

Case Study: JPL Small Body Classification in R

Veneet Bhardwaj

05/01/2021

Contents

| | | |
|----------|---|-----------|
| 1 | Introduction | 3 |
| 1.1 | Objective | 3 |
| 1.2 | A Brief History | 4 |
| 2 | Analysis | 5 |
| 2.1 | Data setup | 5 |
| 2.2 | Outcome Variable | 8 |
| 2.3 | Missing data | 9 |
| 2.4 | Model Variables | 11 |
| 2.5 | Validation and Model datasets | 13 |
| 2.6 | Variable Summary | 14 |
| 2.6.1 | id, class, neo_class | 14 |
| 2.6.2 | H, e, t_jup, a | 16 |
| 2.6.3 | q, ad, moid, moid_jup | 20 |
| 2.6.4 | i, om, w | 24 |
| 2.6.5 | per, n, ma, rms | 27 |
| 2.7 | Variable Means | 31 |
| 2.8 | Distance | 32 |
| 3 | Train & Test sets | 34 |
| 3.1 | Process Data | 34 |
| 3.2 | Inaccuracy Details function | 34 |
| 4 | Model Evaluations | 35 |
| 4.1 | Evaluation metrics | 35 |
| 4.1.1 | Definitions | 35 |
| 4.1.2 | Evaluation Function | 35 |
| 4.2 | PCA | 36 |

| | | |
|----------|-----------------------------|-----------|
| 4.2.1 | Multinom | 39 |
| 4.3 | Naive Bayes | 40 |
| 4.4 | Ensemble | 41 |
| 4.5 | Decision Trees | 44 |
| 4.5.1 | Rpart | 44 |
| 4.5.2 | Random Forest | 47 |
| 4.5.3 | WSRF | 50 |
| 4.5.4 | Ranger | 52 |
| 4.6 | Gradient Boosting | 54 |
| 4.7 | Collated Results | 57 |
| 4.7.1 | Recall | 57 |
| 4.7.2 | F1 | 58 |
| 4.7.3 | F1 vs Recall | 59 |
| 5 | Final Models | 60 |
| 5.1 | Validation | 60 |
| 5.1.1 | Ranger | 60 |
| 5.1.2 | WSRF | 62 |
| 5.1.3 | Rpart | 63 |
| 5.2 | As Holdout | 64 |
| 5.2.1 | WSRF | 64 |
| 5.2.2 | Ranger | 65 |
| 5.2.3 | Gradient Boost | 66 |
| 6 | Conclusion | 67 |
| 6.1 | References | 68 |
| 6.2 | Annexure | 69 |

1 Introduction

A *NEO*, Near Earth Object, is a small Solar System body (an asteroid or comet), which comes in proximity with Earth during its orbit.

A *PHO*, Potentially Hazardous Object, is a NEO with an orbit that intersects that of Earth and hence there is a probability of a collision.

When a *NEO* is detected, like all other small Solar System bodies, its positions and brightness are submitted to the International Astronomical Union (IAU) Minor Planet Center (MPC) for cataloging. The MPC maintains separate lists of confirmed NEOs and potential NEOs.

NEOs are also cataloged by two separate units of the Jet Propulsion Laboratory (JPL) of the National Aeronautics and Space Administration (NASA): the Center for Near Earth Object Studies (CNEOS) and the Solar System Dynamics Group.

This case study is on making a classification model using R, with the JPL dataset.

1.1 Objective

→ The **Objective** of this case study to build a machine learning model to predict the classification of the small solar bodies into *PHO*, *NEO* and *SO* (other small solar bodies).

Target measures :

- Recall for each category above 0.99
- F1 Score for each category above 0.99

The dataset used is the data available at :

https://ssd.jpl.nasa.gov/sbdb_query.cgi

— Downloaded file : Nasa_JPL_SmallBody.csv ~555Mb in size —

— Compressed split files have been uploaded in the folder <https://raw.githubusercontent.com/VeneetBhardwaj/R/> —

1.2 A Brief History

source wikipedia

Comets were the first near-Earth objects to be observed by us.

Their extraterrestrial nature was recognized and confirmed only after Tycho Brahe tried to measure the distance of a comet through its parallax in 1577 and the lower limit he obtained was well above the Earth diameter.

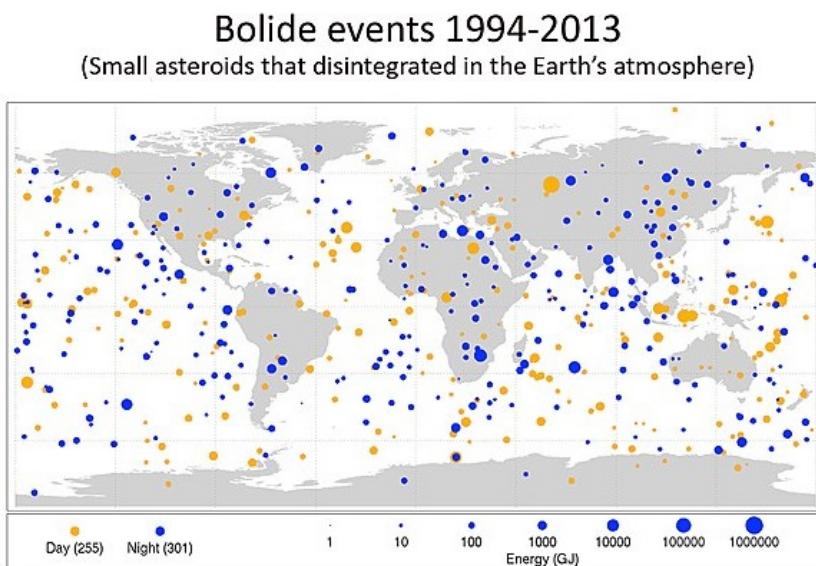
- In 1705, Edmond Halley published his orbit calculations of a NEO, the Halley Comet. The 1758–1759 return of Halley Comet was the first comet appearance that had been predicted.
- It has been said that Lexell comet of 1770 was the first discovered Near-Earth object.
- The first near-Earth asteroid to be discovered was 433 Eros in 1898.
- As of January 2019, five near-Earth comets and five near-Earth asteroids have been visited by spacecraft. A small sample of one NEO was returned to Earth in 2010, and similar missions are in progress.

The first astronomical program dedicated to the discovery of near-Earth asteroids was the Palomar Planet-Crossing Asteroid Survey, started in 1973 by astronomers Eugene Shoemaker and Eleanor Helin.

Several surveys have undertaken the NEO detection.

- Lincoln Near-Earth Asteroid Research (LINEAR)
- Spacewatch, Near-Earth Asteroid Tracking (NEAT)
- Lowell Observatory Near-Earth-Object Search (LONEOS)
- Catalina Sky Survey (CSS)
- Campo Imperatore Near-Earth Object Survey (CINEOS)
- Japanese Spaceguard Association
- Asiago-DLR Asteroid Survey (ADAS)
- Near-Earth Object WISE (NEOWISE).

In 2005, the original USA Spaceguard mandate to detect 90% of near-earth asteroids over 1 km (0.62 mi) diameter, was extended by the George E. Brown, Jr. Near-Earth Object Survey Act, which calls for NASA to detect 90% of NEOs with diameters of 140 m (460 ft) or greater, by 2020.



As of January, 2020, it is estimated that less than half of these have been found. NASA maintains an automated system to evaluate the threat from known NEOs over the next 100 years, which generates the continuously updated Sentry Risk Table.

2 Analysis

```
#load the required libraries
library(knitr)
library(markdown)
options(digits = 5)
if(!require(data.table)) install.packages("data.table", repos = "http://cran.us.r-project.org")
if(!require(caret)) install.packages("caret", repos = "http://cran.us.r-project.org")
if(!require(tidyverse)) install.packages("tidyverse", repos = "http://cran.us.r-project.org")
if(!require(lubridate)) install.packages("lubridate", repos = "http://cran.us.r-project.org")
if(!require(corrplot)) install.packages("corrplot", repos = "http://cran.us.r-project.org")
if(!require(Hmisc)) install.packages("Hmisc", repos = "http://cran.us.r-project.org")
if(!require(e1071)) install.packages("e1071", repos = "http://cran.us.r-project.org")
if(!require(nnet)) install.packages("nnet", repos = "http://cran.us.r-project.org")
if(!require(naivebayes)) install.packages("naivebayes", repos = "http://cran.us.r-project.org")
if(!require(ranger)) install.packages("ranger", repos = "http://cran.us.r-project.org")
if(!require(gbm)) install.packages("gbm", repos = "http://cran.us.r-project.org")
if(!require(rpart)) install.packages("rpart", repos = "http://cran.us.r-project.org")
if(!require(randomForest)) install.packages("randomForest", repos = "http://cran.us.r-project.org")
if(!require(wsrf)) install.packages("wsrf", repos = "http://cran.us.r-project.org")
```

2.1 Data setup

The JPL small body data set is downloaded from https://ssd.jpl.nasa.gov/sbdb_query.cgi

The downloaded file is Nasa_JPL_SmallBody.csv (~555Mb)

```
data <- data.table::fread("Nasa_JPL_SmallBody.csv", na.strings = c(NA, "NA", ""))
dim(data)
```

```
## [1] 1081059      59
```

There are 1081059 observations (rows) across 59 columns in the dataset.

```
names(data)
```

```
##  [1] "id"                  "full_name"           "pdes"                "neo"
##  [5] "pha"                 "H"                   "G"                   "diameter"
##  [9] "extent"              "albedo"              "rot_per"              "orbit_id"
## [13] "epoch"               "epoch_mjd"           "epoch_cal"            "equinox"
## [17] "e"                   "a"                   "q"                   "i"
## [21] "om"                 "w"                   "ma"                  "ad"
## [25] "n"                   "tp"                  "tp_cal"              "per"
## [29] "per_y"               "moid"                "moid_ld"              "moid_jup"
## [33] "t_jup"               "sigma_e"              "sigma_a"              "sigma_q"
## [37] "sigma_i"              "sigma_om"             "sigma_w"              "sigma_ma"
## [41] "sigma_ad"             "sigma_n"              "sigma_tp"              "sigma_per"
## [45] "class"                "producer"             "data_arc"              "first_obs"
## [49] "last_obs"             "n_obs_used"          "n_del_obs_used"       "n_dop_obs_used"
## [53] "condition_code"        "rms"                 "two_body"              "A1"
## [57] "A2"                  "A3"                 "DT"
```

Brief description on the columns of the data set are shown below. These have been *collated from the <https://ssd.jpl.nasa.gov> website.*

```
#load the descriptions csv.
data$id <- str_trim(data$id)
desc_d <- as.data.frame(data.table::fread("JPLdesc.csv"))
desc_d[,2:4] %>% knitr::kable()
```

| column_name | Description | Units |
|-------------|---|-------|
| id | Object Internal Database Id | |
| full_name | Object Full Name/Designation | |
| pdes | Object Primary Designation | |
| neo | Near-Earth Object (Neo) Flag | |
| pha | Potentially Hazardous Asteroid (Pha) Flag | |
| H | Absolute Magnitude Parameter | |
| G | Magnitude Slope Parameter (Default Is 0.15) | |
| diameter | Object Diameter (From Equivalent Sphere) | km |
| extent | Object Bi/Tri-Axial Ellipsoid Dimensions | km |
| albedo | Geometric Albedo | |
| rot_per | Rotation Period | h |
| orbit_id | Orbit Solution Id | |
| epoch | Epoch Of Osculation In Julian Day Form | TDB |
| epoch_mjd | Epoch Of Osculation In Modified Julian Day Form | TDB |
| epoch_cal | Epoch Of Osculation In Calendar Date/Time Form | TDB |
| equinox | Equinox Of Reference Frame | |
| e | Eccentricity | |
| a | Semi-Major Axis | au |
| q | Perihelion Distance | au |
| i | Inclination; Angle With Respect To X-Y Ecliptic Plane | deg |
| om | Longitude Of The Ascending Node | deg |
| w | Argument Of Perihelion | deg |
| ma | Mean Anomaly | deg |
| ad | Aphelion Distance | au |
| n | Mean Motion | deg/d |
| tp | Time Of Perihelion Passage | TDB |
| tp_cal | Time Of Perihelion Passage | ET |
| per | Sidereal Orbital Period | d |
| per_y | Sidereal Orbital Period | years |
| moid | Earth Minimum Orbit Intersection Distance | au |
| moid_ld | Earth Minimum Orbit Intersection Distance | LD |
| moid_jup | Jupiter Minimum Orbit Intersection Distance | au |
| t_jup | Jupiter Tisserand Invariant | |
| sigma_e | Eccentricity (1-Sigma Uncertainty) | au |
| sigma_a | Semi-Major Axis (1-Sigma Uncertainty) | au |
| sigma_q | Perihelion Distance (1-Sigma Uncertainty) | deg |
| sigma_i | Inclination (1-Sigma Uncertainty) | deg |
| sigma_om | Long. Of The Asc. Node (1-Sigma Uncertainty) | deg |
| sigma_w | Argument Of Perihelion (1-Sigma Uncertainty) | deg |
| sigma_ma | Mean Anomaly (1-Sigma Uncertainty) | deg |
| sigma_ad | Aphelion Distance (1-Sigma Uncertainty) | au |
| sigma_n | Mean Motion (1-Sigma Uncertainty) | deg/d |
| sigma_tp | Time Of Peri. Passage (1-Sigma Uncertainty) | d |
| sigma_per | Sidereal Orbital Period (1-Sigma Uncertainty) | d |

| column_name | Description | Units |
|----------------|--|--------|
| class | Orbit Classification | |
| producer | Name Of Person (Or Institution) Who Computed The Orbit | |
| data_arc | Number Of Days Spanned By The Data-Arc | d |
| first_obs | Date Of First Observation Used In The Orbit Fit | UT |
| last_obs | Date Of Last Observation Used In The Orbit Fit | UT |
| n_obs_used | Number Of Observations (All Types) Used In Fit | |
| n_del_obs_used | Number Of Delay-Radar Observations Used In Fit | |
| n_dop_obs_used | Number Of Doppler-Radar Observations Used In Fit | |
| condition_code | Orbit Condition Code (Mpc 'U' Parameter) | |
| rms | Normalized Rms Of Orbit Fit | arcsec |
| two_body | 2-Body Dynamics Used Flag | T/F |
| A1 | Non-Grav. Radial Parameter | |
| A2 | Non-Grav. Transverse Parameter | |
| A3 | Non-Grav. Normal Parameter | |
| DT | Non-Grav. Peri.-Maximum Offset | d |

Observations on the variables (columns) property in the dataset:

- The intent of this case study is to classify the small body objects into the appropriate flags of neo Near-Earth Object (Neo) Flag and pha Potentially Hazardous Asteroid (Pha) Flag.
- Some columns are descriptive of the object like *Object full name* and *name of the person who computed the orbit*.
- Some are regarding objects observation properties, like the *number of observations used for orbit fit*, *date of first and last observation* etc..
- There are also the 1-Sigma Uncertainty of the values in the data set.
- There are variables which are orbital observations and the observations of the object properties. These are the variables which will be used in making the classification model.

2.2 Outcome Variable

The **Outcome Variable** (to be predicted by the model) needs to be setup from a combination of 2 columns - *pha* and *neo*. Both these have binary outcomes of “Y” and “N”.

However there are 12054 missing values in *pha* and 3539 in *neo*, and these observations (rows) are being removed.

```
#check for missing values
cat("pha missing values: ", sum(is.na(data$pha)),
    "\nneo missing values : ", sum(is.na(data$neo)))

## pha missing values: 12054
## neo missing values : 3539
```

The *neo_class* will have three possible values, which are defined as:

$$PHO \rightarrow Potential\ Hazard$$

$$NEO \rightarrow Near\ Earth\ Object(not\ Potential\ Hazard)$$

$$SO \rightarrow Other\ (Solar)\ Object(not\ Near\ Earth\ object\ \&\ not\ Potential\ Hazard)$$

Setting up the Outcome Variable :

```
#This is the outcome variable
data$pha <- as.character(data$pha)
data$neo <- as.character(data$neo)
#remove the missing values.
data <- data[complete.cases(data$neo),]
data <- data[complete.cases(data$pha),]
#convert character to numeric,
data$pha <- ifelse(data$pha == "Y", 1, 0)
data$neo <- ifelse(data$neo == "Y", 1, 0)
#define the classes
data <- data %>%
  mutate(neo_class = as.factor(ifelse(pha == 1, "PHO",
                                       ifelse(pha == 0 & neo == 1, "NEO",
                                              ifelse(pha == 0 & pha == 0, "SO", "Error")))))
describe(data$neo_class)

## data$neo_class
##      n  missing distinct
## 1069002      0       3
##
## Value      NEO      PHO      SO
## Frequency   23544    2163 1043295
## Proportion  0.022    0.002   0.976
```

- With only the complete cases of the flags being used, there is no missing data of the outcome variable.
- The proportions of the PHO and the NEO are very low in the dataset, with 97.6% of the objects being classified as SO.

2.3 Missing data

Variables with missing data:

```
#list features with less than 100% data
m_chk <- tibble()
data_f <- data.frame(data)
for(iter in 1:(ncol(data_f))){
  x <- sum(is.na(data_f[,iter]))
  cname <- as.character(colnames(data_f[iter]))
  uni <- as.character(n_distinct(data_f[iter]))
  perc <- round(((dim(data)[1]-x)/(dim(data_f)[1]))*100),1)
  if (x > 0) {
    m_chk <- bind_rows(m_chk,data.frame(Variable=cname,
                                          Distinct=uni,
                                          Missing=as.character(x),
                                          percentage=str_c(perc,"%"))))
  }
m_chk %>% knitr::kable()
```

| Variable | Distinct | Missing | percentage |
|----------------|----------|---------|------------|
| pdes | 547967 | 521036 | 51.3% |
| H | 9070 | 4007 | 99.6% |
| G | 48 | 1068883 | 0% |
| diameter | 16782 | 929002 | 13.1% |
| extent | 19 | 1068984 | 0% |
| albedo | 1055 | 930153 | 13% |
| rot_per | 12801 | 1045918 | 2.2% |
| ma | 1069001 | 1 | 100% |
| moid_jup | 213322 | 1 | 100% |
| sigma_e | 274855 | 1 | 100% |
| sigma_a | 290958 | 1 | 100% |
| sigma_q | 272278 | 1 | 100% |
| sigma_i | 232282 | 1 | 100% |
| sigma_om | 233199 | 1 | 100% |
| sigma_w | 277495 | 1 | 100% |
| sigma_ma | 285767 | 1 | 100% |
| sigma_ad | 286819 | 1 | 100% |
| sigma_n | 263808 | 1 | 100% |
| sigma_tp | 307291 | 1 | 100% |
| sigma_per | 300529 | 1 | 100% |
| data_arc | 22611 | 337 | 100% |
| n_del_obs_used | 26 | 1068199 | 0.1% |
| n_dop_obs_used | 21 | 1068162 | 0.1% |
| condition_code | 11 | 4 | 100% |
| rms | 63559 | 1 | 100% |
| two_body | 1 | 1069002 | 0% |
| A1 | 1 | 1069002 | 0% |
| A2 | 1 | 1069002 | 0% |
| A3 | 1 | 1069002 | 0% |
| DT | 1 | 1069002 | 0% |

- $H, G, diameter, albedo, rot_per$ are properties of the object, and useful in building a model. But besides H , the remaining variables have a large number of missing values and hence will not be used.

The observations with missing data, for the variables intended to be used for the model, are removed.

```
dim(data)  
  
## [1] 1069002      60
```

2.4 Model Variables

The original dataset, post cleaning, is now split into two datasets

- The model Variables
- The descriptive dataset (*details descriptive of the objects*)

```
#descriptive and model variables dataset separated
model_variables <- data %>% select(id,class,neo_class,H,e,t_jup,
                                         a,q,ad,moid,moid_jup,i,om,w,per,n,ma,rms)

descriptive_data_full <- data %>% select(setdiff(names(data),names(model_variables)))
descriptive_data_full <- bind_cols(id=data$id,moid=data$moid,descriptive_data_full)
#data.table::fwrite(descriptive_data,'descriptive_data_full.csv')
descriptive_data <- descriptive_data_full %>%
  select(id,full_name,producer,neo,pha,per_y,moid,first_obs,last_obs)
data.table::fwrite(descriptive_data,'descriptive_data.csv')
#use only the complete cases. drop the remaining.
model_variables <- model_variables[complete.cases(model_variables),]
```

To make the classification model the **model_variables** will be used.

