dv = 2x e^{x^2} dx$

$uv - \int v\,du = xe^{x^2} - \int e^{x^2} dx$

3.) $\int (-2x)e^x dx$

can be $-2\int \underset{u}{x}\,\underset{dv}{e^x dx}$

$u = x \qquad v = e^x$

$du = dx \qquad dv = e^x dx$

$$-2\int u\,dv = -2\left(uv - \int v\,du\right) = -2\left(xe^x - \int e^x dx\right)$$

$$= -2(xe^x - e^x) + C$$

$$= \boxed{-2e^x(x-1) + C}$$

4.) $\int 4x \sin x \, dx$

this can be manipulated as

$-1\int -(4x \sin x \, dx)$

$\Rightarrow -4\int -x \sin x \, dx$

and knowing that $-\sin x dx$ is the derivative for $\cos x$

we can arrange as follows

$-4\int \underset{u}{x}(\underset{dv}{-\sin x \, dx})$

$u = x \qquad v = \cos x$

$du = dx \qquad dv = -\sin x \, dx$

$$-4\int u\,dv = -4\left(uv - \int v\,du\right)$$

$$= -4\left(x\cos x - \int \cos x \, dx\right)$$

$$= -4(x\cos x - \sin x) + C$$

$$= 4(-x\cos x + \sin x) + C$$

rearrange:

$$= \boxed{4(\sin x - x\cos x) + C}$$

5.) $\int 3z^2 \ln z \, dz$

rearrange:

$\int \underset{u}{\ln z}\,(\underset{dv}{3z^2 dz})$

$u = \ln z \qquad v = z^3$

$du = \frac{1}{z}dz \qquad dv = 3z^2 dz$

$$\int u\,dv = uv - \int v\,du$$

$$= \ln z(z^3) - \int z^3\left(\frac{1}{z}dz\right)$$

$$= z^3\ln(z) - \int z^2 dz$$

$$= z^3\ln(z) - \frac{z^3}{3} + C$$

$$= \frac{3z^3\ln(z) - z^3}{3} + C$$

$$= \boxed{\frac{z^3}{3}(3\ln(z) - 1) + C = \frac{z^3}{3}(\ln(z^3) - 1) + C}$$

EXERCISE #2

ADVANCE CALCULUS : 2BSCS-2

MARASIGAN, VEM AIENSI

1.) $\displaystyle\int \frac{x^2+2x-10}{x^2(x^2+4x+5)}\,dx$

Decompose into partial fractions

$$x^2(x^2+4x+5)\left[\frac{x^2+2x-10}{x^2(x^2+4x+5)}\right] = \left[\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+4x+5}\right]x^2(x^2+4x+5)$$

$$x^2+2x-10 = A(x)(x^2+4x+5) + B(x^2+4x+5) + (Cx+D)(x^2)$$
$$= Ax^3+4Ax^2+5Ax + Bx^2+4Bx+5B + Cx^3+Dx^2$$
$$= Ax^3+Cx^3 + 4Ax^2+Bx^2+Dx^2 + 5Ax+4Bx+5B$$
$$x^2+2x-10 = (A+C)x^3 + (4A+B+D)x^2 + (5A+4B)x + 5B$$

Coefficients:

$0 = A+C$

$1 = 4A+B+D$

$2 = 5A+4B$

$-10 = 5B$

$\frac{-10}{5}=\frac{5B}{5} \Rightarrow \boxed{B=-2}$

$2 = 5A+4(-2)$

$\frac{5A}{5} = \frac{2+8}{5}$

$A = \frac{10}{5} \Rightarrow \boxed{A=2}$

$0 = A+C$

$C = -A$

$\boxed{C=-2}$

$1 = 4A+B+D$

$D = 1-4A-B$

$D = 1-4(2)+2$

$D = 1-8+2$

$\boxed{D=-5}$

Insert values to original and apply integration

$$\int \frac{x^2+2x-10}{x^2(x^2+4x+5)}\,dx = \int\left(\frac{2}{x}+\frac{-2}{x^2}+\frac{-2x-5}{x^2+4x+5}\right)dx$$

