

# COMP2212 PROGRAMMING LANGUAGE CONCEPTS

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# STRUCTURAL SUBTYPING

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- Relies purely on the structure of the type to define the subtyping relation.
- First, the relationship between base, or primitive types, needs to be specified.
  - For example, one could want **short** <: **int** , **float** <: **double**
- Then structure determines the rest. For example, a subtype of the pair type **T** **x** **U** is a pair of subtypes of **T** and **U** separately.

$$\frac{T_1 <: T_2 \quad U_1 <: U_2}{T_1 \times U_1 <: T_2 \times U_2} \text{SUBPAIR}$$

- Record types are a generalisation of pair types, and the subtype relation on records generalises in an interesting way too.
- Record types don't rely on syntactic positioning in the values for their indexing. This means we can write a record with some fields missing and have it be perfectly well formed as a record value (of another type). We can't do the same with a tuple.

# STRUCTURAL RECORD SUBTYPING

- The rules for subtyping on records can be built up in stages.
- First we have the generalisation of what we saw above for pairs.
- This is called **depth** subtyping for records:

if not talking about parent (supertype) and child (subtype),  
then it's depth

$$\frac{T_i <: U_i \quad 1 \leq i \leq n}{\{l_1 : T_1, \dots, l_n : T_n\} <: \{l_1 : U_1, \dots, l_n : U_n\}} \text{SUBRECDDEPTH}$$

- Then we have the notion of **width** subtyping in which there may be extra fields in the subtype:

$$\frac{}{\{l_1 : T_1, \dots, l_n : T_n, l_{n+1} : T_{n+1}\} <: \{l_1 : T_1, \dots, l_n : T_n\}} \text{SUBRECWIDTH}$$

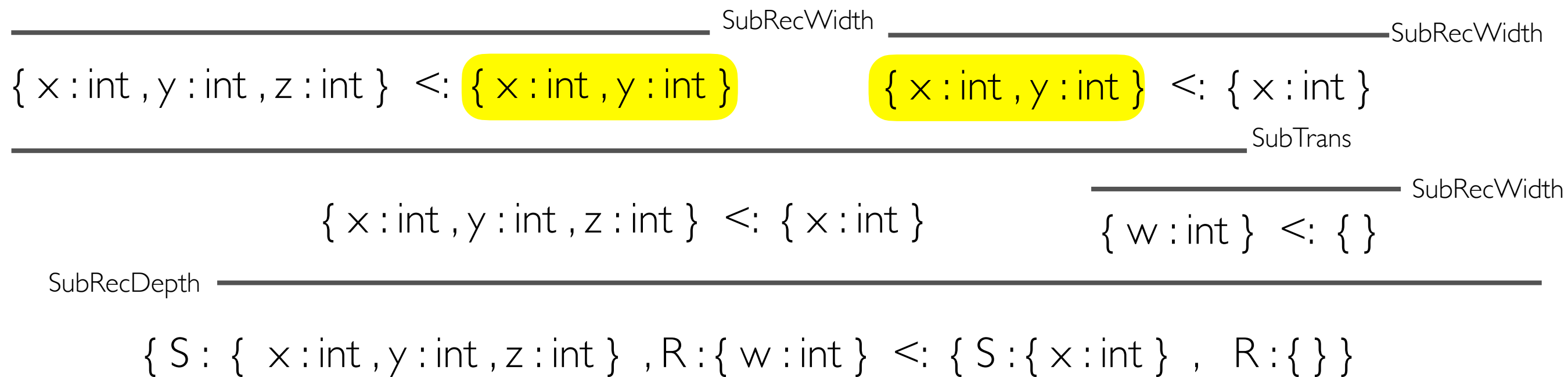
- And finally, we allow re-ordering of the listed fields:

$$\frac{\sigma \text{ a permutation of } 1 \dots n}{\{l_1 : T_1, \dots, l_n : T_n\} <: \{l_{\sigma(1)} : T_{\sigma(1)}, \dots, l_{\sigma(n)} : T_{\sigma(n)}\}} \text{SUBRECPERM}$$

- Not all languages adopt all of these principles. Indeed, Java does not allow depth subtyping. Methods or fields in a subclass cannot have subtypes of that with which they are declared in the supertype.

# EXAMPLE OF RECORD SUBTYPING

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# STRUCTURAL SUBTYPING FOR VARIANTS

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- For sum types  $\mathbf{T} + \mathbf{U}$ , it is intuitive that any value of type  $\mathbf{T}$  can be considered as also being of type  $\mathbf{T} + \mathbf{U} + \mathbf{V}$  as the value of type  $\mathbf{T}$  is still just injected in to the sum, albeit in to a larger sum.
- For generalised, variant, types. We have the same notions of width, depth and permutation subtyping as we do for records.
- There is however, a subtle inversion in the rule for width subtyping:

$$\frac{}{\langle l_1 : T_1, \dots, l_n : T_n \rangle <: \langle l_1 : T_1, \dots, l_n : T_n, l_{n+1} : T_{n+1} \rangle} \text{SUBVARWIDTH}$$

- We see that we can inject in to a larger variant type
- The notions of depth and permutations are identical though.

$$\frac{T_i <: U_i \quad 1 \leq i \leq n}{\langle l_1 : T_1, \dots, l_n : T_n \rangle <: \langle l_1 : U_1, \dots, l_n : U_n \rangle} \text{SUBVARDEPTH}$$

$$\frac{\sigma \text{ a permutation of } 1 \dots n}{\langle l_1 : T_1, \dots, l_n : T_n \rangle <: \langle l_{\sigma(1)} : T_{\sigma(1)}, \dots, l_{\sigma(n)} : T_{\sigma(n)} \rangle} \text{SUBVARPERM}$$

NEXT LECTURE: VARIANCE