The list of the columns in the dataset are :

```
# description of the features to be used in the model
x <- ifelse(c(desc_d$column_name) %in% c(names(model_variables)),1,0)
m_chk <- tibble()
for(iter in 1:59){ if (x[iter]==1) {
  m_chk <- bind_rows(m_chk,data_frame(Variable=desc_d[iter,2],
                                         Description=desc_d[iter,3],
                                         Units=desc_d[iter,4]))}
m_chk %>% knitr::kable()
```

| Variable | Description | Units |
|----------|---|--------|
| id | Object Internal Database Id | |
| H | Absolute Magnitude Parameter | |
| e | Eccentricity | |
| a | Semi-Major Axis | au |
| q | Perihelion Distance | au |
| i | Inclination; Angle With Respect To X-Y Ecliptic Plane | deg |
| om | Longitude Of The Ascending Node | deg |
| w | Argument Of Perihelion | deg |
| ma | Mean Anomaly | deg |
| ad | Aphelion Distance | au |
| n | Mean Motion | deg/d |
| per | Sidereal Orbital Period | d |
| moid | Earth Minimum Orbit Intersection Distance | au |
| moid_jup | Jupiter Minimum Orbit Intersection Distance | au |
| t_jup | Jupiter Tisserand Invariant | |
| class | Orbit Classification | |
| rms | Normalized Rms Of Orbit Fit | arcsec |

```
dim(model_variables)
```

```
## [1] 1064993      18
```

There are 1064993 observations (rows) across 18 columns (including the Outcome Variable) in the dataset to be used for making the Classification Model.

2.5 Validation and Model datasets

```
# Validation set will be 10% of model_variables data
set.seed(131825431, sample.kind = "Rounding")

validation_index <- createDataPartition(model_variables$neo_class,
                                         times = 1, p = 0.1, list = FALSE)

JPL_data <- model_variables[-validation_index,]
JPL_validation <- model_variables[validation_index,]
```

```
dim(JPL_data)
```

```
## [1] 958492      18
```

Model dataset has 851993 observations (rows) across 18 columns (including the predictor).

```
dim(JPL_validation)
```

```
## [1] 106501      18
```

Validation dataset has 213000 observations (rows) across 18 columns (including the predictor).

2.6 Variable Summary

2.6.1 id, class, neo_class

- *id* : The unique id for each object.
- *class* : classification of the objects according to their orbits.
- *neo_class* : The model Outcome Variable.

```
JPL_SmallBody_data <- data.frame(JPL_data)
summary(JPL_SmallBody_data[1:3])
```

```
##      id          class        neo_class
##  Length:958492    Length:958492    NEO: 21183
##  Class :character Class :character  PHO: 1946
##  Mode  :character Mode  :character   SO :935363
```

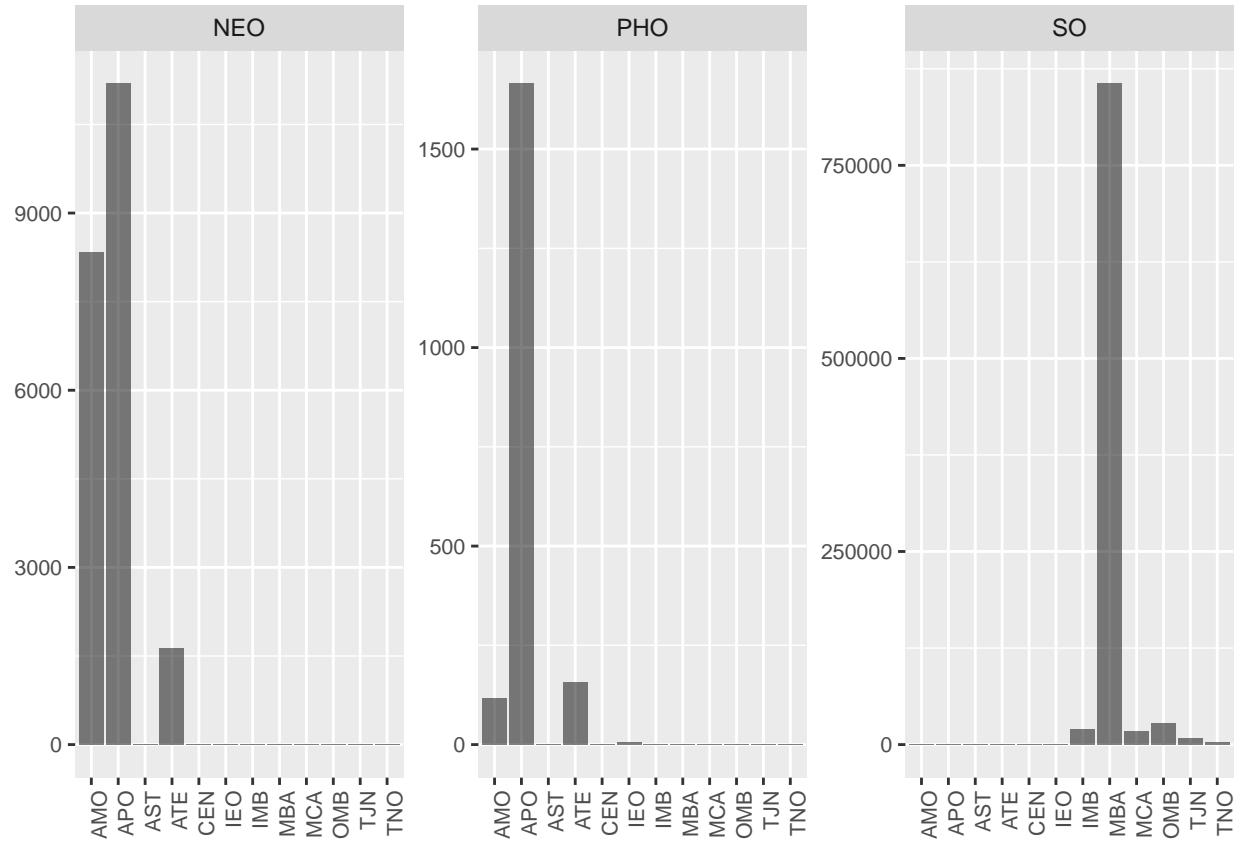
The table below shows the distribution of the Outcome Variable by each orbital class.

```
tab <- table(JPL_SmallBody_data[2:3])
tab %>% knitr::kable()
```

| | NEO | PHO | SO |
|-----|-------|------|--------|
| AMO | 8337 | 118 | 0 |
| APO | 11193 | 1665 | 0 |
| AST | 0 | 0 | 62 |
| ATE | 1637 | 157 | 0 |
| CEN | 0 | 0 | 488 |
| IEO | 16 | 6 | 0 |
| IMB | 0 | 0 | 20477 |
| MBA | 0 | 0 | 855885 |
| MCA | 0 | 0 | 17986 |
| OMB | 0 | 0 | 28370 |
| TJN | 0 | 0 | 8895 |
| TNO | 0 | 0 | 3200 |

A visual representation of the different class by the neo_class.

```
tab %>% as.data.frame() %>%
  ggplot(aes(class,Freq)) +
  geom_bar(stat="identity", fill = "#9999FF", alpha=0.5) +
  facet_wrap(~neo_class, ncol=3,scales = "free") +
  theme(axis.text.x = element_text(angle=90,size=8, hjust=1),
        axis.text.y = element_text(size=8, lineheight = 1.5),
        axis.title.x = element_blank(),
        axis.title.y = element_blank(),
        legend.position = "bottom")
```



- The PHO and NEO have evidently a more similar orbit as compared to the SO. It might be possible to categorise our solar objects into these categories based on the observations that are available.

However, the “class” variable will not be used for further analysis.

2.6.2 H, e, t_jup, a

- H : Absolute Magnitude (M) is a measure of the luminosity of a celestial object, on an inverse logarithmic astronomical magnitude scale. In other words, the magnitude of an asteroid at zero phase angle and at unit heliocentric and geocentric. The more luminous an object, the smaller the numerical value of its absolute magnitude.
- e : Eccentricity e is the ratio of half the distance between the foci c to the semi-major axis $e = \frac{c}{2a}$. An orbit with $e = 0$ is circular, $e = 1$ is parabolic and e between 0 and 1 is elliptic.
- t_{jup} : Tisserand Invariant is a value calculated from several orbital elements(semi-major axis, orbital eccentricity and inclination) of a relatively small object and a larger perturbing body, in this case Jupiter. This is used to distinguish asteroids (typically $T_J > 3$) from Jupiter-family comets (typically $2 < T_J < 3$) and the minor planet groups of damocloids (typically $T_J \leq 2$).
- a : Semi-Major Axis of an orbit ellipse is half the length of the major axis. For solar system bodies, this value is denoted by α

Summary

```
k <- 24576
summary(JPL_SmallBody_data[4:7])
```

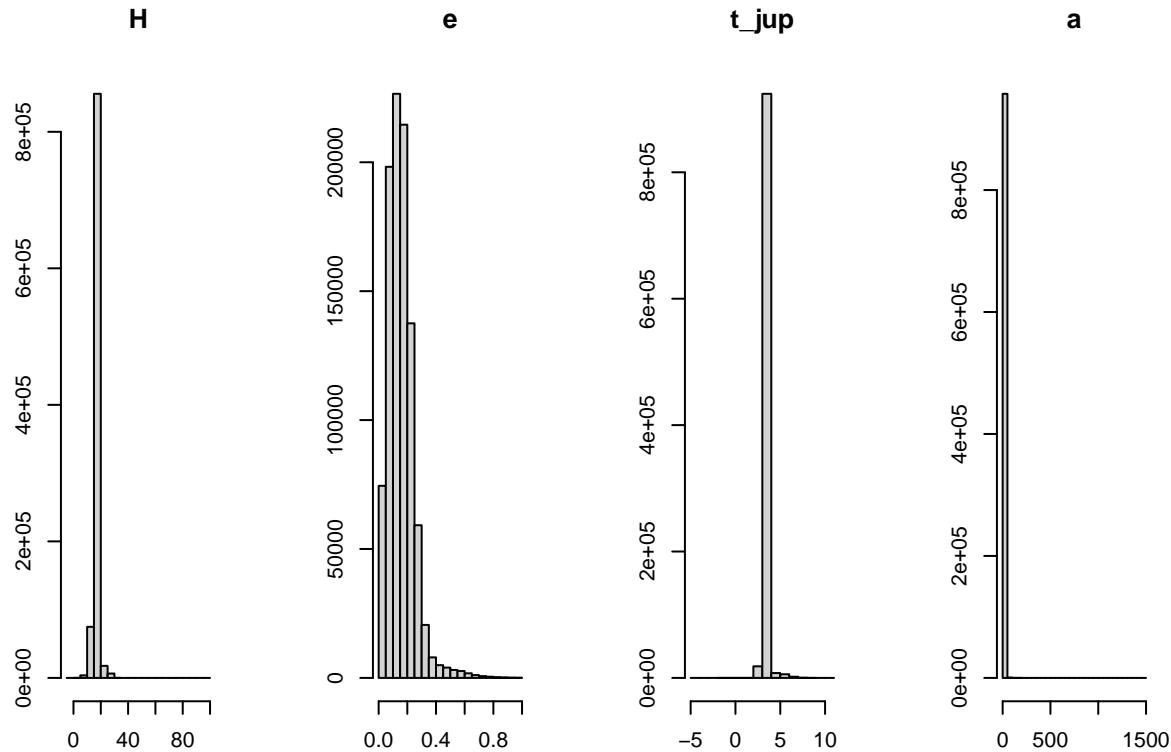
```
##          H              e             t_jup            a
##  Min.   :-0.13   Min.   :0.0000   Min.   :-4.90   Min.   : 0.56
##  1st Qu.: 16.20  1st Qu.:0.0924  1st Qu.: 3.21  1st Qu.: 2.39
##  Median  : 17.01  Median :0.1456  Median : 3.34  Median : 2.65
##  Mean    : 17.04  Mean   :0.1565  Mean   : 3.38  Mean   : 2.87
##  3rd Qu.: 17.90  3rd Qu.:0.2013  3rd Qu.: 3.50  3rd Qu.: 3.01
##  Max.    : 99.99  Max.   :0.9969  Max.   :10.19  Max.   :1455.27
```

- The range of H is between -1.11 (more luminous) and ~ 100 (less luminous).
- The range of e , as expected, is between 0 and 1.
- The range of t_{jup} is between -4.90 (Damocloids) and 10.19 (Asteroids).

```

par(mfrow=c(1,4))
for(iter in 4:7) {
  hist(JPL_SmallBody_data[,iter], main=names(JPL_SmallBody_data)[iter], xlab='', ylab='')}

```

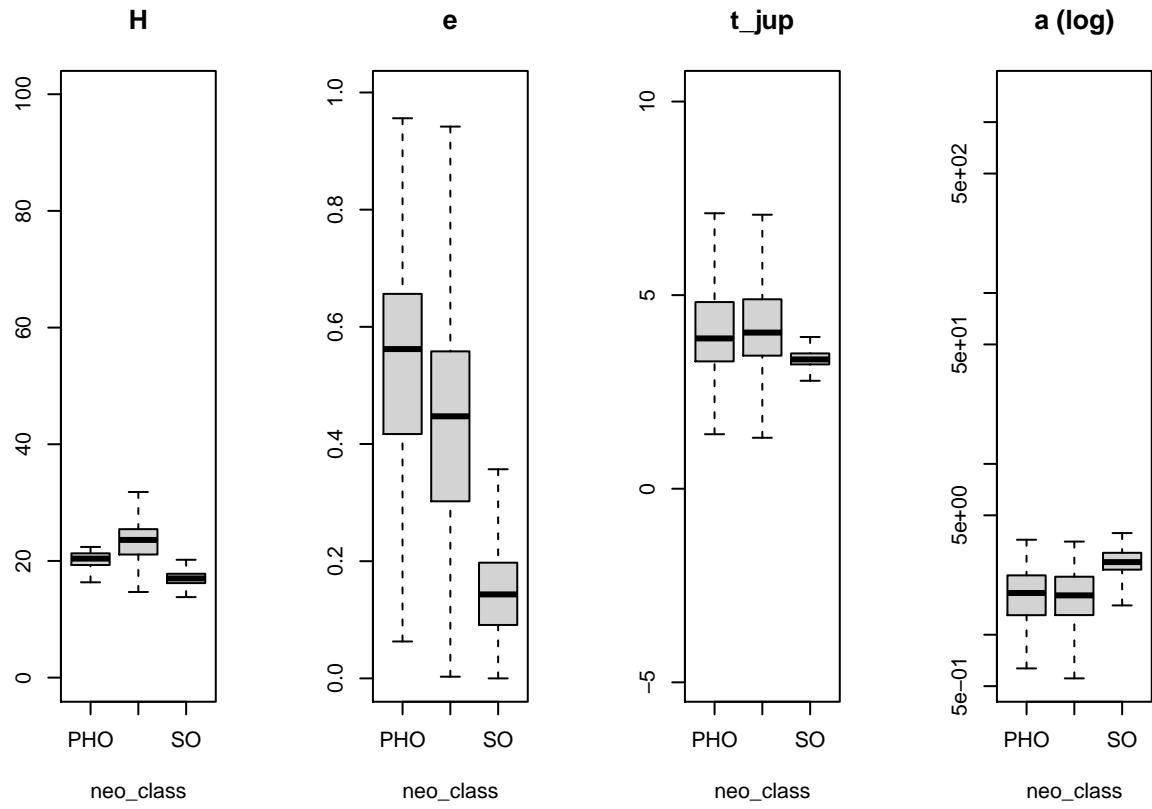


Variations of H , e , t_{jup} by the neo_class (Outcume Variable) :

```

par(mfrow=c(1,4))
X <- JPL_SmallBody_data[,3:7]
for(iter in 2:5) {dat <- list(PHO = X[X$neo_class == "PHO",iter],
                               NEO = X[X$neo_class == "NEO",iter], SO = X[X$neo_class == "SO",iter])
ifelse(iter != 5, boxplot(dat,xlab="neo_class", cex=0, main=str_c(names(X)[iter], "")),
      boxplot(dat,xlab="neo_class", log="y", cex=0, main=str_c(names(X)[5]," (log)")))}

```

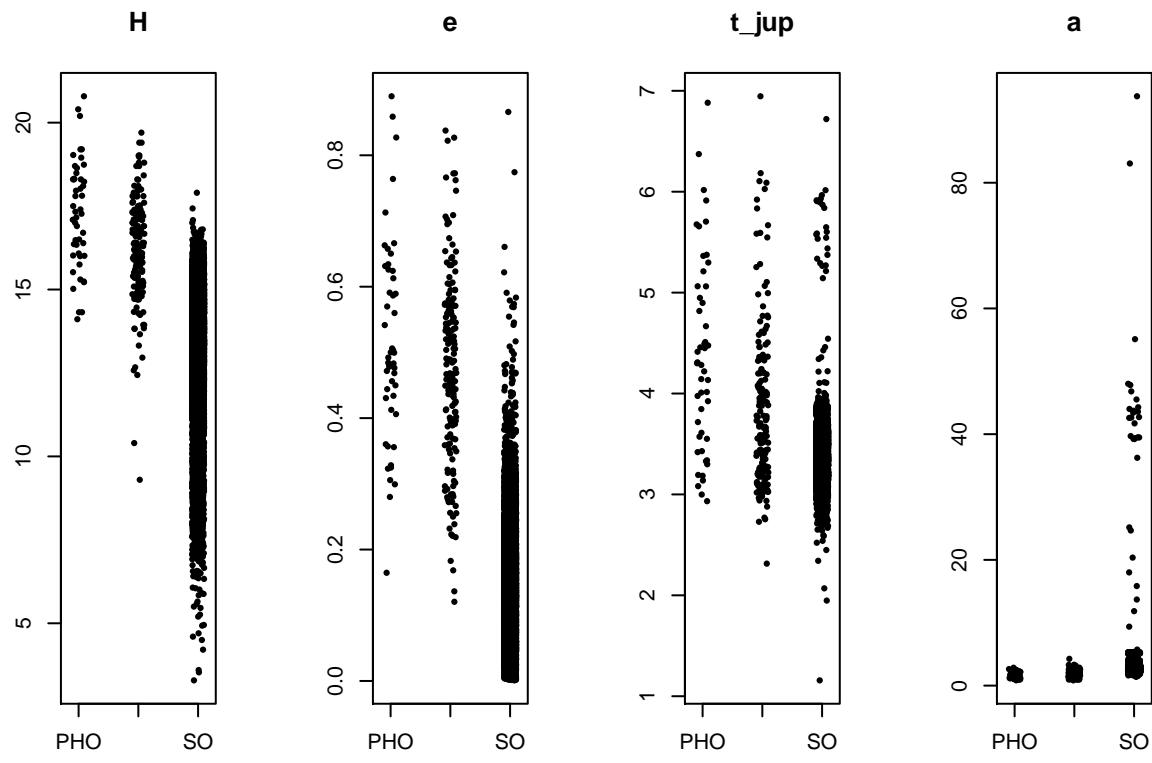


Observations:

- H , e and t_{jup} show some distinction (given a few outliers within) between the three prediction classes (PHO, NEO, SO).
- a shows distinction between the SO and the NEO, PHO groups in terms of range.