$$\int \frac{2}{x}dx \Rightarrow 2\int\frac{1}{x}dx \Rightarrow 2\ln(x)$$

$$\int -\frac{2}{x^2}dx \Rightarrow -2\int\frac{1}{x^2}dx \Rightarrow -2\int x^{-2}dx \Rightarrow -2\left(\frac{x^{-2+1}}{-2+1}\right) \Rightarrow \frac{2x^{-1}}{1} \Rightarrow \frac{2}{x}$$

$$\int\frac{(-2x-5)}{x^2+4x+5}dx \Rightarrow \int\left(\frac{-2x-4-1}{x^2+4x+5}\right)dx \Rightarrow \int\left(\frac{-2x-4}{x^2+4x+5}-\frac{1}{x^2+4x+5}\right)dx$$

$$\int\frac{-2x-4}{x^2+4x+5}dx - \int\frac{1}{x^2+4x+5}dx$$

$$\int\frac{-2x-4}{x^2+4x+5}dx \Rightarrow -\int\frac{(2x+4)dx}{x^2+4x+5} \quad \text{let } u=x^2+4x+5,\ du=(2x+4)dx \quad \Rightarrow -\int\frac{du}{u} \Rightarrow -\ln(u) = -\ln(x^2+4x+5)$$

$$-\int\frac{1}{x^2+4x+5}dx \Rightarrow -\int\frac{1}{x^2+4x+4+1}dx \Rightarrow -\int\frac{1}{(x+2)^2+1}dx \quad \text{let } u=x+2,\ du=dx$$
$$= -\int\frac{1}{u^2+1}du = -\arctan(u)$$
$$= -\arctan(x+2)$$

Combining all will be...

$$\int\frac{x^2+2x-10}{x^2(x^2+4x+5)}dx = \int\left(\frac{2}{x}+\frac{-2}{x^2}+\frac{-2x-5}{x^2+4x+5}\right)dx$$
$$= \int\frac{2}{x}dx + \int\frac{-2}{x^2}dx + \int\frac{-2x-4}{x^2+4x+5}dx - \int\frac{1}{x^2+4x+5}dx$$
$$= 2\ln(x) + \frac{2}{x} + (-\ln(x^2+4x+5)) - \arctan(x+2) + C$$
$$= \boxed{2\ln(x) + \frac{2}{x} - \ln(x^2+4x+5) - \arctan(x+2) + C}$$
$$= \ln(x^2) - \ln(x^2+4x+5) + \frac{2}{x} - \arctan(x+2) + C$$

OR...

$$= \boxed{\ln\left(\frac{x^2}{x^2+4x+5}\right) + \frac{2}{x} - \arctan(x+2) + C}$$

2.) $\displaystyle\int\frac{x^3+x+2}{x(x^2+1)^2}dx = \int\left(\frac{2}{x}+\frac{-2x+1}{x^2+1}+\frac{-2x}{(x^2+1)^2}\right)dx$

$$\int\frac{2}{x}dx \Rightarrow 2\int\frac{1}{x}dx \Rightarrow 2\ln(x)$$

$$\int\frac{-2x+1}{x^2+1}dx \Rightarrow \int\left(\frac{-2x}{x^2+1}+\frac{1}{x^2+1}\right)dx \Rightarrow \int\frac{-2x}{x^2+1}dx + \int\frac{1}{x^2+1}dx$$

$$\int\frac{-2x}{x^2+1}dx \Rightarrow -\int\frac{2x}{x^2+1}dx \quad \text{let } v=x^2+1,\ du=2x\,dx \quad \Rightarrow -\int\frac{du}{u} \Rightarrow -\ln(v) = -\ln(x^2+1)$$

