1 Optimal Control of Pitch/Travel without Feedback

1.1 Derivation of a continous time state space model

In this part of the exercise we will disregard elevation, therefore we assume $e = 0$ and do include it in the model.

The state-vector, $\boldsymbol{x}$ is defined as:

$$\boldsymbol{x} = \begin{bmatrix} \lambda & r & p & \dot{p} \end{bmatrix}^T, \tag{1}$$

where λ is travel, r is speed of travel (travelrate), p is pitch and $\dot{p}$ is pitchrate.

The dynamic equations for the system was given in the problem description. The following equations were given:

$$\dot{\lambda} = r \tag{2a}$$

$$\dot{r} = -K_2 p, \quad K_2 = \frac{K_p l_a}{J_t} \tag{2b}$$

$$\dot{p} = \dot{p} \tag{2c}$$

$$\ddot{p} = -K_1 K_{pd} \dot{p} - K_1 K_{pp} p + K_1 K_{pp} pc, \quad K_1 = \frac{K_f l_h}{J_p} \tag{2d}$$

The state-space form of the system therefore becomes:

$$\underbrace{\begin{bmatrix} \dot{\lambda} \\ \dot{r} \\ \dot{p} \\ \ddot{p} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -K_2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K_1 K_{pp} & -K_1 K_{pd} \end{bmatrix}}_{\boldsymbol{A}_c} \begin{bmatrix} \lambda \\ r \\ p \\ \dot{p} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ K_1 K_{pp} \end{bmatrix}}_{\boldsymbol{B}_c} \underbrace{p_c}_{u} \tag{3}$$

1.1.1 Stability and eigenvalues

The properties of this system is dependent on physical constants $(l_a, J_t, ...)$ and control parameters (K_{pp}, K_{pd}).

Symbolic expressions in Matlab shows that the eigenvalues of A are:

$$\lambda = \pm\frac{1}{2}\left(\sqrt{-K_1(-K_1 K_{pd}^2 + 4K_{pp})} - K_1 K_{pd}\right) \tag{4}$$

The eigenvalues of the continous model, with $K_{pp} = 0,1, K_{pd} = 0,4$ are:

$$\begin{bmatrix} 0 \\ 0 \\ -0.26 + 0.24i \\ -0.26 - 0.24 \end{bmatrix} \tag{5}$$

1.2 Discretizing the continous time model

A discretized model is required for generating an optimal trajectory. *[...] continous time models require quite different solution methods* [?]

We will discretize the model using the forward Euler method, which is given by:

$$\boldsymbol{x}[k+1] = \boldsymbol{I}\boldsymbol{x}[k] + T\boldsymbol{A_c}\boldsymbol{x}[k] + T\boldsymbol{B}_c, \tag{6}$$

where T is the timestep

timestep in the discretization??

Add reference to linsys slides

. Reformulating this, we can write:

$$\boldsymbol{x}_{k+1} = \underbrace{(\boldsymbol{I} + T\boldsymbol{A}_c)}_{\boldsymbol{A}_d} \boldsymbol{x}_k + \underbrace{T\boldsymbol{B}_c}_{\boldsymbol{B}_d} u_k \tag{7}$$

1.2.1 Checking stability

The stability condition for eq. (7) is:

$$|1 + T\lambda| \leq 1 \tag{8}$$

, where λ is an eigenvalue of $\boldsymbol{A}_c$ in eq. (7).

Where is this equation from? I believe that it works, but we should either derive it or reference where we found it. ANSWER: From linsys slides, see above

Using MATLAB we found the eigenvalues of $\boldsymbol{A}$ to be ...

Add Matlab appendix

find eigenvalues

1.3 The open loop optimization problem

How is it formulated?

The optimization problem is to calculate an optimal trajectory for moving the helicopter from $\lambda_0 = \pi$ to $\lambda_f = 0$ by manipulating the input, p_k, of the pitch controller.

We also have a constraint on the input to the pitch controller:

$$|p_k| \leq \frac{30}{180}\pi, k \in \{1, ..., N\} \tag{9}$$

this constraint says that the pitch-reference, p_k can not exceed 30 degrees in either direction.

Our objective function (the cost function we wish to minimize) is given as:

$$\phi = \sum_{i=0}^{N-1} (\lambda_{i+1} - \lambda_f)^2 + qp_{ci}^2, q \geq 0 \tag{10}$$

where q is the weight of input-usage.

Add the steps in our formulation. How we get to the quadprog-formulation:

- How the model is formulated
- How we formulate it as a QP problem with z
- What out constraints are (equality $A_{eq}z = B_{eq}$, inequality: $u_{low} < ...$)

1.4 The weights of the optimization problem

Try using the values 0.1, 1 and 10 as weights q. Plot the manipulated variable and the output. Comment the results with respect to the different weights chosen.

Weighing the input higher by increasing the value of q means that we are placing a higher cost of input - reducing the input usage. This will in turn mean that the cost of deviation in λ is in relation to the input, cheaper. The result is lower input usage and a slower response. This is exactly what is seen in fig. 1.

1.5 The objective function

Furthermore, discuss the objective function (15) (in the lab assignment text) in particular the term $(\lambda_i - \lambda_f)^2$. For instance, could any unwanted effects arise from steering the helicopter to $\lambda = \lambda_f$ with this objective function?

1.6 Experimental results

Printouts of data from relevant experiments (plots). Discussion and analysis of the results. Answer 10.2.2.7 here.

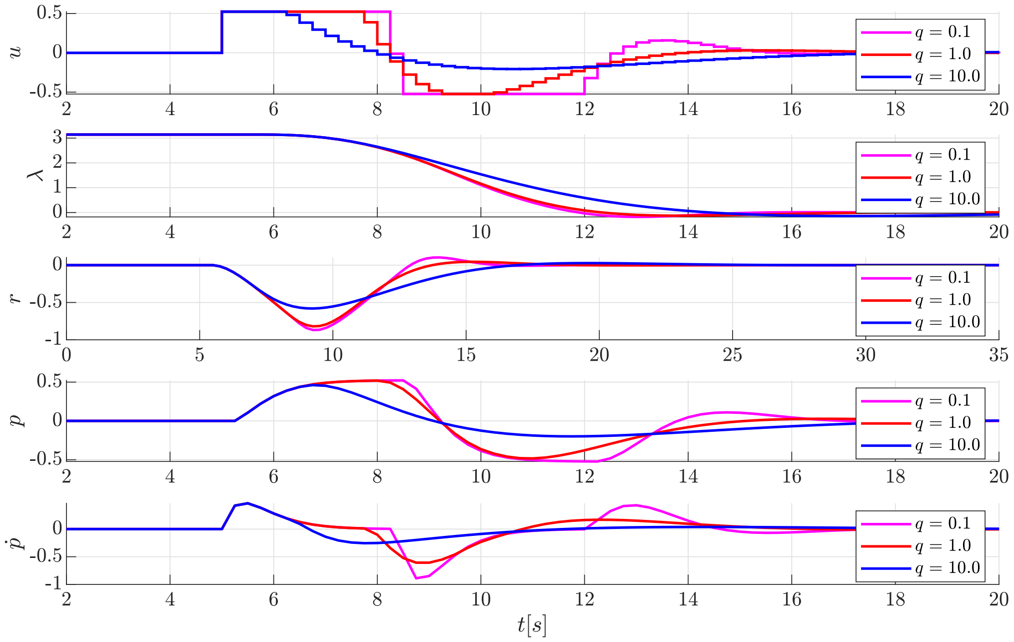

Figure 1: Manipulated variable and outputs with different values of q.

1.7 MATLAB and Simulink

Code and diagrams go here