The stripchart below shows the spread of H, e, t_{jup} (of $\sim 25,000$ observations) in the neo_class :

```
par(mfrow=c(1,4))
X <- JPL_SmallBody_data[1:k,3:7]
for(iter in 2:5) { dat <- list(PHO = X[X$neo_class == "PHO",iter],
                                NEO = X[X$neo_class == "NEO",iter],SO = X[X$neo_class == "SO",iter])
  stripchart(dat,vertical=TRUE,method="jitter", pch=16, col=1, main=str_c(names(X)[iter], ""))}
```



Observations: ($\sim 25,000$ observations)

- H has a smaller range in NEO, PHO groups as compared to SO
- a has a lower range in NEO, PHO groups as compared to SO

2.6.3 q, ad, moid, moid_jup

- q : Perihelion Distance is the distance between the orbiting body and the sun at its closest approach.
- ad : Aphelion Distance is the distance between the orbiting body and the sun when it is furthest away.
- $moid$: Earth Minimum Orbit Intersection Distance. It is the distance between the closest point of osculating orbits of two bodies, in this case with Earth and the other small body object.
- $moid_{jup}$: Jupiter Minimum Orbit Intersection Distance. It is the distance between the closest point of osculating orbits of two bodies, in this case Jupiter and the other small body object.

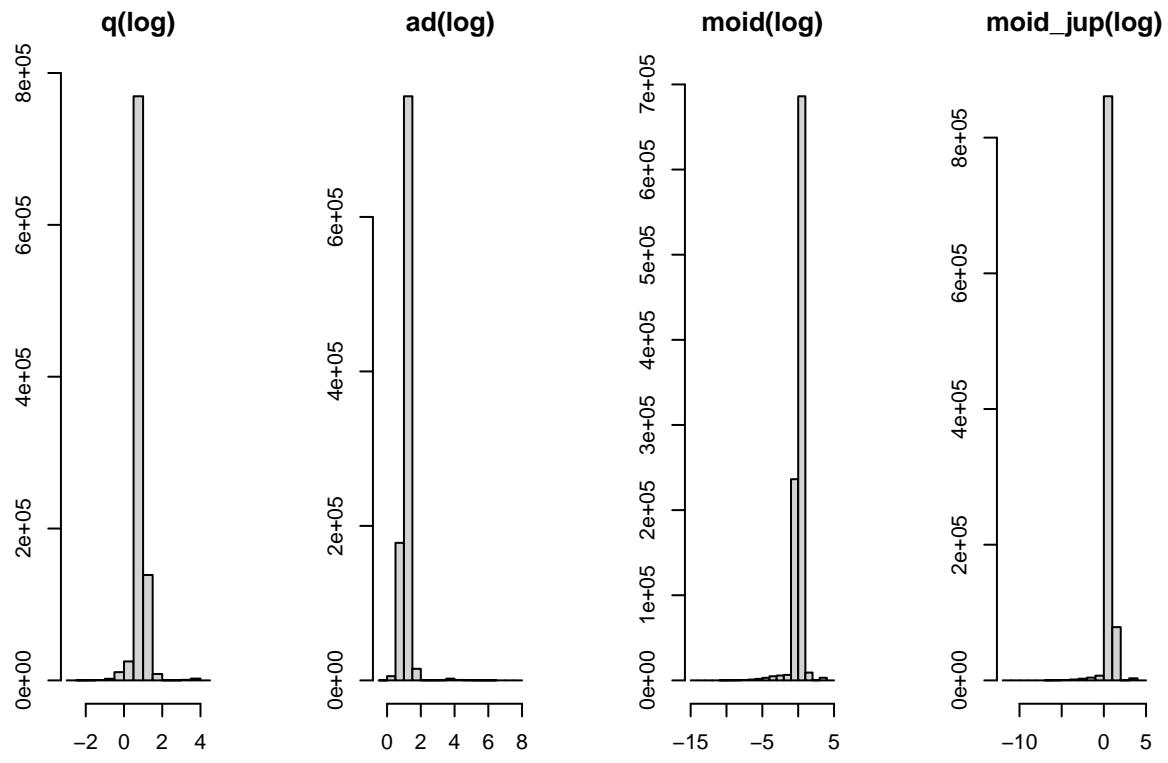
These variables are in au (astronomical unit) : 1 $au = 148,597,870.7\ km$

```
summary(JPL_SmallBody_data[8:11])
```

```
##          q              ad              moid            moid_jup
##  Min.   : 0.07   Min.   : 0.65   Min.   : 0.000   Min.   : 0.00
##  1st Qu.: 1.97   1st Qu.: 2.79   1st Qu.: 0.979   1st Qu.: 1.91
##  Median : 2.23   Median : 3.06   Median : 1.244   Median : 2.20
##  Mean   : 2.39   Mean   : 3.35   Mean   : 1.406   Mean   : 2.27
##  3rd Qu.: 2.58   3rd Qu.: 3.37   3rd Qu.: 1.595   3rd Qu.: 2.45
##  Max.   :80.40   Max.   :2906.08  Max.   :79.480   Max.   :75.78
```

- The range of $a, q, ad, moid, moid_{jup}$ show the spread of the distance.

```
par(mfrow=c(1,4))
for(iter in 8:11) { hist(log(JPL_SmallBody_data[,iter]),
                           main=str_c(names(JPL_SmallBody_data)[iter], "(log)"), xlab='', ylab='')}
```



- The distribution chart shows that $a, q, ad, moid, moid_jup$ have a very large number of values close to zero. (*PS: 0.01 au is still over a million kilometers*)

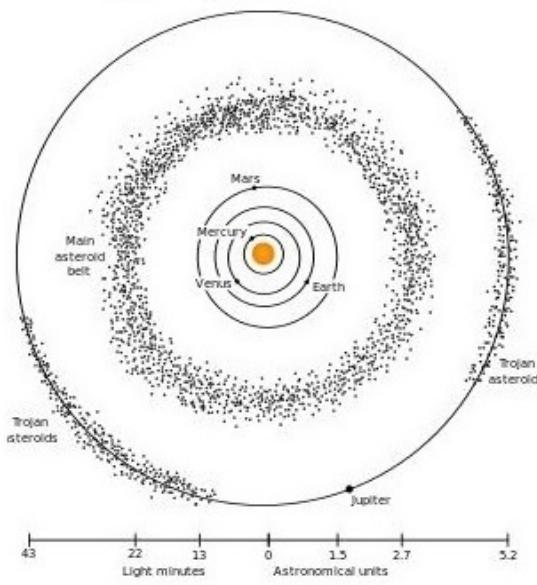
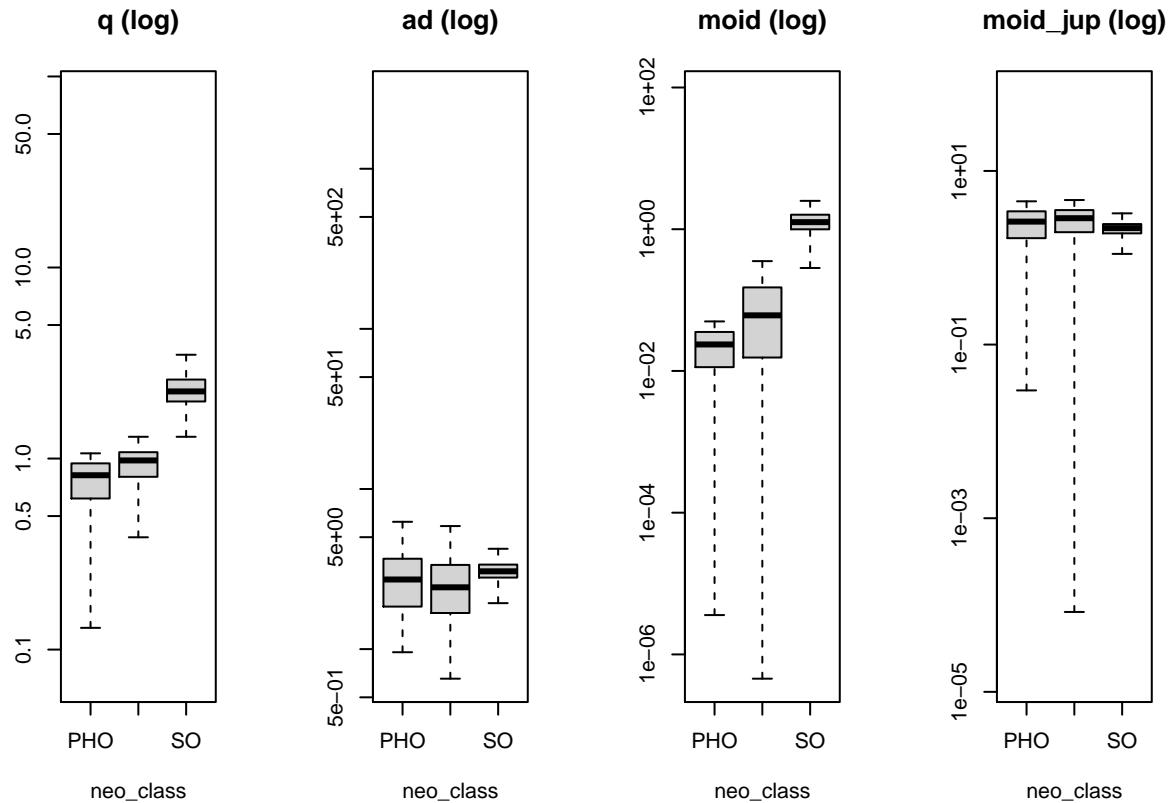


image source: wikipedia

```

par(mfrow=c(1,4))
X <- JPL_SmallBody_data %>% select(neo_class,q,ad,moid,moid_jup)
for(iter in 2:5) { dat <- list(PHO = X[X$neo_class == "PHO",iter],
                                NEO = X[X$neo_class == "NEO",iter],SO = X[X$neo_class == "SO",iter])
  boxplot(dat,xlab="neo_class",log='y',cex=0, main=str_c(names(X)[iter], " (log)"))
}

```



Observations:

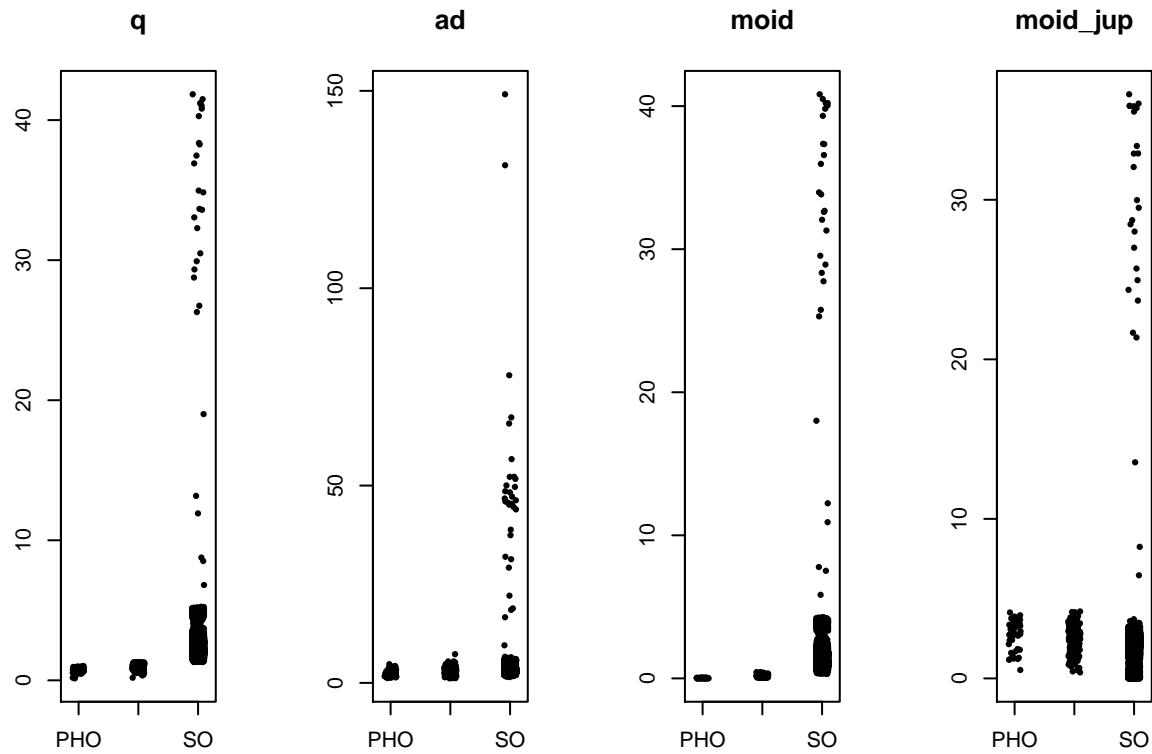
- q does show distinction between the SO and the NEO, PHO groups in value terms, with those in SO having higher values.
- ad shows distinction between the SO and the NEO, PHO groups in terms of range.
- $moid$ shows some distinction between SO and NEO, PHO in value terms, with those in SO having higher $moid$.

The stripchart below shows the spread of $a, q, ad, moid, moid_jup$ (of ~25,000 observations) in the neo_class :

```

par(mfrow=c(1,4))
X <- JPL_SmallBody_data %>% select(neo_class,q,ad,moid,moid_jup)
X <- X[1:k,]
for(iter in 2:5) { dat <- list(PHO = X[X$neo_class == "PHO",iter],
                                NEO = X[X$neo_class == "NEO",iter],SO = X[X$neo_class == "SO",iter])
  stripchart(dat,vertical=TRUE,method="jitter", pch=16, col=1, main=str_c(names(X)[iter], ""))
}

```



Observations: ($\sim 25,000$ observations)

- $q, ad, moid$ have lower range in NEO, PHO groups as compared to SO

Derived Variables

A derived variable from $moid, moid_{jup}$ is defined as:

- m_d is the difference of **Earth Minimum Orbit Intersection Distance** ($moid$) and **Jupiter Minimum Orbit Intersection Distance** ($moid_{jup}$).

$$m_d = moid_{jup} - moid$$

The new variable will reflect in the Process Data chapter.

2.6.4 i, om, w

- i : Inclination is the angle between the vectors normal to the object orbit plane and the specified reference plane (typically elliptic and the equatorial plane).
- om : Longitude of the ascending node is the angle between the X-direction (typically the vernal equinox) and the point at which the body passes up (north) through the reference plane. Often denoted by Ω .
- w : Argument of perihelion is the angle (in the object orbit plane) between the ascending node line and perihelion measured in the direction of the object orbit. Often denoted by ω .

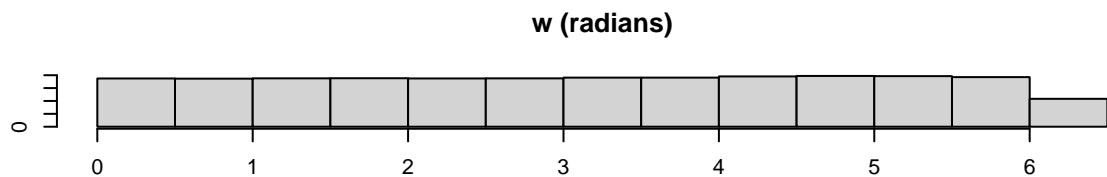
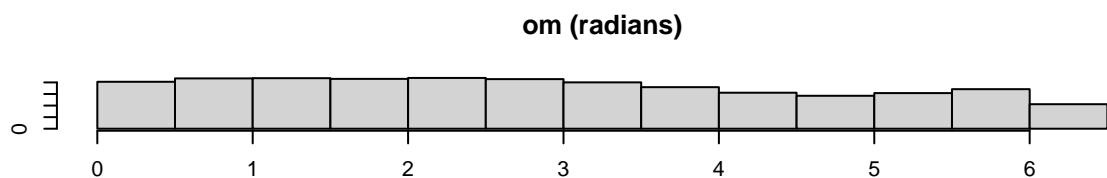
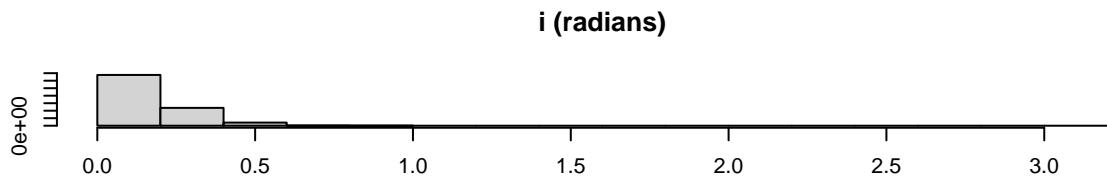
These variables are in degrees : $\pi \text{ rad} = 180 \text{ degrees}$

```
summary(JPL_SmallBody_data[12:14]*pi/180)
```

```
##          i              om              w
##  Min.   :0.00014   Min.   :0.00   Min.   :0.00
##  1st Qu.:0.07365   1st Qu.:1.41   1st Qu.:1.60
##  Median :0.13089   Median :2.79   Median :3.19
##  Mean   :0.15853   Mean   :2.94   Mean   :3.17
##  3rd Qu.:0.21697   3rd Qu.:4.41   3rd Qu.:4.74
##  Max.   :3.07137   Max.   :6.28   Max.   :6.28
```

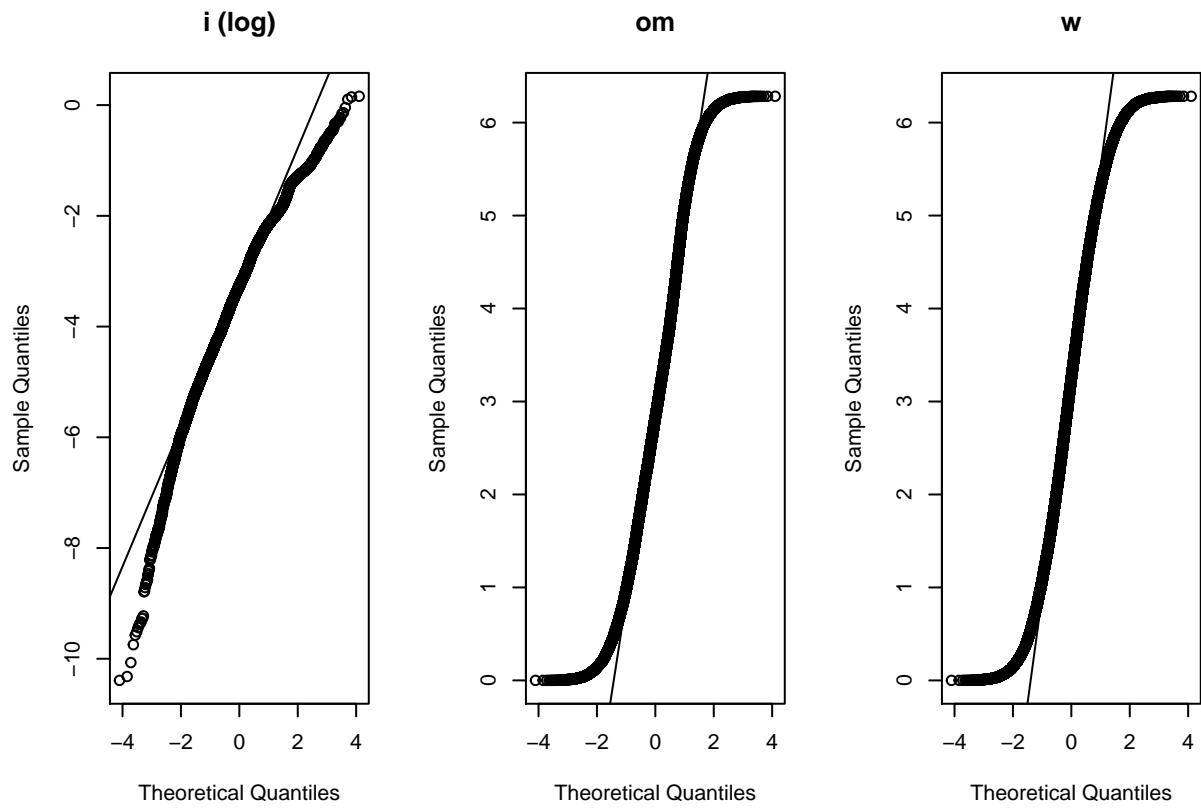
- i has a smaller range (lower maximum value) than om, w .

```
par(mfrow=c(3,1))
for(iter in 12:14) {
  hist((JPL_SmallBody_data[,iter]*pi/180),
    main=str_c(names(JPL_SmallBody_data)[iter], " (radians)", xlab='', ylab='')}
```



- i is mostly distributed in the ~ 0.5 to 0 range.

```
par(mfrow=c(1,3))
qqnorm(log2(JPL_SmallBody_data[1:k,12]*pi/180),
       main=str_c(names(JPL_SmallBody_data)[12]," (log)"))
qqline(log2(JPL_SmallBody_data[1:k,12]*pi/180))
for(iter in 13:14) {
  qqnorm((JPL_SmallBody_data[1:k,iter]*pi/180), main=names(JPL_SmallBody_data)[iter])
  qqline((JPL_SmallBody_data[1:k,iter]*pi/180))}
```



Derived Variables

- **Inclination (i), Longitude of the ascending node (om) and Argument of perihelion (w)** is being converted to radians

$$\{i = i * \frac{\pi}{180}\}, \{om = om * \frac{\pi}{180}\}, \{w = w * \frac{\pi}{180}\}$$

These changes will reflect in the Process Data chapter.