$$\int\frac{1}{x^2+1}dx \quad \text{let } x=\tan(u),\ dx=\sec^2(u) \quad \Rightarrow \int\frac{1}{\tan(u)^2+1}(\sec^2(u)\,du) \Rightarrow \int\frac{\sec^2(u)\,du}{\sec^2(u)}$$
$$\Rightarrow \int du \Rightarrow u$$

from $x=\tan(u)$ *we get* $\arctan(x)=u$

$u=\arctan(x)$

$$\int\frac{-2x}{(x^2+1)^2}dx \quad \text{let } u=x^2+1,\ du=2x\,dx \quad \Rightarrow -\int\frac{2x}{(x^2+1)^2}dx \Rightarrow -\int\frac{du}{u^2} \Rightarrow -\int u^{-2}du$$
$$= -\left(\frac{u^{-2+1}}{-2+1}\right) \Rightarrow -\left(\frac{u^{-1}}{-1}\right) \Rightarrow \frac{1}{u}$$
$$= \frac{1}{x^2+1}$$

Combining all will be...

$$\int\frac{x^3+x+2}{x(x^2+1)^2}dx = \int\frac{2}{x}dx + \int\frac{(-2x+1)dx}{x^2+1} + \int\frac{-2x}{(x^2+1)^2}dx$$
$$= \int\frac{2}{x}dx + \int\frac{-2x}{x^2+1}dx + \int\frac{1}{x^2+1}dx + \int\frac{-2x}{(x^2+1)^2}dx$$
$$= \boxed{2\ln(x) - \ln(x^2+1) + \arctan(x) + \frac{1}{x^2+1} + C}$$

OR...

$$= \boxed{\ln\left(\frac{x^2}{x^2+1}\right) + \arctan(x) + \frac{1}{x^2+1} + C}$$

EXERCISE # 3

ADVANCE CALCULUS: 2BSCS-2

MARASIGAN, VEM AIENSI

1.) $\int_1^{\infty} \frac{4}{x}\,dx$; $\int \frac{4}{x}\,dx = 4\ln x + C$

$\lim_{t\to\infty} \int_1^{t} \frac{4}{x}\,dx$

$= \lim_{t\to\infty} 4\ln(x)\Big|_1^t$

$= \lim_{t\to\infty} \left[4\ln(t) - 4\ln(1)\right]$

$= \lim_{t\to\infty} 4\ln(t) - \cancel{\lim_{t\to\infty} 4\ln(1)}$

this cancels as ln(1) is equal to 0

$= \lim_{t\to\infty} 4\ln(t)$

*logarithm of something that approaches ∞ will be infinity

$= \boxed{\infty}$

the integral is divergent.

2.) $\int_{-\infty}^{4} \frac{1}{\sqrt{x}}\,dx$; $\int x^{-\frac{1}{2}}dx = 2\sqrt{x} + C$

$\lim_{t\to-\infty} \int_t^{4} \frac{1}{\sqrt{x}}\,dx$

$= \lim_{t\to-\infty} 2\sqrt{x}\Big|_t^4$

$= \lim_{t\to-\infty} \left[2\sqrt{4} - 2\sqrt{t}\right]$

$= \lim_{t\to-\infty} 2\sqrt{4} - \lim_{t\to-\infty} 2\sqrt{t}$

there are no squareroot of a negative expression

Hence, the limit does not exist

3.) $\int_{-\infty}^{\infty} e^{-x}dx$; $\int e^{-x}dx = -\frac{1}{e^x} + C$

$\lim_{t\to-\infty} \int_t^{\infty} e^{-x}dx$

$= \lim_{t\to-\infty} -\frac{1}{e^x}\Big|_t^{\infty}$

$= \lim_{t\to-\infty} \left[-\frac{1}{e^{\infty}} - \left(-\frac{1}{e^t}\right)\right]$

