FYS4150
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Abstract

Introduction

Method

The implicit schemes are differential equations that can be rewritten as a set of linear equations. The Euler forward, Euler backward and Crank-Nicolson scheme are all implicit and can therefore be written in term of linear equations after scaling:
Euler backwards, first derivative:

$$u_t \approx \frac{u(x_i, t_j) - u(x_i, t_j - \Delta t)}{\Delta t}$$

Second derivative:

$$u_{xx} \approx \frac{u(x_i + \Delta x, t_j) - 2u(x_i, t_j) + u(x_i - \Delta x, t_j)}{\Delta x^2}$$

One can scale the above equation by $\alpha = \Delta t / \Delta x^2$, so the equation only depends on one scaled variable such that:

$$u_{i,j-1} = -\alpha u_{i-1,j} + (1 + 2\alpha) u_{i,j} - \alpha u_{i+1,j}$$

Now the differential equation can be written as a set of linear equations with a matrix A times a vector V_j such that: $AV_j = V_{j-1}$, where A defined from the above differential equations:

$$A = \begin{bmatrix} 1+2\alpha & -\alpha & 0 & 0 & \cdots \\ -\alpha & 1+2\alpha & -\alpha & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\alpha & 1+2\alpha \end{bmatrix}$$

It is now possible to find the previous vector V_{j-1} when we already know what V_j, which we already know due to our initial conditions. A more generalized equation can be written as:

$$A^{-1}(AV_j) = A^{-1}(V_{j-1})$$

and if we keep multiplying by A^{-1} we get the implicit scheme:

$$V_j = A^{-j} V_0$$

A very similar process can be applied to the Euler forward method:

$$u_t = \frac{u(x_i, t_j + \Delta t) - u(x_i, t_j)}{\Delta t}$$

$$u_{xx} =$$

Results

Discussion

Concluding remarks

Reference list