2.6.5 per, n, ma, rms

- *per* : Orbit period is the time required for an object to make a complete revolution along its orbit. A typical main belt asteroid has an orbit period of 4 years. Orbit period in earth days is used here.
- *n* : Mean Motion is the angular speed required for a body to make one orbit around an ideal ellipse with a specific semi-major axis. $\frac{(2\pi)}{\text{Orbital period}}$
- *ma* : Mean Anomaly is the product of an orbiting object mean motion and time past the perihelion passage.
- *rms* : Normalized RMS Of Orbit Fit.

```
summary(JPL_SmallBody_data[15:18])
```

| | per | n | ma | rms |
|------------|----------|-----------------|---------------|---------------|
| ## Min. | 151 | Min. :0.00002 | Min. : 0.0 | Min. : 0.0 |
| ## 1st Qu. | 1351 | 1st Qu.:0.18884 | 1st Qu.: 81.4 | 1st Qu.: 0.5 |
| ## Median | 1578 | Median :0.22820 | Median :171.2 | Median : 0.6 |
| ## Mean | 2306 | Mean :0.23614 | Mean :175.3 | Mean : 0.6 |
| ## 3rd Qu. | 1906 | 3rd Qu.:0.26656 | 3rd Qu.:269.0 | 3rd Qu.: 0.6 |
| ## Max. | 20277431 | Max. :2.38103 | Max. :369.1 | Max. :18129.0 |

- *rms* value rises after the 3rd quartile, (might be an outlier.)
- *per* value rises after the 3rd quartile, (might be an outlier.)

Outliers for *rms* :

```
JPL_SmallBody_data[JPL_SmallBody_data$rms > 100, 18]
```

```
## [1] 18129.00 402.83
```

Outliers for *per* :

```
table((JPL_SmallBody_data[JPL_SmallBody_data$per > (1000 *365), 3]))
```

```
##
## NEO PHO SO
##   1   0 128
```

There are 111 SmallBody Objects with an orbit period of *greater than 10 Centuries*, and one of them is also a NEO! (just for curiosity, which one?)

```
X <- (JPL_SmallBody_data[JPL_SmallBody_data$per > (1000 *365) &
                           JPL_SmallBody_data$neo_class == "NEO", 1])
descriptive_data <- as.data.frame(read_csv('descriptive_data.csv'))
descriptive_data %>% filter(id==X) %>% knitr::kable()
```

| id | full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|----------|-------------|------------|-----|-----|-------|---------|------------|------------|
| bK17U52R | (2017 UR52) | Otto Matic | 1 | 0 | 5787 | 0.31949 | 2017-10-29 | 2017-11-24 |

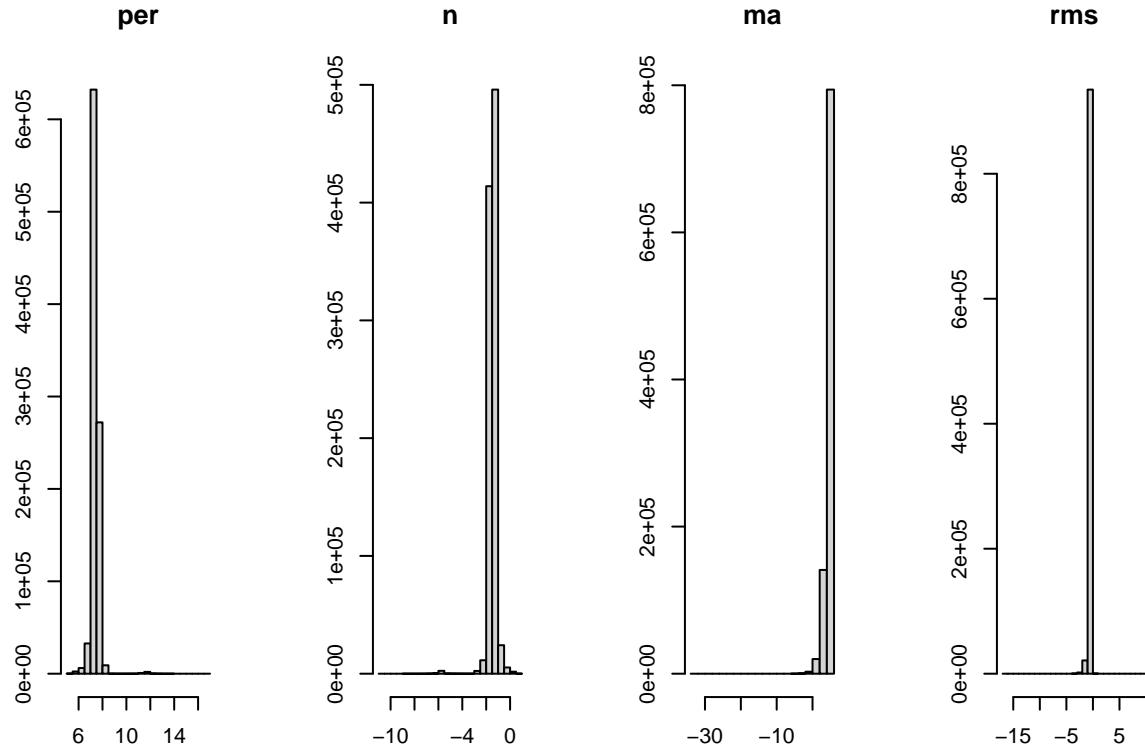
```
JPL_data[JPL_data$id == X, 1:11] %>% knitr::kable()
```

| id | class | neo_class | H | e | t_jup | a | q | ad | moid | moid_jup |
|----------|-------|-----------|------|---------|--------|--------|--------|--------|---------|----------|
| bK17U52R | AMO | NEO | 21.2 | 0.99614 | -0.416 | 322.33 | 1.2426 | 643.41 | 0.31949 | 2.2985 |

Interestingly, *2017UR52* has an Earth Minimum Orbit Intersection Distance (*moid*) of 0.31949 *au* and has an Aphelion distance *ad* of 643.41 *au*.

→ As a general rule, the outlier observations are not kept in the dataset, but since there are more variables the outliers of *per* > 10 *centuries* have been kept as other variables might be able to explain the outcome. Also, even a small body with a huge (in our time frames) orbital period might be a potential NEO (as in the case of *2017UR52*) or a PHO.

```
par(mfrow=c(1,4))
for(iter in 15:18) {
  hist(log(JPL_SmallBody_data[,iter]), main=names(JPL_SmallBody_data)[iter], xlab='', ylab='')}
```

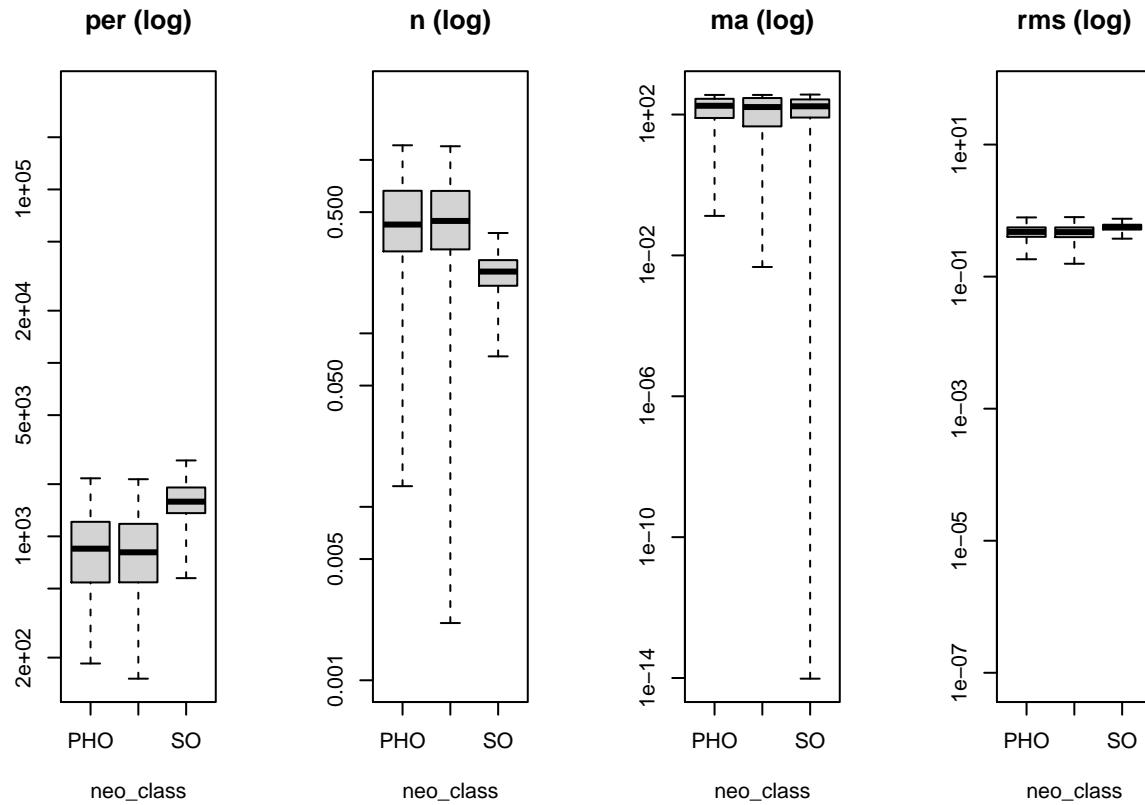


```
par(mfrow=c(1,4))
X <- JPL_SmallBody_data %>% filter(rms <= 100, per <= 1000*365) %>%
```

```

select(neo_class,per, n, ma, rms)
for(iter in 2:5) {
  dat <- list(PHO = X[X$neo_class == "PHO",iter],
              NEO = X[X$neo_class == "NEO",iter],SO = X[X$neo_class == "SO",iter])
boxplot(dat,xlab="neo_class",log='y',cex=0, main=str_c(names(X)[iter]," (log)"))
}

```



Observations:

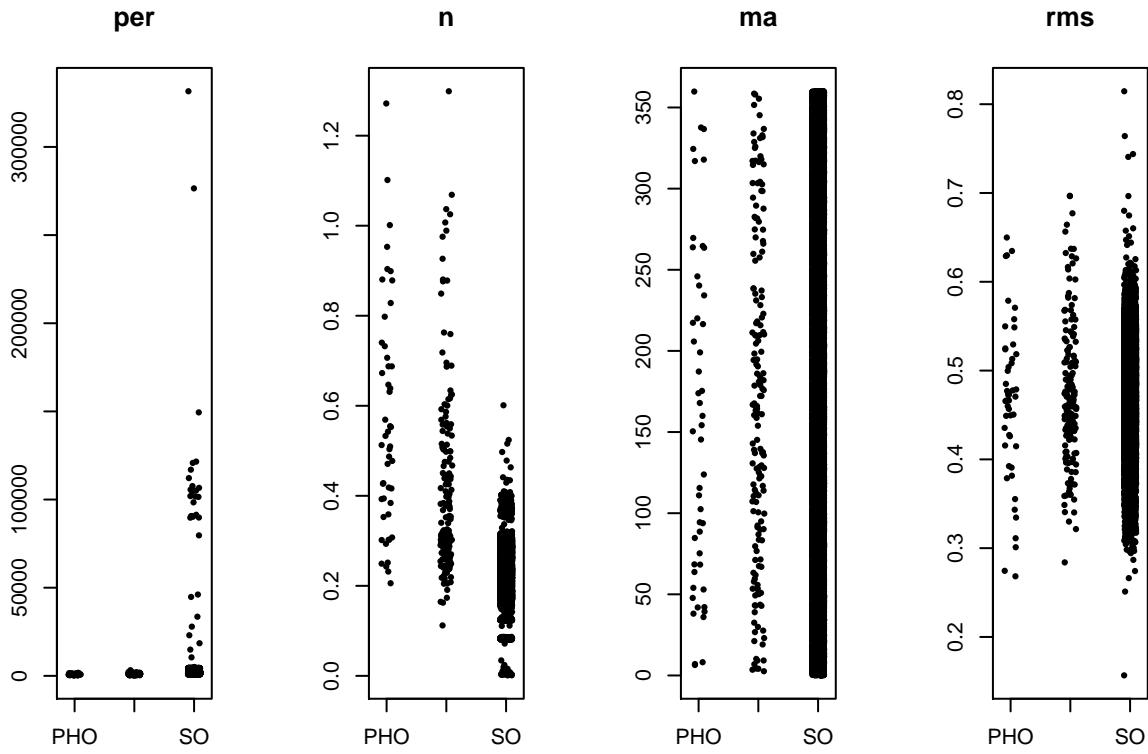
- *per* for most *NEO,PHO* objects are smaller than *SO* and might imply that lower the orbit period (*per*) less the likelihood of the object being and *NEO,PHO*.

The stripchart below shows the spread of *per, n, ma, rms* (of ~25,000 observations) in the *neo_class* :

```

par(mfrow=c(1,4))
X <- JPL_SmallBody_data %>% select(neo_class,per, n, ma, rms)
X <- X[1:k,]
for(iter in 2:5) {
  dat <- list(PHO = X[X$neo_class == "PHO",iter],
              NEO = X[X$neo_class == "NEO",iter],SO = X[X$neo_class == "SO",iter])
  stripchart(dat,vertical=TRUE,method="jitter", pch=16, col=1, main=str_c(names(X)[iter], ""))
}

```



Derived Variables

- Time past the perihelion passage $t_{\text{perihelion}}$ can be derived from Mean Anomaly (ma) and Mean Motion. (reference to time is in days)

$$t_{\text{perihelion}} = \frac{ma}{n}$$

- Time past the Aphelion passage (t_{aphelion}) is the difference between Orbital period (per) and Time past the perihelion passage (t_p). (reference to time is in days)

$$per = t_{\text{aphelion}} + t_{\text{perihelion}}$$

$$t_{\text{perihelion}} = per - t_{\text{aphelion}} \quad \text{or} \quad t_{\text{aphelion}} = per - \frac{ma}{n}$$

- Ratio of perihelion period (T_p) can thus be derived as

$$\begin{aligned} T_p &= \frac{t_{\text{perihelion}}}{per} \\ \Rightarrow T_p &= 1 - \frac{t_{\text{aphelion}}}{per} \quad \text{or} \quad T_p = \frac{\frac{ma}{n}}{per} \end{aligned}$$

These new variables will reflect in the Process Data chapter.

2.7 Variable Means

Comparison of variable means by *neo_class* :

```
pp <- JPL_SmallBody_data[,3:18]
m_comp <- data_frame()
for(iter in 2:15){
  Mean_PHO <- round(mean(pp[pp$neo_class=="PHO",iter]),2)
  Mean_NEO <- round(mean(pp[pp$neo_class=="NEO",iter]),2)
  Mean_SO <- round(mean(pp[pp$neo_class=="SO",iter]),2)
  m_comp <- bind_rows(m_comp,
    data_frame(Variable=colnames(pp[iter]),
               PHO = Mean_PHO, NEO = Mean_NEO, SO = Mean_SO))
}
m_comp %>% knitr::kable()
```

| Variable | PHO | NEO | SO |
|----------|--------|--------|---------|
| H | 20.11 | 23.30 | 16.89 |
| e | 0.54 | 0.43 | 0.15 |
| t_jup | 4.18 | 4.27 | 3.36 |
| a | 1.79 | 1.77 | 2.90 |
| q | 0.76 | 0.93 | 2.43 |
| ad | 2.81 | 2.62 | 3.37 |
| moid | 0.02 | 0.10 | 1.44 |
| moid_jup | 2.54 | 2.71 | 2.26 |
| i | 13.98 | 12.28 | 9.00 |
| om | 174.47 | 172.90 | 168.44 |
| w | 180.27 | 182.38 | 181.56 |
| per | 914.54 | 992.27 | 2338.50 |
| n | 0.52 | 0.52 | 0.23 |
| ma | 179.22 | 171.02 | 175.38 |

Observations:

- *per* group means in descending order - *SO,NEO,PHO*.
- *ma* group means in descending order - *PHO,SO,NEO*.
- *H* group means in descending order - *NEO,PHO,SO*.
- *q* group means in descending order - *SO,NEO,PHO*.
- *i* group means in descending order - *PHO,NEO,SO*.

These observations show that the variables display a pattern in their values.

2.8 Distance

But, a picture is worth a thousand words.