$= \cancel{\lim_{t\to\infty} -\frac{1}{e^{\infty}}} + \lim_{t\to-\infty} \frac{1}{e^t}$

e^{∞} is infinity so it cancels as $-\frac{1}{\infty}$ results to 0

$= \lim_{t\to-\infty} \frac{1}{e^t} = \boxed{\infty}$

Hence, e raise to infinity would be infinity and the integral is divergent

since t approaches $-\infty$, e would be at the numerator

4.) $x = r\cos\theta$ $\quad y = r\sin\theta$

We know that in trigonometric identities, $\cos^2\theta + \sin^2\theta = 1$ we can try manipulating x and y given the following equations as:

$x^2 = (r\cos\theta)^2 \quad y = (r\sin\theta)^2$

combining we get:

$x^2 + y^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta)$

$x^2 + y^2 = r^2(1)$

$x^2 + y^2 = r^2$

* part of question 4:

now we got: $x^2 + y^2 = r^2$

getting r will be:

$r = \sqrt{x^2 + y^2}$

so, in polar coordinates

$\left(r = \sqrt{x^2+y^2},\ \theta = ?\right)$

to get θ, we can also look at identities whereas

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

based on $x = r\cos\theta$ and $y = r\sin\theta$

we can have $\tan\theta = \frac{y}{x}$ or $\frac{r\sin\theta}{r\cos\theta}$

so getting the θ from $\tan\theta = \frac{y}{x}$

we can use inverse tangent

$\tan\theta\left(\frac{1}{\tan}\right) = \frac{y}{x}(\tan^{-1})$

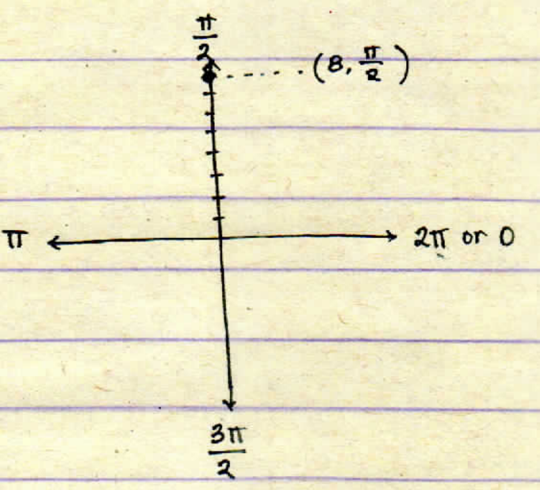
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Now we get:

$\left(r = \sqrt{x^2+y^2},\ \theta = \tan^{-1}\left(\frac{y}{x}\right)\right)$

to convert rectangular coordinates to polar coordinates

5.) $\left(8, \frac{\pi}{2}\right)$ Polar coordinate



for rectangular coordinates

* It must look the same

Conversion:

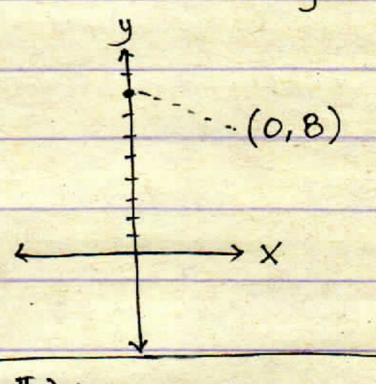
$r = 8 \quad \theta = \frac{\pi}{2}$

$x = r\cos\theta$ $\quad y = r\sin\theta$

$x = 8\left(\cos\left(\frac{\pi}{2}\right)\right)$ $\quad y = 8\left(\sin\left(\frac{\pi}{2}\right)\right)$

$x = 8(0)$ $\quad y = 8(1)$

$x = 0$ $\quad y = 8$



$\left(8, \frac{\pi}{2}\right)$ in Polar Coordinates is equal to $(0, 8)$ in Rectangular Coordinates