The JPL_SmallBody_data dataset is normalized by columns, for each *neo_class*, and then ordered according to the results of the clustering algorithm for a visual cue.

```
x <- JPL_SmallBody_data[,3:18]

#extract and sweep different categories
ph <- x[x$neo_class == "PHO",-1] %>% as.matrix() %>% sweep(.,2, colMeans(.))
ne <- x[x$neo_class == "NEO",-1] %>% as.matrix() %>% sweep(.,2, colMeans(.))
so <- x[x$neo_class == "SO",-1] %>% as.matrix() %>% sweep(.,2, colMeans(.))

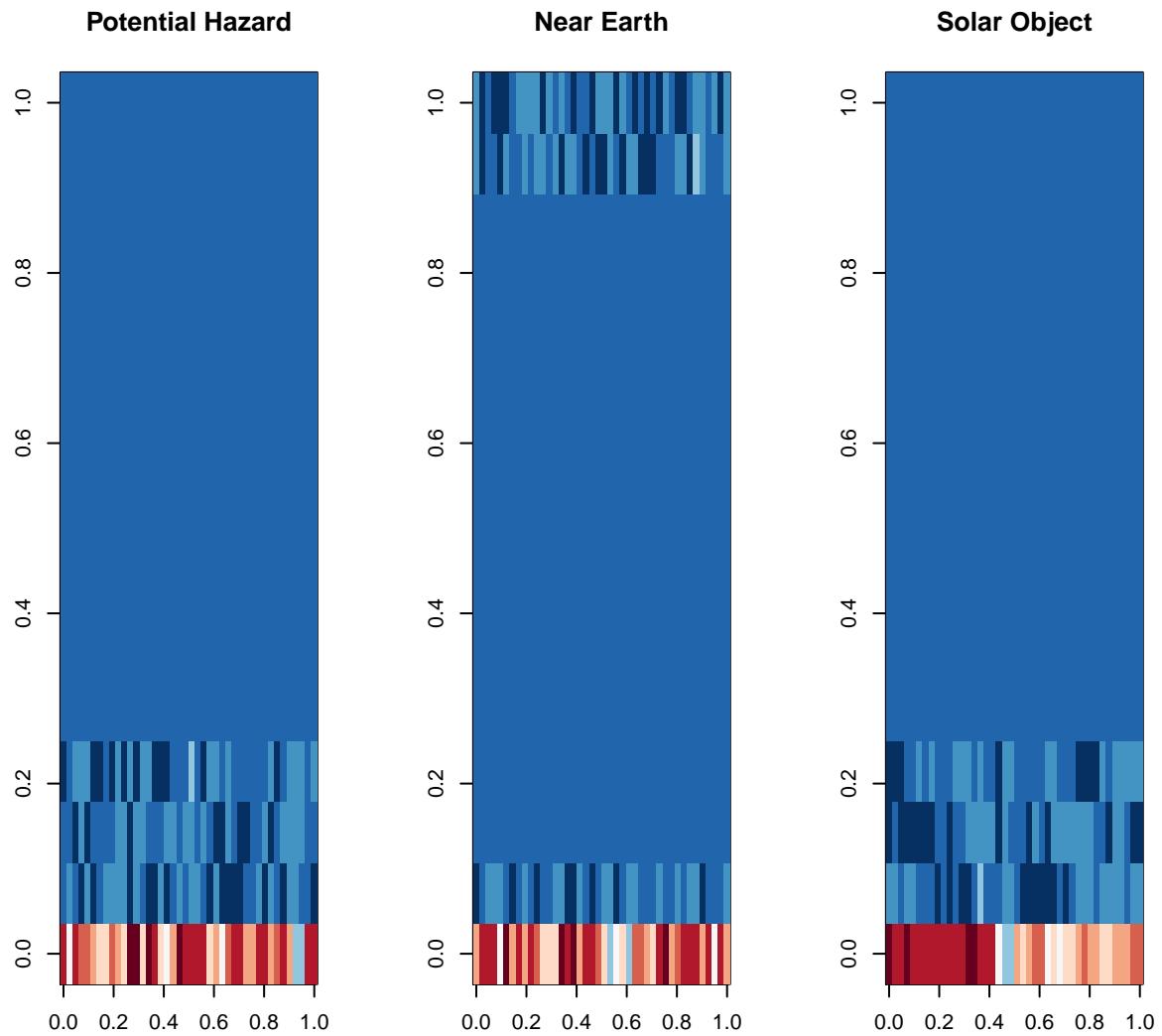
#sweep the data matrix
x <- as.matrix(x[,-1])
x <- sweep(x,2, colMeans(x))

k <- 42
#hclusters based on data and the transpose
ph_1 <- hclust(dist(ph[1:k,]))
ph_2 <- hclust(dist(t(ph[1:k,])))
ne_1 <- hclust(dist(ne[1:k,]))
ne_2 <- hclust(dist(t(ne[1:k,])))
so_1 <- hclust(dist(so[1:k,]))
so_2 <- hclust(dist(t(so[1:k,])))

#image of the hclust
par(mfrow=c(1,3))
image(x[ph_1$order, ph_2$order],
      col=RColorBrewer::brewer.pal(11,'RdBu'), main = "Potential Hazard")

image(x[ne_1$order, ne_2$order],
      col=RColorBrewer::brewer.pal(11,'RdBu'), main = "Near Earth")

image(x[so_1$order, so_2$order],
      col=RColorBrewer::brewer.pal(11,'RdBu'), main = "Solar Object")
```



Observations:

- The image gives a clear visual cue on how the variables in the data provide a clear distinction between the individual categories of the Outcome variable *neo_class*.

3 Train & Test sets

```
set.seed(131825431, sample.kind = "Rounding")
# test set will be 10% of training set.
test_index <- createDataPartition(JPL_data$neo_class, times = 1, p = 0.1, list = FALSE)
JPL_train <- JPL_data[-test_index,]
JPL_test <- JPL_data[-test_index,]
```

3.1 Process Data

The variables are modified as mentioned above for the datasets to be used for training and testing. This data processing will also be done for the validation set.

```
data_process <- function(preData, tChoice){
  #convert to radians and add additional derived variables
  preData <- preData %>%
    mutate(om=om*(pi/180), w=w*(pi/180), i=as.numeric(i*(pi/180)),
          t_to_p = as.numeric((per - (ma/n))), a=log2(a),
          m_d = as.numeric(moid_jup-moid))
  preData <- preData %>% mutate(T_p = as.numeric(round(1-(t_to_p/per),4)))
  preData <- preData %>% select(neo_class,H,q,moid,m_d,e,t_jup,a,i,T_p)
  preData$neo_class <- as.factor(preData$neo_class)

  #return as matrix format
  if(tChoice == 1){
    preData <- data.frame(preData)
    x_matrix <- as.matrix(preData[,-1])
    y_matrix <- factor(preData[,1])
    processData <- list(X = x_matrix, Y = y_matrix)}
  #return as data table
  if(tChoice == 2) processData <- preData
  return (processData)
}
```

3.2 Inaccuracy Details function

```
descriptive_data <- data.table::fread("descriptive_data.csv", na.strings = c(NA,"NA","",""))
descriptive_data <- as.data.frame(descriptive_data)

#function to collate inaccurately classified observations.
inaccurate_details <- tibble()
inaccuracy_check <- function(incorrect_p,method,dat){
  idetails <- data.frame()
  for(each in incorrect_p){
    a <- (dat[each,1])
    idetails <- rbind(idetails,data.frame(model_name = method,
                                            descriptive_data[descriptive_data$id==str_trim(a),]))}
  return(idetails)}
```

4 Model Evaluations

4.1 Evaluation metrics

4.1.1 Definitions

The confusion matrix

| | Actual Positive | Actual Negative |
|--------------------|---------------------|---------------------|
| Predicted Positive | True Positive (TP) | False Positive (FP) |
| Predicted Negative | False Negative (FN) | True Negative (TN) |

Recall, 1-FPR, and Precision

| | | | Definition | Probability representation |
|-------------|-----|-----------|--------------------|----------------------------|
| sensitivity | TPR | Recall | $\frac{TP}{TP+FN}$ | $\Pr(\hat{Y} = 1 Y = 1)$ |
| specificity | TNR | 1-FPR | $\frac{TN}{TN+FP}$ | $\Pr(\hat{Y} = 0 Y = 0)$ |
| specificity | PPV | Precision | $\frac{TP}{TP+FP}$ | $\Pr(Y = 1 \hat{Y} = 1)$ |

F1 Score

This is the harmonic average of Precision and Recall and is given as :

$$2 \times \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

4.1.2 Evaluation Function

In this case study the confusionmatrix function of the caret package is used. The following function takes the confusion matrix and adds the Recall and the F1 scores to a tibble for the results.

```
#function to collate the results
results_process<- function(x,name){
  x1 <- data_frame(model_name=name, Object="NEO",
                     Recall=round(x[1,6],3), F1=round(x[1,7],3))
  x2 <- data_frame(model_name=name, Object="PHO",
                     Recall=round(x[2,6],3), F1=round(x[2,7],3))
  x3 <- data_frame(model_name=name, Object="SO",
                     Recall=round(x[3,6],3), F1=round(x[3,7],3))
  return(bind_rows(x1,x2,x3))
}
```

4.2 PCA

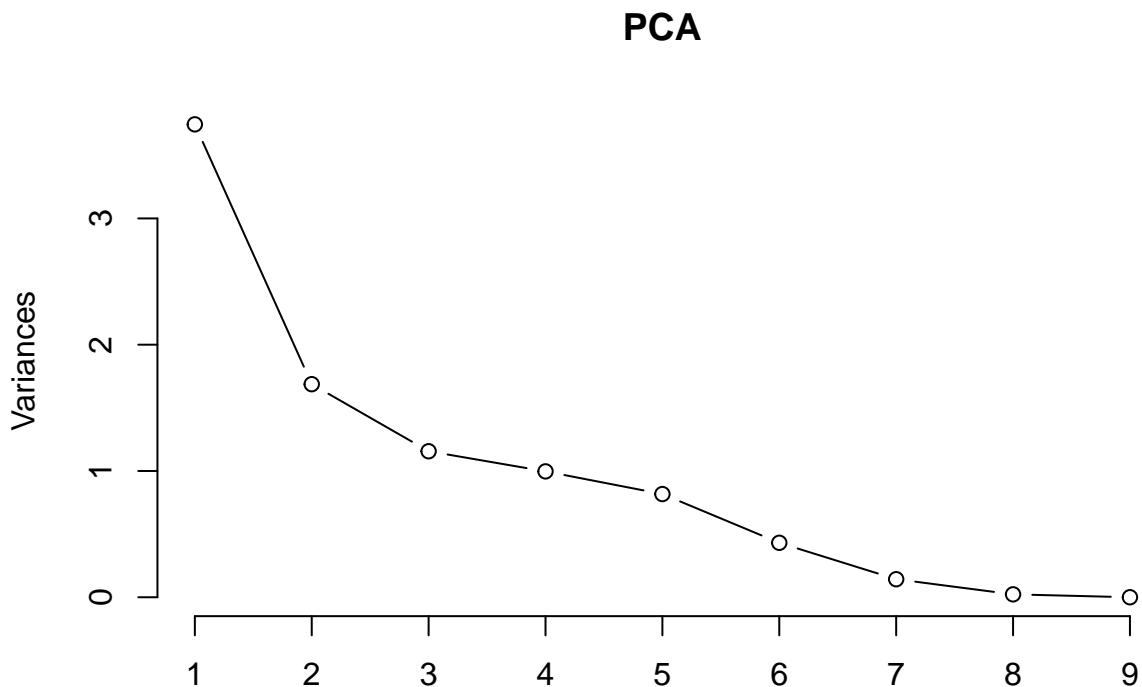
The principal component Analysis assists in reducing the complexity of model being fit by transforming the data, with the distance between the rows preserved but with the variance of the variables in decreasing order.

```
mat_train <- data_process(JPL_train,1)
mat_test <- data_process(JPL_test,1)
set.seed(131825431, sample.kind = "Rounding")
pca <- prcomp(mat_train$X, center = TRUE, scale. = TRUE)
summary(pca)

## Importance of components:
##              PC1    PC2    PC3    PC4    PC5    PC6    PC7    PC8
## Standard deviation     1.935 1.299 1.075 0.998 0.9040 0.6568 0.3781 0.15187
## Proportion of Variance 0.416 0.187 0.128 0.111 0.0908 0.0479 0.0159 0.00256
## Cumulative Proportion  0.416 0.604 0.732 0.843 0.9336 0.9815 0.9974 0.99998
##                                     PC9
## Standard deviation      0.01258
## Proportion of Variance 0.000002
## Cumulative Proportion  1.00000
```

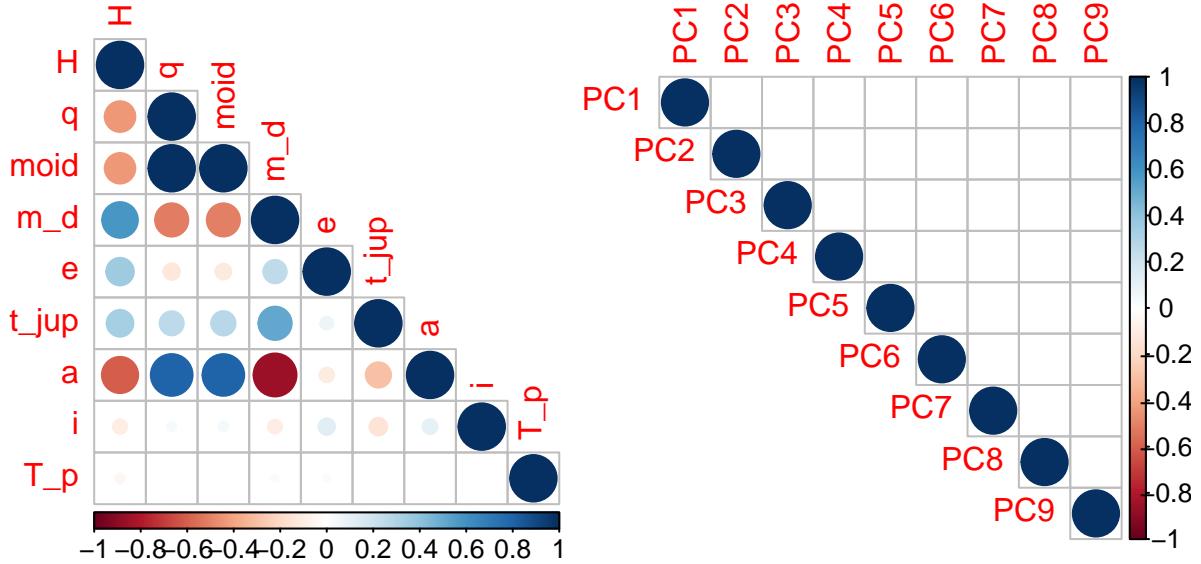
- The first six principal components account for about 80% of the variance in our training set.
- The plot below shows the corresponding variances for the PCAs.

```
par(mfrow=c(1,1))
screeplot(pca,type = "line", main="PCA")
```



Correlation Plot shows the correlation between the variables.

```
par(mfrow=c(1,2))
corrplot(cor(mat_train$x), method="circle", type="lower")
corrplot(cor(pca$x), method="circle", type="upper")
```



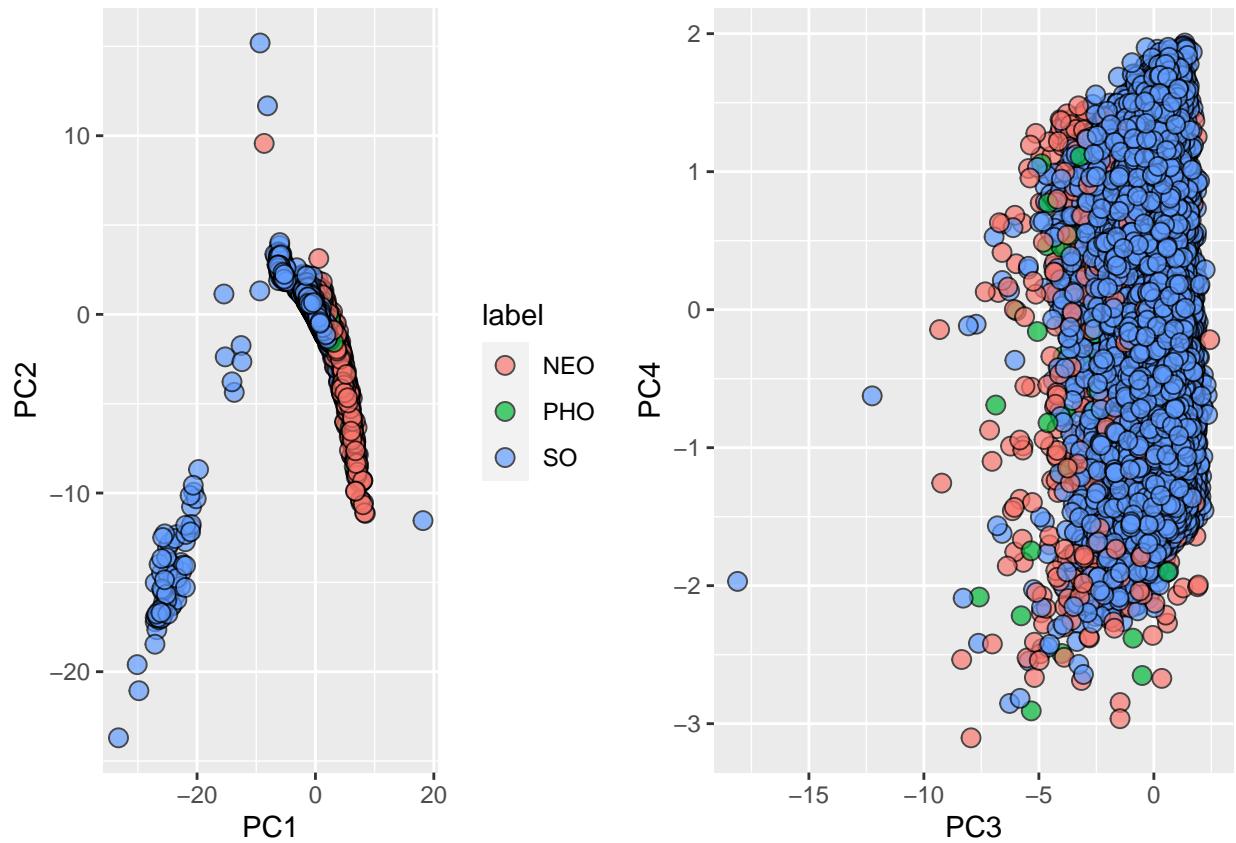
Observations:

- The qa_d variable, which is the difference between the Perihelion Distance q and the Aphelion Distance a , has a correlation of 0.92 with the Semi-Major Axis a .
- There is no correlation in the pca components.

Plot of the first 4 components of the PCA together even on a random sample, gives a visual cue on the information on the *neo_class*.

Note: random sample of 24,576 observations used.

```
set.seed(131825431, sample.kind = "Rounding")
par(mfrow=c(1,1))
p1 <- data.frame(PC1 = pca$x[,1], PC2 = pca$x[,2], label= mat_train$Y) %>%
  sample_n(24576) %>% ggplot(aes(PC1,PC2, fill = label)) +
  geom_point(cex=3,pch=21,alpha=0.7) + theme(legend.position = "right")
p2 <- data.frame(PC3 = pca$x[,3], PC4 = pca$x[,4],label= mat_train$Y) %>%
  sample_n(24576) %>% ggplot(aes(PC3,PC4, fill = label)) +
  geom_point(cex=3,pch=21,alpha=0.7) + theme(legend.position = "hide")
gridExtra::grid.arrange(p1,p2, ncol=2, nrow=1)
```



4.2.1 Multinom

The first model that is tried is a multinom model using the nnet library. It fits multinomial log-linear models via neural networks.

- The first 7 components of the PCA are used, which account for 98.6% of the variance.
- The test set is rotated using the rotation matrix of PCA.

Model Fitting :

```
#first 7 components
k <- 7
train_pca <- data.frame(pca$x[,1:k], neo_class= as.factor(mat_train$Y))
test_pca <- as.matrix(sweep(mat_test$X, 2, colMeans(mat_test$X))) %*% pca$rotation
test_pca <- data.frame(test_pca[,1:k], neo_class= as.factor(mat_test$Y))
set.seed(131825431, sample.kind = "Rounding")
fit_pca_multinom <- multinom(neo_class ~ ., data = train_pca, sumn=1)
```

Model Prediction and the confusion matrix:

```
y_hat_pca_multinom <- predict(fit_pca_multinom,test_pca, type = "class")
cm <- confusionMatrix(y_hat_pca_multinom, test_pca$neo_class)
results <- results_process(as.data.frame(cm$byClass),"PCA Multinom")
cm$table
```

```
##          Reference
## Prediction    NEO     PHO     SO
##      NEO     989     59     0
##      PHO     102     61     0
##      SO    17973   1631 841826
```

```
cat("Accuracy : ", cm$overall["Accuracy"], "\n")
```

```
## Accuracy : 0.97709
```

- The multinom model was able to identify some of the categories, however since the 7 PCA components used explain about 99.7% of the variablily. The accuracy that we have is almost 98%, but it is the Sensitivity in individual categories (SO has a Recall of 1) that matters as well in this case.

```
results %>% filter (model_name == "PCA Multinom") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|--------------|--------|--------|-------|
| PCA Multinom | NEO | 0.052 | 0.098 |
| PCA Multinom | PHO | 0.035 | 0.064 |
| PCA Multinom | SO | 1.000 | 0.988 |

The PCA components, though explain the variablily with lesser variables (components) will not be used as the Recall of 0.99 is the objective, and though accuracy is high the requiremnt of Recall is not being met.

4.3 Naive Bayes

The second model is the Naive Bayes model, a general generative model.

- In a binary case, the smallest true error achieved is determined by the Bayes rule, which is a decision rule that is based on true conditional probability, and is given by $p(X) = Pr(Y = 1|X = x)$.
- Predictors are assumed to be independent within each class label.
- Numeric (metric) predictors are handled by assuming that they follow Gaussian distribution, given the class label, and are modelled with Poisson distribution.
- Prediction is done by the type class.

Model Fitting, Prediction and the confusion matrix:

```
set.seed(131825431, sample.kind = "Rounding")
fit_nb <- naive_bayes(mat_train$X, mat_train$Y, usekernel=FALSE, usepoisson = TRUE)
y_hat_nb <- predict(fit_nb, mat_test$X, type = "class")
cm <- confusionMatrix(y_hat_nb, mat_test$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Naive Bayes"))
cm$table
```

```
##             Reference
## Prediction    NEO      PHO      SO
##       NEO  18774      540    4390
##       PHO    246    1211     172
##       SO      44      0  837264
```

- This model does a bit with a higher F1 and Recall and is able to classify NEO and PHO better.

```
results %>% filter (model_name == "Naive Bayes") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|-------------|--------|--------|-------|
| Naive Bayes | NEO | 0.985 | 0.878 |
| Naive Bayes | PHO | 0.692 | 0.717 |
| Naive Bayes | SO | 0.995 | 0.997 |

- The Recall for all three categories is above 0.9 but the F1 Score (the harmonic average of Precision and Recall) for PHO is below 0.5

4.4 Ensemble

In order to get the right direction on the model which will fit well with the data set, an Ensemble of 6 models is run on a smaller sample.

- LDA : Linear discriminant Analysis. The assumption here is correlation structure is the same for all classes, which thus reduces the number of parameters that are required to estimate.
- QDA : Quadratic discriminant Analysis. This is a version of Naive Bayes in which the assumption is that the distributions $p_{X|Y=1}(x)$ and $p_{X|Y=0}(x)$ are multivariate normal.
- KNN : k-Nearest Neighbor Classification. The k nearest (in Euclidean distance) training set vectors are found, and the classification is decided by majority vote.
- rpart : Recursive Partitioning and Regression Trees. These are decision trees that work by predicting an outcome variable Y by partitioning the predictors.
- svmRadialCost : Support Vector Machine with radial cost. In simple terms, the SVM constructs a linear model with the largest possible margin given to the data points.
- wsrf : Forest of Weighted Subspace Decision Trees. C4.5-based trees (Quinlan (1993)) are grown by wsrf, where binary split is used for continuous predictors (variables) and k-way split for categorical ones.

Note: *sample of 16,384 observations for training and 1638 obeservations for testing is used.*

Model Fitting :

```
set.seed(131825431, sample.kind = "Rounding")
ensemble_train <- data_process(JPL_train[1:16348,],1)
ensemble_test <- data_process(JPL_test[1:1638,],1)
results_ensemble <- data.frame()

models <- c("lda", "qda", "knn", "rpart", "svmRadialCost", "wsrf")

fits <- lapply(models, function(model){
  cat(":- ",model, "\n")
  if (model %in% c("lda","qda")){
    train(ensemble_train$X,ensemble_train$Y, method=model, prior = c(1,1,1)/3)
  }
  else train(ensemble_train$X,ensemble_train$Y, method=model,
             trControl = trainControl(method = "cv", number=5))
})
```

Prediction and the confusion matrix :

```
names(fits) <- models
pred <- sapply(fits,function(object){ predict(object, ensemble_test$X) })

for (each in 1:ncol(pred)){
  cm <- confusionMatrix(factor(pred[,each]), ensemble_test$Y)
  results_ensemble <- bind_rows(results_ensemble,
                                results_process(as.data.frame(cm$byClass), colnames(pred)[each]))
}
```

Ensemble Results:

```
results_ensemble %>% filter (model_name == "lda") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| lda | NEO | 0.400 | 0.480 |
| lda | PHO | 0.667 | 0.500 |
| lda | SO | 0.998 | 0.997 |

```
results_ensemble %>% filter (model_name == "qda") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| qda | NEO | 1.000 | 0.732 |
| qda | PHO | 1.000 | 1.000 |
| qda | SO | 0.993 | 0.997 |

```
results_ensemble %>% filter (model_name == "knn") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| knn | NEO | 0.600 | 0.643 |
| knn | PHO | 0.000 | NA |
| knn | SO | 0.999 | 0.998 |

```
results_ensemble %>% filter (model_name == "rpart") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|----|
| rpart | NEO | 1 | 1 |
| rpart | PHO | 1 | 1 |
| rpart | SO | 1 | 1 |

```
results_ensemble %>% filter (model_name == "svmRadialCost") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|---------------|--------|--------|-------|
| svmRadialCost | NEO | 1.000 | 0.938 |
| svmRadialCost | PHO | 0.333 | 0.500 |
| svmRadialCost | SO | 1.000 | 1.000 |

```
results_ensemble %>% filter (model_name == "wsrf") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|----|
| wsrf | NEO | 1 | 1 |
| wsrf | PHO | 1 | 1 |
| wsrf | SO | 1 | 1 |

- The qda model has a Recall of 1 for the PHO category.
- The rpart and the wsrf models fit well with this subset of the data.

Thus, from the ensemble it seems that the decision tree based models will be the ones which achieve the objective of 0.99 Recall and 0.99 F1.

4.5 Decision Trees

4.5.1 Rpart

Recursive Partitioning and Regression Trees.

These are decision trees that predict an outcome variable Y by partitioning the predictors.

The idea is to build a decision tree and, at the end of each node, obtain a predictor \hat{y} . The prediction space is partitioned into J non-overlapping regions, R_1, R_2, \dots, R_J and then for any predictor x that falls within the region R_j , estimate $f(x)$ with the average of the training observations y_i for which the associated predictor x_i is also in R_j . Regression trees create partitions recursively. Description of how the partition is picked for further partitions and when to stop is given below:

$$R_1(j, s) = \{x | x_j < s\} \text{ and } R_2(j, s) = \{x | x_j \geq s\}$$

- x is the partition to be split.
- j is the predictor.
- s is the value that defines the 2 partitions.

j and s are picked up by finding the pair that minimizes the residual sum of squares (RSS):

$$\sum_{i: x_i \in R_1(j, s)} (y_i - \hat{y}_{R_1})^2 + \sum_{i: x_i \in R_2(j, s)} (y_i - \hat{y}_{R_2})^2$$

This is then applied recursively to the new regions R_1 and R_2 . After partitioning the predictor space into regions, in each region a prediction is made using the observations in that region.

Model Parameters :

- Complexity Parameter : The residual sum of squares must improve by a factor of cp for the new partition to be added. Large values of cp will therefore force the algorithm to stop earlier which results in fewer nodes.

Complexity Parameter :

```
set.seed(131825431, sample.kind = "Rounding")
train_rpart <- train(mat_train$X, mat_train$Y, method="rpart",
                      tuneGrid = data.frame(cp=seq(0.0,0.05, len=25)),
                      trControl = trainControl(method = "cv", number=5))
rpart_cp <- train_rpart$bestTune
rpart_cp
```

```
##          cp
## 20 0.039583
```

Variable Importance :

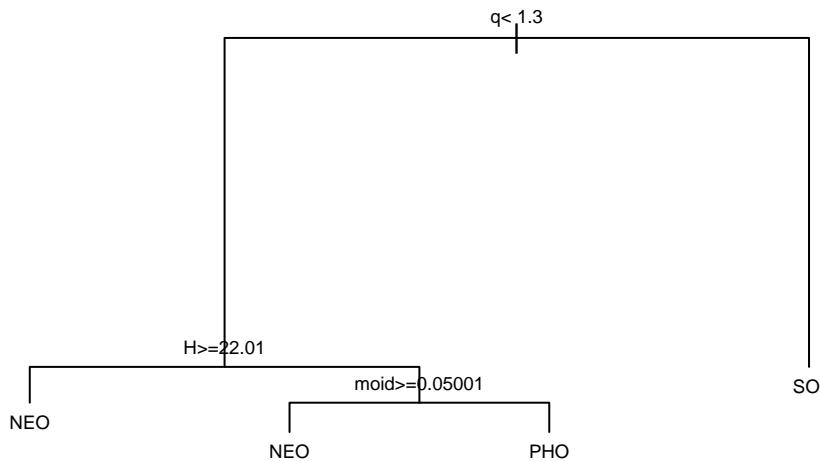
```
train_rpart$finalModel$variable.importance
```

```
##      moid         q         H         a         m_d         e         i       t_jup
## 38705.79 37569.11 26177.31 19022.00 18035.70 15214.81   190.64    59.35
```

- $moid, q, H, a, m_d, e, i, t_jup$ are used in the rpart fit.

Visual cue on the decision tree:

```
plot(train_rpart$finalModel, margin=0.1)
text(train_rpart$finalModel, cex=0.6)
```



Model Fitting, Prediction and the confusion matrix:

```
rtrain <- data_process(JPL_train, 2)
rtest <- data_process(JPL_test, 2)
set.seed(131825431, sample.kind = "Rounding")
fit_rpart <- rpart(neo_class ~ ., data=rtrain, parms = list(split = "gini"),
control = rpart.control(cp = rpart_cp))
predcit_rpart <- predict(fit_rpart, rtest)
labels <- colnames(predcit_rpart)[apply(predcit_rpart, 1, which.max)]
cm <- confusionMatrix(factor(labels), rtest$neo_class)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "rpart"))
cm$table
```

```
##             Reference
## Prediction    NEO      PHO      SO
##           NEO  19026      15      0
##           PHO     38   1736      0
##           SO      0       0  841826
```

Results :

```
results %>% filter (model_name == "rpart") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| rpart | NEO | 0.998 | 0.999 |
| rpart | PHO | 0.991 | 0.985 |
| rpart | SO | 1.000 | 1.000 |

- The Recall is ≥ 0.99 , but the F1 score for *PHO* is 0.985

However, the rpart is known to not be very flexible and is unstable to changes in the training data. Random Forest is used next to overcome some of these shortcomings.

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  labels != rtest$neo_class ), "rpart", JPL_test))
```

4.5.2 Random Forest

Random forests are a popular machine learning approach that addresses the shortcomings of decision trees and improving prediction performance and reduce instability by *averaging* multiple decision trees (a forest of trees constructed with randomness).

This is done by

- Bagging : Generate many predictors, each using regression or classification trees.
- Bootstrap : To induce randomness of individual trees bootstrap is used.

The individual trees **randomly** different, and the combination of trees is the **forest**.

`randomForest` implements Breiman random forest algorithm (based on Breiman and Cutler original Fortran code) for classification and regression.

Tuning Function :

```
gettuned <- function(method_name,grid,dat){
  set.seed(131825431, sample.kind = "Rounding")
  control <- trainControl(method='cv', number = 5)
  train(x = dat$X[2048:8192,], y = dat$Y[2048:8192], nTree=50,
        method=method_name, tuneGrid = grid, trControl = control)$bestTune}
```

Model Parameters :

- `mtry` : Number of variables randomly sampled as candidates at each split. The function `gettuned` above gets the `mtry` for the bestTuned `randomForest` of a small sample size of 6,144 observations.
- `strata` : A (factor) variable that is used for stratified sampling.
- `sampsize` : Size of sample to draw.
- `nodesize` : Minimum size of terminal nodes. For classification it is 1.
- `nTree` : Number of trees to grow.

```
rf_mtry <- gettuned("rf",data.frame(mtry=c(2,3,4,5,6)),mat_train)
```

Model Fitting :

```
set.seed(131825431, sample.kind = "Rounding")
rf_train <- data_process(JPL_train,2)
rf_test <- data_process(JPL_test,2)
fit_rf <- randomForest(neo_class ~ .,data=rf_train, mtry=rf_mtry$mtry,
                       nodesize = 1, strata = neo_class, nTree=rf_mtry$mtry*50,
                       sampsize = 100000, replace=FALSE)
```

Gini Index is defined as

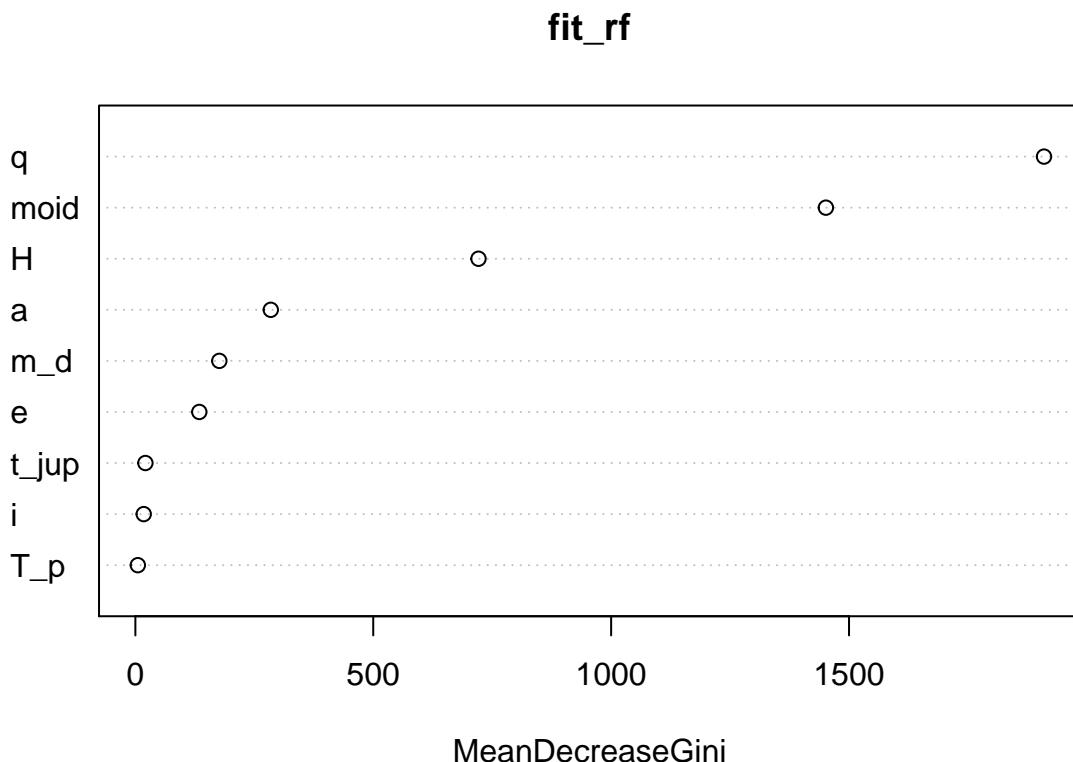
$$\text{Gini}(j) = \sum_{k=1}^K \hat{p}_{j,k} (1 - \hat{p}_{j,k})$$

- $\hat{p}_{j,k}$ as the proportion of observations in partition
- j that are of class k .

GINI importance measures the gain of purity by partitions of a given variable and assists in the decision function of the random forest to select from available partitions.

Variable Importance of the last measure:

```
varImpPlot(fit_rf)
```



- $q, moid, H$ achieved the highest meanDecrease, in the last measure.

Prediction and the confusion matrix:

```
y_hat_rf <- predict(fit_rf, rf_test)

cm <- confusionMatrix(y_hat_rf, rf_test$neo_class)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Random Forest"))
cm$table
```

```
##           Reference
## Prediction    NEO     PHO      SO
##   NEO    19035     19      1
##   PHO      29    1732      0
##   SO        0      0  841825
```

Results :

```
results %>% filter (model_name == "Random Forest") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|---------------|--------|--------|-------|
| Random Forest | NEO | 0.998 | 0.999 |
| Random Forest | PHO | 0.989 | 0.986 |
| Random Forest | SO | 1.000 | 1.000 |

- The Recall is ≥ 0.99 , but the F1 scores for *PHO* is 0.986
- In the rpart NEO had 38 misses and PHO had 15 misses. In the Random Forest though the NEO misses have been lesser the PHO misses have gone up.

This might be caused due to the structure of the data which has over 97.6% of SO. The shortcoming of not being able to associate weights to categories of importance is approached with the WSRF model.

```
incorrect_p <- which(y_hat_rf != rf_test$neo_class )
inaccurate_details <- rbind(inaccurate_details,inaccuracy_check(
  incorrect_p,'Random Forest',JPL_test))
```

4.5.3 WSRF

Forest of Weighted Subspace Decision Trees

Model Parameters :

- mtry : number of trial predictors for a split (mtry).
- nTree : the number of trees to train.
- minNode : minimum number of distinct row references to split a node.
- weights : TRUE for weighted subspace selection.
- importance : Importance of predictors.

```
wsrf_mtry <- gettuned("wsrf", data.frame(mtry=c(2,3,4,5,6)), mat_train)
```

Model Fitting:

```
fit_wsrf <- wsrf(x = mat_train$X, y = mat_train$Y,
                    ntree = wsrf_mtry$mtry*100, mtry=wsrf_mtry$mtry+1,
                    weights=TRUE, importance = TRUE, parallel = TRUE)
```

Variable Importance:

```
fit_wsrf$importance
```

```
##          1          2          3 MeanDecreaseAccuracy MeanDecreaseIGR
## H 4.3853e-01 -0.00601504 0.000000 9.6773e-03 1.60424
## q 9.7405e-01  0.95160149 0.024147 4.7017e-02 2.15759
## moid 2.0908e-03 0.96285192 0.000000 1.9989e-03 2.34837
## m_d 1.5502e-03 0.00228858 0.000000 3.8873e-05 1.02320
## e 2.2230e-03 0.00486232 0.000000 5.9164e-05 1.03253
## t_jup 9.9007e-04 0.00059395 0.000000 2.3076e-05 0.88126
## a 2.1911e-04 0.00070127 0.000000 6.2228e-06 1.04245
## i 5.4128e-05 0.00053115 0.000000 2.2687e-06 1.75072
## T_p -7.2673e-06 -0.00060043 0.000000 -1.3859e-06 1.68448
```

Prediction and confudsion matrix :

```
y_hat_wsrf <- predict(fit_wsrf, mat_test$X)
y_hat_wsrf_m <- as.matrix(y_hat_wsrf$class)
y_hat_wsrf_m <- ifelse(y_hat_wsrf_m == 1, 'NEO', ifelse(y_hat_wsrf_m == 2, 'PHO', 'SO'))

cm <- confusionMatrix(as.factor(y_hat_wsrf_m), mat_test$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "WSRF"))
cm$table
```

```
##          Reference
## Prediction    NEO     PHO     SO
## NEO      19059      8      0
## PHO        5    1743      0
## SO         0      0  841826
```

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  mat_test$Y != y_hat_wsrf_m), "WSRF", JPL_test))
inaccurate_details %>% filter (model_name == "WSRF") %>% .[,3:10] %>% tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|--------------|------------|-----|-----|---------|---------|------------|------------|
| (2000 UQ30) | Otto Matic | 1 | 1 | 2.30864 | 0.01241 | 2000-10-31 | 2000-12-22 |
| (2000 WC1) | Otto Matic | 1 | 1 | 0.82478 | 0.00383 | 2000-11-16 | 2014-11-16 |
| (2001 QJ96) | Otto Matic | 1 | 1 | 2.00941 | 0.00073 | 2001-08-23 | 2015-08-17 |
| (2001 XP31) | Otto Matic | 1 | 1 | 1.22747 | 0.01139 | 2001-12-11 | 2018-02-04 |
| (2003 HN16) | Otto Matic | 1 | 1 | 1.46226 | 0.02121 | 2003-04-27 | 2015-11-06 |
| (2004 HG12) | Otto Matic | 1 | 1 | 1.73195 | 0.02671 | 2004-04-21 | 2004-05-04 |
| (2012 KN11) | Otto Matic | 1 | 0 | 3.03543 | 0.02116 | 2012-05-19 | 2012-06-13 |
| (2014 WR367) | Otto Matic | 1 | 1 | 3.77863 | 0.01492 | 2014-11-21 | 2014-12-15 |
| (2015 RE36) | Otto Matic | 1 | 0 | 1.49899 | 0.00052 | 2012-09-19 | 2018-10-03 |
| (2017 WX12) | Otto Matic | 1 | 1 | 3.20251 | 0.02155 | 2017-11-20 | 2018-03-18 |
| (2018 PH22) | Otto Matic | 1 | 0 | 2.12302 | 0.04166 | 2018-07-13 | 2021-03-24 |
| (2019 KD7) | Otto Matic | 1 | 0 | 0.91890 | 0.03411 | 2019-05-26 | 2019-06-02 |
| (2020 WD5) | Otto Matic | 1 | 0 | 2.45773 | 0.04348 | 2020-11-18 | 2020-12-01 |

Results :

```
results %>% filter (model_name == "WSRF") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| WSRF | NEO | 1.000 | 1.000 |
| WSRF | PHO | 0.995 | 0.996 |
| WSRF | SO | 1.000 | 1.000 |

- The Recall and F1 scores are all > 0.99
- Using a model which implements weights does increase the sensitivity on the outcome prediction.
- However there still are a total of 8 misses in PHO and 5 misses in NEO.

This might be partially explained by the incapability to attach weights manually, only a logical value is accepted in this model.

Thus the shortcoming of not being able to associate weights manually to each category according to their importance is approached with the Ranger model.

4.5.4 Ranger

Ranger is a fast implementation of random forests (Breiman 2001) or recursive partitioning, particularly suited for high dimensional data. Classification and regression forests are implemented as in the original Random Forest (Breiman 2001). Includes implementations of extremely randomized trees (Geurts et al. 2006) and quantile regression forests (Meinshausen 2006).

Model Parameters :

- classification is set to TRUE and accordingly min.node.size is set to 1.
- importance : Variable importance mode.
- splitrule : For classification the default “gini”.
- regularization.usedepth : The depth in regularization.
- num.threads : Number of threads / number of CPUs.
- class.weights : Weights for the outcome classes (in order of the factor levels) in the splitting rule (cost sensitive learning). For classification the weights are also applied in the majority vote in terminal nodes.

Model Fitting :

```
set.seed(131825431, sample.kind = "Rounding")
fit_ranger <- ranger(x = mat_train$X, y = mat_train$Y,
                      classification = TRUE,
                      min.node.size = 1,
                      mtry=rf_mtry$mtry+1,
                      num.trees = rf_mtry$mtry*50,
                      splitrule = 'gini',
                      importance = 'impurity',
                      regularization.usedepth = TRUE,
                      # additional weight of ~ 40:60:0 ~ 2:3:0 ratio from prime numbers.
                      class.weights = c(7,11,0),
                      num.threads = 6, verbose = TRUE)
```

```
## Growing trees.. Progress: 90%. Estimated remaining time: 3 seconds.
```

Variable Importance :

```
fit_ranger$variable.importance
```

```
##          H         q      moid       m_d        e      t_jup        a        i
## 28286.37 67546.53 44144.09  2240.86  1995.23   172.15  4308.85  836.59
##          T_p
##     157.08
```

Prediction and the confusion matrix :

```
y_hat_ranger <- predict(fit_ranger, mat_test$X)

cm <- confusionMatrix(y_hat_ranger$predictions, mat_test$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Ranger"))
cm$table

##           Reference
## Prediction    NEO     PHO      SO
##   NEO    19064      0      0
##   PHO      0    1751      0
##   SO       0      0  841826
```

- **Unity is achieved.** No misses.

```
cm$byClass
```

```
##           Sensitivity Specificity Pos Pred Value Neg Pred Value Precision
## Class: NEO          1            1          1            1          1            1
## Class: PHO          1            1          1            1          1            1
## Class: SO           1            1          1            1          1            1
##           Recall F1 Prevalence Detection Rate Detection Prevalence
## Class: NEO    1 1  0.0220996    0.0220996    0.0220996
## Class: PHO    1 1  0.0020298    0.0020298    0.0020298
## Class: SO     1 1  0.9758706    0.9758706    0.9758706
##           Balanced Accuracy
## Class: NEO          1
## Class: PHO          1
## Class: SO           1
```

- **100% accuracy on the training set**
- Prevalance shows the ditribution of the *neo_class* factors as determined on the orginal data (a 97.6% for SO, 0.002 for PHO and 0.022 for NEO)

Results :

```
results %>% filter (model_name == "Ranger") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------|--------|--------|----|
| Ranger | NEO | 1 | 1 |
| Ranger | PHO | 1 | 1 |
| Ranger | SO | 1 | 1 |

- This will be one of the models that will be used for the Final validation set.

4.6 Gradient Boosting

Generalized Boosted Regression Modeling (GBM)

Boosting is the process of iteratively adding basis functions in a greedy fashion so that each additional basis function further reduces the selected loss function. This implementation closely follows Friedman Gradient Boosting Machine (Friedman, 2001).

The **GBM** implements the generalized boosted modeling framework.

Model Parameters :

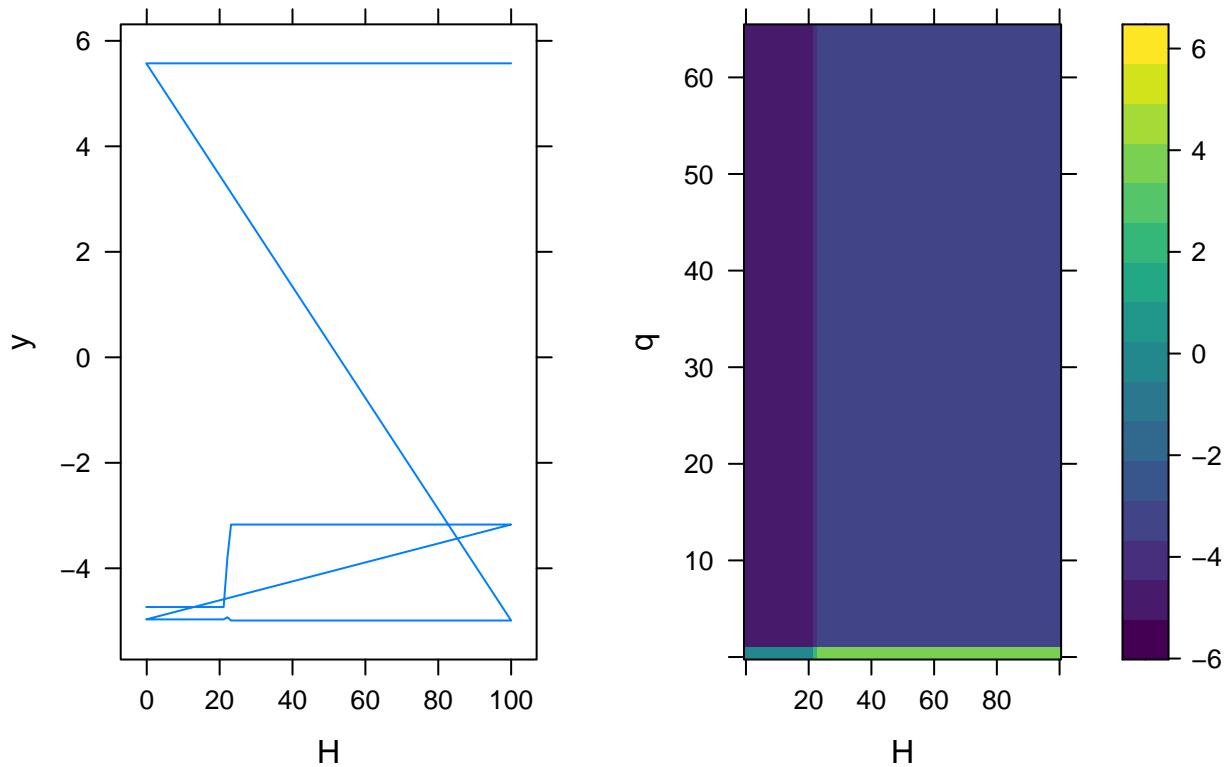
- n.trees : Integer specifying the total number of trees to fit.
- class.stratify.cv : Logical indicating whether or not the cross-validation should be stratified by class. The purpose of stratifying the cross-validation is to help avoiding situations in which training sets do not contain all classes.
- bag.fraction : The fraction of the training set observations randomly selected to propose the next tree in the expansion. This introduces randomness into the model fit.
- train.fraction : The first train.fraction * nrow(data) observations are used to fit the gbm and the remainder are used for computing out-of-sample estimates of the loss function.
- shrinkage : A shrinkage parameter applied to each tree in the expansion. Also known as the learning rate or step-size reduction.
- keep.data : a logical variable indicating whether to keep the data and an index of the data stored with the object.
- interaction.depth : Integer specifying the maximum depth of each tree (i.e., the highest level of variable interactions allowed). A value of 1 implies an additive model. A value of 3 implies a model with up to 3-way interactions.
- n.cores : The number of CPU cores to use. The cross-validation loop will attempt to send different CV folds off to different cores.

Fitting the Model :

```
gbm_train <- data_process(JPL_train,2)
gbm_test <- data_process(JPL_test,2)
set.seed(131825431, sample.kind = "Rounding")
fit_gbm <- gbm(neo_class~, data = gbm_train, n.trees = 50,
                 distribution = "multinomial", class.stratify.cv = TRUE,
                 bag.fraction = 0.3, train.fraction = 0.7, shrinkage = 0.1,
                 interaction.depth = 3, keep.data = FALSE, verbose = FALSE, n.cores = 6)
```

Visual representation of the model:

```
par(mfrow=c(1,1))
best.iter <- gbm.perf(fit_gbm, plot.it = FALSE)
p1 <- plot(fit_gbm)
p2 <- plot(fit_gbm, i.var = 1:2, n.trees = best.iter)
gridExtra::grid.arrange(p1,p2, ncol=2, nrow=1)
```



Prediction and the confusion matrix:

```
y_hat_gbm <- predict(fit_gbm, gbm_test, n.trees = best.iter, type='response')
labels <- colnames(y_hat_gbm)[apply(y_hat_gbm, 1, which.max)]

cm <- confusionMatrix(factor(labels), gbm_test$neo_class)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "gbm"))
cm$table
```

| | Reference | | |
|---------------|-----------|------|--------|
| ## Prediction | NEO | PHO | SO |
| ## NEO | 19030 | 15 | 0 |
| ## PHO | 34 | 1736 | 0 |
| ## SO | 0 | 0 | 841826 |

Results:

```
results %>% filter (model_name == "gbm") %>% knitr::kable()
```

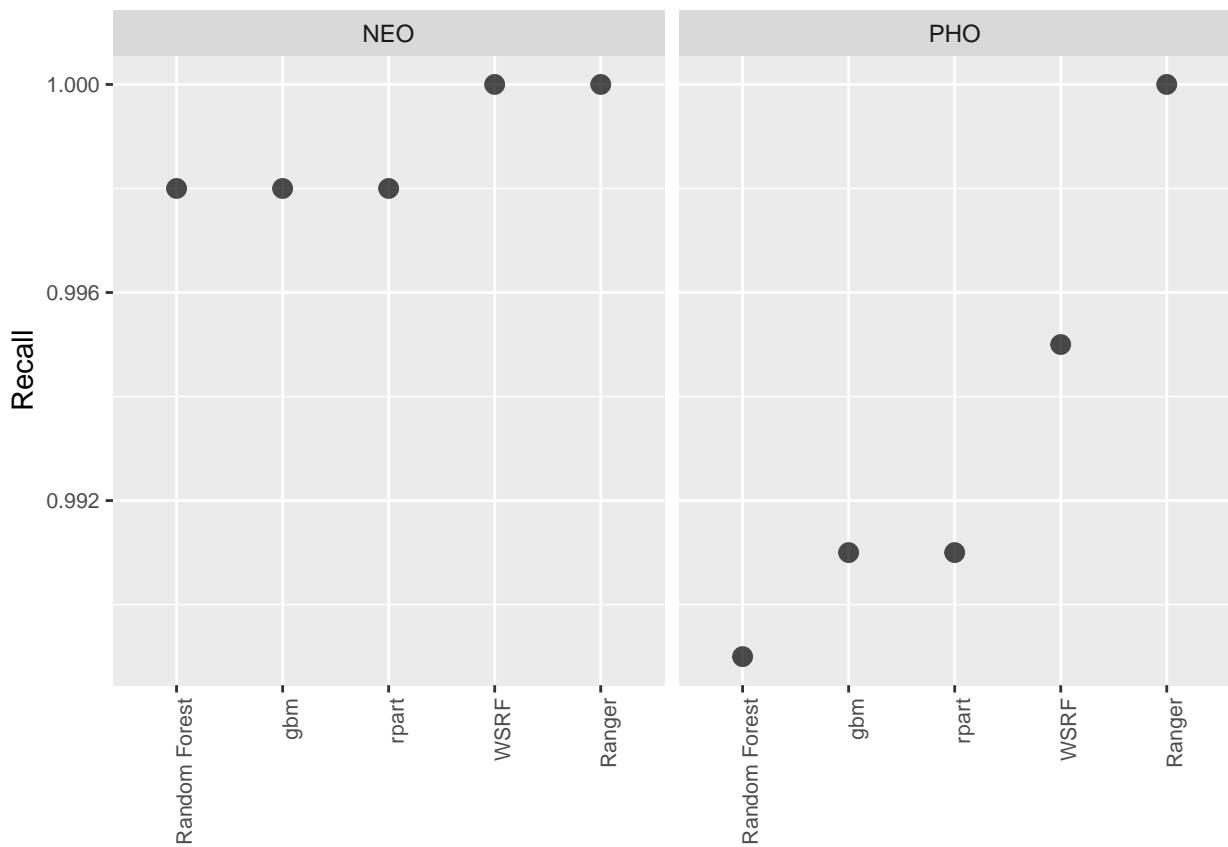
| model_name | Object | Recall | F1 |
|------------|--------|--------|-------|
| gbm | NEO | 0.998 | 0.999 |
| gbm | PHO | 0.991 | 0.986 |
| gbm | SO | 1.000 | 1.000 |

- The misses on PHO and NEO increased.
- The Recall and F1 scores for PHO are 0.991 and 0.986

4.7 Collated Results

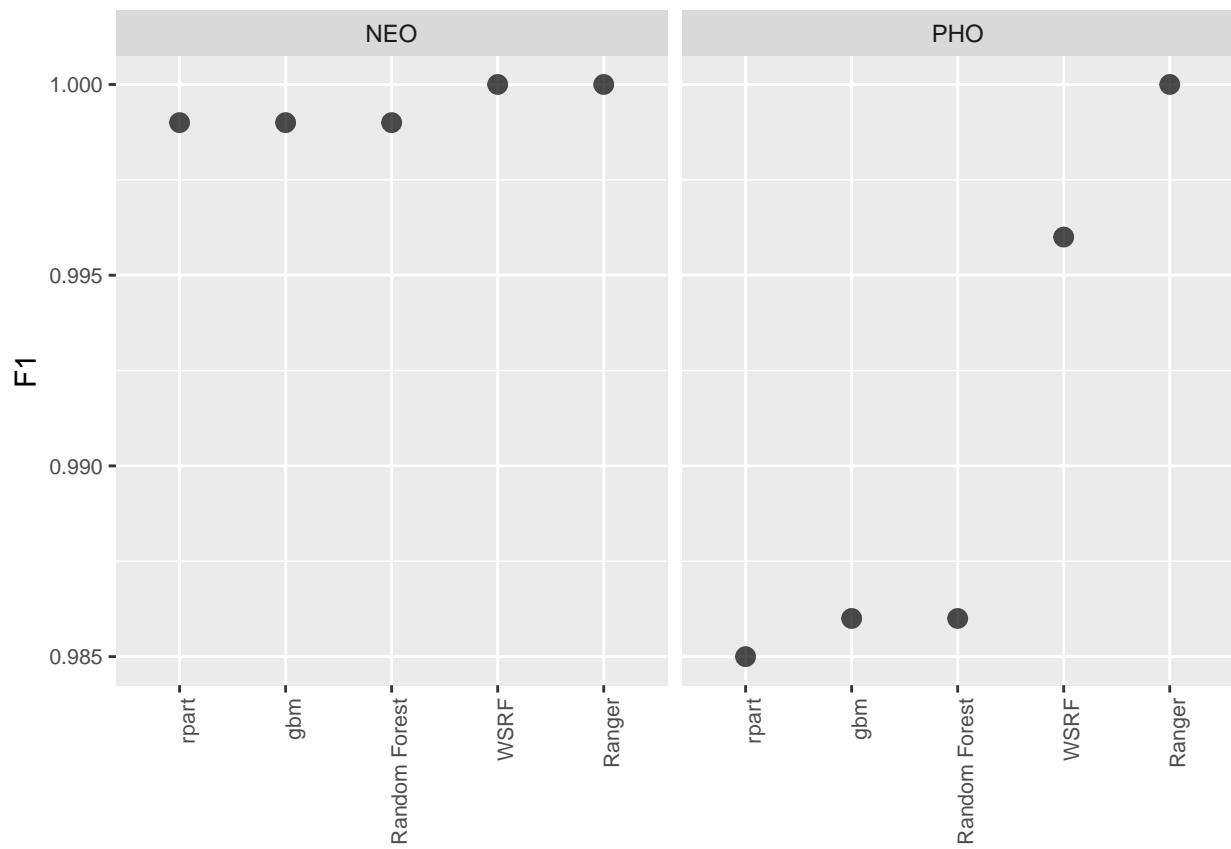
4.7.1 Recall

| model_name | NEO | PHO | SO |
|---------------|-------|-------|-------|
| gbm | 0.998 | 0.991 | 1.000 |
| Naive Bayes | 0.985 | 0.692 | 0.995 |
| PCA Multinom | 0.052 | 0.035 | 1.000 |
| Random Forest | 0.998 | 0.989 | 1.000 |
| Ranger | 1.000 | 1.000 | 1.000 |
| rpart | 0.998 | 0.991 | 1.000 |
| WSRF | 1.000 | 0.995 | 1.000 |

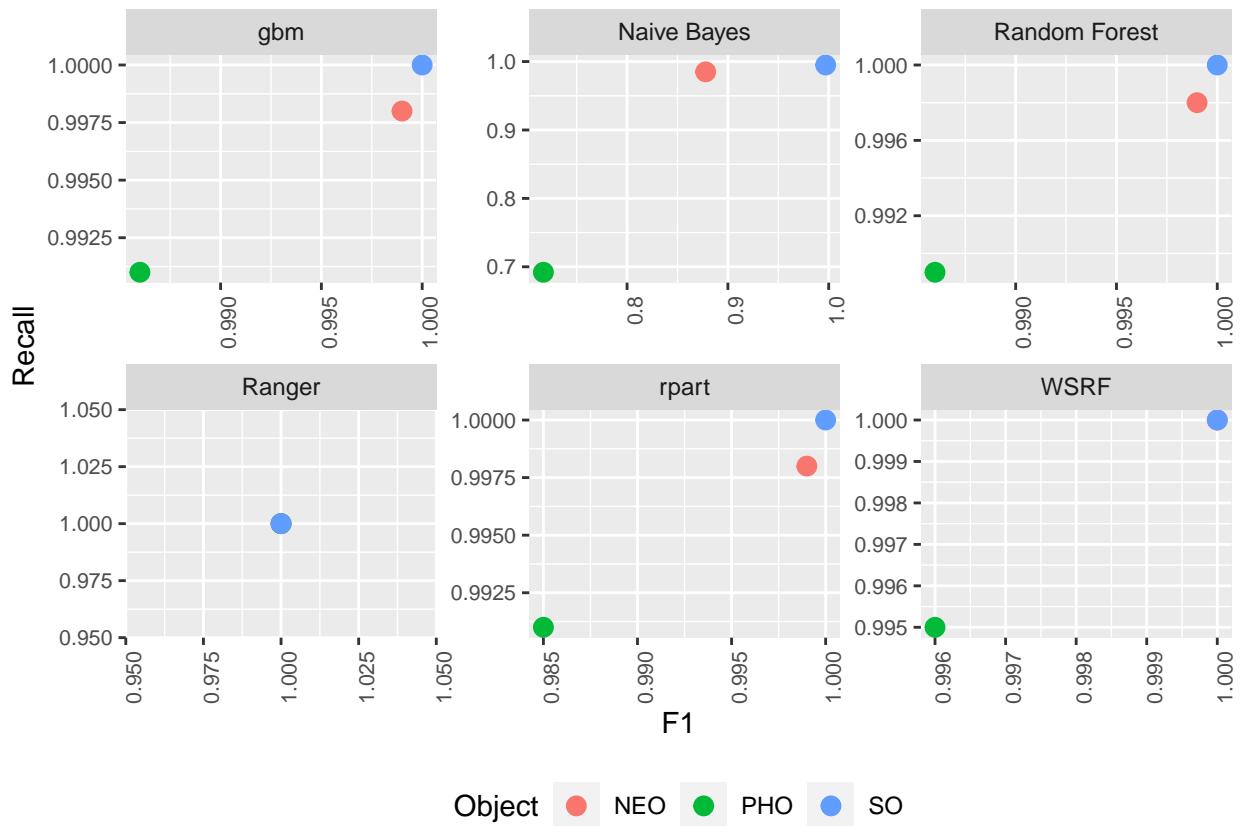


4.7.2 F1

| model_name | NEO | PHO | SO |
|---------------|-------|-------|-------|
| gbm | 0.999 | 0.986 | 1.000 |
| Naive Bayes | 0.878 | 0.717 | 0.997 |
| PCA Multinom | 0.098 | 0.064 | 0.988 |
| Random Forest | 0.999 | 0.986 | 1.000 |
| Ranger | 1.000 | 1.000 | 1.000 |
| rpart | 0.999 | 0.985 | 1.000 |
| WSRF | 1.000 | 0.996 | 1.000 |



4.7.3 F1 vs Recall



- The Rpart, randomForest, wsrp, ranger and gbm models have a Recall > 0.99
- Based on the scores on the predictions from the training and test sets, the final validation models will be on Ranger and WSRP as these models have both the Recall and $F1 \geq 0.99$ for all the three categories.
- Rpart is also tested on the validation set.

5 Final Models

5.1 Validation

5.1.1 Ranger

Validation set matrix:

```
#process as matrix for the model
validation <- data_process(JPL_validation,1)
JPL_F <- data_process(JPL_data,1)
```

Model Fitting, Prediction and the confusion matrix :

```
set.seed(131825431, sample.kind = "Rounding")
#fit the model for validation
final_ranger <- ranger(x=JPL_F$X, y=JPL_F$Y,
                        classification = TRUE, min.node.size = 1,
                        num.trees = rf_mtry$mtry*50, mtry=rf_mtry$mtry+1,
                        splitrule = 'gini', importance = 'impurity',
                        num.threads = 6, regularization.usedepth = TRUE,
                        verbose = FALSE, class.weights = c(7,11,0))

#predict and the confusion matrix
pred_ranger <- predict(final_ranger, validation$X)
cm <- confusionMatrix(pred_ranger$predictions, validation$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Validation : Ranger"))
cm$table

##          Reference
## Prediction    NEO     PHO      SO
##      NEO    2350      0      0
##      PHO       4    217      0
##      SO        0      0 103930
```

Results :

```
results %>% filter (model_name == "Validation : Ranger") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|---------------------|--------|--------|-------|
| Validation : Ranger | NEO | 0.998 | 0.999 |
| Validation : Ranger | PHO | 1.000 | 0.991 |
| Validation : Ranger | SO | 1.000 | 1.000 |

- The Recall and the F1 for the Ranger model on the validation dataset, as validation is > 0.99

Inaccurate Predictions :

```

inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  pred_ranger$predictions != validation$Y), 'Validation : Ranger', JPL_validation))
inaccurate_details %>% filter (model_name == "Validation : Ranger") %>% .[,3:10] %>%
  tibble() %>% knitr:::kable()

```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|---------------------|------------|-----|-----|--------|---------|------------|------------|
| 494975 (2009 WO106) | Otto Matic | 1 | 0 | 1.6110 | 0.05000 | 2009-11-26 | 2018-01-13 |
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

5.1.2 WSRF

Model Fitting, Prediction and the confusion matrix :

```
set.seed(131825431, sample.kind = "Rounding")
#fit the model for validation
final_wsrdf <- wsrdf(x=JPL_F$X, y=JPL_F$Y,
                       ntree = wsrdf_mtry$mtry*100, mtry=wsrfd_mtry$mtry+1,
                       weights=TRUE, importance = TRUE, parallel = TRUE)
#predict and the confusion matrix
pred_wsrdf <- predict(final_wsrdf, validation$X)
pred_wsrdf_m <- as.matrix(pred_wsrdf$class)
pred_wsrdf_m <- ifelse(pred_wsrdf_m == 1, 'NEO', ifelse(pred_wsrdf_m == 2, 'PHO', 'SO'))

cm <- confusionMatrix(as.factor(pred_wsrdf_m), validation$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Validation : WSRF"))
cm$table
```

```
##          Reference
## Prediction    NEO      PHO      SO
##      NEO    2351      0      0
##      PHO      3     217      0
##      SO      0      0 103930
```

Results :

```
results %>% filter (model_name == "Validation : WSRF") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|-------------------|--------|--------|-------|
| Validation : WSRF | NEO | 0.999 | 0.999 |
| Validation : WSRF | PHO | 1.000 | 0.993 |
| Validation : WSRF | SO | 1.000 | 1.000 |

- The Recall for the PHO and SO is 1, with that of NEO > 0.99
- The F1 for all the categories is > 0.99

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  validation$Y != pred_wsrdf_m), "Validation : WSRF", JPL_validation))
inaccurate_details %>% filter (model_name == "Validation : WSRF") %>% .[,3:10] %>%
  tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|--------------|------------|-----|-----|--------|---------|------------|------------|
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

5.1.3 Rpart

Model Fitting, Prediction and the confusion matrix :

```
#process as data tables for the model
validation <- data_process(JPL_validation,2)
JPL_F <- data_process(JPL_data,2)
#fit model
Final_rpart <- rpart(neo_class ~. , data=JPL_F,
                      control = rpart.control(cp = rpart_cp, minsplit = 10, minbucket=1))
#predict and confusion matrix
pred_rpart <- predict(Final_rpart, validation)
labels <- colnames(pred_rpart)[apply(pred_rpart, 1, which.max)]
cm <- confusionMatrix(factor(labels), validation$neo_class)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Validation : rpart"))
cm$table
```

```
##             Reference
## Prediction    NEO     PHO      SO
##           NEO  2350      0      0
##           PHO     4    217      0
##           SO      0      0 103930
```

Results :

```
results %>% filter (model_name == "Validation : rpart") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|--------------------|--------|--------|-------|
| Validation : rpart | NEO | 0.998 | 0.999 |
| Validation : rpart | PHO | 1.000 | 0.991 |
| Validation : rpart | SO | 1.000 | 1.000 |

- The Recall and the F1 for all the categories is > 0.99

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  labels != validation$neo_class), "Validation : rpart", JPL_validation))
inaccurate_details %>% filter (model_name == "Validation : rpart") %>% .[,3:10] %>%
  tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|---------------------|------------|-----|-----|--------|---------|------------|------------|
| 494975 (2009 WO106) | Otto Matic | 1 | 0 | 1.6110 | 0.05000 | 2009-11-26 | 2018-01-13 |
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

5.2 As Holdout

5.2.1 WSRF

- Illustration : Using the fit from the train set and treating the validation data as a holdout data.

Validation set matrix:

```
#process as matrix for the model
validation <- data_process(JPL_validation,1)
JPL_F <- data_process(JPL_data,1)
```

Prediction and the confusion matrix :

```
y_f_wsrdf <- predict(fit_wsrdf, validation$X)
y_f_wsrdf_m <- as.matrix(y_f_wsrdf$class)
y_f_wsrdf_m <- ifelse(y_f_wsrdf_m == 1, 'NEO', ifelse(y_f_wsrdf_m == 2, 'PHO', 'SO'))
cm <- confusionMatrix(as.factor(y_f_wsrdf_m), validation$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Holdout : WSRF"))
cm$table
```

```
##             Reference
## Prediction    NEO     PHO     SO
##      NEO    2351      0      0
##      PHO      3    217      0
##      SO       0      0 103930
```

Results :

```
results %>% filter (model_name == "Holdout : WSRF") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|----------------|--------|--------|-------|
| Holdout : WSRF | NEO | 0.999 | 0.999 |
| Holdout : WSRF | PHO | 1.000 | 0.993 |
| Holdout : WSRF | SO | 1.000 | 1.000 |

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  y_f_wsrdf_m != validation$Y ), 'Holdout : WSRF', JPL_validation))
inaccurate_details %>% filter (model_name == "Holdout : WSRF") %>% .[,3:10] %>%
  tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|--------------|------------|-----|-----|--------|---------|------------|------------|
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

5.2.2 Ranger

- Illustration : Using the fit from the train set and treating the validation data as a holdout data.

Prediction and the confusion matrix :

```
set.seed(131825431, sample.kind = "Rounding")
#predict using the training fitted model against the validation set.
pred_0_ranger <- predict(fit_ranger, validation$X)
cm <- confusionMatrix(pred_0_ranger$predictions, validation$Y)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Holdout : Ranger"))
cm$table
```

```
##             Reference
## Prediction    NEO     PHO     SO
##      NEO  2350      0      0
##      PHO      4   217      0
##      SO       0      0 103930
```

Results :

```
results %>% filter (model_name == "Holdout : Ranger") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|------------------|--------|--------|-------|
| Holdout : Ranger | NEO | 0.998 | 0.999 |
| Holdout : Ranger | PHO | 1.000 | 0.991 |
| Holdout : Ranger | SO | 1.000 | 1.000 |

- The Recall and the F1 for the Ranger model on the validation dataset, as a holdout is > 0.99

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  pred_0_ranger$predictions != validation$Y ), 'Holdout : Ranger', JPL_validation))
inaccurate_details %>% filter (model_name == "Holdout : Ranger") %>% .[,3:10] %>%
  tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|---------------------|------------|-----|-----|--------|---------|------------|------------|
| 494975 (2009 WO106) | Otto Matic | 1 | 0 | 1.6110 | 0.05000 | 2009-11-26 | 2018-01-13 |
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

5.2.3 Gradient Boost

- Illustration : Using the fit from the train set and treating the validation data as a holdout data.

Validation set data table:

```
#process as matrix for the model
validation <- data_process(JPL_validation, 2)
```

Prediction and the confusion matrix:

```
y_f_gbm <- predict(fit_gbm, validation, n.trees = best.iter, type='response')
labels <- colnames(y_f_gbm)[apply(y_f_gbm, 1, which.max)]
cm <- confusionMatrix(factor(labels), validation$neo_class)
results <- bind_rows(results, results_process(as.data.frame(cm$byClass), "Holdout : gbm"))
cm$table
```

```
##          Reference
## Prediction    NEO     PHO      SO
##      NEO    2351      0      0
##      PHO      3    217      0
##      SO       0      0 103930
```

Results :

```
results %>% filter (model_name == "Holdout : gbm") %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|---------------|--------|--------|-------|
| Holdout : gbm | NEO | 0.999 | 0.999 |
| Holdout : gbm | PHO | 1.000 | 0.993 |
| Holdout : gbm | SO | 1.000 | 1.000 |

Inaccurate Predictions :

```
inaccurate_details <- rbind(inaccurate_details, inaccuracy_check(which(
  labels != validation$neo_class), 'Holdout : gbm', JPL_validation))
inaccurate_details %>% filter (model_name == "Holdout : gbm") %>% .[,3:10] %>%
  tibble() %>% knitr::kable()
```

| full_name | producer | neo | pha | per_y | moid | first_obs | last_obs |
|--------------|------------|-----|-----|--------|---------|------------|------------|
| (2009 QS) | Otto Matic | 1 | 0 | 3.0708 | 0.04234 | 2009-08-17 | 2009-09-16 |
| (2015 BW310) | Otto Matic | 1 | 0 | 4.2435 | 0.02712 | 2014-07-31 | 2015-02-18 |
| (2018 HA1) | Otto Matic | 1 | 0 | 1.5291 | 0.02622 | 2018-04-19 | 2018-05-19 |

6 Conclusion

The JPL small body dataset was downloaded from the https://ssd.jpl.nasa.gov/sbdb_query.cgi website, cleaned and initial variable assessment is done using various techniques.

Additional derived variables were also included for model evaluations. t-tests to check the statistical significance of the variations in each variable was also done separately (*summary included in R code file*). However, the details have not been included here in order to maintain concision of the case study document.

In the model evaluation the multinom (PCA), Naive Bayes, rpart, randomForest, wsrf, ranger and gbm models were evaluated. An ensemble of lda, qda, knn, rpart, svmRadialCost and wsrf was used for direction for a suitable model.

The classification of Small Bodies model was built using only 9 variables $H, q, moid, m_d, e, t_jup, a, i, T_p$, of which 2 variables m_d and T_p are derived variables.

The Final validation was done with The Ranger model (a fast implementation of random forests), WSRF (Forest of Weighted Subspace Decision Trees) and rpart (Recursive Partitioning and Regression Trees).

The Final Scores:

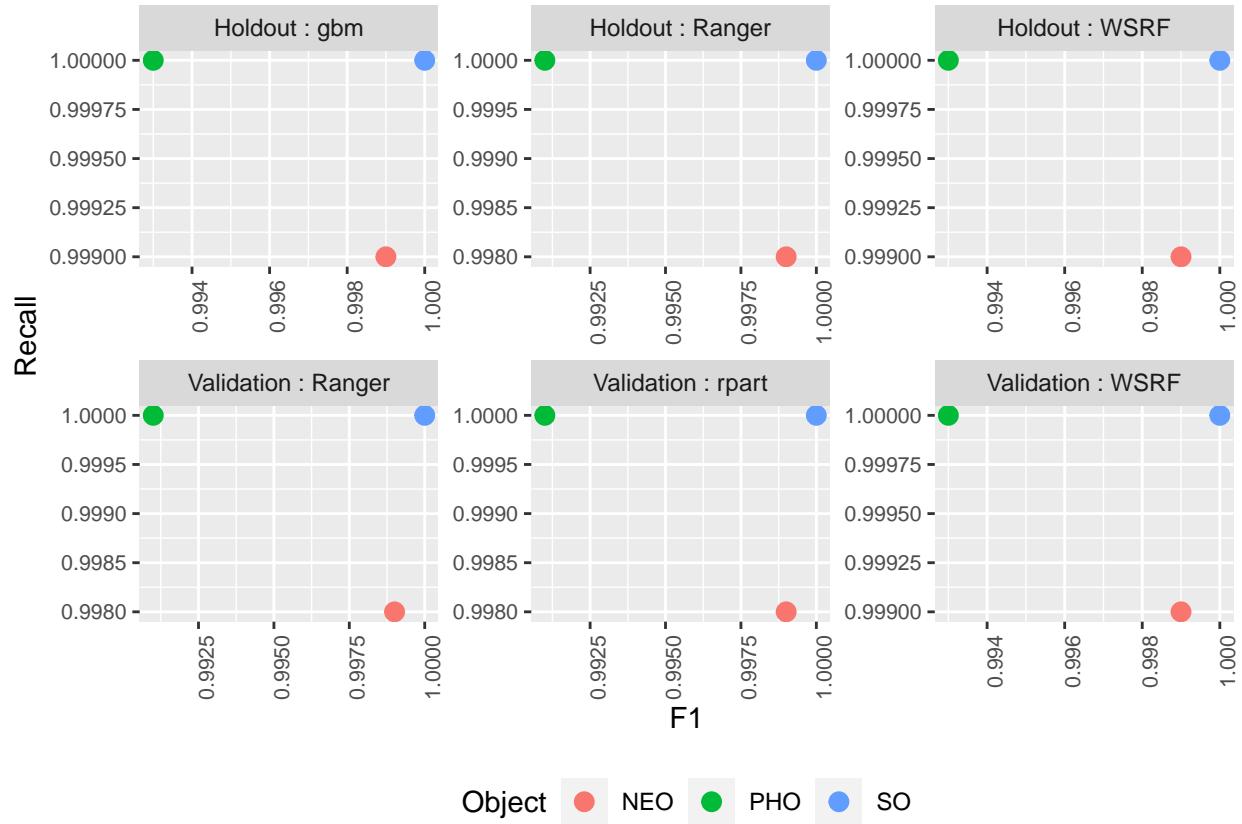
Recall

| model_name | NEO | PHO | SO |
|---------------------|-------|-----|----|
| Holdout : gbm | 0.999 | 1 | 1 |
| Holdout : Ranger | 0.998 | 1 | 1 |
| Holdout : WSRF | 0.999 | 1 | 1 |
| Validation : Ranger | 0.998 | 1 | 1 |
| Validation : rpart | 0.998 | 1 | 1 |
| Validation : WSRF | 0.999 | 1 | 1 |

F1

| model_name | NEO | PHO | SO |
|---------------------|-------|-------|----|
| Holdout : gbm | 0.999 | 0.993 | 1 |
| Holdout : Ranger | 0.999 | 0.991 | 1 |
| Holdout : WSRF | 0.999 | 0.993 | 1 |
| Validation : Ranger | 0.999 | 0.991 | 1 |
| Validation : rpart | 0.999 | 0.991 | 1 |
| Validation : WSRF | 0.999 | 0.993 | 1 |

The Objective of the case study, to build a machine learning model to predict the classification of the small solar bodies into *PHO*, *NEO* and *SO* (other small solar bodies), with the target measures of Recall for each category above 0.99 and F1 Score for each category above 0.99 was achieved.



Further tweaking and tuning of the models and reassessing variables, both observed and derived might achieve an absolute Recall and F1 score of 1.

6.1 References

- Rafael A. Irizarry Introduction to Data Science: <https://rafalab.github.io/dsbook/>
- NASA JPL website <https://ssd.jpl.nasa.gov/>
- Articles on <https://en.wikipedia.org> for the brief history details and the two images.

veneet.bhardwaj@gmail.com

6.2 Annexure

Recall and F1 Scores of each model:

```
results %>% knitr::kable()
```

| model_name | Object | Recall | F1 |
|---------------------|--------|--------|-------|
| PCA Multinom | NEO | 0.052 | 0.098 |
| PCA Multinom | PHO | 0.035 | 0.064 |
| PCA Multinom | SO | 1.000 | 0.988 |
| Naive Bayes | NEO | 0.985 | 0.878 |
| Naive Bayes | PHO | 0.692 | 0.717 |
| Naive Bayes | SO | 0.995 | 0.997 |
| rpart | NEO | 0.998 | 0.999 |
| rpart | PHO | 0.991 | 0.985 |
| rpart | SO | 1.000 | 1.000 |
| Random Forest | NEO | 0.998 | 0.999 |
| Random Forest | PHO | 0.989 | 0.986 |
| Random Forest | SO | 1.000 | 1.000 |
| WSRF | NEO | 1.000 | 1.000 |
| WSRF | PHO | 0.995 | 0.996 |
| WSRF | SO | 1.000 | 1.000 |
| Ranger | NEO | 1.000 | 1.000 |
| Ranger | PHO | 1.000 | 1.000 |
| Ranger | SO | 1.000 | 1.000 |
| gbm | NEO | 0.998 | 0.999 |
| gbm | PHO | 0.991 | 0.986 |
| gbm | SO | 1.000 | 1.000 |
| Validation : Ranger | NEO | 0.998 | 0.999 |
| Validation : Ranger | PHO | 1.000 | 0.991 |
| Validation : Ranger | SO | 1.000 | 1.000 |
| Validation : WSRF | NEO | 0.999 | 0.999 |
| Validation : WSRF | PHO | 1.000 | 0.993 |
| Validation : WSRF | SO | 1.000 | 1.000 |
| Validation : rpart | NEO | 0.998 | 0.999 |
| Validation : rpart | PHO | 1.000 | 0.991 |
| Validation : rpart | SO | 1.000 | 1.000 |
| Holdout : WSRF | NEO | 0.999 | 0.999 |
| Holdout : WSRF | PHO | 1.000 | 0.993 |
| Holdout : WSRF | SO | 1.000 | 1.000 |
| Holdout : Ranger | NEO | 0.998 | 0.999 |
| Holdout : Ranger | PHO | 1.000 | 0.991 |
| Holdout : Ranger | SO | 1.000 | 1.000 |
| Holdout : gbm | NEO | 0.999 | 0.999 |
| Holdout : gbm | PHO | 1.000 | 0.993 |
| Holdout : gbm | SO | 1.000 | 1.000 |

veneet.bhardwaj@gmail.com